Implications for High Speed Research:
The Relationship Between Sonic Boom
Signature Distortion
and Atmospheric Turbulence

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Presentation at NASA HSR Sonic Boom Workshop
NASA Ames Research Center
May 12-14, 1993
Good morning. My name is Dr. Victor W. Sparrow of the Penn State University Graduate Program in Acoustics. The Graduate Program in Acoustics is a Department in the Penn State College of Engineering. My co-author is Tom Gionfriddo, a graduate student at Penn State who finished up his M.S. degree in Acoustics early in the Fall of 1992. Much of the work I will be presenting today is the result of Tom's effort on his master's thesis.

The topic I will be discussing today is Implications for High Speed Research: The Relationship Between Sonic Boom Signature Distortion and Atmospheric Turbulence. But before we get to these implications, let us review a little history concerning previous research on sonic boom waveform distortion.

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In 1968 Dr. Allan Pierce hypothesized that the cause of sonic boom distortion, which takes the form of spiked or rounded waveforms, was due to atmospheric turbulence. This was a theoretical result, and was not widely accepted at the time due to the lack of experimental evidence.

In 1973 Ribner, Morris, and Chu performed laboratory experiments which showed that one could cause sonic boom shaped waves to spike or become rounded, if the waves were propagated through a turbulent jet. This laboratory result gave some evidence that turbulence could, in fact, be the cause of sonic boom waveform distortion in the atmosphere. Others also performed similar laboratory experiments.

In the mid to late 1970's, however, the role of molecular relaxation absorption in sonic boom propagation had not yet been established. The relative importance of molecular relaxation and atmospheric turbulence for sonic boom distortion was not clear.

- In 1968 Pierce hypothesized (Ref. 1) that the cause of sonic boom distortions, such as spiked or rounded waveforms, was due to atmospheric turbulence.

- In 1973 Ribner, Morris, and Chu found in the laboratory (Ref. 2) that sonic boom shaped acoustic waves indeed were distorted in a turbulent jet, producing both spiked and rounded waveforms.

- However, the relative importance of atmospheric turbulence and molecular relaxation effects had yet to be established.
By the early 1980's the theory for molecular relaxation absorption in the atmosphere was fairly well understood. The two dominant processes are Oxygen and Nitrogen relaxation, with humidity (water vapor) being the next most important process. The result of the theory is that molecular relaxation cannot cause the spikes on sonic boom waves, although they can round sonic boom waves somewhat. It is thought that the rounding effect is insufficient to explain observed distorted waveforms, however.

After the molecular relaxation theory was understood, it became the common notion that atmospheric turbulence is primarily responsible for sonic boom distortion. This is an assumption which most workers in sonic boom propagation have adopted, since the spiking and rounding could not be due to molecular relaxation. Most of the talks during the rest of this session make this assumption.

- By the early 1980's molecular relaxation was fairly well understood. Molecular relaxation cannot cause spikes on sonic boom waves.

- Thus, most researchers in sonic boom propagation have assumed that turbulence must be responsible for sonic boom distortion.

- Most of the talks in this session make such an assumption.
Until recently this assumption has not been tested statistically. Such a test would provide a firm foundation for much of the ongoing work on sonic boom propagation through turbulence at a number of NASA Contractor sites, including The University of Mississippi, The University of Texas at Austin, Penn State University, Wyle Laboratories, etc.

One supposes here that an originally undistorted sonic boom propagating through turbulence should, on average, be more distorted as it propagates through more turbulence.

However,

until recently this assumption has not been tested statistically with real sonic boom data and real atmospheric turbulence.

One supposes that a sonic boom propagating through more turbulence should, on average, be more distorted.
Therefore, the purpose our research study is was to test the above hypothesis rigorously. That is, the specific purpose is to see if increasing travel distances through turbulence is correlated with increasing sonic boom wave distortion. This paper documents the results of our study.

In this study it is assumed that the strength of the atmospheric turbulence is somewhat uniform, and it is the travel distance of booms through the turbulence that is important. This assumption is necessary due to the absence of direct turbulence measurements to complement the sonic boom experimental data which will be used to test the hypothesis.

The Purpose of this study is to test the above hypothesis rigorously.

