Gaussian Beam and Physical Optics Iteration Technique for Wideband Beam Waveguide Feed Design

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The Gaussian beam technique has become increasingly popular for wideband beam waveguide (BWG) design. However, it is observed that the Gaussian solution is less accurate for smaller mirrors (approximately < 30\lambda in diameter). Therefore, a high-performance wideband BWG design cannot be achieved by using the Gaussian beam technique alone. This article demonstrates a new design approach by iterating Gaussian beam and BWG parameters simultaneously at various frequencies to obtain a wideband BWG. The result is further improved by comparing it with physical optics results and repeating the iteration.

I. Introduction

Geometrical optics (GO) is a well-known technique used in the design of many beam waveguide (BWG) feed systems. A BWG feed system is composed of one or multiple feedhorns with a series of flat and curved mirrors arranged so that power can be propagated from the horn through the mirrors with minimum losses. Horns and equipment can thus be located in a large, stable enclosure at an accessible location. While GO is useful for designing high-frequency or electrically large mirrors (approximately > 50\lambda in diameter with ~20 dB edge taper), some BWGs may be operated at low frequency and have a mirror size of only about 20\lambda in diameter. Due to diffraction effects, the characteristics of a field propagated between small BWG mirrors (approximately < 20\lambda in diameter) will be substantially different from the GO field. Therefore, the GO design is not suitable for a high-performance wideband BWG antenna with small BWG mirrors. The Gaussian beam technique has become increasingly popular for wideband BWG design. The Gaussian beam mode is an approximate solution of a wave equation describing a beam of radiation that is unguided but effectively confined near an optical axis. The zero-order mode is normally used in the design. A major advantage of the Gaussian technique is the simplicity of the Gaussian formula, which is easy to implement with negligible computer time.

G. Goubau gave the first mathematical expression of Gaussian modes derived from the solution of Maxwell's equations described by a continuous spectrum of cylindrical waves [1]. T. S. Chu developed the Fresnel zone imaging principle of the Gaussian beam to design a pseudofrequency-independent BWG feed [2]. S. Betsudan, T. Katagi, and S. Urasaki used a similar imaging tech-
The design goal is to have \( R \) and \( D \) be constant over the field amplitude has fallen 18 dB from its maximum value. — 18-dB-beam diameter is defined as the diameter at which the Gaussian beam at the subreflector, respectively. The radius of curvature and the — 18-dB-beam diameter of the subreflector, respectively. \( R \) and \( D \) are the wavefront radius of curvature and the — 18-dB-beam diameter of the Gaussian beam at the subreflector, respectively. The — 18-dB-beam diameter is defined as the diameter at which the field amplitude has fallen 18 dB from its maximum value.

**II. Design Geometry**

The basic goal is to design a BWG feed system with good performance from 2 to 32 GHz by utilizing mirror sizes of 20\( \lambda \) in diameter with edge taper —23 dB at 2.295 GHz (other frequencies may have different edge tapers). It is also required that all feedhorns be placed in a basement room below the azimuth wheel and track. The feed system must provide a simultaneous operation capability and a fast response feed selection system. The geometry of the BWG antenna is shown in Fig. 1, where \( M_2, M_3, \text{and} M_5 \) are curved mirrors and \( M_1, M_4, \text{and} M_6 \) are flat plates. If \( M_6 \) is a dichroic plate, an additional reflector and horn, beyond \( M_6 \), can provide dual frequency simultaneous operation. All the flat plates are assumed to be sufficiently large and are excluded from the design. The BWG in Fig. 1 can be reduced to a horn and three curved mirrors (Fig. 2) with curved mirrors replaced by thin lenses. Let \( f_1 \) be the focal length and let \( D_i \) be the diameter of lens \( M_i \) with edge taper —\( T_i \) dB (\( i = 2, 3, \text{and} 5 \)). The horn and mirrors are separated by distances \( L_i \) (\( i = 0, 1, 2, 3, \text{and} 5 \)), with \( L_H \) representing the length of the horn. \( D_H \) and \( D_{SUB} \) are diameters of the horn and the subreflector, respectively. \( R \) and \( D \) are the wavefront radius of curvature and the —18-dB-beam diameter of the Gaussian beam at the subreflector, respectively. The —18-dB-beam diameter is defined as the diameter at which the field amplitude has fallen 18 dB from its maximum value.

