

**ON THE AGING
OF SONIC BOOMS**

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Slide 1 —

This is the familiar formulation of sonic boom propagation. The near-field signature strength is defined by the F-function. There is an amplitude factor, the inverse of root-B, which is a generalization of cylindrical spreading. Nonlinear steepening appears as an adjustment to arrival time. This definition of the age parameter is very convenient, since once it's been computed for given flight conditions it can be applied to any F-function.

SONIC BOOM AMPLITUDE AND AGING

$$\delta p = \frac{P_o}{\sqrt{B}} F(\tau)$$

$$\frac{1}{\sqrt{B}} = \frac{\gamma M^2}{(2\beta r_a)^{1/2}} \left(\frac{\rho_o a_o^3 A_a}{\rho_a a_o^3 A} \right)^{1/2}$$

$$\tau = t - \underbrace{\int_0^s \frac{ds}{a_o}}_{\text{Acoustic Propagation}} + \underbrace{\frac{\gamma+1}{2\gamma} \int_0^s \frac{ds}{a_o \sqrt{B}}}_{\text{Nonlinear Steepening}} F(t)$$

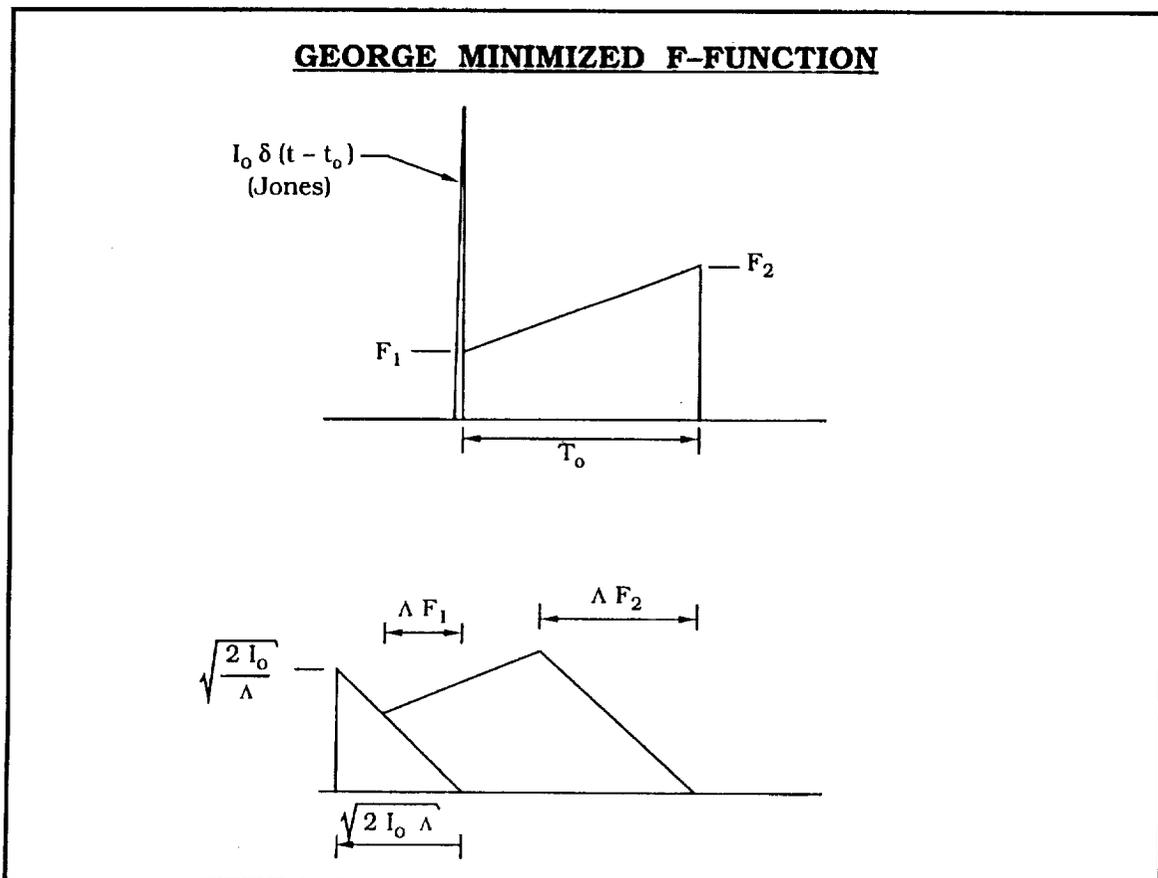
Age Parameter:

$$\Lambda(s) = \frac{\gamma+1}{2\gamma} \int_0^s \frac{ds}{a_o \sqrt{B}}$$

Signature at τ advances by $\Lambda(s) F(\tau)$

Slide 2 —

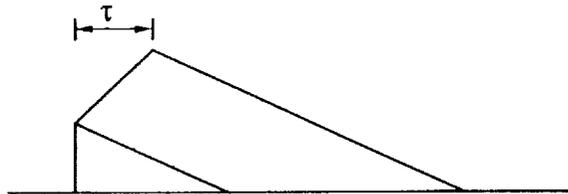
The kind of F-function we're most interested in is a shaped minimized one. This is George's F-function for a minimum-shock boom. (For simplicity, I'm only discussing the forward half, hence George's original form rather than the George/Seebass or Seebass/George extension to front and rear shocks.) As it ages, the initial delta-function impulse (Jones's asymptotic optimum) ages as an N-wave, while the isentropic compression behind follows. Everything is a very simple function of age parameter.



Slide 3 —

The target ground boom occurs when the N-wave from the impulse just coalesces into the ramp, and the ramp still has some rise time which is slow enough to be not audible. Everything — including the matching value of the initial impulse — is related in simple ways. It's worth looking at the age for which the ramp turns into a shock, as well as the age for the design condition.

FINAL MINIMUM SHOCK BOOM



Target τ at Design Condition:

$$\Lambda F_2 - \Lambda F_1 = T_0 - \tau$$

$$\Lambda = \frac{T_0 - \tau}{F_2 - F_1}$$

$$\frac{\Lambda(\tau)}{\Lambda(o)} = \frac{\Lambda}{\Lambda_{\text{shock}}} = \frac{T_0 - \tau}{T_0}$$

Bow Shock Blends at Design Condition:

$$\sqrt{2 I_0 \Lambda} = \Lambda F_1$$

$$I_0 = \frac{\Lambda F_1^2}{2}$$

Slide 4 —

These values are idealizations of the various low-boom designs which have been discussed over the past few years, for minimum boom Mach 1.6 to 1.8 and flight altitude of 40 to 50 kfeet. I've idealized the duration and slope of the isentropic compressions, and assumed a perfectly matched nose impulse. The design age parameter is about 0.8, with full shock coalescence occurring around 1.3. For this signature's initial and target ramp durations, if the signature ages by perhaps 30 to 60 percent more than the optimum age it will degenerate into a very noisy full shock. Lesser degrees of "overaging" will not increase its loudness as drastically.

The 50 msec ramp is somewhat arbitrary. With Leatherwood, Sullivan, and Shepherd's excellent results from NASA—Langley's boom box experiments, it would be appropriate to establish formal target values for the ramp slope based on optimizing loudness.

TYPICAL VALUES

$$M = 1.7$$

$$T_0 = 0.12 \text{ sec } (L \approx 200 \text{ ft})$$

$$F_1 = 0.06 \sqrt{\text{ft}}, \quad F_2 = 0.15 \sqrt{\text{ft}}$$

$$\tau = 0.05 \text{ sec}$$

Resultant Design Condition:

$$\Lambda = 0.8 \text{ sec } \sqrt{\text{ft}}$$

Shock Coalescence Condition:

$$\Lambda_{\text{shock}} = 1.3 \text{ sec } \sqrt{\text{ft}}$$

Slide 5 —

In a uniform atmosphere, the age parameter grows as the square root of distance. A convenient model for parametric analysis is an isothermal exponential atmosphere, with straight ray paths and fairly simple complete expressions. The real atmosphere can reasonably be approximated by a scale height in the range indicated. The isothermal-exponential atmosphere age parameter has an asymptotic limit, which equals the uniform atmosphere value at a radius (in the uniform atmosphere) which is about a scale height. This asymptote leads to the concept of freezing.

