Review of Sonic Fatigue Technology

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# List of Contents

1. **Introduction**  

2. **Environment**  
   2.1 Jet and Rocket Pressure Loads  
   2.2 Turbulent Boundary Layers  
   2.3 Separated Flow and Oscillating Shocks  
   2.4 Thermal Environment  

3. **Response and Fatigue**  
   3.1 Early Experimental Work  
   3.2 Development of a Standard Test-Specimen  
   3.3 Fatigue Aspects  
      3.3.1 Low Amplitude rms stress  
      3.3.2 Effect of Nonlinear response of composite structure  
      3.3.3 Effect of temperature on fatigue life  
      3.3.4 Crack propagation  

4. **Thermal Effects**  
   4.1 Early Studies  
   4.2 Recent Studies  

5. **Theory and Design Procedures for Metallic Structures**  
   5.1 The Single Degree of Freedom Model  
   5.2 Development of the Model and Nomographs  
   5.3 Comparison of Experimental Results with Theory and Nomographs  
   5.4 Conclusions from the early Experimental work  

6. **Studies of Composite Structures**  
   6.1 Studies in the U.K.  
   6.2 Studies in the U.S.A.  

7. **Recent Developments**  
   7.1 Experimental  
   7.2 Theoretical  
      7.2.1 Normal Mode Method  
      7.2.2 Methods for Nonlinear Response  
   7.3 The Statistical Energy Analysis Method  

8. **Conclusion**  

9. **Recommendations for Future Studies**  

10. **References**
1. Introduction

In the late nineteen fifties incidents were reported in which aircraft structures close to high intensity jet exhausts suffered minor damage. Skin cracking was noticed and failures of small cleats and internal support structure occurred in a few cases. These incidents alerted industry and the research centres to the possibility of problems as the performances of aircraft and engines increased. Measurements were made on some full scale aircraft and also tests were made with large sections of typical structure. Theoretical work and tests on simple structures such as flat and curved plates were begun. As the power of the engines increased in the next generation of aircraft designs further problems arose and gave added impetus to the work. Because of the form of the failures the majority of the problems were of a nature which caused excessive maintenance rather than endangering the life of the aircraft. However there was always the fear that a catastrophe was just 'waiting to happen'.

Several aircraft manufacturers and the research centres set up comprehensive tests on large parts of aircraft structure in which it was possible to reproduce representative structure around the region of maximum noise intensity. They generally used an actual jet engine to provide realistic acoustic excitation and measurements of the pressures in the nearfield region of the jet exhaust were made together with strain measurements on the structure. Observations of fatigue crack initiation and endurance were recorded.

In parallel with the experience being built up in the test programmes, theoretical studies were initiated. Jet noise studies aimed primarily at reducing noise nuisance were extended into the nearfield region of the jet to give an increased understanding of the pressure fluctuations on the structure. Simple theoretical models of the structure and the excitation process were made. It soon became apparent that the complete process from random excitation of a built-up structure to final fatigue failure was far too complicated for the theoretical and computational tools available at the time. Powell (1958) developed the normal mode formulation for linear response and Miles' (1954) seminal paper gave the result for the response in one mode as well as going on to consider the fatigue aspects. Simple models were developed from this early work in an attempt to help designers construct aircraft which could resist the acoustic loads.

The lack of a complete theoretical treatment which could be applied to aircraft designs lead to the development of design guides and data sheets for industry. These took as their basis the theoretical work on simple models and used the results of specially designed tests to derive empirical modifications to the simple formulae. An extensive series of studies was sponsored by the USAF which lead to design nomographs for every type of structure in common use. A parallel activity sponsored by AGARD was based on a wider range of test results but was limited to primary structure. A guide to good design practice evolved and then together with the nomographs it was generally possible for aircraft designers to produce satisfactory structures.
This was the state of affairs towards the end of the nineteen sixties and the early seventies. Although the power of the engines was still increasing dramatically, the pressure levels were not increasing because of the use of higher bypass configurations needed to reduce the community noise. This leveling off of the increase in pressure loading on the structure coupled with the use of established design procedures and more general awareness of the likely problems lead to a great reduction in the research and development activity on sonic fatigue.

New interest was not aroused until the early to mid nineteen eighties when it became apparent that the widespread use of composites posed new problems. Higher operating temperatures for some aerospace configurations also raised the probability of failures from a combination of the thermal and the acoustic environment. The major feature of composites is their high structural efficiency and hence relatively high displacement under working loads. The response to acoustic loads now becomes nonlinear and, when coupled with high temperatures, thermal buckling can occur. New theoretical work is now in progress.

In the early work it was not possible to get better agreement than to within a factor of two. Predictions always seemed to overestimate the response levels. In the new work attempts are being made to bridge the gap between the nomographs, analytical results and finite element computations.

The purpose of this report is to review existing sonic fatigue technology and in particular to assess recent developments. It will also suggest a plan for co-ordinated programme of theoretical and experimental work to meet the anticipated needs of future aerospace vehicles.
2. Environment

High frequency random pressure fluctuations of the acoustic type on aerospace structure are generally caused by the turbulent mixing in a high speed jet efflux or a turbulent boundary layer. Additional pseudo 'acoustic' loads occur under an oscillating shock wave. Shock-turbulence interactions in jet streams can also cause instabilities which radiate high intensity sound waves. A guide for use in estimating the pressure fluctuation on the surface of aerospace vehicles was produced by Ungar, Wilby and Bliss in 1976 (1).

2.1 Jet and Rocket Pressure Loads

The early work was concerned primarily with sound radiation and near field pressures caused by high velocity jet efflux. A mathematical framework for mixing noise in a jet was formulated by Lighthill (2,3) who first showed theoretically the dependence of the intensity of sound radiated on a high power of the jet velocity. Many workers used and extended this theory and many measurements were made at model and full scale to investigate thoroughly this dependence. A semi empirical prediction method for the near field pressure fluctuation around a jet engine was produced by the Engineering Sciences Data Unit (ESDU) of the Royal Aeronautical Society and others (4). The general characteristics of noise from a rocket engine are similar to those of the jet engine efflux. In this case however the velocities are much greater, the diameter may be greater and there may be marked shock interaction effects in the supersonic flow.

The statistical description of the pressure field close to a jet or rocket efflux is required for use in the theoretical methods. The power spectrum of the pressures is relatively smooth, peaking at a frequency in the 100 to 600 Hz range. The value of the frequency of the peak is inversely proportional to the nozzle diameter. The magnitude is proportional to approximately the fourth power of the jet velocity. The fuller description of the loading actions requires a knowledge of the spatial relationships of the pressures on the surface of the structure. This is most conveniently expressed in the form of the cross spectral density of the pressures over distance. The variation in the real part of the cross spectral density along two lines parallel to the jet axis but at different distances away is shown for a typical jet in figure 1 which is taken from reference 5. In the usual formulation of the forcing function there is a double area integral of the product of the cross spectral density function and the mode shape. The integral of the imaginary part over the area is zero thus we are only interested in the real part. This can be represented as a decaying cosine function. The rate of decay varies for different physical situations and positions in the flow field.

In certain aircraft configurations the jet exhaust impinges on the structure or is reflected from a nearby ground surface. In these situations the pressure loading on the surface is increased considerably and in the extreme it can be doubled. A review of the pressure load enhancement due to impingement is given by Lansing et al. (6) and the effect of ground reflections is discussed by Scholton (7) who also gives data measured on an experimental VSTOL aircraft.
2.2 Turbulent Boundary Layers

The pressure fluctuations under a turbulent boundary layer are also a potential source of structural damage. The overall pressure level is not usually as high as that for the extreme jet cases and the spectrum is relatively flat out to a high frequency, as shown in the lowest curve in figure 2. In this composite curve produced by Coe (8, 9) the spectral density of surface pressures is non-dimensionalised and plotted against a non-dimensional frequency parameter. The early definitive measurements were made by Bull (10) in a subsonic boundary layer. Subsequent measurements under supersonic flow indicated that the overall levels expressed as a percentage of the freestream dynamic pressure are somewhat lower than those for subsonic flow. A discussion of more recent work is given by Mixson and Roussos (11) who emphasises that most of the existing data is for speeds less than Mach 2.5. The limited data at higher speeds shows considerable scatter but confirm the trend of a decrease in pressures with increase in Mach No.

2.3 Separated Flow and Oscillating Shocks

In addition to areas of structure which are subjected to widely distributed acoustic pressure loads there are parts of the vehicle which are subjected to local high intensity pressure fluctuations. These occur in the regions of separated flow, cavities and oscillating shocks. There has not been any extensive study of these phenomena although a very good study at model scale has been made by Coe and Chyu (8, 9). This can be most conveniently summarised in figure 2 which is taken from Coe's report. These spectral shapes show that as the normal boundary layer flow is disturbed the mean pressures increase and the form of the spectrum changes. In the case of separated flows the spectral density at low frequency is increased by three orders of magnitude and beneath oscillating shock waves there is a further increase of an order of magnitude. Although the mean square levels are not increased so dramatically because of the changes in the shape of the spectrum the structural response is more heavily influenced by the low frequency components if the pressures are correlated over distances approaching the spacing of the structural supports.

In each of the cases shown in figure 2, except that of the attached boundary layer, the basic mechanism causing the random pressure fluctuations is an instability in the flow. These mechanisms are now reasonably well understood but the amplitude of the disturbance cannot be predicted. Thus we must rely on measurements and empirical formulae to arrive at reasonable estimates on new designs of vehicle.

2.4 Thermal Environment

In the 'traditional' acoustic fatigue loading case the maximum acoustic pressures occur for a very short time just before start of roll of an aircraft or launch of a rocket vehicle. As soon as the vehicle begins to move the relative velocity of the jet stream in the stationary air reduces steadily and hence the surface pressures reduce because they are proportional to the relative velocity raised to a power of about 4. The period of maximum pressures is usually no more than 10-20 seconds during which time the hot gases do not have
time to heat up the structure significantly. However in vertical take-off aircraft the build up of vehicle velocity is much slower and heating of the structure does occur. A sketch of a probable time history of build up of acoustic pressures and the associated temperature for conventional and vertical take-off is shown in figure 3.

New designs for suborbital flight vehicles and reusable launchers have structures which must withstand very high acoustic loads at take-off/launch. They also reach high transient temperatures during ascent and re-entry which will be coupled with significant acoustic loads associated with the high dynamic pressures at these points in the flight path. An estimate of the temperature time history of such vehicles made by NASA is shown in figure 4. The combined acoustic thermal variations are indicated broadly in figure 3. The earlier practice of testing at maximum acoustic pressure levels combined with maximum temperature would result in a serious overtest in this case because the two extremes do not occur at the same time.

An aerospace structure will be designed to minimise the effect of steady temperatures but the transient heating and cooling will cause thermal stresses in the structure which will interact with any acoustically induced stresses. In any testing programme it will be vitally important, therefore, to try to reproduce the correct rates of temperature rise and fall not just the correct maximum levels. In fact the rates of heating and cooling are likely to be more important than the absolute maximum levels for the types of structures now in the early design stage.

This section has only considered the thermal and acoustic loading acting on the surface structure. However there will be other areas of internal structure such as engine bays where high temperatures will be reached for long periods of time. Normally the acoustic loading is lower in these regions but in some of the new designs this may not be the case.
3. Response and Fatigue

This section gives a brief description of the forms of response that occur in a typical structure before a more detailed discussion of the theoretical methods and experimental results is treated in later sections. The fatigue aspects are also reviewed.