**III. Design Procedure**

Different frequencies usually have different values of \( D \). The design goal is to have \( R \) and \( D \) be constant over the design frequency range with acceptable spillover losses at all mirrors and \( D = D_{SUB} \). Three frequencies: 2.295 GHz (S-band), 8.45 GHz (X-band), and 32.0 GHz (Ka-band) are used in the design. Curved mirrors are arranged so that the mirror system has a low cross-polarization. For a different design frequency range, the S-, X-, and Ka-bands can be replaced by low, middle, and high frequencies of the band. Input parameters are operating frequencies, \( D_{SUB}, D_1, D_3, D_5, \text{and maximum allowable spillover loss (or dB edge taper) at each mirror. The relationship between} D_H \text{and} R_H \text{is known. The rest of the parameters in Fig. 2 are unknown and are to be determined during the iteration process. The desired ranges of some of the input and output parameters need to be established. The initial values to start the iteration can be obtained by the GO design that roughly fits to antenna structures or other requirements.}

The design procedure can be described as follows:

**Step 1.** The radius of curvature \( R \) and beam diameter \( D \) at the subreflector are calculated starting from the horn and proceeding through mirrors \( M_5, M_3, \text{and} M_2 \) to the subreflector by using the zero-order Gaussian mode. Details are shown in the Appendix. Let \( R_s, R_x, \text{and} R_{ka} \) (\( D_s, D_x, \text{and} D_{ka} \)) be the radii of curvature (—18-dB-beam diameters) at the subreflector calculated at S-, X-, and Ka-bands, respectively. The unknown parameters are iterated so that \( R_s = R_x = R_{ka} \) and \( D_s = D_x = D_{ka} \). It is quite easy to have \( R_x = R_{ka} \) (as well as \( D_x = D_{ka} \)). However, in many cases (due to structure constraints, size of mirrors, etc.), the iteration cannot converge to the condition \( R_s = R_x = R_{ka} \). Instead, \( R_s \) is usually greater than \( R_x \) and \( R_{ka} \). Therefore, one might have to accept \( R_s > R_x = R_{ka} \).

**Step 2.** The \((R_s, R_x, R_{ka})\) and \((D_s, D_x, D_{ka})\) are re-calculated by PO, a more accurate technique, with BWG parameters obtained from Step 1. Recall that the Gaussian solution predicts that \( R_s = R_x = R_{ka} \) in Step 1, while PO results show that \( R_s > R_x > R_{ka} \). The beam diameters from PO calculations are slightly smaller than the Gaussian results at all frequencies (but the trend may not be consistent for other cases). It is noted that the differences of \( R \) calculated from PO and Gaussian software are larger at electrically smaller mirrors.

**Step 3.** In order to offset the discrepancy between Gaussian and PO results, as indicated in Step 2, Step 1 is repeated and the unknown parameters are iterated so that \( R_s < R_x < R_{ka} \), which are approximately the same amounts as indicated in Step 2 but in the opposite sense (“larger” in Step 2 results in “smaller” in Step 3). A nu...
merical example will be provided shortly. A similar adjustment procedure is also applied for the beam diameters $D_s$, $D_x$, and $D_{ka}$.

Steps 2 and 3 can be repeated until an acceptable result is achieved. For simplicity, only radii of curvature at X- and Ka-bands are considered as examples here. In Step 1, after millions of iterations, one obtains $R_x = R_{ka} = 476$ in. The calculation in Step 2 by PO software gives $R_x = 488$ in. and $R_{ka} = 478$ in. with $\Delta R_x = 488 - 476 = 12$ in. and $\Delta R_{ka} = 478 - 476 = 2$ in. In Step 3, the goal is to iterate the unknown parameter so that $R_x = 476 - 12 = 464$ in. and $R_{ka} = 476 - 2 = 474$ in. When Step 2 is repeated with parameters recently obtained from Step 3, the results are $R_x = 477$ in. and $R_{ka} = 476$ in. The radius of curvature $R = 476.5$ in. is chosen for a dual-shaped reflector synthesis.