AMPLITUDE AND AGE PARAMETERS

Uniform Atmosphere:

$$\frac{1}{\sqrt{B}} = \frac{\gamma M^2}{\sqrt{2\beta r}}$$

$$\Lambda_u = \frac{(\gamma+1) M^3}{2 \sqrt{2\beta} \beta a_a} 2 r^{1/2}$$

Isothermal, Exponential Atmosphere:

$$a_o = \text{constant}$$

$$\rho_o = \rho_g e^{-z/H} ; \quad H = 20,000 \text{ ft} - 30,000 \text{ ft}$$

$$\frac{1}{\sqrt{B}} = \frac{\gamma M^2}{\sqrt{2\beta r}} e^{-r/2H}$$

$$\Lambda_i = \frac{(\gamma+1) M^3}{2 \sqrt{2\beta} \beta a_a} \sqrt{2\pi H} \operatorname{erf}(\sqrt{r/2H})$$

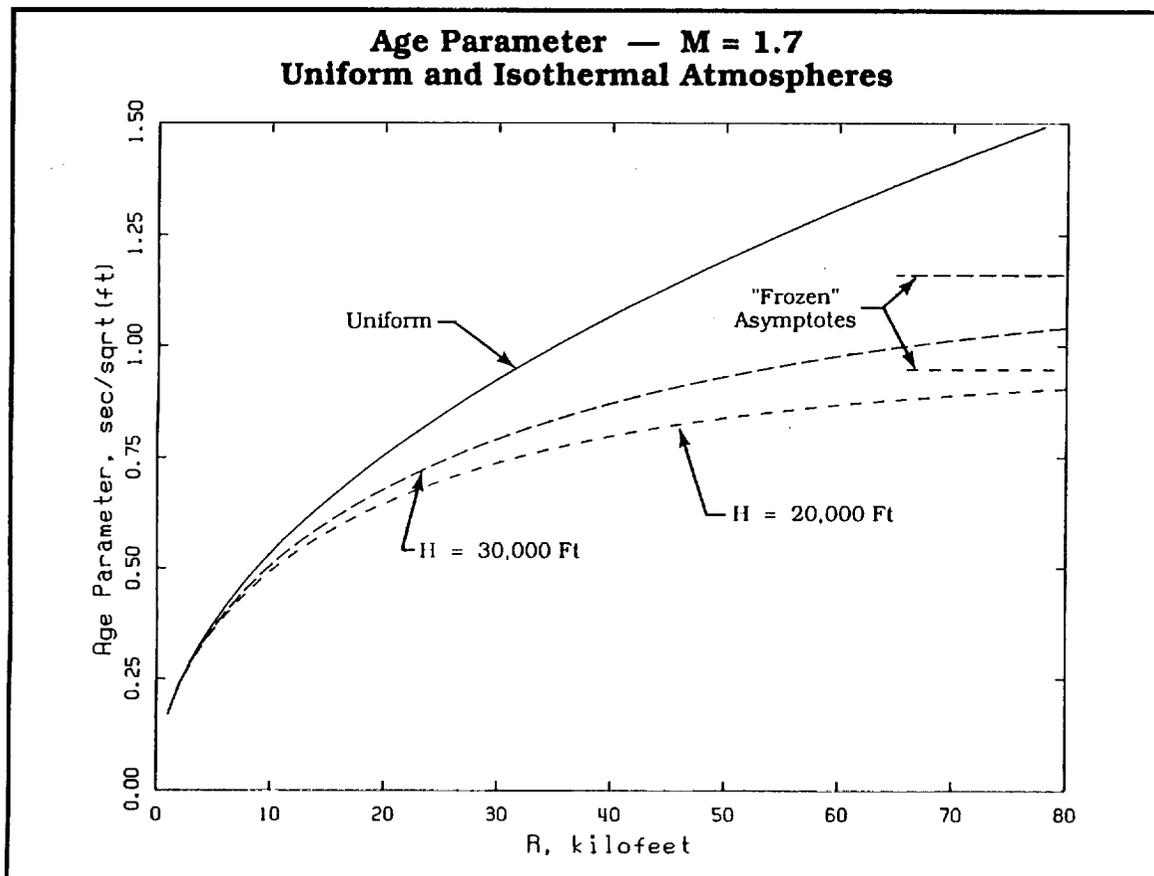
$$\text{Freezing: } \Lambda_i(\infty) = \Lambda_u(\pi H/2)$$

Slide 6 —

Here are uniform and isothermal-exponential atmosphere age parameters at Mach 1.7. The asymptotic "frozen" limits are indicated on the right edge. Notice that these limits are not reached, or even approached, until very high altitudes — perhaps double the 40 to 60 kfeet of interest for HSCT. The term "freezing" has been used fairly often, and this familiarity has led to some common misconceptions. Atmospheric gradients clearly slow the aging process — to the point that McLean's midfield signature concept is practical — but we are not close to the freezing regime. The isothermal-exponential age parameters in this figure, in the altitude range of interest, look like diminished versions of the uniform age parameter.