3.1. Early Experimental Work

The major aircraft companies began to discover incidents of acoustic fatigue on their new aircraft designs in the mid nineteen fifties. The structure was predominantly of the form of aluminium skins with aluminium ribs, frames and stiffeners. The frames usually contained lightening holes and ribs usually had lightening holes and stiffeners. When early problems were experienced with flaps close to jet flows the conventional stiffened skin wedge structures were replaced with honeycomb structures. The enhanced stiffness of honeycomb plates and full depth wedges for the trailing edge produced a design which had high resistance to acoustic loads. Several companies set up test rigs to reproduce the structure and jet engine excitation and observations of the types of failure and a limited amount of strain measurement were made. Unfortunately the data analysis systems available for such low amplitude random signals were rather rudimentary at the time. Thus although practical experience was gained, which in turn lead to better design practice, little quantitative information of lasting value was acquired.

These early ad hoc tests were followed by more comprehensive investigations in which, for example, a section of the Caravelle fuselage (12) or a flap (13) were fully instrumented. Tests also began on simple structures (14) to investigate the mechanism of the response and fatigue failure. The single panel tests suffered from the disadvantage of not reproducing the flexible boundary conditions which exist on aircraft structures. A systematic study of panel arrays was made by Nelson (15).

From these tests and further theoretical studies it became clear that the response to jet noise excitation was very localised in nature. The lower frequency overall modes of the shell structure were not excited by the sound field to any considerable extend. There were two reasons for this: one was that the pressures were usually lower at these frequencies (up to 80 Hz or so) and secondly the pressures were not correlated over the whole structure. At the higher frequencies where the pressure spectra peaked (200 → 500 Hz) the structural wavelengths were approaching the typical separation distances of stiffeners. Small changes in these stiffener spacings and other non-uniformities in actual aircraft structures caused by attachment points etc. lead to modes of vibration in this frequency range which showed a few panels predominating.

The single panel tests exhibited a response pattern which was dominated by the fundamental mode of vibration. The edge stresses were highest in this mode and fatigue failure usually began at rivet lines along the supports. Response in the second plate mode (at about twice the frequency - depending on aspect ratio) was usually about an order of magnitude lower in amplitude. This was partly because the spectral density was usually lower.
at this higher frequency and also because the sound pressure field was an inefficient exciter of this asymmetric mode. The response in the third mode was further reduced as both of these effects continued to reduce the excitation efficiency of the sound field.

The effect of surrounding structure is to add more modes to the response in between the frequencies which would have been calculated for the single plate. The effect of a panel array was studied by Lin (16,17) theoretically and Clarkson experimentally (18,19). Further refinements were made by Mead and Sen Gupta (20). Because of the non-uniformity of panel sizes at the rear of an aircraft (where the structure is tapering) several low order modes are excited in which one panel will have much greater response that its neighbours. This effect is reduced as the sizes of the panels become more uniform. The response of the skins and ribs of horizontal and vertical stabilizers and control surfaces also exhibits similar phenomena. This is also true of the honeycomb plates often used for the outer skins of control surfaces which are very close to jet effluxes.

3.2 Development of a Standard Test Specimen

It is clear from the early work that tests on a single panel would not be good enough to reproduce the realistic response of an aircraft structure to high intensity noise. At the other extreme it is impracticable to reproduce large sections of structure to test the endurance of locally responding panels. Thus a study was done by Nelson (15) to investigate the effect of surrounding structure on the response of the central (test) panel. In this series of tests four panel configurations were used:

- Single plate
- 3 x 3 bays
- 5 x 5 bays
- 7 x 7 bays

There were thus an equal number of stringers and frames.

The tests were carried out in a travelling wave acoustic facility. Strain gauges and accelerometers were used to measure the response of the structure to sinusoidal and random loadings. As would be expected an initial sinusoidal sweep showed that the response spectra exhibited more and more modes as the number of bays in the test specimen was increased. Unfortunately, when the fatigue tests were carried out, a random sound pressure spectra of only 100 Hz bandwidth centred on the major resonance was used. Thus the wider band response of the larger panels was inhibited by the form of excitation used. Under this band limited excitation the strain gauge at the edge of the panel and directly above the web of the Z section stringer (the highest strain point in the plate) showed a single peak in the first three configurations. The forty nine-bay configuration showed a secondary peak at about 70% of the amplitude of the main peak. The results of the fatigue tests then showed that there was no significant difference in the lifetime of the multi-bay panels. A representative wide band excitation would have produced a higher rms stress in the multi-bay panels and consequently a lower fatigue life. No conclusion or recommendation for the size of array is
made in this report but nearly all subsequent work on which the design nomographs are based used a nine-bay configuration.

We have to conclude that the multimodal response features of real structures were not reproduced in the fatigue tests and therefore the results have limited value in guiding us as to the size of array which can be realistically representative. The early work by Lin and also by Clarkson had shown that the coupling across the stringer was the most important effect in the response of a skin panel and thus it would have been better to have many more stringers than frames. A more suitable test structure would be one having four or more frames and eight or more stringers. The acoustic loading spectrum should be wide enough to cover the coupled response of the panels i.e. typically 100 - 1000 Hz.

3.3 Fatigue Aspects

3.3.1 Low amplitude rms stress

The high frequency response of structures to wide band random acoustic loading means that the material will be subjected to a large number of strain reversals in a normal aircraft life time. Even allowing for only about one minute of maximum excitation per flight an endurance of $10^5 - 10^6$ cycles is required. As in all fatigue tests there will be significant scatter in the results and so it is not practicable to test the required number of realistic specimens to get a good S-N curve. Thus the policy adopted has been to separate out the stress estimation process from the fatigue life estimation. Stress prediction procedures have been used to estimate the stress at the fatigue critical points in the structure, and then simple coupon tests have been used to determine the endurance of the different aircraft materials at stress levels giving lives in the range $10^4 - 10^6$ cycles. This two step process allows independent improvements to be made in the two procedures separately. A comprehensive series of S-N curves is given in the ESDU Data Sheets (4).

A compromise has to be made in the reproduction of the complex time history of the response. For a predominantly uni-modal response the time history takes the form of a sine wave whose amplitude is randomly modulated. This is an example of the classic case of the response of a single degree of freedom system to a white noise form of excitation and the probability distribution of the peaks has been shown by Crandall and Mark (21), to follow a Rayleigh Distribution. This would be the form of response of a single plate and would also be characteristic of some actual structural plates where the response is dominated by a single mode. For multi-modal response the wave form begins to show secondary peaks. The effect of these peaks is being studied by Miles (22) who has shown that in a simple structure (base excited beam) the life can be reduced by an order of magnitude if all peaks resulting from white noise excitation are included in the Palmgren-Milner cumulative damage assessment. In practice the excitation is not white noise and so the bandwidth limitation of jet noise will reduce the number of secondary peaks and hence reduce this additional damage. Schederup and Galef (23) made an analysis of time histories from many tests on representative
structures and showed that even where the response was multi-modal in character the peak to trough distribution in this wave-form approximated to a Rayleigh Distribution.

Based on this evidence and the need for a relatively simple and quick test procedure, narrow band random coupon testing was adopted. The test specimen is usually a single or double cantilever which is mounted on a vibration table. In some tests it may be mass loaded at the tip. The specimen is driven by a narrow band force centred on the first bending frequency of the cantilever. The root of the cantilever can be designed to reproduce the rivet or bonded joint at the edge of a typical panel. Data from these tests are then used to construct a random S-N curve. The ordinate is rms stress and the endurance is based on the number of zero crossings (19) as derived by Rice (24) from a Gaussian white noise signal. This form of test procedure was used to produce the ESDU Data Sheets (4).

3.3.2 Effect of Nonlinear response of composite structures

The early work on metals in the endurance range $10^6 - 10^8$ cycles is at low stress amplitudes and therefore linear behaviour of the structure is a reasonable assumption. However when composite construction is used the deflections are much greater at levels which give fatigue failures. Thus nonlinear behaviour is an important feature of the response. In metals the effect of nonlinearity in response is to superimpose a membrane stress onto the bending stress. For a given surface stress, membrane stresses which have the same magnitude throughout the depth of the plate will do more damage than bending stresses which reduce to zero at the centre. In fibre reinforced composites however the mechanism of failure is usually delamination. Further work is needed to link delamination failure to the surface strain measurements.

3.3.3 Effect of temperature on fatigue life

Section 4 discusses the effect of temperature on the response of structures to wide band noise excitation. The most dramatic effect is when the critical temperature is reached and the structure buckles. In this condition the frequency of vibration of the lowest mode is reduced to a minimum and violent oil-canning or snap-through motion occurs. Fatigue damage in this phase is at its maximum but if the temperature is increased above this critical value, the structure stiffens, its frequency increases and the damage rate fails. Normally a structure would be designed so that it did not operate in this critical region although it may pass through it as the structure heats up in the response to the external environment changes.

The effect of temperature on material properties has been investigated in coupon type tests (see for example the work of Schneider (25) and it has usually been found to decrease the life at a given rms stress level. Later work by Soovere (26) shows that the life of composite laminate specimens is also reduced somewhat by an increase in the ambient temperature. In the majority of this test work the upper temperature has been usually 150-200°C but a few tests have been made up to 600°C.
3.3.4 Crack propagation

The fail-safe design philosophy for parts of an aerospace structure is based on the assumption that an incipient crack is present after construction. It is then necessary to ensure that the crack does not propagate to a critical or dangerous length between major inspections. The experience of acoustic fatigue failures in such structural elements as thin skins suggests that the initial phase of crack initiation is long and that once a crack has become visible propagation occurs relatively rapidly. However in certain structural components, such as pressurized fuselages, the skin materials is likely to be thicker than that which would give rise to crack initiation from acoustic loads. But if a crack has been initiated by the main cabin pressurisation, loading conditions can arise where the acoustic loads cause additional crack propagation. This was studied by Clarkson (27) and Jost (28) in some of the early series of tests on the Concorde design.

These early tests were made on a flat plate under steady tension simulating the hoop stress in the fuselage. A central crack (normal to the tensile stress field) was cut in the plate and observed to propagate under acoustic loading. As the crack reached the critical length, which caused local buckling of the free edge, an elliptical area in the centre of the panel became unstable and violent snap-through motions occurred. This is similar to the oil canning phenomena described in the next section when a plate is buckled by temperature increases. The rate of crack propagation is at a maximum at this stage, but as the length of the crack increases the buckling becomes well established and further increases of length cause a stiffening of the plate. The frequency of the first mode of vibration falls to a minimum at the snap-through point and then rises again. A finite element analysis of this vibration was made by Petyt (29,30) and Byrne (31) obtained some of the basic fatigue information required for predicting the rate of growth of the crack. Recently work on this topic has been done by Fujimoto and Sumi (32,33,34) who develop further the Finite Element approach.
4. Thermal Effects

4.1 Early Studies

In the early work referred to in the preceding section there was little increase in temperature of the structure during the brief exposure to high intensity noise. However in some of the newer military designs the jet grazes or impinges on the structure and there can be a large increase in the temperature of the surface plates. In the early nineteen seventies a series of studies of the response of plate structures to high acoustic loads and high temperatures was made (25, 35, 36) under the sponsorship of the Air Force Flight Dynamics Laboratory. Summaries were later presented to an AIAA meetings (37, 38). This early work was concerned with a study of the basic effects and went on to propose design nomographs (25).

