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Progress in Modeling Atmospheric Propagation of Sonic Booms

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The improved simulation of sonic boom propagation through the real atmosphere requires greater understanding of how the transient acoustic pulses popularly termed sonic booms are affected by humidity and turbulence. A realistic atmosphere is invariably somewhat turbulent, and may be characterized by an ambient fluid velocity v and sound speed c that vary from point to point. The absolute humidity will also vary from point to point, although possibly not as irregularly. What is ideally desired is a relatively simple scheme for predicting the probable spreads in key sonic boom signature parameters. Such parameters could be peak amplitudes, rise times, or gross quantities obtainable by signal processing that correlate well with annoyance or damage potential. The practical desire for the prediction scheme is that it require a relatively small amount of knowledge, possibly of a statistical nature, concerning the atmosphere along the propagation path from the aircraft to the ground. The impact of such a scheme, if developed, implemented, and verified, would be that it would give the persons who make planning decisions a tool for assessing the magnitude of environmental problems that might result from any given overflight or sequence of overflights.

Realistic Sonic Boom Propagation Problem

- Turbulent atmosphere
  - diffraction by smaller turbulent eddies
  - focusing and defocusing by larger turbulent eddies
- Molecular relaxation important
  - Humidity controls molecular relaxation
- Nonlinear distortion
  - tendency toward waveform steepening
  - stretching of waveform
  - more rapid elimination of very narrow spikes
  - overtaking of closely-spaced shocks

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The technical approach that has been followed by the author and some of his colleagues is to formulate a hierarchy of simple approximate models based on fundamental physical principles and then to test these models against existing data.

For propagation of sonic booms and of other types of acoustic pulses in nonturbulent model atmospheres, there exists a basic overall theoretical model that has evolved as an outgrowth of geometrical acoustics. This theoretical model depicts the sound as propagating within ray tubes in a manner analogous to sound in a waveguide of slowly varying cross-section. Propagation along the ray tube is quasi-one-dimensional, and a wave equation for unidirectional wave propagation is used. A nonlinear term is added to this equation to account for nonlinear steepening, and the formulation has been carried through to allow for spatially varying sound speed, ambient density, and ambient wind velocities. The model intrinsically neglects diffraction, so it cannot take into account what has previously been mentioned in the literature as possibly important mechanisms for turbulence-related distortion. The model as originally developed could predict an idealized N-waveform which often agrees with data in terms of peak amplitude and overall positive phase duration. It is possible, moreover, to develop simple methods based on the physics of relaxation processes for incorporating molecular relaxation into the quasi-one-dimensional model of nonlinear propagation along ray tubes.

Simpler Propagation Models

- Propagation along ray tubes
  - stratified atmosphere
  - turbulence ignored
  - spiking and roundening effects not predicted
- Model can account for
  - gross magnification and demagnification
  - nonlinear distortion
  - molecular relaxation contribution to rise times

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There are currently two methods that are in use for carrying out such an incorporation of relaxation phenomena into propagation predictions; one is a numerical algorithm that arose out of the 1973 doctoral dissertation by Pestorius, which carries forward the propagation along a ray tube as an alternating sequence of two basic types of steps. In one step one has linear propagation of a Fourier superposition of frequency components, and each frequency component is shifted in phase and attenuated in an appropriate manner with propagation along a given distance interval. In the other step, the nonlinear distortion is carried out according to inviscid weak shock theory through the same distance interval.

The author and his colleagues, on the other hand, have been working with an explicit set of approximate partial differential equations analogous to Burgers' equation, an early version of which can be found in the author's 1981 textbook. One very simple model that has been used by the author and his colleagues is what is termed the asymptotic quasi-steady theory of sonic boom waveforms.

Solution Techniques

- Transient evolution using Pestorius algorithm
  - split-step algorithm
  - nonlinear distortion step
  - molecular relaxation step using Fourier transforms
- Asymptotic quasi-steady theory
  - basic waveform shape predicted with neglect of molecular relaxation
  - rise-phase corrected for molecular relaxation
  - correction based on local humidity, temperature, and net pressure jump in shock

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The asymptotic quasi-steady theory predicts an explicit waveform shape near the leading shock, given the waveform peak amplitude and the local humidity and temperature. The model incorporates modecular relaxation, which is slower for dryer air and consequently a cause of sharper bangs in humid air.