Having the correct age parameter is essential even for predicting N-wave sonic booms. The overwhelming success of that endeavor suggests that there is no need to conduct elaborate experiments to prove that we know how sonic booms age, and certainly no need to become obsessed with asymptotic freezing.

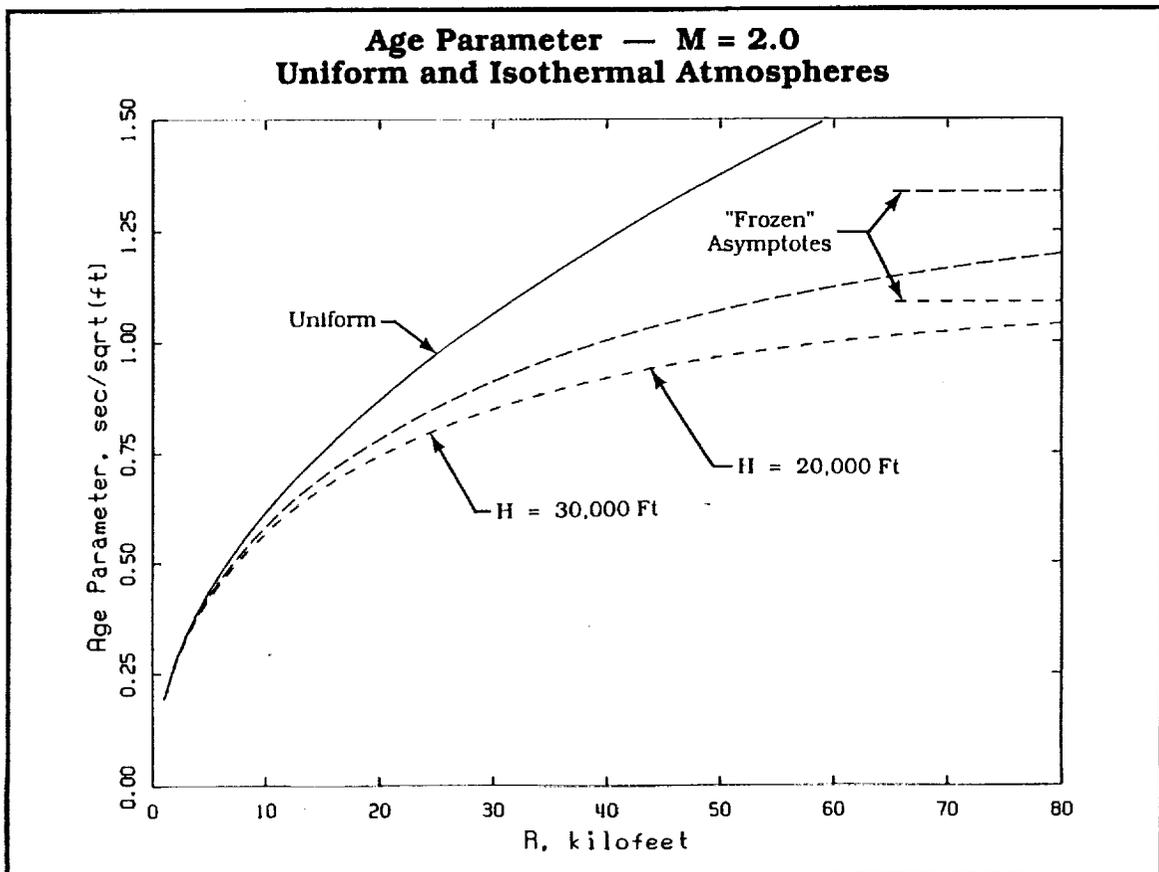
What certainly does need attention is determining whether mid-field signature aircraft designs are practical, and whether the signatures will survive under real-world conditions. Wind tunnel tests confirming configuration concepts have been successful, but wind tunnels are not large enough to allow aging to the midfield. Flight tests using RPVs and modified existing aircraft are being planned, and are necessary. The rest of this paper concentrates on proper scaling, with regard to age parameter, of these reduced-scale flight tests. Some phenomena which do not scale are identified, and emphasis is placed on pre-test analysis of elements which may not be intuitively obvious.