Hieken's report (35) begins by developing the equation for the response of a structure to distributed random pressure loads. It follows Powell's normal mode method (39) and pays particular attention to the acoustic characteristics of the test enclosure. A factor called the 'Participation Factor' is introduced to describe the coupling between the acoustic field and the structural mode shapes. This is the same as Powell's Joint Acceptance function. The final part of the section on facility effects is that dealing with the cooling effect of the airflow in the progressive wave tube when the sound generator is operating. The heating energy requirements increased by a factor of 3 when the maximum air flow was used. The theory for the thermal stresses in the structure is developed from the heat flux equations assuming steady state conditions. A finite difference method of solution of the equations is given. There is a good description and treatment of the edge effects both from a structural and a thermal part of view. The theory as developed is limited to two dimensions therefore cannot predict the temperature gradients across the thickness of a test specimen. This should be satisfactory for thin metal plates but not for honeycomb. The computer program given calculates the temperature and stress distribution across the panels.

The report by Jacobson and Finwall (36) gives a good discussion of the mechanism of the thermal loading of structures. This is an experimental study with no theoretical development. Thermal acoustic tests are made on beams, uniform plates and plates carrying 2 stiffeners which are mounted rigidly at their edges (or ends in case of beams). A very full description of the tests is given. Three elevated temperatures are used and the material chosen appropriately:

- 300 °F Aluminium alloy (2024)
- 600 °F Titanium alloy (64 4V)
- 1000 °F René 41

Vibrator tests at elevated temperatures were made to provide fatigue information and the analysis of the strain measurements gives both the membrane and the bending components of the strain.
In plates restrained from in-plane motion at their edges there builds up a compressive in-plane stress as the temperature increases. A point is then reached when the plate buckles. If the plate is subject to intense acoustic pressures during this phase the plate will buckle outwards or inwards and move violently between these positions. This phenomena is known as oil-canning or snap-through. As the temperature continues to rise the plate becomes permanently buckled in one direction. Further rises in temperature cause the curvature of the buckled plate to increase and thus its fundamental frequency of vibration increases.

There is a good discussion of the oil-canning effect and the shaker tests are designed to reproduce this buckling. The fatigue specimens are in the form of beams held rigidly at their ends and excited by a shaker at their centres. Heat is supplied by the lamps used in the progressive wave tube. There is no theoretical development but a lot of experimental data is given. A semi empirical criterion for oil-canning is given in terms of the buckling amplitude $A_0$ and the rms acoustic displacement $w$. For fully fixed boundaries oil-canning occurs if

$$\frac{A_0}{w} \text{ in the range }$$

$$1.5 \leq \frac{A_0}{w} \leq 6; \text{ for } \frac{w}{h} \geq 0.3$$

where $h$ is the plate thickness. In addition to giving the onset of oil-canning this also gives the termination i.e. when the plate is well buckled. A similar relationship is likely to hold for flexibly restrained edges. There is no measurement of the frequency of vibration and so the frequency change associated with the development of oil-canning is not brought out in this work.

The report by Schneider (25) forms the basis of the design procedure adopted by the industry. The analysis considers the heating of a single uniform plate simply supported at its edges and restrained from in-plane motion. It assumes a spatially uniform temperature across the panel and hence derives the simple relationship for thermal stress:

$$\sigma_x = \sigma_y = \frac{E \alpha T}{1 - \nu}$$

The next step is to compute the buckling temperature assuming no in-plane motion of the edges. After buckling is established the bending stresses are computed to add to the membrane stresses given by the formula above. The effect of in-plane compressive stress is to reduce the natural frequency of the fundamental mode of the plate. The frequency decreases as the temperature increases until buckling occurs. After buckling is developed the plate is now stiffer and further buckling increases the curvature of the plate and the frequency rises. A simple relationship for this is given:
\[ f(r) = f_0 \left[ 1 - r \right]^{1/2} \quad 0 \leq r \leq 1 \]

\[ f(r) = f_0 \left[ 2(r - 1) \right]^{1/2} \quad r \geq 1 \]

where \( r = \frac{T}{T_c} \) and \( f_0 \) = fundamental plate natural frequency

\( T_c \) = buckling temperature

Measurements show that the frequency does change in a similar way to that predicted although it does not reduce to zero.

Coupon tests are used to provide fatigue data at temperatures up to 600°F and the fatigue data show a reduction in life with increase in temperature. 3 bay and 9 bay panels with fully fixed edges were tested in a travelling wave facility to provide stress response data and fatigue life checks. The simpler theory is then used to derive semi empirical relationships and design data is presented in Nomograph form. Extreme caution is advised in using this simple data sheet.

The errors in using this work are likely to arise from the simplifying assumptions made about the panel edge supports. Any in-plane motion of the supports will affect the temperature at which buckling will take place. The supports in the more realistic 9 bay test panels will provide a thermal discontinuity and so give rise to non uniform temperature in the plate. There will also be stresses induced by the thermal gradients at the edge.

The three reports taken together give a good study of the problem of the acoustic thermal excitation of skin structures. A more realistic estimate of thermal buckling of a single plate could be made by assuming that the edge members bend in the in-plane direction but remain fixed at the corners. This, however, would not be an appropriate assumption for a panel array where the restraint would depend crucially on the method of attachment of the stringers to the supporting frames. This is likely to be such as would allow the stringers to move in the in-plane direction in order to minimise thermal stresses.

### 4.2 Recent Studies

An experimental study of the effects described in these three reports was made by Ng and Clevenson (40). In these tests a single aluminium plate (12" x 15" x 0.063") was subjected to a broad band sound field at levels up to 160dB overall SPL and temperature up to 250°F. In the first series of tests the plate was clamped rigidly in a massive steel frame and in the second series a fibre glass insulating layer (10 layers of tape) was inserted between the plate and the mounting. Strain gauges were attached to the plate but it was not
possible to provide thermal compensation for the strain gauge bridge and so the static thermal stresses could not be measured. Thermocouples were used to measure the temperature distribution across the plate. In both test series the maximum temperature of 250°F was achieved at the centre but the edge temperature was down to about 130°F in the uninsulated case and 200°F in the insulated. There was no attempt to calculate these distributions. In the fully clamped case, onset of thermal buckling occurred at about 100°F and calculations based on Schneider's result (25) and using an average plate temperature show reasonable agreement. This is surprising because the Schneider formula assumes simple supports. It would be interesting to do a more accurate calculation using the actual temperature distribution and fully fixed supports. This could be done with the new EAL Finite Element programme (41). In the insulated support case there is a slightly higher buckling temperature but a marked reduction in central deflection at equivalent centre temperatures. The changes in central deflection are probably due to some in-plane motion at the supports. The snap-through phenomena was apparent in the strain response at the buckling temperature. The rms strains were highest at this point and the spectrum showed higher low frequency content. For the insulated support the buckling effects were much lower and snap-through did not appear to take place. Calculations of the buckling temperature using the method given in reference 25 did not give good agreement in this case.

This work suggests that the extreme effects of snap-through with their associated large chaotic motion and subsequent high rate of fatigue damage may only occur when a plate is rigidly supported at its edges. When some in-plane motion is allowed (as will be in the case in built up structures) the most severe effects are mitigated.
5. Theory and Design Procedures for Metallic Structures

The incidents which triggered off the theoretical studies occurred on aluminium alloy stiffened skin structures. Thus the early work was concentrated on metallic configurations and design procedures were developed. These also formed the basis for much more recent work on composite construction.

5.1 The Single Degree-of-Freedom Model

One of the earliest theoretical studies was made by Miles (42) in which he modelled the structure as a single panel. The jet noise excitation was represented by a uniform pressure field having a spectral density $G_p(f)$ at the fundamental resonance frequency of the panel, $f_n$. The spectral density of the response was then integrated over the frequency domain to produce the now much used expression for the mean square response:

$$\bar{y}^2(t) = \frac{\pi}{4 f_n G_p(f)} \left( \frac{y_0}{F_0} \right)^2$$

The stress equivalent of this result is:

$$\bar{\sigma}^2(t) = \frac{\pi}{4 f_n G_p(f)} \left( \frac{\sigma_0}{F_0} \right)^2$$

where $y_0$ is the static displacement produced by the uniformly distributed force $F_0$ and $\sigma_0$ is the static stress, and $\eta$ is the modal damping ratio.

The form of the response of the single degree-of-freedom system to white noise is a randomly modulated sine wave as discussed in section 3. Miles used Miner’s cumulative damage hypothesis (43) to obtain an estimate for the fatigue life of the panel.

The next major step is the theoretical studies was the work of Powell (39). He developed the normal mode approach to formulate the response of a structure to random pressure loads. The statistical properties of the pressures were described fully by their cross spectral density. Neglecting cross coupling between the modes leads to a series solution for the spectral density of displacement:

$$G_y(x_1, \omega) = \frac{\phi_r^2(x_1)}{\sum_{r=1}^{\infty} A_r \int A_r(\omega_2) \phi_r(x_1) G_p(x_2, x_3, \omega) dA dA}$$

$$M_r^2 \left[ (\omega_r^2 - \omega_r^2)^2 + \eta_r^2 \omega_r^2 \right]$$
If it can be assumed that each mode is relatively lightly damped and separated in frequency, the mean square displacement can be obtained by integrating over each mode separately and using the Miles result. Thus we have the approximate result for the mean square displacement as a summation of the response in each mode:

\[
y^2(t) = \sum_{r=1}^{\infty} \frac{\pi}{2 \omega_r \eta_r} \int_{A} \int_{A} \phi_r(x_2) \phi_r(x_3) G_p(x_2, x_3, \omega) \text{d}A \text{d}A
\]

To obtain a relatively simple formulation which could be used in the design of aircraft panels Clarkson (44) suggested a considerable simplification of this result. If we assume that the response is dominated by one mode and that at the natural frequency of that mode the pressures are approximately in phase over the area of the panel, the static displacement \(y_0(x_1)\) at \(x_1\) is given by

\[
y_0(x_1) = \frac{\int_{A} \phi_r(x_2) \text{d}A}{\omega_r M_r} \phi_r(x_1)
\]

Then the equation for \(y^2(t)\) reduces to Miles equation (eqn.1) given at the beginning of this section.

In using this to produce a design procedure we need to make some assumption about the boundary conditions of the panel. Clarkson (44) assumed fully fixed conditions and then was able to propose a series of steps to constitute a design process i.e.

1. Estimate the natural frequency of the panel \(f_n\).

2. Estimate the spectral density of the acoustic load at this frequency \(G_p(f_n)\).

3. Assume a damping ratio at this frequency \(\eta\).

4. Compute the static stress \(\sigma_0\) at the edges and centre of the panel induced by a uniform force of unit magnitude. (The effect of edge doublers etc. could be allowed for in this calculation).

5. From the equation 1 estimate the rms stress.

6. From a knowledge of the rms stress and frequency, estimate the time to failure at the edge or centre using the results of coupon tests such as those described in section 3.3.1.
If we look at the assumptions made we see that some lead to an overestimate and some to an underestimate. The assumption that the pressure is in phase over the whole surface of the panel and the assumption of fully fixed edges will both give over estimates of the stress. The neglect of the contribution to the rms stress from the higher modes to the overall rms stress will give an underestimate of the stress.

Separate Data Sheets have been produced for each of these items by the Engineering Sciences Data Unit taking over from the early work produced for AGARD (45). The individual items are continuously updated as new data becomes available and they are extended to include new materials and configurations. A list of the current titles is given in the Appendix to the Reference Section at the end of this report. In addition to presenting a design procedure, the data sheets also show comparisons of estimated and measured stresses for a wide range of structures. The designer is thus able to see the likely scatter and hence make allowance for this in choosing his structural parameters. The sheets offer the most up-to-date information available and the step-by-step process allows the designer to use his own information for any of the components if he wishes. In spite of all these continuous improvements the assumptions made mean that in general the data is scattered through a band which extends from half to twice the estimated value.