More specifically, is it true that

the further a boom travels through turbulence

⇒

increased waveform distortion

?
In this study there were two primary tasks. The first was to develop an algorithm for quantifying the distortion in a sonic boom. Such an algorithm should be somewhat automatic, with minimal human intervention. Once the algorithm was developed, it was used to test the previously mentioned hypothesis. This hypothesis testing was the second task. Using readily available sonic boom data, we statistically tested whether there was a correlation between the sonic boom distortion and the distance a boom traveled through atmospheric turbulence.

In this study we

A. Developed an algorithm to quantify the distortion in a sonic boom waveform.

B. Tested the correlation between this distortion and the distance a boom traveled through atmospheric turbulence.
The terminology that is used in our paper is described here. The booms have a maximum shock overpressure after some rise time. This maximum shock overpressure is called the bow shock. The duration is then defined as the time as waveform slopes off to the minimum shock overpressure at the tail shock. For most of the booms examined, the duration was between 75 and 200 milliseconds, and the amplitudes varied between 30 and 200 pascals. Most booms have a subsonic fundamental frequency in the range of 6 to 10 hertz. Our definition of rise time is from 10% to 90% of the maximum shock overpressure.

**SONIC BOOM WAVEFORM TERMINOLOGY**

- **Duration**
- **Maximum shock overpressure**
- **Rise time**
- **Bow shock**
- **Rise phase**
- **Minimum shock overpressure**
- **Tail shock**

Duration typically 75-200 ms, amplitude 30-200 Pa

Subsonic fundamental ~ 6-10 Hz

Rise time is time from 10% to 90% of maximum shock overpressure

Why annoying? Rise phase structure important
Sonic booms can be distorted in many ways as they propagate through the atmosphere. Here a large number of sonic booms were collected into categories, and a representative waveform example is shown from each category. The waves which were the most undistorted were called Classic N. Waveforms showing one large peak were called Peaked. Many of waveforms have two distinct peaks, and were called Double-peaked. Some waveforms had many peaks, and these were called Multi-peaked. The U-wave category was defined as those waves having very large spikes on both the bow and tail shocks, the spikes dominating all features. The Rounded waveform category had rounded bow and tail shocks. All other waveforms, which could not be classified in one of the previous categories, were called Messy (for lack of a better term).

## DATA CLASSIFICATION BY WAVEFORM SHAPE

- **Classic N**: Waves having very large spikes on both the bow and tail shocks, the spikes dominating all features.
- **Peaked**: Waveforms showing one large peak.
- **Double-peaked**: Waveforms with two distinct peaks.
- **Multi-peaked**: Waveforms with many peaks.
- **U-wave**: Waves having very large spikes on both the bow and tail shocks.
- **Rounded**: Waveforms with rounded bow and tail shocks.
- **Messy**: All other waveforms, which could not be classified in one of the previous categories.
Because of the great variability in the distortions a sonic boom wave can undergo, an automatic numerical procedure was developed for quantifying the distortion in a sonic boom wave. The first component of this quantification is to find a basis for comparison. Since one usually puts microphones on the ground, and measures the sonic boom wave only after it has been distorted, it is necessary to estimate the wave shape of the sonic boom before it was distorted.

The assumption made here is that before any waveform distortions occurred that the sonic boom wave had the shape of a perfect N-wave with zero rise times on both bow and tail shocks. The energy in the distorted sonic boom is measured, and then it is assumed that the undistorted ideal N-wave has the same energy. Obviously, this is an approximation.

Given this information the proper maximum overpressure, duration, and start time offset of the ideal N-wave is automatically computed. Additional details on the elaborate algorithm used to calculate the parameters for the ideal N-wave, given the distorted sonic boom wave, are available in the M.S. thesis of Gionfriddo (Ref. 3).

**WAVEFORM DISTORTION ANALYSIS: COMPARE DATA TO IDEAL N-WAVE MODEL**

![Diagram of a sonic boom signature](Image)

Model of the sonic boom signature just prior to entering the TBL.

Ideal N-wave and the recorded data have equal acoustic energy.

The ideal N-wave is superimposed on the same time axis as the recorded data.

The proper maximum overpressure, duration, and start time offset of the ideal N-wave must be determined.

It is desired to have Classic N data with the lowest mean-squared deviation for any waveshape. Therefore, Classic N data are used as a reference for superimposing the ideal N-wave correctly over the recorded data.
Here is a typical recorded sonic boom wave with its ideal N-wave superimposed.