Table 1 shows a numerical comparison between Gaussian and PO techniques of the BWG configuration shown in Fig. 3. If the PO software is not available, one could use data from Table 1, provided that the new BWG configuration closely resembles the one in Fig. 3. It is noted that discrepancies between PO and Gaussian results are larger for electrically smaller diameters. The discrepancies are less sensitive to the distance between mirrors as long as they are in the Fresnel zone. In a design with the minimum mirror diameter $> 50\lambda$, reasonably good performance can be achieved by implementing only Step 1.

IV. Conclusion

The result from this design technique is shown in Fig. 3, with $f_2 = f_3 = 2580$ in. and $f_5 = 220$ in. The mirror diameters are $D_2 = D_3 = 105$ in., $D_5 = 131.5$ in., with their average edge tapers shown in Table 1. It is noted that the design result shown in Fig. 3 is close to the optimum performance. Some small performance sacrifices are made for cost, structure retrofit, maintenance, and accessibility. Also, some S-band performance is sacrificed in order to achieve very good performances at X- and Ka-bands. Spillover loss at each mirror is listed in Table 2. The spillover loss is calculated by integrating a scattered field calculated by the PO software, with the assumption that there is no tube effect. It is observed from Table 2 that energy is more confined well inside the BWG at higher frequencies since the spillover loss is lower at higher frequencies. Higher gain horns are needed for higher frequencies. Aperture diameters of S-, X-, and Ka-band horns are 4.57$\lambda$, 10.02$\lambda$, and 15.44$\lambda$, respectively. One operating mode for this antenna is simultaneous S-/X-band, with the dichroic surface $M_6$ reflecting S-band and passing X-band. In another operation mode, $M_6$ is flipped out of the beam path, allowing Ka-band (32.0 GHz; as well as X-band) to propagate to $M_7$. Given that $M_7$ is a dichroic surface, simultaneous X-/Ka-band operation is achieved. By simply rotating $M_6$, extra frequency bands can be used. In 1993, the NASA Deep Space Network (DSN) will begin construction of three 34-m BWG antennas based on this high-performance design concept.

References


Table 1. Radius of curvatures and beamwidths of fields at the subreflector and average edge tapers of $M_2$, $M_3$, and $M_5$, as shown in Fig. 3

<table>
<thead>
<tr>
<th>Frequency band</th>
<th>$R$, in.</th>
<th>$D$, in.</th>
<th>Average edge taper, dB</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Gaussian</td>
<td>PO</td>
<td>Gaussian</td>
</tr>
<tr>
<td>S</td>
<td>462.2</td>
<td>515.0</td>
<td>138.5</td>
</tr>
<tr>
<td>X</td>
<td>464.1</td>
<td>477.0</td>
<td>136.9</td>
</tr>
<tr>
<td>Ka</td>
<td>473.7</td>
<td>476.0</td>
<td>133.7</td>
</tr>
</tbody>
</table>

Table 2. Spillover loss of each mirror at 2.295, 8.45, and 32.0 GHz

<table>
<thead>
<tr>
<th>Mirrors</th>
<th>Spillover losses, dB</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2.295 GHz</td>
</tr>
<tr>
<td>$M_1$</td>
<td>0.004</td>
</tr>
<tr>
<td>$M_2$</td>
<td>0.051</td>
</tr>
<tr>
<td>$M_3$</td>
<td>0.018</td>
</tr>
<tr>
<td>$M_4$</td>
<td>0.017</td>
</tr>
<tr>
<td>$M_5$</td>
<td>0.075</td>
</tr>
<tr>
<td>$M_6$</td>
<td>0.025</td>
</tr>
<tr>
<td>$M_7$</td>
<td>-</td>
</tr>
<tr>
<td>$M_8$</td>
<td>-</td>
</tr>
<tr>
<td>Total</td>
<td>0.190</td>
</tr>
</tbody>
</table>
Fig. 1. Beam waveguide design configuration.