5.2 Development of the Model and Nomographs

Many attempts have been made to improve the accuracy of the estimation procedure. Early ones of note were by Bozich, Jacobs and Sen Gupta. Bozich (46) looked at the double area integral of the mode shape and the cross spectral density of pressure in the joint acceptance function of Powell (39). He computed this function for a range of structural wavelength and pressure flow field characteristics. This could then be used as a factor (always less than 1) applied to the simple result given by equation 1. Barnoski and Maurer (47) work out examples for this function for different forms of pressure field. Mead and Sen Gupta (48,49,50) developed the periodic structure model to give an alternative representation of stiffened skin structures. Lindberg and Olson (51, 52) developed a finite element model of flat and curved stiffened plates but made no comparison with experimental results. Jacobs and Lagerquist (53, 54, 55) used a finite element model of a skin-stringer panel to compute the response to random pressure loads. Hay (56) surveyed all the data available to produce a compendium of damping factors. A very thorough review of the role of damping and a listing of a wide range of results has been presented more recently by Soovere (57). The proceedings of the workshops held by the USAF (58, 59) contain many papers which address all aspects of damping and its influence on structural response and fatigue. Unfortunately none of these improvements were able to guarantee an agreement between theory and measurement better than to within a factor of 2.

Early experimental studies of the response of single panels to high intensity acoustic loads were made by NASA (14). In these tests the panels were subjected to both random and sinusoidal loads. Fatigue lives were obtained for the two types of loading and also for curved as well as flat panels but there was no attempt to compute the response stress levels.
Early test work on aircraft structures in the USA not only reported on full scale experience and special tests on representative structure but went on to propose suitable design configurations. Belcher and his colleagues (60) were among the first to present the design information in the form of a nomograph. In its simplest form a nomograph consists of a single diagram containing several series of curves. Each series represents one feature of the design i.e. panel thickness, width, aspect ratio, curvature, damping, material properties. Starting with the sound pressure level one moves through the diagram from curve to curve until one reaches the final fatigue life. If this is not great enough the process is repeated but this time a different thickness and or width could be chosen.

This represented a different approach to design. In the European case the procedure uses data sheets separately to determine the loading action, stress response and finally the fatigue life, and thus allows the designer greater flexibility in the use of the most recent data as explained earlier in this section. In the USA the nomograph approach was developed during the nineteen sixties from the early work of Belcher et al. (60) McGowan (61) and then Ballentine et al. (62) made successive developments of the procedure and extended it to a wider range of structures. This is a completely empirical approach based on tests on representative structures. For skin structures a 9-bay panel configuration evolved as a standard form of test specimen following the work of Nelson (15) described earlier. A series of structures are tested in a high intensity noise environment and stresses and fatigue life measured for each specimen. The basic single degree-of-freedom analysis is used to give the form of the result but empirical modifications are made in order to fit the data. These 'best fit' curves are then used to construct nomographs of the form shown in figure 5. Extensive tests were carried out to derive the information necessary to produce comprehensive design guides for all types of structural configuration and support details. The work of Rudder, Plumblee and Jacobson (63, 64, 65) together with that of Schneider (25) discussed in the previous section now forms the basic design guide for the US aerospace industry. Unfortunately these guides are not updated regularly as is the case of the ESDU data sheets.

The most comprehensive guide is that of Rudder and Plumblee (63). It considers the following types of structure:

- Stiffened skin
  - rivetted
  - + temperature effects
- Box structures
  - skins and internal ribs, lightening holes
- Wedge structures
- Honeycomb plates
  - flat
  - curved
Types of construction
chemically milled
corrugated
bonded/beaded
laminated - see Holehouse (100)

Stress concentration factors
holes
rows of holes
\{ tension and bending \}

Random rms. S-N curves
Al. alloys, stainless steels, titanium, nickel, and glass fibre.

All the Nomographs are based on tests on simple structures and therefore the accuracy in use depends on how closely the behaviour of the actual structure resembles that of the multibay test structure. Experience of using the Nomograph is discussed in some of the reports which will be dealt with later.

Many failures occur in the support structure rather than in the skin. Rudder (64) considers this in the next design guide which gives a good description of the problem and proposes a simple calculation procedure. This computes the shear loading around the edges of a panel responding in its fundamental mode. This is then applied as a lateral load to the support structure. It is regarded as a preliminary study and there is no check with full scale experience given. There does not appear to have been any further development of this important aspect of design. Jacobson (65) made a comprehensive series of tests on 10 panels (5 different designs) which each had J section stiffeners bonded to the skin. The stress at the maximum stress point in the panel was about one fifth of that at the rivet line in an equivalent rivetted panel. The lives of the panel were thus considerably increased. A revised nomograph is produced for use with the standard design procedure (63). There is one feature of these tests which is difficult to understand - that is that there were twice as many failures at the centre of the short side as at the centre of the long side. The aspect ratio of the central test panel was 2 and so one would expect the stress at the centre of the long side to about 45% higher that that at the centre of the short side in the fundamental mode. This figure comes from the ratio of stresses at the edges of a fully fixed plate subjected to a uniform loading (as given by Timoshenko and Woinowsky-Krieger (66).

A series of studies was made to investigate the effect of detail design changes such as joints, edge close-out effects in honeycomb plates and different materials. The results of these are used together with the more comprehensive design guides to produce a satisfactory structure.

Wentz and Wolfe (67) made a series of tests on flat and curved stiffened panels in which the joints were fabricated with different jointing techniques and some further results are given in a more recent paper by Wolfe and Holehouse (68). Some typical aircraft
structures were also tested and two jointing methods used were: weld bonding and adhesive bending. Coupon tests were also carried out to provide supplementary fatigue data. Failures occurred either in the skin above the web of the stiffener or by peeling of the joint. The life for the skin failure mode is longer than that for a rivetted joint because of the reduced stress concentration at the joint. The adhesion failures are due to 'peeling' of the joint starting from the bond edge. The peel stress cannot be measured readily and so skin surface strain measurements are used. For a given surface strain thicker skins give higher peel stresses and therefore fail earlier. A better indicator of failure was found to be the bending moment at the joint section and when this parameter was used the fatigue data collapsed onto a single design curve. Considerations such as these will also be important in considering the failure mechanisms in composites where failure usually takes the form of delamination.

The main design guide (63) contains the nomographs for flat aluminium honeycomb panels produced by Ballentine et al. (62). This work does not study the important effect of edge close-out design. One specific design is used and all data refer to that. Soovere (69) has made a comprehensive study of the effect of the edge design in an attempt to explain the measurements which show that there is a higher strain on the inside faceplate than on the outer. A method is given for the prediction of the surface strains which allows for the asymmetry of the edge fixture and predicts a membrane strain on the inner face plate which is superimposed on the bending strain. This gives better agreement with measured surface stresses and hence fatigue life predictions can be improved.

The data on diffusion bonded titanium honeycomb panels included in the main design guide is taken from the work of Holehouse (70). The response of the panels used in the tests showed a predominant peak at the fundamental frequency however the measured overall stresses were consistently lower than those predicted by the single mode equation of Miles (42). Estimated stresses at the centre of the panels were 1/2 to 1/4 of those measured. An empirical modification was made to the theoretical result to provide the best fit with the data from which a design nomograph was produced.

Jacobson and van der Hyde (71) describe a large series of tests aimed at producing design information for honeycomb structures with glass or boron fibre reinforced faceplates. Some aluminium honeycomb panels were tested to provide a check with earlier data. Coupon tests were conducted to provide additional fatigue information. Faceplate stresses were measured but there is no comparison with any theoretical results. The so called theoretical stresses in their paper are deduced from the fatigue data. Such a deduction involves too much scatter to be considered comparable to other theoretical methods. The strain responses showed marked nonlinearities at overall noise levels about 155dB.

5.3 Comparison of Experimental Results with Theory and Nomographs

In several of the reports already discussed experimental results are quoted in support of the theory or as a means to develop the empirical design relationships. There are several
other reports where experimental results are described and attempts are made to use one or more theoretical methods. Some of these comparisons will now be reviewed. Arcas (72) has proposed a modification to Clarkson's method (44) and also compared the results with Ballentine's (62) method. This comparison is shown in figure 6. This shows that with the simple analyses currently available there is considerable scatter in the results. Some of the discrepancies may be due to experimental error but this cause would only count for a proportion of the scatter. It is likely that the agreement shown here is the best that can be expected from the simple theory. It also shows that although the design nomographs were 'tailored to fit' their experimental data closely, when they are applied to results from tests by other workers the comparison is little better than the other methods.

The experiments of Coe (8) compare measured displacement and strain spectra on a single fully fixed panel responding to a turbulent boundary layer and separated flow. The normal mode method described above is used to compute the response but although the theory includes the full description of the cross spectra the computation assumed the flow fields to be uniform and homogeneous. A comparison of measured and calculated responses is shown in Figures 7 and 8. The displacement peak predictions follow the measurements closely but are high by a factor of about 3 to 4. The agreement at each of the stress peaks is more variable but the predicted overall rms stress would be about a factor of 2 greater than the measured value. Part of the discrepancy is likely to be due to the simplification in the representation of the pressure field. Measured values of damping were used.

Jacobs and Lagerquist (53, 54, 55) have used the finite element method to compute the response of a stiffened panel and have compared the results with measurements. This was one of the first pieces of work to use the finite element method to model a structural section which was typical of fuselage construction. The model of the structure requires the force to be specified at each node point connecting the individual structural plate elements. The authors develop a method to derive the cross spectral density of the force on the nodes from the distributed pressures under a turbulent boundary layer. A similar formulation could be used for jet noise excitation. The method was used to compute the response stresses in the single panel tests performed by Maestrello (73) in a turbulent boundary layer. Using a 72 element structural model of the plate results were obtained for the response over the frequency range which included the first five normal modes. Using the measured damping ratios produced good agreement with the measured deflection spectral density. Stress comparisons were not made. A three bay stiffened skin panel is modelled by Jacobs and Lagerquist (54) and rms deflection is computed. This result is shown in figure 9 and the single measurement point is given for comparison. The skin is of 0.032 in. thickness, while the measured deflection is 0.064 in. This indicates that the panel is having nonlinear response. There is then a discussion of the application of this method to a stiffened skin fuselage structure but no computations of the stress are given. This is because many more elements would be required than could be handled on the computers of the day. However, a decade later the large finite element programs such as NASTRAN began to be used to give more accurate models of larger areas of structure. The greatest problem remains that of the
modelling of the region of high stress gradients to produce estimates of the maximum stresses. In these regions a large number of elements are required.

A description of tests on major structural components in the B1 aircraft development programme is given by Belcher (74, 75). The first report gives measurements of the acoustic loads on the surface of the rear fuselage and the horizontal stabiliser. It goes on to describe structural acoustic tests on a large section of the rear fuselage (fuel bay) and several box test sections to represent the principal features of the stabiliser design. The second report documents the failures which occurred giving much more detail and many experimental results. It is valuable as a guide to good design practice for structural details such as attachments, joints, choice of rivets etc. The rear fuselage test specimen is a conventional stiffened skin structure attached to deep ribs which have their unsupported flanges connected together by metal straps. The stabilizer structure has thick aluminium skins attached to titanium ribs which have a sine-wave web form of construction. Only a few strain measurements are reported and there is no comparison with any theoretical results.

