

FAULT DETECTION AND ISOLATION

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SUMMARY

In order for a current satellite-based navigation system (such as the Global Positioning System, GPS) to meet integrity requirements, there must be a way of detecting erroneous measurements, without help from outside the system. This process is called Fault Detection and Isolation (FDI). Fault detection requires at least one redundant measurement, and can be done with a parity space algorithm. The best way around the fault isolation problem is not necessarily isolating the bad measurement, but finding a new combination of measurements which excludes it.

BACKGROUND

The objective of fault detection and isolation is to use inconsistencies in redundant sensor measurement data to detect and isolate sensor malfunctions. If a given single measurement is in error, it will cause the navigation solution to be in error, possibly greater than the allowable error threshold. Outside sources may not be able to broadcast in a timely manner that a signal is in error; for instance, if a single GPS satellite malfunctions, it could be from 15 minutes to several hours before the information is made public in the satellite broadcast data. Therefore, it is imperative for FDI algorithms to be able to detect and isolate instrument errors using only data from the instruments themselves.

FDI can be implemented in any multisensor navigation system with redundant measurements. Current work is focusing on satellite navigation using GPS, along with hybrid systems such as GPS/Loran-C (Long Range Navigation - C) or GPS/IRS (Inertial Reference System) [3]. FDI used specifically with GPS is also known as RAIM, or Receiver Autonomous Integrity Monitoring [4].

To detect step errors or fast growing ramp errors, a Kalman filter will work well. However, it will not detect a slow growing ramp error, such as might be caused by a GPS satellite clock drift. To detect slow growing errors, the Kalman filter algorithm should be used in parallel with a parity space algorithm.

PARITY SPACE AND ESTIMATION SPACE

Estimation space contains the actual horizontal measurement error and the alarm threshold for a given error. However, actual positions and actual errors are not known, given that all of the measurement data is coming from imperfect sensors. Therefore, the work of detecting and isolating errors is done in parity space. Parity space is a mathematical tool where measurement noise and biases are used to create a parity vector. The parity vector determines the detection statistic, d_k , which is compared to a detection threshold, T_D , in order to determine whether an alarm condition exists.

Errors and biases in parity space and estimation space are related, but it is not a one to one correspondence. The exact correspondence will be determined by measurement geometries. For instance, with a good geometry, a large measurement error (parity space) will result in only a small position error (estimation space). The reverse can also be true. Figure 1 illustrates two different slow growing ramp errors plotted in parity space versus estimation space. In case I, the detection threshold is crossed before the alarm threshold, yielding a false alarm. As the error continues to grow, the alarm threshold is crossed, turning it into a correct fault detection. In case II, the alarm threshold is crossed before the detection threshold, resulting in a missed detection. As the error continues to grow, the detection threshold is crossed, turning it into a correct fault detection. An ideal algorithm would minimize both the number of false alarms and missed detections.

LEAST SQUARES ESTIMATOR ALGORITHM

In a least-squares approach to fault detection, the relationship between the measurements and the user state (position) is given by:

$$\mathbf{y} = \mathbf{H}\boldsymbol{\beta} \quad (1)$$

where: \mathbf{y} = measurement vector (n-by-1)
 \mathbf{H} = data matrix (n-by-m)
 $\boldsymbol{\beta}$ = user state vector (m-by-1)

\mathbf{y} is a vector of n measurements, one from each instrument. In the case of using only GPS satellites, it would consist of the pseudoranges. $\boldsymbol{\beta}$ is the m-element user state vector, consisting of the user position coordinates and other navigation state elements such as clock offset with respect to GPS time. \mathbf{H} is an n-by-m matrix which relates the measurements to the user states.

There are three possible cases:

- 1) $n < m$: Underdetermined system
- 2) $n = m$: Exactly determined system

3) $n > m$: Overdetermined system

In the underdetermined case, a navigation solution is not possible. In the exactly determined case, a navigation solution is possible, but fault detection is not.