Fig. 2. Beam waveguide design parameters.
Fig. 3. Detailed dimensions of the beam waveguide configuration.
Appendix

The purpose of this appendix is to show zero-order Gaussian mode expressions used to compute fields from a horn through a series of mirrors and ending at a subreflector.

A. From a Horn to a Mirror (a Thin Lens)

The beam radius and phase front radius of curvature at the aperture of a corrugated horn are

\[ \omega_0 = 0.32 \, D_H \quad (A-1) \]

and

\[ r_0 = \sqrt{L_H^2 + \frac{D_H^2}{4}} \quad (A-2) \]

respectively [2]. The beam radius is defined as the radius at which the field amplitude has fallen 1/e of its maximum values, where \( D_H \) and \( L_H \) are the diameter and length of the corrugated horn, as shown in Fig. A-1. The radius of curvature on the lefthand side of the thin lens \( M_1 \) is [5]:

\[ r'_1 = \frac{L_0}{1 - \frac{1 + L_0/r_0}{(1 + L_0/r_0)^2 + \left(\lambda L_0/\pi \omega_0^2\right)^2}} \quad (A-3) \]

The beam radius at \( M_1 \) (which is the same on both sides of the thin lens) is [5]:

\[ \omega_1 = \omega_0 \sqrt{\left(1 + L_0/r_0\right)^2 + \left(\lambda L_0/\pi \omega_0^2\right)^2} \quad (A-4) \]

where \( L_0 \) is the distance from the horn aperture to the center of the lens (or mirror), and \( \lambda \) is a wavelength of an operating frequency.

The diameter and spillover loss of the lens \( M_1 \) with \(-T_1\)-dB edge taper are each [3]

\[ D_1 = 0.6786 \, \omega \sqrt{T_1} \quad (A-5) \]

and

\[ P_1 = 10 \log_{10}(1 - e^{-0.2303T_1}) \quad (A-6) \]

Spillover loss in Eq. (A-6) is reasonably accurate for design purposes. After the design is completed, more accurate predictions of the spillover loss will be obtained by direct integration of a scattered field computed by PO and spherical wave expansion software. The radius of curvature on the right-hand side of the lens \( M_1 \) can be obtained from a thin lens relation

\[ \frac{1}{r_1} = \frac{1}{f_1} - \frac{1}{r'_1} \quad (A-7) \]

where \( f_1 \) is the focal length of the lens \( M_1 \), and \( r_1 \) (as well as \( r'_1 \)) is defined to be positive when its phase front is convex toward the lens surface.

B. Between Mirrors

Similar to Section A, the radius of curvature and beamwidth at the left-hand side of \( M_2 \) are

\[ r'_2 = \frac{L_1}{1 - \frac{1 - L_1/r_1}{(1 - L_1/r_1)^2 + \left(\lambda L_1/\pi \omega_1^2\right)^2}} \quad (A-8) \]

and

\[ \omega_2 = \omega_1 \sqrt{\left(1 - L_1/r_1\right)^2 + \left(\lambda L_1/\pi \omega_1^2\right)^2} \quad (A-9) \]

where \( L_1 \) is the distance between the two lenses. \( \omega_1 \) and \( r_1 \) are given in Eqs. (A-4) and (A-7), respectively. The differences between signs in Eqs. (A-3) and (A-8), and also in Eqs. (A-4) and (A-9) are due to a definition that a radius of curvature of a Gaussian beam is negative when it is concave toward the direction of propagation (+Z), and \( r_1 \) is concave toward +Z as in Fig. A-2. The value of \( r_1 \) obtained from Eq. (A-7) can be directly substituted into Eqs. (A-8) or (A-9) without changing any of the signs. The diameter and spillover loss of the lens \( M_2 \) for \(-T_2\)-dB taper can be obtained from Eqs. (A-5) and (A-6), with \( T_1 \) replaced by \( T_2 \). It is noted that \( M_2 \) can be a subreflector. The calculations in Section B are repeated for all the rest of the mirrors and the subreflector.
Fig. A-1. The geometry of a circular aperture corrugated horn illuminating a thin lens.

Fig. A-2. The geometry of a Gaussian beam between two lenses.