Several papers by Groen, for example, reference 76 and 77 review the experience on the vertical take off fighter. This paper gives the results of a test programme on a section of the fuselage and on a composite flap. The fuselage is of metallic stiffened skin design. Measurements of thermal and acoustically induced stresses were made on the skin panels but no measurements of temperature are reported. Estimates of temperature are used to predict thermal stresses by Schneider's method given in reference (25). There is no agreement at all with the thermal stress measurements. The measured spectra of strains at the centre of the long edges of the individual panels show three or four predominant peaks. The measured overall response was compared with the Miles prediction, the design method (63) and a NASTRAN analysis. Because the first two predictions are based on a unimodal assumption the rms level for each spectral peak was computed separately and compared with the predictions. This is not the correct way to use the predictions because as has been explained earlier in this section the assumption on the pressure field, edge fixing and single mode response taken together are intended to give estimates of the multimodal response. The total rms of the measured spectra should be used. If this is done the measured rms stress of about 1,100-1,200 lbs/in$^2$ compares with estimates from the Miles type of prediction of 4,080 lbs/in$^2$ and the empirical design method of 1,870 lbs/in$^2$. The NASTRAN analysis used a full multibay model in which the substructure was also included but only modelled as bending bars rather than as a series of connected plates. Even so the agreement with the measurements was good (490, and 1010 lbs/in$^2$ rms for the response in the first two modes).

The most recent paper by Groen (78) gives a very comprehensive design procedure for new STOVL aircraft. There is a detailed description of the near field noise characteristics of the multiple nozzle engine configurations. The new higher powered engines show screech tones in the spectra plus broad band shock noise. This gives greatly increased high frequency content to the normal jet mixing noise spectrum. The stress response measurements and calculations are repeated from the earlier report (76). There is also a good survey of candidate materials for the higher temperature regions where the maximum temperature is
expected to reach $370^\circ$F. New structural designs for these regions to minimise thermal stresses are described.

The measurements of the acoustic response of the composite skin of the flap structure showed again that the actual stresses (374 lb/in$^2$) are lower than the predictions made from the two design methods [Jacobson (79): 1450 lb/in$^2$. Holehouse (100): 480 lb/in$^2$.] A NASTRAN analysis of a single panel having fixed edges gave a predicted stress of 560 lb/in and a more elaborate modelling as a multi bay structure gave 520 lb/in$^2$.

These experimental results from a full scale structure suggest that a good modelling of the support structure by a computer programme such as NASTRAN will give reasonable estimates of structural response to acoustic loads. The model should be improved further by modelling the support structure with a configuration of interconnected plates not just a single beam in bending. The thermal stress measurement indicates that the present design method (25) is very inaccurate for a real structure.

5.4 Conclusion from the Early Experimental Work

The first tests on single panels showed a response in which the individual modes could be clearly identified. They were well separated in frequency and the maximum response occurred in the first mode. The response level dropped by at least an order of magnitude in moving from the first mode to second, second to third etc. Calculations of the response levels in each mode showed a similar pattern of falling peak level with increase in mode number but the calculated levels were usually high by a factor of about 2.

Tests on multibay panels or stiffened skin structure which is more representative of actual aircraft constructions showed a markedly different response pattern. For a uniform spacing of stiffeners and frames the individual panels are now coupled together and respond as a complete unit. The lowest mode is usually one in which adjacent panels vibrate out of phase and the frequency (usually in the region 200-400 Hz) is one at which the acoustic pressures are in phase over the panels. Thus the effective excitation of the coupled mode is very low. The modes in the first response band have combinations of in and out of phase displacements of individual panels with only the highest mode in the band having all panels in phase. At this higher frequency the correlation length of the pressures is smaller and so the effective excitation is reduced. Calculations using this more realistic model still give over predictions of the stress levels.

In actual aircraft construction the stiffened skin designs at the rear of the aircraft (which is usually subjected to the highest acoustics pressure loading) have a non-uniform spacing of stiffeners, as the taper of the structure leads to stiffeners running out at different frame stations. The effect of this non-uniformity is to disturb the patterns described in the previous paragraph. Mode shapes now often show one panel responding predominantly. There are still many modes present in the response pattern but the more uniform response is changed to one showing only a few peaks. The single mode model gives a
better prediction in this case as the two major simplifying assumptions have opposite
effects.

The nomographs have been developed from tests on 9-bay structures - mostly with uniform
spacing of stiffeners and frames. The empirical results from these give better agreement
with the uniform panel arrays but not with the non-uniform.

In the single case (76) where a finite element model used extensive modelling of the
support structure a reasonable estimate of the response of the stiffened skin was achieved.
The computed mode shapes showed the more local nature of the response for non-uniform
stiffer spacing described in the previous paragraph. This suggests that the problem with
the early attempts to correlate theory with experiment was the boundary condition of the
panels.

In box structures such as horizontal and vertical stabilisers the two skin surfaces are
connected together by ribs. The acoustic loading on the two surfaces will generally be
uncorrelated in the frequency range of interest. The response spectra of the strain in the
skins and ribs show that very many modes contribute to the overall level. It is
impracticable to use enough elements in the Finite Element models to give a reasonable
estimate of the stress response. The ESDU design method, which uses the single skin response
calculations and divides by 3 to allow the vibration energy in the ribs and opposite skins,
gives predictions that are usually within a factor of 2 of the measured result.

The next uncertainty is the value of the damping. Approximations have to be made in
the theory which are adequate for typical built-up structures. However in new types of
construction such as integrally machined skins there can be a significant reduction in the
damping in the structure.

When trying to make comparisons between theory and experiment one should also bear in
mind the likely experimental errors. The sound level itself cannot generally be determined
better that ± 1/2 dB and the effect of reflection at the structural surface will vary with
the angle of incidence of the sound waves. The low level of strain in the structure and
possible thermal effects make the strain measurement difficult and the subsequent analysis of
the random signal can also introduce errors. All of these errors except the reflection
effect are unbiased and so there would be an equal probability of over or under prediction.

In stiffened skin construction the failure occurred almost always either along the
rivet line or along a line above the edge of the support member of doubler. These stress
concentrations are reproduced well in coupon tests such as those conducted by Byrne (80, 81)
which were made to provide the basic fatigue date.
6. Studies of Composite Structures

Early work on composite panel arrays is reported by Wolfe and Jacobson (82). This also summarises the work of Jacobson (79) which develops the general theory. The first paper (82) describes tests on a series of multi-bay panels of boron-epoxy and graphite-epoxy skin construction. Tests on the multi-bay graphite-epoxy panel subjected to a broad band acoustic loading give a single predominant resonant peak at 139 dB SPL but a very broad response at 166 dB indicating that the panel is behaving in a very nonlinear manner. Two methods are used to estimate the stresses in the panel. The first is essentially a unimodal Rayleigh Ritz method in which a series of beam functions are used to represent the displacement function. In the second method the Finite Element procedure is used with 25 elements. Both theoretical methods give similar predictions which in themselves are 2 or 2.5 times higher than the measured data.

6.1 Studies in the UK

In the United Kingdom complementary work was concentrated on gaining a fundamental understanding of the behaviour of composites and many of the tests were confined to simple beams and single plates. Much of the early experimental work was done by Adams and White and their co-workers. Adams (see for example references 83,84) studied the dynamic properties of stiffness and damping and their variation with frequency in a wide range of composite configurations and different fibre materials. The majority of the work was on glass fibre and carbon fibre. White (85) also made similar tests but concentrated on carbon fibre. This paper also reported a series of tests on short (chopped) fibre configurations with random and aligned fibres in studies aimed at increasing the inherent damping in the composite. A fuller account of the work on matrices of short aligned fibres together with an associated theoretical study is given in the most recent paper by White and Abdin (86). The random orientation of short fibres which was also studied in the earlier paper gives a higher damping ratio than that of the aligned fibres and is much simpler to manufacture but the degradation in strength is too great to make it a feasible practical configuration. At high fibre volume fractions (65%) the damping can be increased by a factor of about 3 by reducing the fibre length from 3mm (which behaves in a manner very similar to that of the continuous fibre) to 0.25mm. The reduction in the modulus of elasticity associated with this reduction in length is about 40%. More modest increases in damping with a smaller reduction in stiffness can be achieved with fibres of intermediate length.

One of the first studies of the response of composite plates to random acoustic loads in comparison with the response of aluminum alloy plates was made by White (87). This produced the often quoted strain spectra showing the nonlinear behaviour of the panel which is shown in figure 10. At low excitation levels (130dB) the spectrum shows several clear peaks associated with the response in the first six plate modes but as the excitation level increases the frequencies of the peaks increase and the peaks become much wider as the overall level increases. The probability distribution also shows marked nonlinear characteristics for the cases corresponding to the higher excitation levels. More recent
experimental work by White and Mousley (88) compares overall plate response levels to predictions made using the simple formula (equation 1). Mode shapes were also measured and in later tests the plate was subject to in-plane compressive loads. As the in-plane load increases the plate approaches the buckling condition and 'snap-through' occurs (89). A study of the snap-through phenomena on aluminium buckled plates has also been reported by Ng (90). The paper by White (91) reviews this work and goes on to discuss fatigue failure of the composite plates. The coupon tests which have been used successfully to develop basic S-N data for metal structures are much less reliable for composites. This is because of the edge peeling problem which makes it difficult to reproduce the kind of failure which takes place in actual structures. This is usually a delamination failure within the plate at some distance from the edge. To overcome this problem White has developed an extra wide coupon test which uses a half sine wave clamp for the cantilevered plate. This yields a representative delamination failure within the plate.

The conclusion to be drawn from the series of studies by White and his co-workers is that, at response levels lower than those which produce pronounced nonlinearities, the response of single plates is dominated by the response in one or two of the lower modes and that the overall level is close to that predicted by the simple formula (equation 1). When in-plane loads or higher excitation levels produced marked nonlinear response the estimations are higher than the measured overall levels.

A further investigation of damage initiation and propagation was made by Drew and White (92). In this work a finite element model was used to compute the stresses in the region of delamination and measurements were made of the changes in static stiffness, frequency and damping as damaged progressed. This confirmed the work of others (i.e. Soovev (93)) who had also detected significant changes in damping and natural frequency of plate specimens.

Ferguson (94) is developing micro mechanics equations to describe the dynamic behaviour of composites and shows how the method can be used to study the effect of moisture and fibre breakage and debonding. This can then be used to build up a theory of fatigue damage mechanics.

An investigation of the effect of temperature on the material behaviour has been made by Galea and White (95). The test plate and surrounding structure were heated up to the test temperature before the edge clamps were tightened. Thus thermal buckling effects were eliminated. It was found that the natural frequencies of the CFRP plates decreased slightly with increasing temperature. This decrease was somewhat greater than could be explained by reductions in material properties. Reductions in residual thermal stresses may have been responsible for this change. The higher temperatures (up to 120°C) did not change the bending strain response significantly. There was no significant increase in nonlinearity over the room temperature result as sound intensity levels increased. The response in the first mode was predominant and therefore the prediction method based on equation 1 gave good agreement with the experimental results.
The design procedure developed by the Engineering Sciences Data Unit is continuously under review and new results are incorporated as soon as they have been validated. The latest information available on properties is given in reference 96. The measured damping properties are collated in 97 and the natural frequency information is given in reference 98. This is all put together in the estimation procedure described in reference 99. This Data Sheet also gives comparisons of the measured and estimated stresses obtained from a range of tests (figure 11). This shows the amount of scatter of the results and hence the likely accuracy of the simple linear prediction theory. It also shows clearly the onset of nonlinear behaviour.