To quantify the distortion in the measured waveform, the notion of a mean-squared deviation is used, defined below. The functions $p_{\text{ideal}}[n]$ and $p_{\text{recorded}}[n]$ are both assumed to be digitized data. For the example waveform shown here, the mean-squared deviation is 0.075.

**EXAMPLE OF RECORDED BOOM WITH IDEAL N-WAVE SUPERIMPOSED**

![Graph showing an example of recorded boom with ideal N-wave superimposed.](image)

**MEASURE OF WAVEFORM DISTORTION: MEAN-SQUARED DEVIATION**

The *mean-squared deviation* (MSD) is defined as:

$$\text{MSD} = \frac{\sum (p_{\text{ideal}}[n] - p_{\text{recorded}}[n])^2}{\sum (p_{\text{ideal}}[n])^2}$$
Here are two more sonic boom waveforms with superimposed ideal N-waves. The upper waveform is a Peaked sonic boom wave, and it shows a mean-squared deviation of 0.095. The lower waveform is a Rounded waveform, having a much larger mean-squared deviation of 0.23.
The particular sonic boom data we analyzed was taken near Edwards Air Force Base in the late summer of 1987 by the U. S. Air Force. Autonomous Boom Event Analyzer Recorder (BEAR) systems took data over several days from a wide variety of supersonic aircraft: F-4, F-14, F-15, F-16, F-18, SR-71, T-38, AT-38, and F-111D. The recorders were placed at the mile markers along a road in the area. The aircraft were to fly perpendicular to the road over a specific flight track. From 44 aircraft flights, over 500 data files were obtained for analysis. The specific position of where the aircraft overflew the road was recorded, and this information has been taken into account in our analysis.

MOJAVE DESERT SONIC BOOM ACQUISITION SCHEMATIC

F-4, F-14, F-15, F-16, F-18, SR-71, T-38, AT-38, and F-111D aircraft

Lateral array of thirteen Boom Event Analyzer Recorder (BEAR) systems

Flight track perpendicular to array

44 Flights ~ 500 data files

ground flight track
To determine the path length a sonic boom will traverse through the turbulent boundary layer near the earth's surface, a short exercise in three-dimensional solid geometry is needed. Knowing the altitude of the aircraft, its Mach number, the lateral ground distance of the receiving microphone from the aircraft's actual flight track, and the thickness of the boundary layer, this path length can be obtained. The path length is shown as a dark solid line in the diagram.

Function of:
- altitude
- Mach number
- lateral ground distance
- TBL thickness
The turbulent boundary layer is defined as the thickness of the mixing layer of the planetary boundary layer. One may assume that the turbulence in the mixing layer is somewhat evenly distributed and homogeneous.

To determine the thickness of this layer, a numerical model by A. K. Blackadar was employed. The numerical model takes into account information from rawinsonde launches, surface weather data, satellite cloud photos, and soil parameters for the site of the sonic boom tests. Blackadar's model provides daily profiles to 2000 m height for temperature, water content, wind, and boundary layer thickness.

PATH LENGTH THROUGH THE TURBULENT BOUNDARY LAYER (CONTINUED)

TBL thickness estimated using numerical model by A. K. Blackadar (PSU meteorology)

Input atmospheric information from rawinsonde launches, surface weather station, satellite cloud photos, soil parameters

Model provides diurnal profiles to 2000 m for temperature, water content, wind, and TBL thickness
For the days of the tests, 3 - 8 August 1987, profiles of the turbulent boundary layer thickness were obtained from the Blackadar model. One can see the thickness of the boundary layer generally grew during the day between 7:00 AM and 2:00 PM. Because of the meteorological conditions present, the boundary layer grew much more on 3 August and 4 August than it did on the other days of the tests.
Before continuing, the analysis procedure will be summarized. On one track the BEAR sonic boom data was obtained and calibrated. From this data pressure versus time plots were obtained, which were subsequently sorted into waveshape categories. Given these plots, the sonic boom distortion quantification algorithm was run, and mean-squared deviations from the computer generated corresponding ideal N-waves were obtained for all the waveforms.

On the other track, the aircraft flight parameters and geometry were combined with the meteorological data and subsequent predictions of the boundary layer height from Blackadar's model. From this information the path length through the turbulence was found for each recorded sonic boom waveform.

It was then possible to determine if a statistical correlation existed between the mean-squared deviation and the path length the sonic boom traveled through the turbulence.