Algorithms for managing the redundant measurements in an overdetermined system form the basis of fault detection. A parity equation can be derived from equation 1, starting with a mathematical manipulation called the QR factorization on the data matrix H (ref. 2):

$$H = QR \quad (2)$$

H is factored into an n -by- n orthonormal matrix Q ($Q^T Q = I$) and an n -by- m upper triangular matrix R . R contains $(n-m)$ rows of zeros along the bottom, due to the $n-m$ redundant measurements in H . Substituting QR for H in equation (1) gives:

$$\begin{aligned} \mathbf{y} &= QR\boldsymbol{\beta} \\ Q^T \mathbf{y} &= Q^T QR\boldsymbol{\beta} \\ Q^T \mathbf{y} &= R\boldsymbol{\beta} \end{aligned} \quad (3)$$

Now partition R into an m -by- m upper triangular matrix U and $(n-m)$ rows of zeros, denoted by 0 . Similarly, partition Q^T into Q_1 (m -by- n) and Q_2 ($(n-m)$ -by- n rows).

$$\begin{pmatrix} Q_1 \\ \text{---} \\ Q_2 \end{pmatrix} \begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix} = \begin{pmatrix} U \\ \text{---} \\ 0 \end{pmatrix} \begin{pmatrix} \beta_1 \\ \vdots \\ \beta_m \end{pmatrix} \quad (4)$$

The least squares navigation state solution is:

$$\boldsymbol{\beta} = U^{-1} Q_1 \mathbf{y} \quad (5)$$

U is an upper triangular matrix. Due to the nature of the QR factorization, all matrix elements on the diagonal must be non-zero. Therefore, U is always non-singular and this equation always has a solution.

The parity equation is:

$$Q_2 \mathbf{y} = 0 \quad (6)$$

The measurement vector \mathbf{y} contains noise (\mathbf{e}) and measurement biases (\mathbf{b}). If \mathbf{y} is replaced by

($y - \underline{e} - \underline{b}$), the 0 in equation (6) can be replaced by the parity vector \underline{p} .

$$\begin{aligned}\underline{p} &= Q_2 y - Q_2 \underline{e} - Q_2 \underline{b} \\ \underline{p} &= -Q_2 \underline{e} - Q_2 \underline{b}\end{aligned}\tag{7}$$

Thus, a parity vector will be determined by the noise and bias errors. From the parity vector, it can be determined whether an instrument is in error and an alarm should be raised.

PARITY SPACE AND DETECTION PROBABILITIES

Consider a situation with one redundant measurement. In this case, the parity vector will be reduced to a scalar, and the detection statistic reduces to the absolute value of the scalar. In the case where no measurement bias exists, figure 2 shows the distribution of the parity scalar. Since there is no bias error, the position error is definitely under the alarm threshold and the system is either in the normal operation condition or the false alarm condition. The probability of a false alarm (P_{FA}) is obtained by integrating the areas outside of T_D . For noise having a normal distribution (generally a good assumption), this integral is a standard Gaussian function.

Figure 3 illustrates the case where a large measurement bias exists, making the position error larger than the alarm threshold. In this case the system is either in the correct fault detection condition or in the missed detection condition. The probability of a missed detection (P_{MD}) is the integral of the area inside T_D . Again, if Gaussian noise is assumed, this is a standard Gaussian function.

PROTECTION RADIUS

The above example uses detection threshold, measurement noise, and measurement bias error as parameters to find P_{FA} and P_{MD} . Accuracy requirements are stated in a form like "the probability of exceeding 100 meters accuracy is no greater than 0.05". In order to compare FDI results with such specifications, it helps to rearrange the procedure. This means using the parameters alarm threshold, measurement noise, P_{FA} , and P_{MD} to determine the protection radius, which is the smallest horizontal position error that is guaranteed to be detected with the given probabilities. If all parameters are kept constant, the protection radius will vary only as a function of satellite geometry.

The method resulting in the best protection radius uses all satellites in view. However, many receivers are limited to six channels and are incapable of using more than six measurements. A way around this is to search all possibilities of combinations of 5 or 6 satellites for the set with the best geometry, and use that set to find the protection radius.

Figure 4 shows a comparison of each method for a given location over the span of one day. The parameters used to generate these plots are: $\sigma = 32$ meters, $P_{FA} = 6.67 \times 10^{-5}$, and $P_{MD} = 3.3 \times 10^{-9}$.

Since all aircraft carry a baroaltimeter, this can be used as another instrument to improve the algorithm. The altimeter adds another measurement without requiring more channels. The altimeter measurement is weighted according to its accuracy and the phase of flight. Figure 5 shows the effect of altimeter aiding, using the same parameters as before and an altimeter with statistics identical to the GPS satellites.