6.2 Studies in the U.S.A.

The USAF equivalent of this design procedure for composite construction is that developed by Holehouse (100, 101). It complements the work of Rudder and Plumlee (63) for metallic structures. Test were made on a series of panels having Z or J section stiffeners in a standard 9-bay panel array. Coupon tests were made to provide more extensive random S-N data. The NASTRAN finite element program was used to calculate the mode shapes and strains in the fundamental mode. It was found difficult to reproduce the strain gradient at the stiffeners without using a large number of very small elements in that region. The stiffeners had to be modelled by several plate elements - not just a beam. The Miles type of estimation using the Finite Element mode shapes gave estimates of stress which were much higher than the measurements. Design nomographs were produced by 'best fits' to the data. In this case the damping ratio is not included as an independent variable - a typical value has been absorbed into one of the empirical constraints. This is a weakness when applying the method to different forms of construction.

The fatigue tests showed that the composite construction gave a weight saving of approximately 50% when compared with an equivalent aluminium structure for a given life and sound pressure level. They also showed that J section stiffeners were better than Z sections because of their symmetrical attachment to the skin. The design curves are given for Z section stiffeners and a modification procedure is suggested for J sections. Although the time to the appearance of the first crack is longer than in the case of aluminium alloy the fatigue development after that point was very rapid. Once failure has occurred (through delamination) disintegration is rapid whereas in the metal panel there is a much slower rate of crack propagation.

The problem with this series of nomographs is that they are not being regularly updated to incorporate the latest results. However a more recent paper by Holehouse (102) does describe a further analysis of the data used in deriving the design nomographs.

Soovere gives a good account of a development programme for a composite aileron design (95) and a fuller version appeared later in reference 103. The tests on the full scale aileron were backed up by coupon tests on the thin sandwich design of the skin surfaces. These skins are constructed with two 3-ply layers separated by a thin core of epoxy resin.
contain glass micro-balloons. Failure took the form of delamination close to the root. As the cantilever tests progressed there was a relatively small change in the first natural frequency but the damping increased by a factor of 2.4 and the peak response reduced to 0.4 of its initial value by the time failure took place. Noticeable change in damping (about 20%) occurred 10 cycles before failure. The full aileron structure was tested in a progressive wave facility where the response to the maximum test level was observed to be nonlinear. This behaviour was also confirmed by the broad nature of the response spectral density. The measured damping was very low (0.004) but no comparison of measured stress with predicted stress spectra is given. Early tests suggested that there would be a degradation of fatigue life due to exposure to moisture in the operating environment. The second report (103) gives details of coupon tests designed to investigate this phenomena. The test specimens were immersed in water for 19 days at 150°F and then tested at 180°F. No degradation in fatigue life was observed.

Arcas, Parente and Goss (104) report on tests on an aluminium panel and on a graphite-epoxy panel both of which were designed to withstand the same static loads. The test panels were of 5 x 3 bay construction. The aluminium panel had Z cross section stiffeners and 0.04" skin thickness whereas the composite panel had top-hat stiffeners and 0.0525" thick ply skins. A lighter weight composite panel (of skin thickness 0.042) was also made for comparison. Damping was measured in the first 20 resonant modes. The damping ratio for the aluminium panel falls within the band of values suggested by Hay (56) and that for the composite panels agrees with the values used by Holehouse (100). The panels were all tested at normal incidence and the response was linear for all up to the maximum test level of 145 dB 1/3 octave spectrum level. The predicted stress levels were 43% high for the aluminium (using the design method given in reference 63) and 34% high for the equivalent composite panel using the design method given in reference 100. The thinner skinned composite panel had a 20% reduction in weight of skin but the stress only increased by 4%. The life tests on the aluminium panels showed that, after initial tests of 5 hours on all panels at 140 dB (1/3 octave levels), when the levels were increased to 145 dB (1/3 octave level) the rivet heads began to break and after a further 4 hours the skin began to crack. In the composite panel no failures occurred in the test zone (centre 3 panels) at 150 dB (1/3 octave level).

Soovere (26) reports on tests on two designs of composite panels suitable for fuselage construction. Two panels of each design were tested at sound levels up to 167 dB overall and up to temperatures of 254°F. One of the two designs was made of conventional 8-ply skin with J section stiffeners which have been shown in other work to be better than those having a Z cross-section. The second panel design had a thin sandwich construction for the skin in which two 3-ply layers were separated by a thin core of epoxy resin filled with glass micro balloons - a form of construction which had proved successful in the aileron tests (93, 105). Rectangular cross section, or blade, stiffeners were used on these two panels. Measurements on the two designs showed that the damping of the sandwich skin was about an order of magnitude greater than that of the more conventional structure. Soovere attributes this to the difference in the acoustic radiation but it is possible that the sandwich type of construction contributes some of the extra damping. If it is due to the mode shape alone
(i.e. acoustic radiation) this may not be reproduced in a full scale structure because the mode shapes would be different from those of the 3-bay test panels. At 157 dB sound pressure level there were marked nonlinearities in the response. Comparison of the measured stresses with those predicted by the Holehouse method (100) shows that the estimates for the conventional panel are about half of those measured whereas the estimates for the thin sandwich skin panel are twice as great as the measurements. Soovere puts down the difference between the two designs to the order of magnitude difference in damping and thus points out one of the weaknesses of the Holehouse method. There is no term to allow for the measured damping - as is normally the case in the ESDU and other Nomographs. This is a particular weakness when considering composites because some of the novel designs possible with such a versatile building material can have relatively large damping built into them. The effect of temperature itself on the material properties was not marked up to these level (245°F).

The next paper by Soovere (105) summarises the above paper and also the ones on the aileron tests (93,103). It adds additional information about static shear load superimposed on the above fuselage panel tests. At buckling the overall dynamic strain is increased by about 30%. Plots show the decrease in frequency as the shear load increases and damping is also increased. Some tests were made at 125°C but the evidence of a temperature effect was inconclusive.

Jacobson (106) describes a similar series of tests on composite panels typical of what might be designed for a VSTOL aircraft fuselage. The panels were 36" x 24" containing 3 bays. Four were flat and four had 100" radius of curvature. Ambient temperatures and 250°F were used to 163 dB SPL. The panels had two top-hat stiffeners and J section frames. Heating was applied from the inside of the structure. Stresses were measured and the Holehouse method (100) was used for the predictions. For the top-hat stiffeners two dimensions were used in the prediction. One used a panel width equal to the distance between the stiffener centrelines and the other used the distance between the edges of the support stiffeners. Even using the smaller width in the calculations yields predictions which generally over estimate the measured stresses. Measurements show that the damping ratio lies in the range 0.014 to 0.042 and Jacobson agrees with Holehouse that damping can be omitted as a specific term in the nomographs because of scatter of the results (this conflicts with Soovere’s findings). Jacobson states that the life was lowered by the high temperatures but the data given in the paper does not support this contention. One pair of tests gave lower life whilst another gave increased life. These tests show some of the problems associated with using the Holehouse design method (100) on structures which differ in design from those on which the nomograph is based.
7. Recent Developments

A review of the recent work at the NASA Langley Research Center is given by Mixon and Roussos (11). These describe the current aims of the research on aircraft developments and on the National Aerospace Plane project.

7.1 Experimental

In an attempt to understand the overestimation of acoustic fatigue stresses by the use of the simple formula or the design nomographs, NASA set up a carefully controlled experiment on a single panel (107). The sound level was of low intensity to avoid nonlinear response complications and one aluminium and three composite panels were used in the tests. The panels were mounted in the centre of a very massive particle board plate with edge conditions which provided elastic restraint against rotation but prevented translation. The measurements showed that the response in the first mode dominated the strain spectrum. The spectral density of the peak response in the second mode was an order of magnitude lower than that in the first. A Rayleigh-Ritz analysis was used to compute a mode shape which gave a ratio of edge to centre strains equal to the measured ratio and the response to a uniform pressure field was calculated. The measurements and predictions of the spectra of the central acceleration were in very good agreement, however the strain calculations were high by a factor of about three.

To investigate this discrepancy further, additional analyses were carried out and reported by Roussos and Brewer (108). In this work the mode shapes were improved by the use of more terms in the Rayleigh-Ritz method and the NASTRAN Finite Element program was also used. There was excellent agreement between the two theoretical predictions but both still overestimated the response by the factor of three. Both centre and edge strains were overestimated by the same amount which suggests that the mode shape model is good. The inclusion of in-plane motion at the boundary in the model did not give any improvement. At first sight the fact that the acceleration predictions were good suggests that the error is in the mode shape. However the extensive studies of this aspect leaves little room for doubt. Other possibilities are errors in the measurements as these are very small strains.

7.2 Theoretical

7.2.1 Normal Mode Method

Blevins (109) starts from the formulation set out by Clarkson (44) and proposes a representation of the pressure distribution which can lead to an approximation which gives better results than the uniform pressure field. He assumes that the pressure load on the panel is equal to the mass inertia load which gives the simplification that the joint acceptance function becomes unity. He goes on to discuss improved approximations and compares his results with those from the ESDU method (4) on an aluminium panel. Good agreement is achieved between the two prediction methods. NASTRAN is then used to calculate
the frequency and stress response in the first two modes of an integrally stiffened titanium panel tested in a travelling wave tube. Estimates of the two peaks in the spectrum of strain responses show qualitative agreement with the measurement but the amplitude is overestimated by a factor of about 3.

7.2.2 Methods for Nonlinear Response

Theoretical methods for treating the physical characteristics of composites are now well established [see for example the review by Reddy (110)]. The main features of these structures in acoustic fatigue applications are the different forms of nonlinearities present in the response. Experimental results by White (87) and others have shown that the effect of nonlinearity on the response to high intensity noise is for the peaks (as seen in the linear response spectrum) to broaden and to move up in frequency. Two groups of workers have made the greatest contribution to developing the theory of the response of composites to high intensity acoustic pressures.

The group lead by Mei at Old Dominion University have considered single plates subjected to a simplified pressure field. In the majority of their work they have used the equivalent linearization method to study several different forms of nonlinearity. The second group whose principal worker is Vaicaitis of Columbia University has studied the effect of more realistic representations of the pressure field but has been more limited in the range of nonlinearities which have been included in the model. The major papers of the two groups are listed in the Bibliography and will now be reviewed.

In the first papers, Mei and Wentz (111) and Mei and Prasad (112) give a comprehensive theoretical study of the response of composite panels. The geometric nonlinearity of large deflection is considered and different lay up angles and numbers of plies are included. This gives a good theoretical development using the equivalent linearization technique and a single mode analysis. The theory is developed for edges which are allowed to move in the in-plane direction. This is a most important feature as typical aerospace structures will have some in-plane freedom of motion. This will be a particular design feature of hot structures where such motion will be needed to minimise quasi-static thermal stresses. Many graphs showing the effect of varying the composite layup construction on rms displacements are given.

The next stage of sophistication is to include nonlinear damping. The papers by Mei and Prasad (113,114,115) aim to explain the observed broadening of the response peak and its increase in frequency by including nonlinear damping as well as large amplitude displacements in the theory. This is a very valuable formulation because damping is inherently nonlinear and its behaviour and magnitude is one of the major unknowns in the work up-to-date. It is clear that once slippage takes place at joints the damping becomes very nonlinear. In this work a single mode analysis is used and the results show the expected broadening and increase in frequency. The broadening itself does not look as great as that observed by White (87).
The first study on a beam is extended to plates (114) and then goes on to include additional modes (115).

A most recent study of the effect of nonlinear damping has been made by Robinson and Mei (116) where the objective is to separate out the effects of large displacement and nonlinear damping. In this work a digital simulation method is used with a time domain representation of the forcing pressures. The pressure distribution itself is taken as spatially uniform over the panel. For a more realistic representation of the cross spectral density of pressures close to a jet or turbulent boundary layer it may be better to use the frequency domain form for the simulation as discussed in the next section. However in this work Robinson is able to show the separate effects of large deflection and nonlinear damping. The interesting result is that the large deflection nonlinear behaviour causes widening of the response peak and an upward shift in frequency whereas nonlinear damping has the opposite effect. Starting with a large displacement response at 150 dB the effect of increasing the nonlinear term in the damping is to reduce the width of the peak and move it downwards in frequency. This unexpected result should be investigated further.