**ANALYSIS PROCEDURE**

![Diagram of the analysis procedure](image-url)
During the analysis procedure it became immediately apparent that those waveforms which were either shaped as a Classic N-waves, or were nearly shaped as such, were primarily manifest only in the early morning hours.

This result leads us to believe that as the turbulent boundary layer grew through the day, that the number of undistorted waveforms decreased. This result is averaging over all of the usable observation data.

**SIMPLE DEMONSTRATION: FRACTION OF RECORDED SONIC BOOM WAVEFORMS THAT ARE LEAST-DISTORTED**

445 recordings from 7/31 - 8/7 overflights during 7AM-1PM

Fraction is number of *classic N* and *nearly classic N* waveforms per number of waveforms recorded during the hour intervals

classic N $\Rightarrow$ least distorted

TBL thickness increasing $\rightarrow$
This is a plot of the mean-squared deviation as a function of altitude (or roughly Mach number, since faster planes generally flew higher). It is apparent that increased altitude and speed imply decreased waveform distortion and a decreased spread of data points. Now higher and faster flying planes generally will have shorter propagation paths through the turbulent boundary layer, which can be shown from simple geometry. Thus, it appears as if longer propagation paths through the turbulence result in larger mean-squared deviations, i.e., more distorted waveforms.

ALTITUDE AND MACH NUMBER INFLUENCE UPON WAVEFORM DISTORTION

Increased altitude and speed $\rightarrow$ decreased waveform distortion and decreased spread of data points

F-18, F-15, SR-71, F-4, and F-16 data
Here are some results listed by plane type of the linear correlation coefficients between the mean-squared deviation and the path length through the turbulent boundary layer. It is seen that there is strong correlation in some cases (F-18 and F-15) and fair correlation in the others. This also gives us some evidence that increasing distortion is correlated with increasing path length through the turbulence.

The F-4 and F-16 data have the lowest correlations and the lowest altitudes. For these cases the sonic boom signatures probably did not have time to develop into an N-wave by the time it began to interact with the turbulence. The other aircraft types flown had too few data points to draw any statistical conclusions.

We also are currently working on obtaining correlation coefficients grouped by altitude and Mach number as well as by plane type, to determine how these factors interrelate.

<table>
<thead>
<tr>
<th>Aircraft type</th>
<th>Correlation coefficient: MSD vs TBL path length</th>
<th>Number of samples</th>
</tr>
</thead>
<tbody>
<tr>
<td>F-18</td>
<td>0.712</td>
<td>62</td>
</tr>
<tr>
<td>F-15</td>
<td>0.591</td>
<td>75</td>
</tr>
<tr>
<td>SR-71</td>
<td>0.398</td>
<td>48</td>
</tr>
<tr>
<td>F-4</td>
<td>0.324</td>
<td>46</td>
</tr>
<tr>
<td>F-16</td>
<td>0.318</td>
<td>65</td>
</tr>
</tbody>
</table>

(Correlation coefficients for mean-squared deviation and TBL path length)

Strong correlation in some cases, fair correlation in others

F-4 and F-16 have lowest correlations and the lowest altitudes
- Sonic boom signature before TBL probably not N-wave
- Less distance for nonlinear steepening to work before TBL
The conclusions of this study are the following: a strong linear correlation exists between the mean-squared deviation and path length through the turbulence for the F-18 and F-15 sonic boom data. Fair correlation exists for the SR-71, F-4, and F-16 data. An increase in altitude and speed results in decreased waveform distortion and a small deviation between distortion values. Looking at the waveform classification results, the large percentage of Classic N-wave data during each day’s early flights seems to correspond with the thin boundary layer at that time.

Conclusions:

- A strong linear correlation exists between mean-squared deviation and path length through the turbulence for F-18 and F-15 sonic boom data.

- Fair correlation exists for SR-71, F-4, and F-16 data.

- An increase in altitude and speed results in decreased waveform distortion and a smaller deviation between distortion values.

- Looking at waveform classification results, the large percentage of classic N wave data during early flights seems to correspond with the thin boundary layer at that time.
The implications of this study for high speed research are the following: Increased interaction between real atmospheric turbulence and actual sonic boom data does imply more distorted waveforms. The common assumption prevailing in the sonic boom propagation community for the last several years has been validated statistically. And most importantly, it is now clear that atmospheric turbulence will determine how well a shaped sonic boom will remain shaped as it propagates to the ground. We are now led to believe that higher and faster aircraft having shaped sonic booms will, on average, have more shaped boom preserved than will aircraft flying at lower altitudes and slower speeds, since flying higher and faster minimizes the path length through the turbulence.