FAULT ISOLATION

The fault isolation problem is very difficult. Previous work explored fault isolation using both a snapshot method and a time history method. Since the objective is to ensure that the aircraft is flying with a set of good measurements, it is not necessary to isolate the bad measurement. It is only required that the bad measurement is not used in the navigation solution. With this in mind, Fault Detection and Exclusion (FDE) was devised.

In FDE, once an alarm is raised, the algorithm discards the present combination of satellites and looks for the combination with the next-best geometry. If this set also raises the alarm, the algorithm goes on to the next best set. Once a set is found that doesn't raise the alarm, that set is used from then on for navigation. In this manner, the bad satellite is not necessarily isolated, but it is excluded.

CONCLUSIONS

A fault detection algorithm for a multisensor navigation system has been presented. A protection radius has been calculated using several different algorithms, with the best-of-six plus altimeter aiding method being chosen as the best method that will work with all receivers. The fault isolation problem has been bypassed by using fault exclusion. The only remaining work for the algorithm is to program it into a receiver and flight test it.

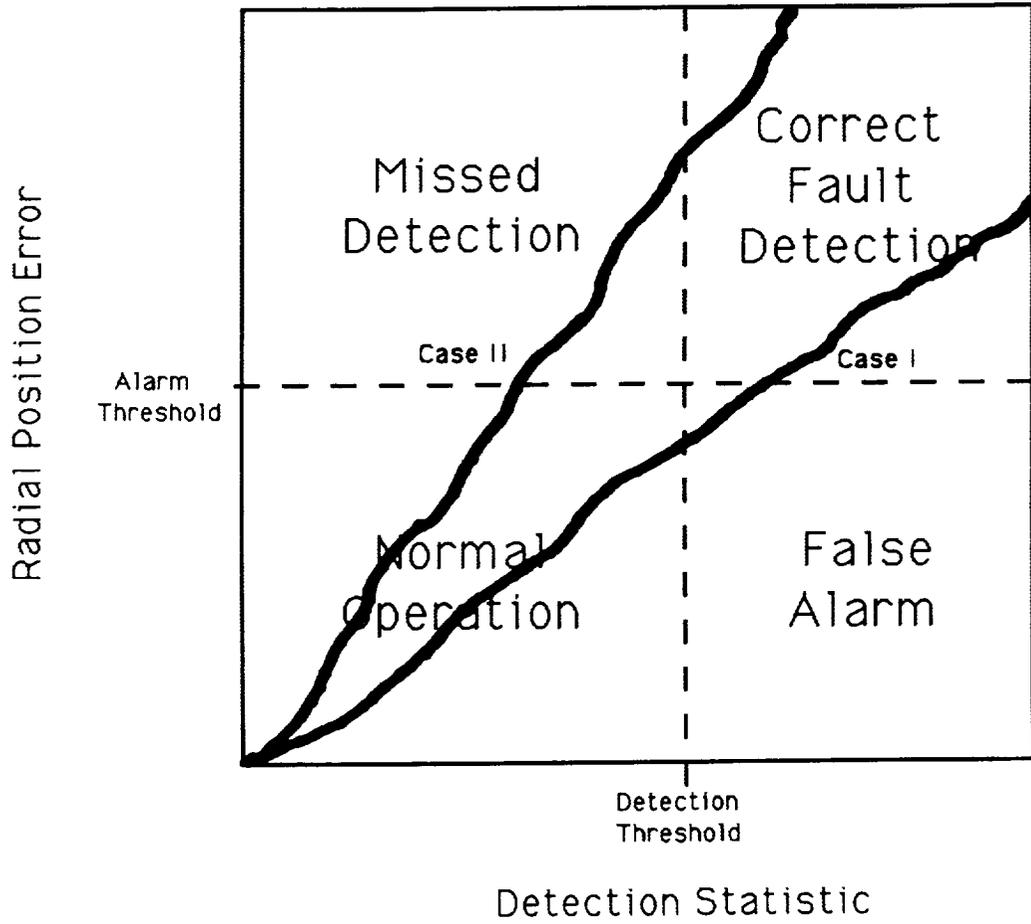
ACKNOWLEDGEMENTS

This work was supported by the Federal Aviation Administration and the National Aeronautics and Space Administration through the Joint University Program for Air Transportation Research (Grant NGR 36-009-17).

