An Interactive Educational Tool for Compressible Aerodynamics

Thomas J. Benson
Lewis Research Center
Cleveland, Ohio

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Thomas J. Benson*
NASA Lewis Research Center
Cleveland, Ohio 44135
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ABSTRACT

A workstation-based interactive educational tool has been developed to aid in the teaching of undergraduate compressible aerodynamics. The tool solves for the supersonic flow past a wedge using the equations found in NACA 1135. The student varies the geometry or flow conditions through a graphical user interface and the new conditions are calculated immediately. Various graphical formats present the variation of flow results to the student. One such format leads the student to the generation of some of the graphs found in NACA-1135. The tool includes interactive questions and answers to aid in both the use of the tool and to develop an understanding of some of the complexities of compressible aerodynamics. A series of help screens make the simulator easy to learn and use. This paper will detail the numerical methods used in the tool and describe how it can be used and modified.

INTRODUCTION

The last decade has witnessed a revolution in the availability, computing power, and memory capacity of personal computers and workstations. Current desk top workstations provide computing power available only on large mainframes in the past. Coupling this hardware growth with new operating systems, windowing capability, and the development of graphical user interfaces (GUI) creates an entirely new computing environment. Interestingly, it is an environment in which young students feel quite at home. Having grown up with video games and personal computers at school or in the home, undergraduates are seldom intimidated by the new technologies. The challenge for educators is to learn, develop and apply the new technology to the traditional problems of fundamental understanding. Some personal computer based software have been developed recently, Refs. 1-3, to assist in the teaching of fluid mechanics and propulsion. The simulator described in this paper is a first attempt at exploiting the power of workstation computer graphics for strictly educational purposes.

The current tool was derived from a supersonic inlet design simulator, Ref. 4, under development at NASA-Lewis. The inlet tool uses the equations from NACA-1135, Ref 5, to quickly solve for the flow conditions across shock waves generated by multiple compression surfaces to determine inlet performance. It also solves for the drag of the inlet, and includes a model for the subsonic diffuser, and the bleed/bypass system. The inlet designer can vary the geometry, free stream conditions, or engine airflow requirements and evaluate changes in the inlet performance parameters. It became evident during development of the inlet simulator that it could be used either for preliminary design or for graduate level education since it modeled most of the important physical processes present in high speed inlets. However the many processes involved in a typical inlet rendered the tool too complex for undergraduate education. By limiting the problem to flow past a single wedge, the single physical process of flow compression (or expansion) could be highlighted. Since much of the coding was already available, the development of a simpler educational tool was initiated.

A guiding principal in the development of this...
simulator is to involve the student in the learning process; to make the student work with the package to achieve a result, not just present the answer to a problem. Some features were added to this tool which are not present in the inlet simulator to promote the student interaction. What has evolved from this work is the creation of a desk top wind tunnel. The student is presented with a physics problem in one view window. In another window is a control panel which the student uses to vary the conditions of the physics problem and in a third window raw output data from the problem is presented to the student. The student can then select which data to analyze in another window, record the data through interaction with the tool, and plot the data on a view screen. A series of “Help” screens are available to explain how each part works.

The desk top wind tunnel concept could be used in many areas of undergraduate education. The current tool can be rather easily modified to consider alternate physics problems, such as a supersonic nozzle, a sting-mounted wedge at angle of attack, a diamond airfoil, or basic shock intersection or reflection problems. The basics of the code can be retained and other classes of fluids problems coded, including potential flow, conformal mapping, and basic airfoil theory. Many basic thermodynamics, magnetics, electrostatics and optics problems also lend themselves to presentation in this way.

ANALYSIS

The inviscid, supersonic flow past a wedge results in one of three possible physical processes; (1) a normal shock detached from the wedge at large positive wedge angles, (2) an oblique shock attached to the wedge at low to moderate positive angles and (3) a Prandtl-Meyer centered expansion for negative angles. The boundary between attached and detached shock waves can be determined as a function of the upstream Mach number. The equations used in the tool are taken from Ref. 5 for each of the flow conditions.

Normal Shock

\[
\frac{\rho_1}{\rho_0} = \frac{(\gamma + 1)M^2}{(\gamma - 1)M^2 + 2} \quad (1)
\]

\[
\xi = \frac{p_1}{p_0} = \frac{2\gamma M^2 - (\gamma - 1)}{(\gamma + 1)} \quad (2)
\]

\[
M_1^2 = \frac{(\gamma - 1)\xi + (\gamma + 1)}{2\gamma \xi} \quad (3)
\]

\[
T_1 \quad \frac{T_0}{T_0} = \frac{\xi (\gamma - 1)\xi + (\gamma + 1)}{(\gamma + 1)\xi + (\gamma - 1)} \quad (4)
\]

\[
\frac{p_1}{p_0} = \left[\frac{(\gamma + 1)\xi + (\gamma - 1)}{(\gamma - 1)\xi + (\gamma + 1)}\right]^{\frac{\gamma + 1}{\gamma - 1}} \quad (5)
\]

where \(M\) is the upstream Mach number, \(M_1\) is the downstream Mach number, \(\gamma\) is the ratio of specific heats, \(\rho\) is the density, \(p\) is the static pressure, \(T\) the static temperature, \(p_t\) the total pressure, with the “0” conditions upstream of the shock and the “1” conditions downstream. For a given upstream Mach number, Eqs. (1-5) can be easily solved.