Implications for High Speed Research:

- Increased interaction between real atmospheric turbulence and actual sonic booms does imply more distorted waveforms.
- This common assumption has been validated statistically.
- Most Important: Atmospheric turbulence primarily will determine how well a shaped sonic boom will remain shaped as it propagates to the ground.
The authors would like to thank Dr. Allan D. Pierce and Dr. Richard Raspet for numerous discussions. This research was supported by NASA Langley Research Center, under grant NAG-1-1365, administered by Dr. G. L. McAninch, Scientific Officer.

References


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The topic I will be discussing today is Implications for High Speed Research: The Relationship Between Sonic Boom Signature Distortion and Atmospheric Turbulence. But before we get to these implications, let us review a little history concerning previous research on sonic boom waveform distortion.

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In the mid to late 1970's, however, the role of molecular relaxation absorption in sonic boom propagation had not yet been established. The relative importance of molecular relaxation and atmospheric turbulence for sonic boom distortion was not clear.

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By the early 1980's the theory for molecular relaxation absorption in the atmosphere was fairly well understood. The two dominant processes are Oxygen and Nitrogen relaxation, with humidity (water vapor) being the next most important process. The result of the theory is that molecular relaxation cannot cause the spikes on sonic boom waves, although they can round sonic boom waves somewhat. It is thought that the rounding effect is insufficient to explain observed distorted waveforms, however.

After the molecular relaxation theory was understood, it became the common notion that atmospheric turbulence is primarily responsible for sonic boom distortion. This is an assumption which most workers in sonic boom propagation have adopted, since the spiking and rounding could not be due to molecular relaxation. Most of the talks during the rest of this session make this assumption.

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- Thus, most researchers in sonic boom propagation have assumed that turbulence must be responsible for sonic boom distortion.

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One supposes that a sonic boom propagating through more turbulence should, on average, be more distorted.
Therefore, the purpose our research study is was to test the above hypothesis rigorously. That is, the specific purpose is to see if increasing travel distances through turbulence is correlated with increasing sonic boom wave distortion. This paper documents the results of our study.

In this study it is assumed that the strength of the atmospheric turbulence is somewhat uniform, and it is the travel distance of booms through the turbulence that is important. This assumption is necessary due to the absence of direct turbulence measurements to complement the sonic boom experimental data which will be used to test the hypothesis.

The Purpose of this study is to test the above hypothesis rigorously.

More specifically, is it true that

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increased waveform distortion

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In this study there were two primary tasks. The first was to develop an algorithm for quantifying the distortion in a sonic boom. Such an algorithm should be somewhat automatic, with minimal human intervention. Once the algorithm was developed, it was used to test the previously mentioned hypothesis. This hypothesis testing was the second task. Using readily available sonic boom data, we statistically tested whether there was a correlation between the sonic boom distortion and the distance a boom traveled through atmospheric turbulence.

In this study we

A. Developed an algorithm to quantify the distortion in a sonic boom waveform.

B. Tested the correlation between this distortion and the distance a boom traveled through atmospheric turbulence.
The terminology that is used in our paper is described here. The booms have a maximum shock overpressure after some rise time. This maximum shock overpressure is called the bow shock. The duration is then defined as the time as waveform slopes off to the minimum shock overpressure at the tail shock. For most of the booms examined, the duration was between 75 and 200 milliseconds, and the amplitudes varied between 30 and 200 pascals. Most booms have a subsonic fundamental frequency in the range of 6 to 10 hertz. Our definition of rise time is from 10% to 90% of the maximum shock overpressure.

SONIC BOOM WAVEFORM TERMINOLOGY

- **Duration**: Typically 75-200 ms, amplitude 30-200 Pa
- **Subsonic fundamental**: ~ 6-10 Hz
- **Rise time**: Time from 10% to 90% of maximum shock overpressure
- **Why annoying?**: Rise phase structure important

![](image.png)
Sonic booms can be distorted in many ways as they propagate through the atmosphere. Here a large number of sonic booms were collected into categories, and a representative waveform example is shown from each category. The waves which were the most undistorted were called Classic N. Waveforms showing one large peak were called Peaked. Many of waveforms have two distinct peaks, and were called Double-peaked. Some waveforms had many peaks, and these were called Multi-peaked. The U-wave category was defined as those waves having very large spikes on both the bow and tail shocks, the spikes dominating all features. The Rounded waveform category had rounded bow and tail shocks. All other waveforms, which could not be classified in one of the previous categories, were called Messy (for lack of a better term).
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The assumption made here is that before any waveform distortions occurred that the sonic boom wave had the shape of a perfect N-wave with zero rise times on both bow and tail shocks. The energy in the distorted sonic boom is measured, and then it is assumed that the undistorted ideal N-wave has the same energy. Obviously, this is an approximation.