This can alternately be termed the "steady-state" model or the "quasi-frozen waveform" model. The terminology is not ideal, and one must first understand the detailed assumptions involved before adopting any conceptions about what the terminology implies. The ideas involved go back to early papers by G. I. Taylor and Richard Becker on the structure of shock waves, only here the mechanisms of interest are molecular relaxation rather than viscosity or thermal conduction. The first tenet of the theory is that molecular relaxation is important only in the rise phase of waveforms. Such seems justified because the characteristic times, such as positive phase duration, associated with other portions of the waveform are invariably much longer than the characteristic relaxation times for molecular relaxation. During most of the time at which the waveform is being received, it is reasonable to assume that the air is in complete quasi-static thermodynamic equilibrium. Molecular relaxation is a nonequilibrium thermodynamic phenomenon and is important only when pressure is changing rapidly, with characteristic times of the order of a few milliseconds or less.

Molecular-relaxation correction

- Rise-phase prediction near $t = t_{\rm sh}$:

$$p = (\Delta p)_{\rm sh} f_{\rm rise}(t - t_{\rm sh}, \text{parameters})$$

$$f_{\rm rise}(\tau, \text{parameters}) \rightarrow 0 \quad \text{as} \quad \tau \rightarrow -\infty$$

$$f_{\rm rise}(\tau, \text{parameters}) \rightarrow (\Delta p)_{\rm sh} \quad \text{as} \quad \tau \rightarrow +\infty$$

- parameters are $(\Delta p)_{\rm sh}$, temperature, and humidity
- Composite expression:

$$\begin{aligned} p =& p_{\rm basic}(t) H(t_{\rm sh} - t) \\ &+ (\Delta p)_{\rm sh} f_{\rm rise}(t - t_{\rm sh}, \text{parameters}) \\ &+ \left[p_{\rm basic}(t) - (\Delta p)_{\rm sh}\right] H(t - t_{\rm sh}) \end{aligned}$$

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A second hypothesis, which is related to the first, but which requires extensive analysis for its justification, is that the rise phase of the waveform is determined solely by the peak overpressure of the shock and the local properties of the atmosphere. Strictly speaking, one expects the waveform received at a local point to be the result of a gradual evolution that took place over the entire propagation path, so it depends in principle on the totality of the atmospheric properties along the path. However, an N-wave shape, or at least the positive phase portion, is often established fairly close to the source (i.e., the flight trajectory in the case of sonic boom generation) relative to the overall propagation distance. With increasing propagation distance, the peak overpressure decreases, but does so very slowly, and the positive phase duration increases, but also does so very slowly. There is a net loss of energy from the wave and the loss takes place almost entirely within the rise phases of the shocks. However, the manner in which the peak overpressure decreases and the positive phase duration increases is virtually independent of the energy loss mechanism.

One should note in particular that the model based on the asymptotic quasi-steady theory predicts rise times.

Rise-time prediction

- $(\Delta t)_{\mathrm{rise}}$ is time interval for $f_{\mathrm{rise}}(t - t_{\mathrm{sh}}, \text{parameters})$ to rise from 0.1 to 0.9.

- parameters are $(\Delta p)_{\mathrm{sh}}$, temperature, and humidity

- $(\Delta t)_{\mathrm{rise}}$ is function of "parameters"

$$(\Delta t)_{\mathrm{rise}} = F([\Delta p]_{\mathrm{sh}}, \text{temperature}, \text{humidity})$$

where F is "universal" function.

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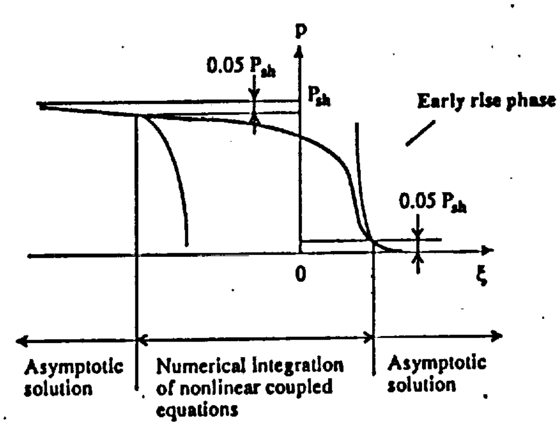
The viewgraph here sketches the principal ideas that are embodied in calculating the rise-phase of a sonic boom profile according to the quasi-static theory. The parameter ξ in the diagram is $x - v_{sh}t$. Ahead of the shock the overpressure p is asymptotically zero, and the theory predicts the manner in which it approaches zero. Behind the shock p asymptotically approaches the net shock overpressure P_{sh}, and here again the theory predicts the manner in which this asymptotic limit is achieve. For points in between one must must numerically solve a set of coupled ordinary nonlinear differential equations. One interesting aspect of the solution is that the nitrogen relaxation is only important is the later portion of this rise phase.