The effect of transverse shear is considered in the next group of papers by Mei and Prasad (117,118). This is a wholly theoretical study. The governing equations are set out clearly and a modal solution developed using the equivalent linearization technique. This shows the range of length/thickness ratios over which the shear deformation is an important contribution to the response. At a ratio of 10 the bending stress is only 60% of the total. At a ratio of 20 transverse shear only contributes 10% and above about 50 the effect is negligible. Thus this phenomenon will only be relevant in special configurations.

The next study of note is that devoted to the effect of elasticity in the rotational restraint at the panel edges and the effect of initial imperfections (119,120). This follows the method of analysis developed in the earlier papers and goes on to consider the effect of simply supported, fully fixed and elastically restrained edges. It also considers an initial imperfection which takes the form of an initial curvature of the plate such as might be induced by the curing process. The shape is assumed to be the same as that of the fundamental mode shape and is defined by the initial central deflection. Theoretical results are given for the changes in frequency and rms central deflection due to variation in the edge rotational stiffness and the initial central deflection. The results show that small increases in edge restraint above the simply supported case have a marked effect on both parameters and indicate that in practice it will be very difficult to get a true simply supported edge condition.

Following earlier work on nonlinearities in uniform beams and plates Mei and his co-workers [Mei and Chiang (121), Chiang and Mei (122), Locke and Mei (123,124)], have also begun to develop the finite element method for use in these nonlinear problems. The first two of these papers develop the equations for the multi-modal representation of a beam and the third paper extends this work to plates. The method is tested on a single plate having the dimensions of those used by Paul (125). It reproduces the buckling temperature,
The rms strain is computed as a function of temperature and sound pressure level.

The earlier part of this theoretical work and associated experiments is described in the chapter by Mei and Wolfe (126).

The project by Moyer (127) was intended to be a theoretical study backed by experimental work on aluminium and graphite-epoxy plates. In the event the work is limited to a theoretical study of the response of a single degree-of-freedom system to a random force. The time domain simulation procedure is used with the Duffing equation representation of the nonlinearity. The response spectra shows the broadening of the spectral peaks and the shift in frequency. Moyer claims that this 'discovery' is unreported prior to this work.

The next distinct group of work is that due to Shinozuka (128,129,130) and used by some of his co-workers. Reference (129) is one of the basic references to this method. A frequency domain simulation is used in which the pressure function $f(x)$ is represented by the series:

$$f(x) = \sqrt{\frac{2}{\pi}} \sum_{k=1}^{N} A(\omega_k) \cos(\omega_k x + \phi_k)$$

$\phi_k$ is the independent random phase which is uniformly distributed from $0 \rightarrow 2\pi$.

$\omega_k$ is also randomly distributed in a small range

$$\pm \frac{\delta\omega}{2} \ll (\omega_k - \omega_k')$$

to avoid periodicities appearing in the simulation. This paper discusses the simulation of multi-variate processes and applications to boundary layer pressure fields etc. are discussed in later papers. Yang (131) discusses the Shinozuka model and looks at the accuracy of simulation for a given number of terms in the series. He concludes that 500 terms would be adequate.

The first paper by Vaicaitis and his co-workers (132) uses this simulation scheme to represent a boundary layer pressure field. It is not clear from the paper exactly how the simulation is implemented but results are given for a typical structural panel. The initial working shows that the number of terms in the series can be reduced from for subsequent numerical integration in the time domain.
The next paper (133) extends the work on boundary layer pressure fields and shows the time history of the generalised random force, its probability density, distribution of peaks and threshold crossings. An example is given of the response of a stiffened panel to boundary layer pressure fluctuations.

The paper by Vaicaitis, Dowell and Ventries (134) gives little information on the simulation but goes on to give results for the nonlinear response of a panel to boundary layer pressures with and without a backing cavity. The interesting result on the wave form of the response is that in the nonlinear case the maximum response amplitude is only $1.6 \times$ rms whereas in the linear case it is $3 \times$ rms. The next paper (135) applies the method to a section of Space Shuttle structure which holds the surface insulation panels. Estimates of natural frequencies agree closely with those obtained by the finite element method. Estimates of the deflection and the stringer stress are also produced. In the two cases where measurements are available from acoustic tunnel tests the predictions are about twice as great as the measured data.

The next paper by Vaicaitis (136) gives a general review of the basic concepts of the simulation procedures, the time domain modal solutions of the nonlinear equations and the Monte Carlo Method as used up to 1986. The time domain representation of the forcing function is rewritten in a form suitable for use with the FFT algorithm. Simulated realisations of the forcing function are applied to the equation of motion and the response time history is obtained for each by numerical integration. Then statistical ensemble averages are obtained for the response at each time interval using the finite element method to solve the structural response equation.

The more general paper on acoustic fatigue by Vaicaitis (137) is a brief review of the applications of the simulation procedures described in earlier papers. Little new material is contained in the paper and there is no comparison with any experimental results.

The next paper by Vaicaitis and Choi (138) introduces the use of the transfer matrix method to produce the response solution for a stiffened plate array. The treatment attempts to include all the forcing functions i.e. cavity pressure, unsteady aerodynamic pressure, thermal loads as well as random pressures. It is therefore unable to give an adequately detailed study of each. No experimental results are given.

The next paper by Vaicaitis and Choi (139) extends the previous work to consider fatigue life. Using the same structural model and computation procedure as in the earlier paper, Miner's linear damage rule is used to estimate fatigue life. This work is also reported in the Conference Proceedings (140).

Maekawa (141) provides the one completely independent check on the method. He applies this to the specimen tests reported by Van der Hyde and Kolb (142) for the three bay stringer panel. He is only able to use a single degree-of-freedom model for this structure and so to allow for the flexibility of the stringer supported edges he considers the case of simply
supported and fully fixed edges. He gives examples of the forcing and response waveform in the unbuckled, snap-through and post buckled phase of vibration of the heated panels. This is one of the most interesting and thorough pieces of analysis. The theoretical results and simulations follow closely the nonlinearity but the magnitude of the response is between 2 or 3 times that of the experiment. This is an excellent piece of work to follow up.

The most recent paper by Vaicaitis and Choi (143) puts together the previous work on the response of a stiffened panel using the transfer matrix method to solve the structural response equation and the simulation method to determine the time history of the response. The structural model used to check the theory was that designed by Van der Hyde and Kolb (142) and used first by Maekawa (141) to check the accuracy of the simulation method. The use of the transfer matrix method gave Vaicaitis and Choi a much better structural model of the test specimen. The results of their computations are also shown in figure 12. Good agreement between the simulation results and the experiment is now achieved. A second paper just published by Choi and Vaicaitis (144) is almost identical to the previous paper (143). This is a most important paper as it represents the closest agreement between theory and experiment that has been achieved for nonlinear response.

7.3 The Statistical Energy Analysis (SEA) Method

An alternative method of modelling the vibration of aircraft structures in high intensity noise fields is to consider the energy flow in the structure. Starting from the energy flow from the sound field an energy balance equation is constructed for each element of the structure. The result is a frequency band average of the total vibrational energy in each component. At first sight this appears to be a very crude way to estimate rms stresses at critical points in randomly excited structures. However there are situations where it is impossible to get good estimates using the normal mode or travelling wave method and therefore it is worthwhile to monitor developments of the SEA method for possible applications in specific cases. The methods already described seem to provide reasonable estimates of stress levels where the response spectrum at the point of interest has only a few modes contributing the majority of the response. Even in this case it is difficult to get better agreement than to within a factor of 2. Where the response shows a broad band form, such as may often be the case for internal structure such as frames and ribs, the standard methods are much less reliable. It is in cases like this that the SEA method may have something to offer. The method itself was developed primarily for use in building acoustics where the main interest is in the noise level inside a room resulting from an external noise field. The transmission of the energy is then via the air and also through the structural elements. The accuracy needed in these studies is usually much less than is expected in structural vibration work. Recent studies have tried to refine the method and consider the application to structural vibration problems. It is thought therefore worthwhile to outline the method and indicate the present state of development.
Lyon suggested the method and his book (145) gives a general description of the main features. A very good recent account has been given by Norton (146) and this also includes an extensive bibliography. Proposals for the use of this method in the study of the response of spacecraft structures to acoustic excitation are described in references (147) and (148). A mathematical model of the structure is built up by considering the average energy of vibration of each structural component. This in turn involves the spatial average mean square velocity of the structure if the mass can be assumed to be distributed in a reasonably uniform manner. An energy balance equation can then be written for each component. The energy input from the environment (acoustic pressure field) and/or from the attached vibrating structures is equal to the energy which is radiated as sound, or dissipated as heat, plus the energy which is transferred to the attached structures. For a multi component structure an energy matrix equation can be written to describe all these transfers.

The energy equations only become sufficiently simple to handle in this way if an average over many natural frequencies is taken. Bands of frequencies are chosen (usually 1/3 octave for room acoustic work) and the total response is assumed to be equal to the response of one typical mode multiplied by the number of modes in the band. The response level is thus approximately proportional to the number of modes in the band. The response level is thus approximately proportional to the modal density.

The radiation of energy from the structural component is described by the dissipation loss factor. Where this loss is caused primarily by hysteretic losses in the material itself and friction at joints etc. empirical data must be used to obtain typical loss values. Part of the loss results from sound radiation and this can be expressed in terms of the radiation efficiency of the structure.

The transfer of energy from one component to another is expressed in terms of the coupling loss factor. By writing this loss in the same form as the hysteretic loss (i.e. $\eta \omega E$) the equations are simplified and a loss factor matrix can be built up.

The SEA Parameters

The power flow balance matrix equation is a very simple one and therefore improvements in accuracy can only come from improvements in the parameters which are inserted into the equation. In the early use of the SEA method in room acoustics sufficient accuracy of estimation of internal sound levels could be obtained by relatively simple models of the structural components. However if we are trying to estimate structural levels, more precise representations of the type of structures commonly used in aerospace construction must be developed.

Modal Density

The analysis procedure described above requires an estimation of the number of modes of vibration occurring in each frequency band.
Analytical results have been derived for several types of uniform structure and a good summary of available results have been prepared by Hart and Shah (149). Honeycomb plates and shells have been studied by Wilkinson (150) and Erikson (151) and this has been extended to parabolic shells by Elliott (152).

For aerospace structures, however, the majority of components are far from being uniform. There are irregular cut outs and stiffeners and irregularly distributed masses of varying sizes. Also some types of structural component such as corrugated plates and shells have not been studied extensively. An experimental technique for the measurement of modal density based on the measurement of the point impedance of the structure has been developed by Clarkson and his co-workers for typical spacecraft components (153,154,155). This work has provided results for:

- Corrugated plates and shells
- Effect of cut outs and stiffeners on honeycomb plates
- Effect of masses on honeycomb plates
- Honeycomb plates with carbon fibre reinforced face plates

**Radiation Resistance**

The radiation resistance of a structure is the measure of its ability to radiate sound and is the important parameter which determines the energy transfer between a radiating structure and the surrounding medium. The size of the structure relative to the wavelength of sound in the medium is a major controlling parameter. Flexural waves in flat or curved structures are the most efficient radiators of sound.

The most recent results for flat isotropic plates are those produced by Broadbent et al. (156). A modification to the simply supported plate results has been produced by Maidanik (157) and a method for the application of these results to honeycomb plates is given by Ferguson (158).