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WAVEFORM DISTORTION ANALYSIS:
COMPARE DATA TO IDEAL N-WAVE MODEL

Model of the sonic boom signature just prior to entering the TBL.

Ideal N-wave and the recorded data have equal acoustic energy.

The ideal N-wave is superimposed on the same time axis as the recorded data.

The proper maximum overpressure, duration, and start time offset of the ideal N-wave must be determined.

It is desired to have Classic N data with the lowest mean-squared deviation for any waveshape. Therefore, Classic N data are used as a reference for superimposing the ideal N-wave correctly over the recorded data.
Here is a typical recorded sonic boom wave with its ideal N-wave superimposed.

To quantify the distortion in the measured waveform, the notion of a mean-squared deviation is used, defined below. The functions $p_{\text{ideal}}[n]$ and $p_{\text{recorded}}[n]$ are both assumed to be digitized data. For the example waveform shown here, the mean-squared deviation is 0.075.

**EXAMPLE OF RECORDED BOOM WITH IDEAL N-WAVE SUPERIMPOSED**

![Graph showing acoustic pressure vs. time with MSD = 0.075]

**MEASURE OF WAVEFORM DISTORTION: MEAN-SQUARED DEVIATION**

The mean-squared deviation (MSD) is defined as:

$$MSD = \frac{\sum (p_{\text{ideal}}[n] - p_{\text{recorded}}[n])^2}{\sum (p_{\text{ideal}}[n])^2}$$
Here are two more sonic boom waveforms with superimposed ideal N-waves. The upper waveform is a Peaked sonic boom wave, and it shows a mean-squared deviation of 0.095. The lower waveform is a Rounded waveform, having a much larger mean-squared deviation of 0.23.
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**MOJAVE DESERT SONIC BOOM ACQUISITION SCHEMATIC**

F-4, F-14, F-15, F-16, F-18, SR-71, T-38, AT-38, and F-111D aircraft

Lateral array of thirteen Boom Event Analyzer Recorder (BEAR) systems

Flight track perpendicular to array

44 Flights  ~ 500 data files

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**Diagram of Flight Track and BEAR Systems**

- N
- Ground flight track
- Highway 395
- BEAR systems
To determine the path length a sonic boom will traverse through the turbulent boundary layer near the earth's surface, a short exercise in three-dimensional solid geometry is needed. Knowing the altitude of the aircraft, its Mach number, the lateral ground distance of the receiving microphone from the aircraft's actual flight track, and the thickness of the boundary layer, this path length can be obtained. The path length is shown as a dark solid line in the diagram.

**Function of:**
- altitude
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The turbulent boundary layer is defined as the thickness of the mixing layer of the planetary boundary layer. One may assume that the turbulence in the mixing layer is somewhat evenly distributed and homogeneous.

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PATH LENGTH THROUGH THE TURBULENT BOUNDARY LAYER (CONTINUED)

TBL thickness estimated using numerical model by A. K. Blackadar (PSU meteorology)

Input atmospheric information from rawinsonde launches, surface weather station, satellite cloud photos, soil parameters

Model provides diurnal profiles to 2000 m for temperature, water content, wind, and TBL thickness
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It was then possible to determine if a statistical correlation existed between the mean-squared deviation and the path length the sonic boom traveled through the turbulence.
During the analysis procedure it became immediately apparent that those waveforms which were either shaped as a Classic N-waves, or were nearly shaped as such, were primarily manifest only in the early morning hours.

This result leads us to believe that as the turbulent boundary layer grew through the day, that the number of undistorted waveforms decreased. This result is averaging over all of the usable observation data.