Early rise phase: O_2 relaxation dominates
Later rise phase: N_2 relaxation dominates

The theoretical rise phase is determined using asymptotic and numerical solution methods:



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Kang carried out detailed comparisons of the predictions of the frozen-profile model with actual waveforms of sonic booms, recorded by the US Air Force in the Mojave Desert in 1987. The original comparison reported in Kang's doctoral thesis, unfortunately, was flawed because the reflection at the ground was incorrectly taken into account. (That such may have been the case was first suggested to the author by Gerry McAninch as a result of a conversation with Alan Wenzel.)

Waveforms measured during flight tests

at ground level

- $(\Delta t)_{\text{rise}}$ can be measured for each data sample.
- Theory predicts

$$(\Delta t)_{\text{rise}} = F([\Delta p]_{\text{sh}}, \text{temperature}, \text{humidity})$$

  where F is "universal" function.
- For comparison of data with theory, what value of $[\Delta p]_{\text{sh}}$ does one use in the "universal function" F?
- Theory is based on idealization of plane wave propagating in one direction through unbounded medium. Measurements were made on the ground

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The corrected procedure takes the ground as rigid and the reflection process as linear, so that the waveform at the ground has the same time dependence as the incident wave, only the amplitude is twice as great. The theoretical predictions based on accumulated nonlinear effects for a unidirectional propagating wave are applicable to the so-inferred incident wave.

Rigid ground idealization:

- $$p_{\text{ground}} = 2p_{\text{inc}}; \qquad (\Delta p)_{\text{sh,grnd}} = 2(\Delta p)_{\text{sh,inc}}$$

- Theory predicts a function F, where

  $$(\Delta t)_{\text{rise}} = F([\Delta p]_{\text{sh,inc}}, \text{temperature}, \text{humidity})$$

  decreases with increasing (Δp) roughly as $(\Delta p)^{-1}$

- Previous comparisons were erroneously based on above with argument taken as $[\Delta p]_{\text{sh,grnd}}$ rather than $(1/2)[\Delta p]_{\text{sh,grnd}}$

- Using a (Δp) that is too large by a factor of 2 means you tend to underpredict the rise time.