**Loss Factor**

There are no theoretical results for the loss factor of structures therefore damping measurements have to be relied upon for estimate of the frequency average values of typical structures. This is discussed by Ranky and Clarkson (159) and typical results given.
Coupling Loss Factor for Structural Joints

There have been some analytical studies of specific junctions such as beam to beam (160) and plate to plate (161). For other types of junctions of direct interest in aerospace structure one must reply on the limited experimental data (162). The development of the experimental techniques is discussed by Norton and Greenhalgh (163). The values of the coupling loss factor for typical joints are about an order of magnitude less than the internal structural loss factor and thus experiments are very difficult and the results must be treated with great caution.

Maximum rms Stress in Plate Structures

As mentioned in the introduction to this section, the major disadvantage of the method from a structural response point of view is that the results are all in the form of statistical averages over time, frequency, and surface area. For room acoustic applications this is usually perfectly adequate but if we are to extend the method successfully to applications to structural fatigue we need to know the rms stress at the points of likely failure. There has been little work on this aspect. Clarkson has suggested that the maximum stress in stiffened plates is about twice the spatial average stress. Ungar (164) made some studies of the effect of structural discontinuities on stress concentration and Stearn (165,166) developed a theoretical analysis based on the random distribution of bending waves in a reverberant field. The second of these papers shows that the highest values of rms stress at a structural discontinuity are about twice the spatial average value. The most recent work of Norton and Fahy (167) gives the results of a much more extensive study of the statistics of the velocity and strain distributions.

Further refinements are being made Clarkson (168) who has used the normal mode formulation to estimate coupling loss factors and Keane and Price (169) are developing the probabilistic models which are used in the SEA method. Langley (170) has also recently provided a theoretical derivation of the SEA approach to vibration prediction. In this latest work expressions for the coupling loss factors are obtained in terms of the frequency and space averaged Green functions of the coupled system.
8. Conclusion

The review of the early work and of the activities which have begun more recently to study the behaviour of composite materials in a high noise level environment has shown that the phenomenon of sonic fatigue is now quite well understood but it is still not possible to predict the response stresses in a new structure to better than an accuracy of about a factor of 2. This possible error implies an error of at least an order of magnitude in fatigue life estimates made in the design stage. It is disappointing that the more sophisticated methods and greater computer power now available have not been able to improve significantly on the early simple results. Sonic fatigue continues to be a problem because of unacceptable inspection and maintenance procedures required to ensure that no minor damage spreads to form a potential catastrophic failure. The fatigue life of any specific piece of structure is very sensitive to the detail design of attachments, fasteners and the edge supports. Experience and testing has lead to good design practice which can do much to alleviate the worst problem.

The reasons for the discrepancies between experimental results and the theories probably vary from one type of structure to another. In the early test work there were very many difficulties and errors were likely in the measurement of the sound field, the low stress levels and the data processing. This produced a scatter of results about the theoretical estimates. In more recent work the experimental errors are reduced considerably and a bias is now appearing which shows estimates to be greater than the experimental results by a factor of about 2. Possible reasons for this are:-

Metallic structures responding in the linear range.

1. Single plates: a single first order mode dominates the response. Calculations of the response in this mode overestimate the strains. This could be the result of an underestimate of the damping (acoustic radiation damping is important in this mode) and lateral motion of the support structure to balance the inertia forces produced in the vibrating plates.

2. Stiffened plate structures with uniformly spaced stiffeners. In these structures many modes exist in bands about the frequency which would have been associated with the fundamental, first harmonic etc. of a single plate. The transfer matrix and travelling wave (periodic structure) methods have been developed with some success but the accuracy is not much improved.

3. Built-up structure typical of aircraft construction. In these structures the stiffener spacing is not usually exactly uniform and the resulting mode shapes are ones which show one panel having a displacement much greater than its neighbours. This now approximates to a single panel with flexible support structure. Recent work by Groen shows that a good finite element model can be made and reasonable estimates of response in the predominant mode achieved.
**Composite Structures**

Tests have shown that a composite structure designed to the same overall requirement as a metallic structure can have a considerably enhanced sonic fatigue life. The design details are important and symmetrical stiffeners have been shown to give better results. In the linear response range the estimates of stress levels can be made in the same manner as for metallic structures and similar agreements are achieved. However, because of the high strength of the materials the behaviour at working stresses becomes nonlinear due to the large displacements. Theories for the nonlinear response have been developed. These are mostly limited to a single mode. However, the recent work using the simulation techniques coupled with the transfer matrix or finite element method to solve the equation of motion is giving good results.

The fatigue mechanism in composite structures is usually one in which delaminations take place between layers within the matrix. This is difficult to detect and once initiated it spreads rapidly.

Temperature does not seem to pose a major problem up to about 250°F. Most materials show some degradation in performance but the almost catastrophic effect which was expected to result from snap-through does not occur on built-up structures where the edges are not rigidly held in the in-plane direction. Much higher temperatures such as those that will be associated with the sub-orbital and re-entry vehicles will cause transient thermal strains which could couple with pseudo-acoustic loads and cause damage to the structure. Much more work will be required before we can have any confidence in any estimates which are made on the basis of existing knowledge.
9. **Recommendations for Future Studies**

To improve conventional sonic fatigue design it is suggested that several carefully controlled experiments be made with representative structures, the results of which can be used to check and develop the theories. The structures would need to exhibit multi-modal response and also show up the differences between metal and composite construction.

The next leap forward into hypersonic vehicles requires many fundamental investigations. I suggest that the first studies to be made in preparation for this next development be on the combination of transient thermal strain effects with acoustic loading.

1. Conventional structures

**Experimental**

Type of test specimen: Multi-bay stiffened skin panel
(1) Uniform stiffener and rib spacing to give uniform mode shapes.
(2) Larger central panel to give non-uniform mode shapes.
   Support : Heavier edge frames (to be included in theoretical mode)
   Materials: (a) Aluminium Alloy
   (b) Carbon fibre reinforced plastic
   (c) Metal matrix composites
   (d) Titanium alloys

Test in high intensity travelling wave facility to get linear and nonlinear response.

**Theoretical**

Develop the simulation method together with different methods of solving the equation of motion. This method will then be able to handle nonlinearities.

Further work on the stresses in internal/support structure following on the early analysis by Rudder (64). Finite Element modelling would make a more comprehensive analysis possible.

**Fatigue**

Extensive studies of the delamination form on sonic fatigue failure of composite plates. Methods of detection of initial failure and monitoring of progress. Link with design surface strains.

Fatigue crack growth for damage tolerant designs.
2. **NASP and Similar Hypersonic Vehicles**

Combination of high temperature with high acoustic loads. As the transient thermal strains will be as important (if not more important) than high soak temperatures, develop pilot facility to reproduce correct rates of changes of temperature rises/falls and co-ordination with the time history of acoustic loads.

As future of project is not certain, the maximum flexibility in the range of parameters should be sought.

3. **Alternative Theoretical Approach**

Monitor developments in the application of the Statistical Energy Method. If the accuracy for structural vibration estimates can be improved this would offer a good method for combined thermal and acoustic environments.
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Appendix to Reference List

Contents of Acoustic Fatigue Data Series (Reference 4)

ESDU Engineering Data: Acoustic Fatigue Series

Volume 1  General
Definition of terms for acoustic fatigue analysis
Bandwidth correction
Combination of levels in dB
The relation between sound pressure level and rms fluctuating pressure
Design information on acoustic fatigue
Estimation of the stiffnesses and apparent elastic properties of laminated lat plates
Design against fatigue. Vibration of structures under acoustic or aerodynamic excitation

Endurance under acoustic loading
Endurance of aluminium alloy structural elements subjected to simulated acoustic loading
Endurance of titanium and titanium alloy structural elements subjected to simulated acoustic loading
Endurance of Fibre-reinforced composite, laminated structural elements subjected to simulated random acoustic loading

Volume 2  Loading Actions
The estimation of near-field sound pressure levels due to jet noise
Estimation of sound pressure levels due to buzz-saw noise within the intake duct of a supersonic fan or compressor
Computer-based estimation procedure for near-field single-stream jet noise
Damping

Damping in acoustically excited structures
Estimation of damping in laminated and fibre-reinforced plated

Volume 3

Stress/strain response to acoustic loading

Estimation of the rms stress in skin panels
The effect of edge panels under uniform pressure
The estimation of rms stress in stiffened skin panels subjected to random acoustic loading
Stress response of flat or singly-curved sandwich panels with cores of zero/ flexural stiffness subjected to random acoustic loading
The estimation of rms stress in skin panels with flexible stiffeners subjected to random acoustic loading
Estimation of rms stress in internal plates of a box structure subjected to random acoustic loading
Estimation of rms strain in laminated skin panels subjected to random acoustic loading
Estimation of rms strain in laminated face plates of a simply supported sandwich panels subjected to random acoustic loading. Including a simplified natural frequency prediction method

Volume 4

Natural Frequencies of Plate Structures

Natural frequencies of built-up periodic skin-stringer structures. I. Stringers rigid in bending
Reference frequency of panel with flexible stiffeners
Natural frequencies of rectangular flat plates with various edge conditions
Natural frequencies of rectangular, specially orthotropic laminated plates (Sheet 83036)

Natural frequencies of rectangular singly-curved plates

Natural frequencies of isotropic and orthotropic rectangular plates under static in-plane loading (Sheet 87029)

Natural frequencies of singly-curved laminated plates with simply supported edges (Sheet 89011)

**Volume 5**

**Natural Frequencies of Shell Structures**

Natural frequencies of thin-walled isotropic circular-cylindrical shells (Sheet 78004)

Free vibration of thin-walled, orthogonally stiffened, circular-cylindrical shells (Sheet 80040)

Natural frequencies of thick-walled isotropic circular-cylindrical shells (Shear diaphragm end conditions) (Sheet 81018)

**Volume 6**

**Natural Frequencies of Sandwich Panels and Box Structures**

Natural frequencies of flat or singly-curved sandwich panels with cores of zero flexural stiffness (Sheet 72016)

Natural frequencies of box structures (Sheet 74025)

Natural frequencies of simply-supported sandwich panels with laminated face plates (Sheet 85027)
Figure 1. Real part of the cross spectral density of pressures at two positions close to a jet efflux (Reference 5).
Figure 2. Typical power spectra of pressure fluctuations underlying supersonic flow (Reference 9).
Figure 3. Time history trends for acoustic loads and temperature.

(a) Aircraft: conventional take off and vertical take off

(b) National Aerospace Plane (NASP)
Figure 6. Comparison of predicted and measured rms panel stress (Reference 72).
Figure 7. Comparison of predicted and measured displacement response spectra (Reference 9).
Figure 8. Comparison of predicted and measured stress response spectra (Reference 9).
Figure 9. RMS Deflection of a stiffened skin panel (Reference 54).
Figure 10. Strain response spectra of carbon fibre reinforced plastic plates (Reference 87).
Figure 11. Comparison of measured and estimated strains in carbon fibre reinforced plastic plates (Reference 99).
Figure 12. Comparison of measured and estimated nonlinear response of a three bay panel (Reference 143).
From the early-1960s until the mid-1980s, there was very little theoretical development for sonic fatigue prediction. Design nomographs based on simple theoretical models and results of specially designed tests were developed for most common aircraft structures. The use of advanced composites in the 1980s, however, generated an increased interest in development of more sophisticated theoretical models because of the possibilities for a much wider range of structural designs. The purpose of this report is to review sonic fatigue technology and, in particular, to assess recent developments. It also suggests a plan for a coordinated program of theoretical and experimental work to meet the anticipated needs of future aerospace vehicles.