**SIMPLE DEMONSTRATION: FRACTION OF RECORDED SONIC BOOM WAVEFORMS THAT ARE LEAST-DISTORTED**

445 recordings from 7/31 - 8/7 overflights during 7AM-1PM

Fraction is number of *classic N* and *nearly classic N* waveforms per number of waveforms recorded during the hour intervals

classic N $\Rightarrow$ least distorted
This is a plot of the mean-squared deviation as a function of altitude (or roughly Mach number, since faster planes generally flew higher). It is apparent that increased altitude and speed imply decreased waveform distortion and a decreased spread of data points. Now higher and faster flying planes generally will have shorter propagation paths through the turbulent boundary layer, which can be shown from simple geometry. Thus, it appears as if longer propagation paths through the turbulence result in larger mean-squared deviations, i.e., more distorted waveforms.

ALTITUDE AND MACH NUMBER INFLUENCE UPON WAVEFORM DISTORTION

Increased altitude and speed $\Rightarrow$ decreased waveform distortion and decreased spread of data points

F-18, F-15, SR-71, F-4, and F-16 data
Here are some results listed by plane type of the linear correlation coefficients between the mean-squared deviation and the path length through the turbulent boundary layer. It is seen that there is strong correlation in some cases (F-18 and F-15) and fair correlation in the others. This also gives us some evidence that increasing distortion is correlated with increasing path length through the turbulence.

The F-4 and F-16 data have the lowest correlations and the lowest altitudes. For these cases the sonic boom signatures probably did not have time to develop into an N-wave by the time it began to interact with the turbulence. The other aircraft types flown had too few data points to draw any statistical conclusions.

We also are currently working on obtaining correlation coefficients grouped by altitude and mach number as well as by plane type, to determine how these factors interrelate.

**CORRELATION COEFFICIENTS FOR MEAN-SQUARED DEVIATION AND TBL PATH LENGTH**

<table>
<thead>
<tr>
<th>Aircraft type</th>
<th>Correlation coefficient: MSD vs TBL path length</th>
<th>Number of samples</th>
</tr>
</thead>
<tbody>
<tr>
<td>F-18</td>
<td>0.712</td>
<td>62</td>
</tr>
<tr>
<td>F-15</td>
<td>0.591</td>
<td>75</td>
</tr>
<tr>
<td>SR-71</td>
<td>0.398</td>
<td>48</td>
</tr>
<tr>
<td>F-4</td>
<td>0.324</td>
<td>46</td>
</tr>
<tr>
<td>F-16</td>
<td>0.318</td>
<td>65</td>
</tr>
</tbody>
</table>

(Linear regression correlation coefficients)

Strong correlation in some cases, fair correlation in others

F-4 and F-16 have lowest correlations and the lowest altitudes
- Sonic boom signature before TBL probably not N-wave
- Less distance for nonlinear steepening to work before TBL
The conclusions of this study are the following: a strong linear correlation exists between the mean-squared deviation and path length through the turbulence for the F-18 and F-15 sonic boom data. Fair correlation exists for the SR-71, F-4, and F-16 data. An increase in altitude and speed results in decreased waveform distortion and a small deviation between distortion values. Looking at the waveform classification results, the large percentage of Classic N-wave data during each day’s early flights seems to correspond with the thin boundary layer at that time.

Conclusions:

- A strong linear correlation exists between mean-squared deviation and path length through the turbulence for F-18 and F-15 sonic boom data.
- Fair correlation exists for SR-71, F-4, and F-16 data.
- An increase in altitude and speed results in decreased waveform distortion and a small deviation between distortion values.
- Looking at waveform classification results, the large percentage of classic N wave data during early flights seems to correspond with the thin boundary layer at that time.
The implications of this study for high speed research are the following: Increased interaction between real atmospheric turbulence and actual sonic boom data does imply more distorted waveforms. The common assumption prevailing in the sonic boom propagation community for the last several years has been validated statistically. And most importantly, it is now clear that atmospheric turbulence will determine how well a shaped sonic boom will remain shaped as it propagates to the ground. We are now led to believe that higher and faster aircraft having shaped sonic booms will, on average, have more shaped boom preserved than will aircraft flying at lower altitudes and slower speeds, since flying higher and faster minimizes the path length through the turbulence.

Implications for High Speed Research:

- Increased interaction between real atmospheric turbulence and actual sonic booms does imply more distorted waveforms.

- This common assumption has been validated statistically.

- Most Importantly: Atmospheric turbulence primarily will determine how well a shaped sonic boom will remain shaped as it propagates to the ground.
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References