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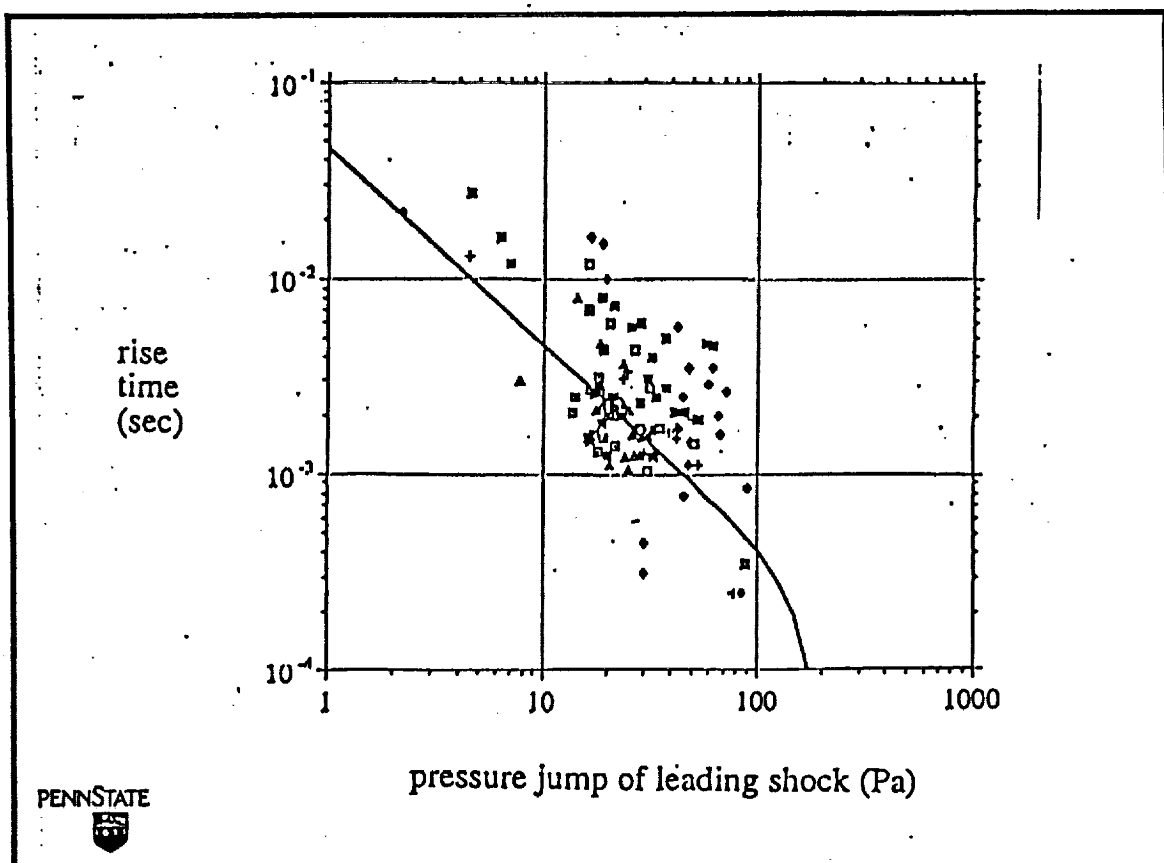
This summarizes the comparison of rise time data with the asymptotic quasi-steady-state theory. The overpressures on the horizontal axis are those actually observed in waveforms recorded at the ground. The theoretical curve is derived assuming that the incident wave's overpressure is one-half of what is measured. All of the data was taken at times when the humidity and temperature were very nearly the same, so one theoretical curve suffices for the entire data set. The data was taken in 1987 in the Mojave desert, with various airplanes, flying at various altitudes and with various Mach numbers. The relative humidity was 24% and the temperature at the ground was 33° C.



The overall result of the comparison, with ground reflection taken into account as here described, is that the theoretically predicted rise times are roughly the same as the average rise time of the experimental waveforms under conditions of the same incident waveform pressure amplitude and the same atmospheric humidity.

Inferences from updated theory-data comparison:

- Relaxation theory predicts rise times of correct order of magnitude
- Theoretical predictions of rise times tend (but not in all cases) to be lower than observed in field data
- Turbulence is major factor in rise times.

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The rise phase structure of the waveform is basically a tug-of-war between nonlinear steepening and molecular relaxation. When the boom passes through a region where the molecular relaxation is weaker, the nonlinear steepening causes the waveform to sharpen up and causes the rise time to decrease until the mechanisms balance each other out. One can associate a characteristic adjustment time with this restoration of the balance between the two mechanisms. The quasi-steady hypothesis used in the simpler models hypothesis rests on the assertion that this characteristic adjustment time is substantially less than any characteristic time it takes for the waveform to propagate over a path segment within which the relevant atmospheric properties (especially the absolute humidity) change appreciably. A current question regarding the closely related and competing effects of molecular relaxation and nonlinear steepening is just how resilient is the steady-state model.

Raspet has referred to the characteristic distance over which recovery from a perturbation to the asymptotic waveform takes place as the healing distance.

Concept of healing distance:

- Suppose rise phase of waveform slightly perturbed from asymptotic quasi-steady-state form
- For further propagation through a homogeneous medium, the perturbation dies out
- Rise phase eventually evolves to asymptotic form that depends on x and t only in combination $x - V_{\mathrm{sh}}t$.
- What is characteristic additional propagation distance for the perturbation to die out?

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The question of the magnitude of the healing distance has been answered tentatively by detailed numerical computations of transient evolution of waveforms over large propagation distances by Raspet and others, with the apparent prediction that it takes propagation distances of several kilometers (the value depending on the peak amplitude) for the waveform to recover from slight perturbations in the steady-state shape. Even more so than is the case for the rise time, there is room for considerable arbitrariness in the definition of this healing distance. The present author suspects that one can devise a meaningful definition for which the numerical value of this healing distance is less than a kilometer for representative cases of interest.