Slide 7 —

This is an age parameter chart similar to the previous one, except that it is at Mach 2.0. Differences between this chart and the previous one exhibit the importance of using the correct Mach number. That the age parameter is bigger at Mach 2 than at 1.7 illustrates one of the reasons why boom minimization is easier at low Mach numbers.

The age parameter for the real atmosphere is somewhat more complicated, depending on both the flight altitude and the ground altitude, rather than just the difference. Flight test design must use the real atmosphere. However, the isothermal-exponential model is adequate for the purposes of the current discussion.



Slide 8 —

A real-world consideration is that the age parameter is larger off track than on. This is the relation for the isothermal-exponential atmosphere. At an azimuth of 45 degrees, the age parameter is about 15 percent bigger, and at 60 degrees it is about 40 percent bigger. There is a favorable benefit with the amplitude factor, but care must be taken to allow a margin before shock formation. In a flight test of an axisymmetric vehicle, on- and off-track measurements can be used to obtain several ages per flight.

OFF-TRACK AGING

- On-Track: $\phi = 0, r = z.$
- Off-Track, Isothermal Model:
 - $r = z / \cos \phi$
 - $H_{\text{effective}} \rightarrow H / \cos \phi$

$$\Lambda = \frac{(\gamma+1) M^3}{2 \sqrt{2\beta} \beta a_a} \sqrt{2 \pi H / \cos \phi} \operatorname{erf}(\sqrt{z/2H})$$

$$\Lambda_{\phi} = \Lambda_{\phi=0} / \sqrt{\cos \phi}$$

Slide 9 —

The first detail to consider for scaling flight test experiments is how the source scales with size. This is a simple representation of the volume and lift components of the F-function. The volume component always remains the same. Increasing the aircraft altitude at fixed weight increases the F-function, as well as the age parameter due to increased distance. Flying at an optimum lift coefficient keeps F fixed, and there is some recovery from the reduced amplitude factor at higher altitudes, so long as the increased aging does not lead to serious shock coalescence. This is why mission profiles with increasing altitude as fuel burns off tend to not show loudness increases, but rather decreases.

For model scaling purposes, it is reasonable to assume that the boom is dominated by volume, or that full-scale lift coefficient will be replicated. The entire F-function then behaves like volume boom.

F-FUNCTION FOR VOLUME AND LIFT (FIXED M)

$$F(x) = \underbrace{\frac{1}{2\pi} \int_0^x \frac{A''(\xi)}{(x-\xi)^{1/2}} d\xi}_{\text{Volume}} + \underbrace{\frac{\beta \cos \phi}{\rho_\infty u_\infty^2} \frac{1}{2\pi} \int_0^x \frac{L'(\xi)}{(x-\xi)^{1/2}} d\xi}_{\text{Lift}}$$

- Volume F-Function Fixed by Geometry.
- Lift Component:
 - Increases if Raise Altitude at Fixed L
(Off-Design Condition)
 - Decreases if L Decreases at Fixed Altitude
(Fuel Burn-Off at Fixed Altitude)
 - Stays Constant at Fixed C_L
(Optimized Cruise Profile)

Slide 10 —

For geometrically similar models, the F-function scales as the square root of length. A model less slender than full scale (larger f) increases F , which would in turn require a smaller propagation distance to retain the same relative aging.

SCALING OF VOLUME BOOM

$$F_v(x) = \frac{1}{2\pi} \int_0^x \frac{A''(\xi)}{(x-\xi)^{1/2}} d\xi$$

Let

$$A(x) = f^2 l^2 \tilde{A}(x/l)$$

l = length

f = fineness ratio

$\tilde{A}(x/l)$ = nondimensional shape

$$F_v = f^2 \sqrt{l} \tilde{F}_v(x/l)$$

Slide 11 —

To achieve proper scaling in a uniform atmosphere simply requires altitude proportional to model length. In a real atmosphere, where the rate of growth of the age parameter is slower at larger distances, a model experiment requires flight altitudes which are smaller than obtained by using the model to full-scale ratio. If, for some reason, the model flight altitude must be proportional to model length, then the model must have a more slender fineness ratio.