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Estimation Space



Parity Space

Figure 1. Two slowly growing measurement errors plotted in parity space versus estimation space.

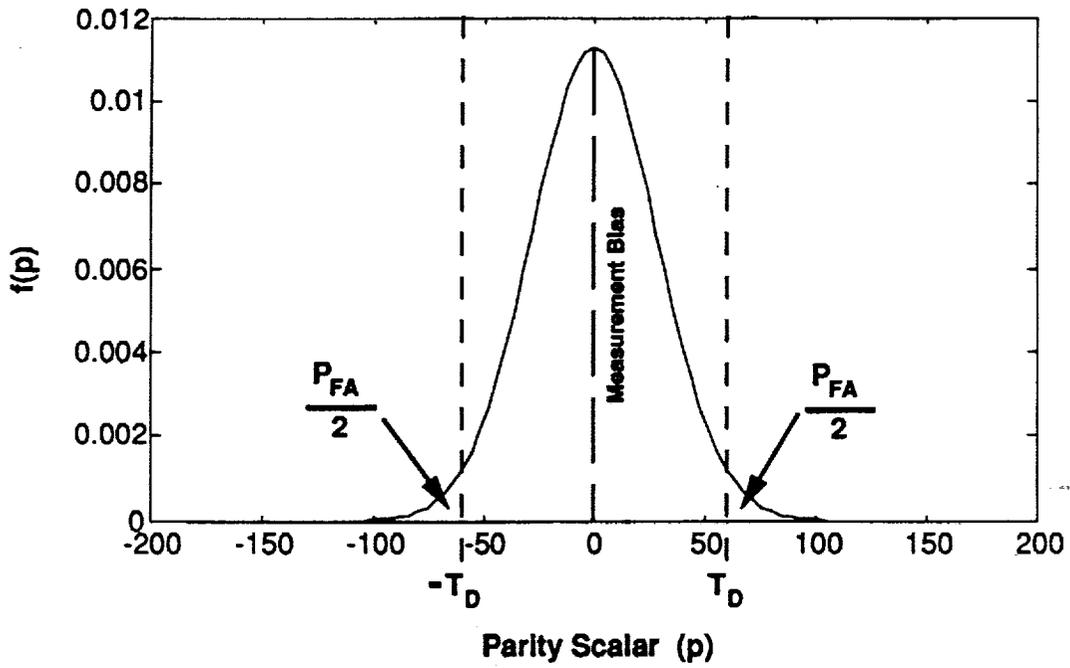


Figure 2. Probability density function for the parity scalar in the absence of a measurement bias error.