That the latter speculation has some credibility can be seen at once when one considers that a typical value for the pertinent relaxation time is about 1 ms (corresponding to the relaxation time of N_2 in air with 50% relative humidity). The waveform moves with roughly the sound speed, which is of the order of 340 m/s, and a pressure amplitude of 50 Pa would move with an additional speed increment of $\beta P/\rho_0 c = (1.2)50/400 = 0.15\,\mathrm{m/s}$. If such a peak lags a zero-crossing by a distance of $(.001)(340)$ m, then the distance for it to overtake the zero-crossing in the absence of any dissipation effect is approximately $(.001)(340)^2/0.15$ or $0.8\,\mathrm{km}$. To put such an estimate in perspective, one can contrast this with a distance of 11 km for a typical height of the tropopause and with a representative distance of 15 km for a ray trajectory from the aircraft flight track to the ground. The numbers sometimes mentioned for the thickness of the atmosphere's turbulent boundary layer, on the other hand, are much smaller, on the order of 1 to 2 km.

Order of magnitude of healing distance:

- Take pertinent relaxation time as 1 ms
- Waveform moves with speed $c \approx 340\,\mathrm{m/s}$
- Pressure amplitude of 50 Pa moves with additional speed increment of

$$\beta(\Delta p)/\rho_0 c = (1.2)(50)/(400) = 0.15\,\mathrm{m/s}$$

- Propagation distance for peak to overtake zero crossing in absence of dissipation:

$$(0.001)(340)^2/0.15 \approx 0.8\,\mathrm{km}$$

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Kang and the author some time ago initiated a systematic study of the tendency toward the steady-state profile in which the analysis was based more on analytic considerations rather than lengthy numerical case studies. In this theory the steady-state profile provides a framework whereby perturbations to the profile can be regarded as a superposition of natural eigenfunctions of fixed shape as seen by someone moving with the nominal shock speed. Each such natural eigenfunction has its own natural decay time which results as an eigenvalue in the theory. The task then emerges of systematically determining these eigenfunctions and the associated eigenvalues.

Eigenfunctions and characteristic healing distances:

- Propagation equations are

$$\frac{\partial u}{\partial t} + c\frac{\partial u}{\partial x} + \beta u\frac{\partial u}{\partial x} - \delta'_{\rm cl}\frac{\partial^2 u}{\partial x^2} + \sum_\nu (\Delta c)_\nu \frac{\partial u_\nu}{\partial x} = 0$$

$$\frac{\partial u_\nu}{\partial t} + \frac{1}{\tau_\nu}u_\nu = \frac{\partial u}{\partial t}$$

where $p = \rho_0 xu$.

- Steady state solution is

$$u = F(\xi); \quad u_\nu = F_\nu(\xi)$$

where $\xi = x - V_{\rm sh}t$.

- Take perturbed solution to be of form

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$$u = F(\xi) + \psi(\xi)e^{-\lambda x}; \quad u_\nu = F_\nu(\xi) + \psi_\nu(\xi)e^{-\lambda x}$$

This summarizes the mathematical problem that ideally should be solved to determine a sequence of healing distances. One expects the so-posed eigenvalue problem to have several solutions that correspond to real eigenfunctions. The structure of the problem is still to be studied, and one does not have any orthogonality theorems as yet regarding different eigenvector functions. However, a crude solution can be found if one replaces the governing equations by Burgers' equation with an effective bulk viscosity.

Equations governing healing eigenfunctions:

- $$u = F(\xi) + \psi(\xi)e^{-\lambda x}; \quad u_\nu = F_\nu(\xi) + \psi_\nu(\xi)e^{-\lambda x}$$

  are inserted into propagation equations; one keeps only linear terms in the ψ's, with result in matrix form

  $$[\mathcal{L}]\{\psi\} = \lambda[\mathcal{M}]\{\psi\} + \lambda^2[\mathcal{N}]\{\psi\}$$

  where $[\mathcal{L}]$, $[\mathcal{M}]$, and $[\mathcal{N}]$ are 3-by-3 matrices made up of linear operators, each possibly involving differentiation with respect to ξ and the steady state profile functions $F(\xi)$, $F_1(\xi)$, $F_2(\xi)$.