MODEL FLIGHT TEST

- Require $(\Lambda F)_{\text{model}} = \frac{1}{L} (\Lambda F)_{\text{full scale}}$
- $F \propto \sqrt{1/L}$, so need $\Lambda \propto \sqrt{1/L}$
- Uniform Atmosphere: $\Lambda \propto \sqrt{r}$,
So Scale $r_{\text{model}} / r_{\text{full}} = 1/L$
- Real Atmosphere: Λ tapers off
 $\Rightarrow r_{\text{model}} / r_{\text{full}} < 1/L$

Slide 12 —

This table shows the scaling for three size models: full, one-third, and one-tenth. This roughly covers the range of proposed tests. The quantity shown as F is the nominal full-scale F -function at the end of the ramp, which was 0.15 in our example. The age parameter is that required so that age times F will scale proportionately to model length; for full scale, the age parameter for 50,000 feet is shown. The value of z shown for the models is the corresponding altitude in the real atmosphere. The altitude ratio is always less than the model size ratio, and for the smallest scale is about half. The model age parameter is also proportionately more sensitive to altitude errors than full scale. It is straightforward to calculate required altitude precision, and the test plans must address this.

The final parameter in the table is the duration of the ramp. A nominal 50 msec full-scale ramp is expected to be clearly discernible. The corresponding model ramp durations are very close to the 3 to 10 msec range normally encountered for shock wave rise times. If we expect shocks to have their typical "full-scale N -wave" rise times, the smaller scale tests are somewhat dubious.

**RELATIVE SCALES FOR ONE-THIRD AND
ONE-TENTH MODEL TESTS**

<u>Scale</u>	<u>F</u>	<u>Λ</u>	<u>Z</u>	<u>Z/Z_{full}</u>	<u>τ</u>
1.0	0.15	0.84	50,000	1.0	50 msec
1/3	0.087	0.48	9,500	0.19	17 msec
1/10	0.047	0.27	2,600	0.05	5 msec

Slide 13 —

Rise times may not, however, be as long as they are at full scale. Bass, Layton, and Bolen (*JASA*, July 1987) measured projectile shock waves which were considerably thinner than expected from steady theory. The reason was that propagation distances were sufficiently short that they had not had time to reach steady state. At last year's sonic boom workshop, Raspet discussed "healing times" and suggested that even full-scale N-wave shocks might not be steady. If full-scale shocks have not achieved steady thickness, then model shocks must certainly be investigated.

The current HSR team has developed several models which are suitable for calculating the evolution of shock structures. This is the time to use them.

POTENTIAL FOR THIN SHOCKS

- Unsteady Shock Formation Over Short Distances
(Bass, Layton, and Bolen - *JASA*, July 1987).
- Healing Time of Weak Shocks (Raspet, 1992).
- Current Analysis Tools Available to Predict.

Slide 14 —

As a final element, I'd like to show some "anything can happen" data. This is of interest because this could mask results of an "aging" flight test, and also because it provides an example of turbulence effects which do not fit into the usual expected pattern.