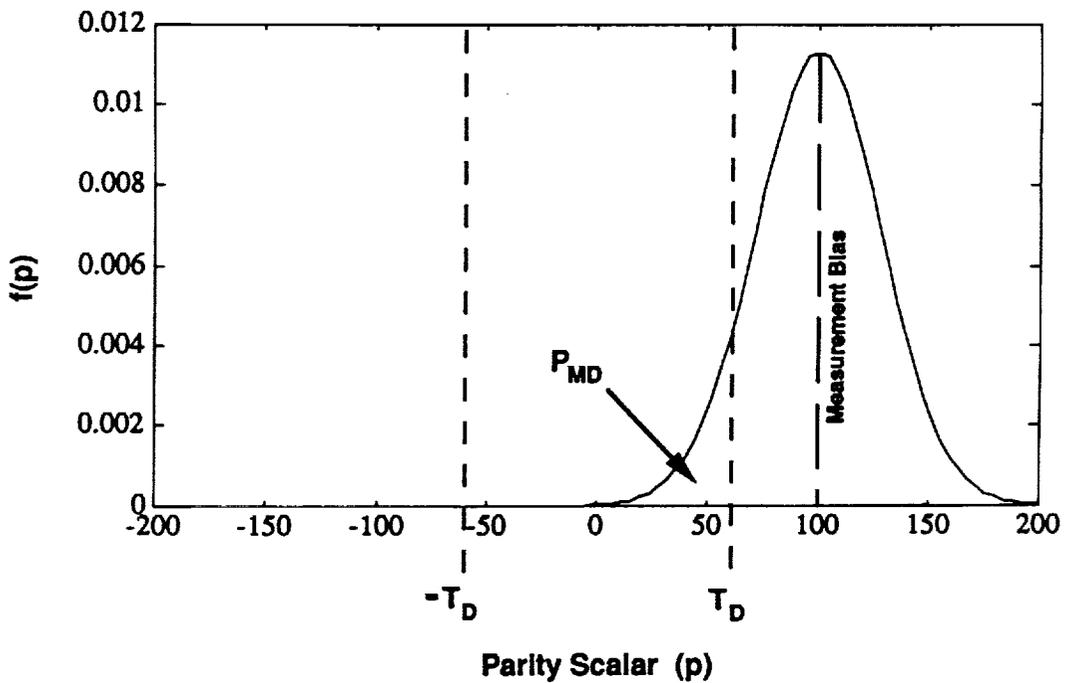


Figure 3. Probability density function for the parity scalar in the presence of a measurement bias error.

Figure 4. Worst Case Protection Radius (36N 140E)

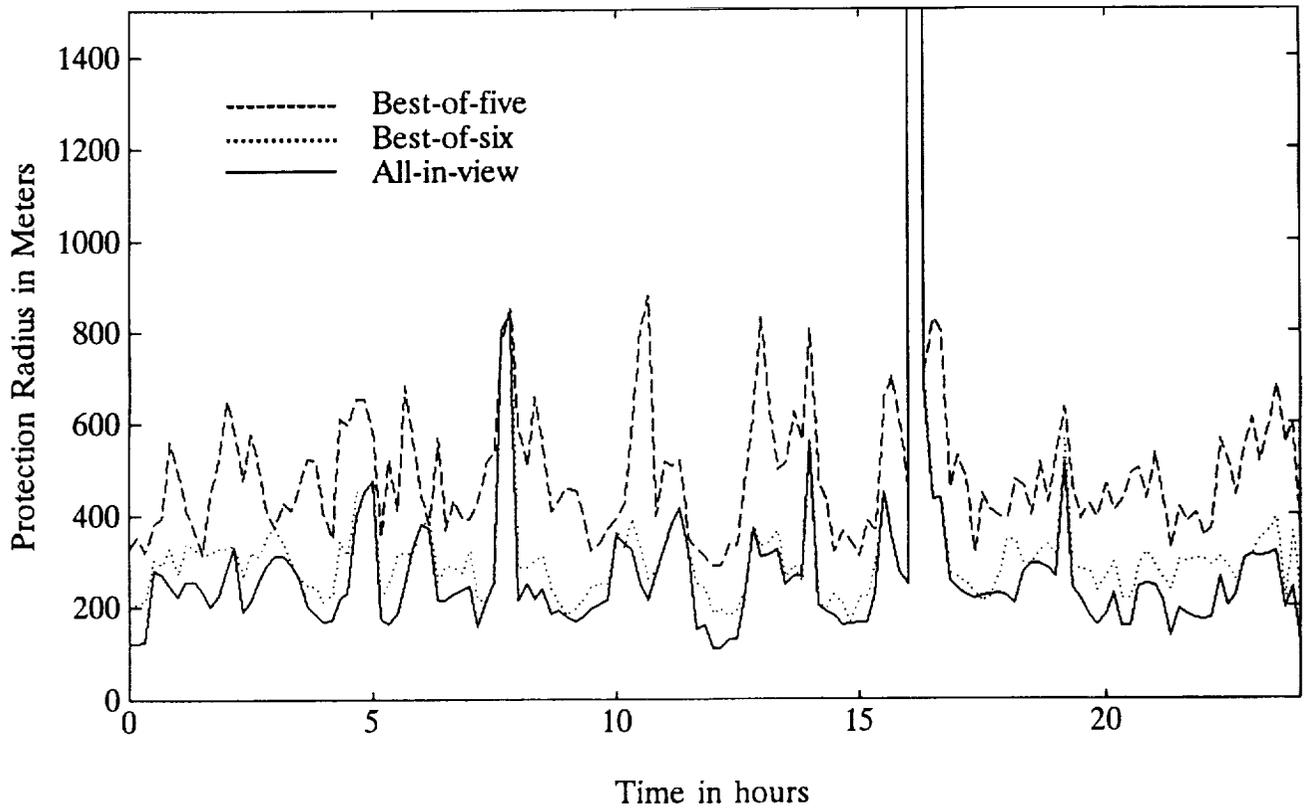


Figure 5. Worst Case Protection Radius (36N 140E)