- $\{\psi\}$ is an eigenfunction array (ψ, ψ_1, ψ_2)

- Nontrivial solution (boundary condition of ψ's equal to zero at $\xi = \pm\infty$) exists only for special values of λ.

- Special values of λ construed as eigenvalues and as reciprocals of characteristic healing distances.

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Predictions of healing distance based on effective bulk viscosity:

- $$L_{\text{heal}} = 16\frac{\delta_{\text{appar}}}{c}\left(\frac{\rho c^2}{\beta(\Delta p)_{\text{sh}}}\right)^2$$
$$\delta_{\text{appar}} = \frac{\mu_{\text{bulk}}}{2\rho} + \cdots$$

- Tisza's equivalence:
$$\mu_{\text{bulk}} \approx 2\rho c(\Delta c)_\nu \tau_\nu$$

- so we infer
$$L_{\text{heal}} = 16c\tau_\nu\left(\frac{(\Delta c)_\nu}{c}\right)\left(\frac{\rho c^2}{\beta(\Delta p)_{\text{sh}}}\right)^2$$

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Representative numerical values based on effective bulk viscosity:

- $$L_{\text{heal}} = 16c\tau_\nu\left(\frac{(\Delta c)_\nu}{c}\right)\left(\frac{\rho c^2}{\beta(\Delta p)_{\text{sh}}}\right)^2$$

- Typical numerical values (O_2 relaxation)
$$(\Delta p)_{\text{sh}} = 50\,\text{Pa}; \quad \frac{\rho c^2}{\beta(\Delta p)_{\text{sh}}} = 2300$$
$$\frac{(\Delta c)_\nu}{c} = 3 \times 10^{-4}; \quad \tau_\nu = 10^{-5}\,\text{s}$$

- so we infer $L_{\text{heal}} \sim 250\,\text{m}$

- But you can get much larger values using parameters for N_2 relaxation.

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The author and his colleagues have recently been exploring various methods for computation of sonic boom propagation through turbulent atmospheres and have obtained a generalization of the Burgers equation which has some similarities to the KZK equation and to the NPE equation of McDonald and Kuperman. The equation can be regarded as a string of "small terms" tacked onto the inviscid linear Burgers equation, with individual terms accounting for nonlinear steepening, viscous attenuation, refraction, molecular relaxation, and diffraction.

The Penn State Univ Propagation Equation (PSUPE):

- Generalization of Burgers equation (which really should be called the Cole equation, as inferred from Cal Tech literature of late 1940's by ADP)
- Term for diffraction by smaller turbulent eddies
- Molecular relaxation term
- Nonlinear steepening term
- Turbulence can be simulated using Fourier transforms (series) with random number algorithms used in selection of coefficients.
- Larger scale turbulence and ambient atmospheric stratification can be incorporated in multiplicative Blokhintzev factor (roughly same as $1/\sqrt{A}$, where A is ray tube area) which varies with distance along central ray.

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Pestorius had a good idea

- Basic Pestorius algorithm alternated between nonlinear distortion (NL) and absorption-dispersion (AD) steps
- Noise-Con 93 paper by ADP shows that this is rigorously correct in limit of small step sizes Δx. Taking limit yields partial differential equation

$$\frac{\partial u}{\partial x} + \frac{1}{c}\frac{\partial u}{\partial t} = \mathcal{M}_{\mathrm{NL}}\{u\} + \mathcal{M}_{\mathrm{AD}}\{u\}$$

  where right-side terms are same as appear in PSUPE.

- Why not use same idea to handle the diffraction term?

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Concluding remarks

- Solving PSUPE to realistically simulate sonic boom statistics will require major theoretical innovations in computational acoustics. (But so what?)
- Two doctoral theses presently in progress at Penn State on alternate approaches to solving PSUPE
  - Kirchhoff integral for the diffraction step.
  - Finite-difference algorithms using flux-corrected transport methods of Boris and McDonald.
- Another idea being pursued by the present author independently is that the healing eigenfunctions are natural basis set for a variational (or Galerkin) formulation which reduces the dimensionality of the problem, and which leads to simpler ways of decomposing messy waveforms encountered in field data.

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