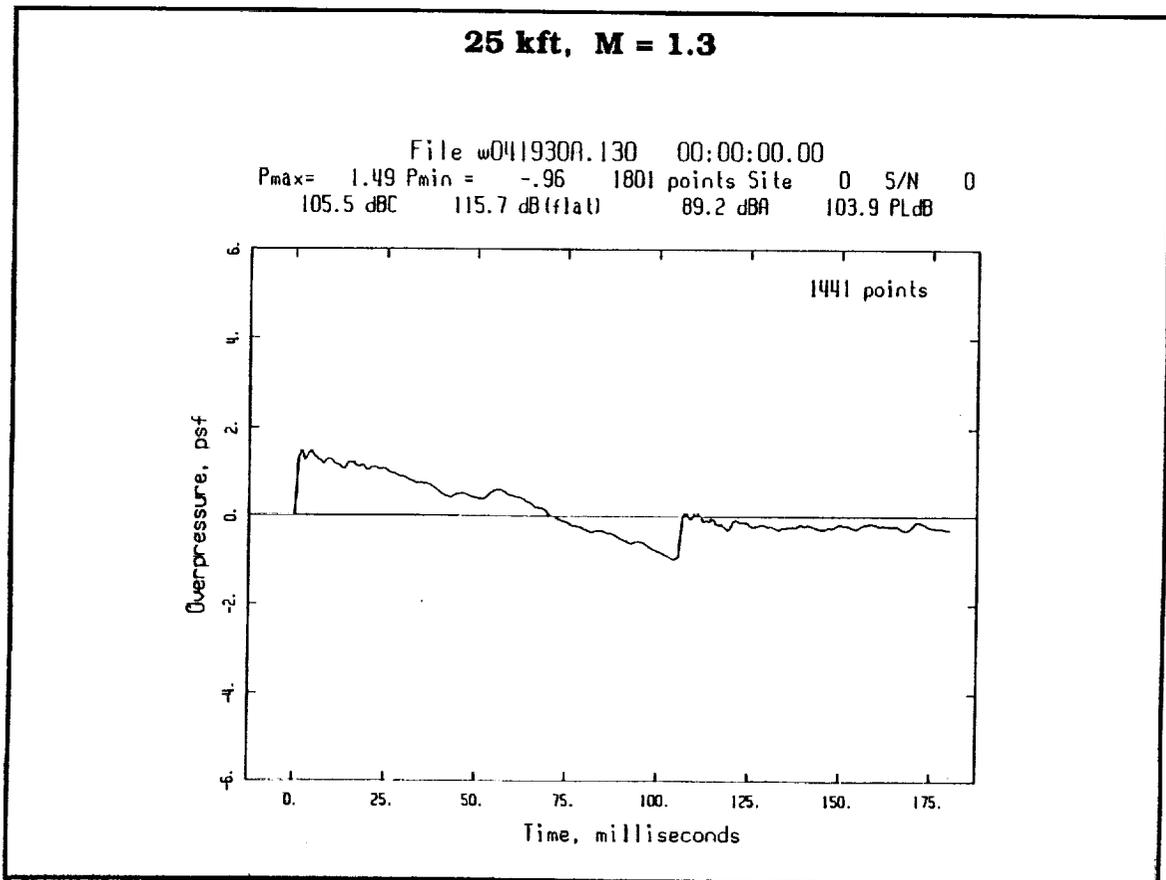
We recently performed what should have been a very dull Boomfile test protocol on an aircraft not yet in the data base. Tests were conducted in the early morning, under overcast skies, so there was no convective turbulence. There was virtually no surface wind. The weather had been changing (it snowed overnight, and was clearing during the tests) and a rawinsonde at a site about 100 miles away indicated wind shears at about 15 to 20 kft and 20 to 25 kft MSL. The site was at about 5 kft MSL.

SUPPLEMENTAL BOOMFILE FLIGHT TEST

- Documentation of Steady Boom for an Aircraft Not Previously Measured.
- Flight Parameters: 10k, 18k, 25k AGL, at Mach 1.2, 1.25, 1.30.
- Recorded On and Near Centerline by USAF BEAR Systems.
- Early Morning, Calm Surface Conditions.
- Rawinsonde During Test, About 100 Miles Away.

Slide 15 —

Here is a typical good measurement. In this test, which yielded about 25 valid recordings from ten passes, about half the records were of this quality. The rest had significant distortion. The next three slides are particularly interesting.



Slides 16, 17, 18 —

These are distorted measurements at 25, 18, and 10 kfeet AGL, respectively. Overpressures and durations were as expected. There was considerable distortion, and rise times were very long. These three booms have a particular characteristic that they look very much like minimum-shock shaped booms. Slides 15 and 16 are the same flight condition, so the nice minimum shock shape in 16 is anomalous.

Through most of this paper I have discussed simulation issues that could lead to what the statisticians call Type I errors — failing to confirm a phenomenon that exists. These three booms raise the possibility of Type II errors — detecting an effect that is not there.

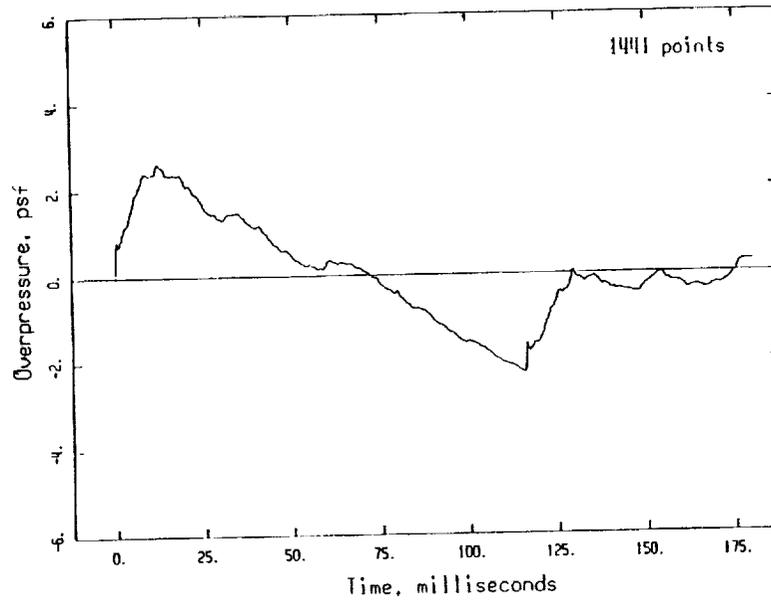
These measurements are also interesting because they have features not fully consistent with the usual characteristics of distorted booms. Distortion was more common for the higher altitude runs, while the low-altitude runs yielded a higher percentage of clean booms. Surface layer turbulence would affect all altitudes. There was a considerable amount of noise after the booms — the kind that sounds like the echoing of distant thunder. This tended to persist for several seconds. Each BEAR would typically record three or four records per boom, with the first being the N-wave and the rest being noise. This was much more than observed during the 1987 Boomfile tests at Edwards AFB.

The site was flat, so the distortion and aftershocks had to be a combination of scattering and multiple paths. The long aftershocks and the substantial distortion are consistent with scattering from the shear layers and weather at higher altitudes, i.e., not in the surface and mixed layer as normally expected. Anecdotal reports of "echoing" in NASA's JAPE II sonic boom propagation experiment should be re-examined in terms of multi-path propagation from higher altitude atmospheric structures.

These measurements underscore that individual test conditions can obscure fine details such as would be seen in model tests of midfield signatures. Both Type I and Type II errors are possible. Care must be taken to design the initial round of shaped boom flight tests so as to avoid these conditions. It is also clear that atmospheric conditions occur which lead to distortions somewhat different from those seen in summertime desert tests. It would be prudent to extend propagation investigations into other geographical areas and atmospheric conditions.

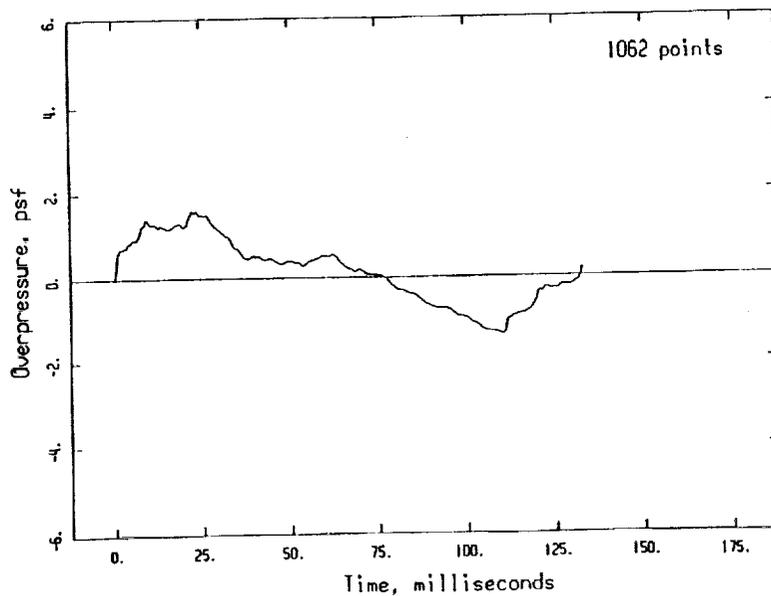
18 kft, M = 1.25

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108.9 dBC 121.6 dB(flat) 89.7 dBA 103.5 PLdB



25 kft, M = 1.3

File w041913A.130 19:13:27.01 January 30 1993
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104.3 dBC 117.1 dB(flat) 84.6 dBA 99.1 PLdB



12.5 kft, M = 1.2

File w042017A.130 20:17:15.18 January 30 1993
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106.2 dBC 121.1 dB(flat) 88.1 dBA 102.4 PLdB