Oblique Shock

\[
\cot \delta = \tan \theta \left[\frac{(\gamma + 1)M^2}{2(M^2\sin^2\theta - 1)} - 1\right] \quad (6)
\]

\[
\frac{\rho_1}{\rho_0} = \frac{(\gamma + 1)M^2\sin^2\theta}{(\gamma - 1)M^2\sin^2\theta + 2} \quad (7)
\]

\[
\xi = \frac{p_1}{p_0} = \frac{2\gamma M^2\sin^2\theta - (\gamma - 1)}{(\gamma + 1)} \quad (8)
\]

\[
M_1^2 = \frac{M^2[(\gamma + 1)\xi + (\gamma - 1)] - 2(\xi^2 - 1)}{\xi[(\gamma - 1)\xi + (\gamma + 1)]} \quad (9)
\]

\[
T_1 \quad \frac{T_0}{T_0} = \frac{\xi (\gamma - 1)\xi + (\gamma + 1)}{(\gamma + 1)\xi + (\gamma - 1)} \quad (10)
\]

\[
\frac{p_1}{p_0} = \left[\frac{(\gamma + 1)\xi + (\gamma - 1)}{(\gamma - 1)\xi + (\gamma + 1)}\right]^{\frac{\gamma + 1}{\gamma - 1}} \quad (11)
\]

where \(\delta\) is the wedge angle and \(\theta\) is the shock angle. With the upstream Mach number and wedge angle given, Eq. (6) is solved iteratively for the shock angle. Knowing the shock angle, Eqs. (7-11) can be easily solved for the flow conditions across the shock.

Centered Prandtl-Meyer Expansion
For negative wedge angles, knowing the upstream Mach number determines the value of the Prandtl-Meyer function as given in Ref. 6:

\[
\nu = \sqrt{\frac{\gamma + 1}{\gamma - 1}} \tan^{-1} \sqrt{\frac{(\gamma - 1)(M^2 - 1)}{\gamma + 1}} - \tan^{-1} \sqrt{M^2 - 1} \quad (12)
\]

The wedge deflection angle is then added to the Prandtl-Meyer angle \( \nu \) to give a new downstream value of the Prandtl-Meyer function:

\[
\nu_1 = \nu_0 + \delta \quad (13)
\]

With the new value of \( \nu \), Eq. (12) is solved iteratively for the downstream Mach number. Since the expansion is isentropic, the total pressure ratio is one and the downstream flow conditions can then be determined by the isentropic relations:

\[
\frac{T_1}{T_0} = \frac{\frac{1 + \frac{\gamma - 1}{2} M_0^2}{1 + \frac{\gamma - 1}{2} M_1^2}} {\frac{1 + \frac{\gamma - 1}{2} M_0^2}{1 + \frac{\gamma - 1}{2} M_1^2}} \quad (14)
\]

\[
\frac{p_1}{p_0} = \frac{\frac{1 + \frac{\gamma - 1}{2} M_0^2}{1 + \frac{\gamma - 1}{2} M_1^2}} {\frac{1 + \frac{\gamma - 1}{2} M_0^2}{1 + \frac{\gamma - 1}{2} M_1^2}} \quad (15)
\]

\[
\frac{\rho_1}{\rho_0} = \left( \frac{p_1}{p_0} \right)^{\frac{1}{\gamma}} \quad (16)
\]

In summary, the simulator checks the wedge angle to determine if it is positive or negative. If the wedge angle is negative, Eqs (12-16) are used to compute the downstream flow conditions. If the angle is positive, the simulator must determine if the shock has detached. The equation defining this condition is derived from Eq. (6). For a given Mach number, the equation is differentiated with respect to \( \theta \) to find the conditions for which \( \frac{\partial \theta}{\partial \theta} \) is equal to zero. This produces the following relationship:

\[
2\gamma M^4 \sin^4 \theta_{\text{max}} - [(\gamma + 1) M^4 - 4M^2] \sin^2 \theta_{\text{max}} - (\gamma + 1) M^2 + 2 = 0 \quad (17)
\]

This quadratic equation is solved for the value of \( \theta_{\text{max}} \), the maximum shock angle for which the shock wave is oblique. The corresponding maximum wedge angle can be determined from Eq. (6). The simulator compares this value to the chosen wedge angle; as long as the chosen angle is smaller, Eqs (6-11) are used for flow conditions, if the chosen angle is larger, Eqs (1-5) are used.

**DESCRIPTION OF SIMULATOR**

Some examples of the results obtained using the simulator are given in Figs. 1-6 which are screen dumps from the latest version. Fig. 1 shows the basic layout of the simulator which is divided into four main sections: the main view window is in the upper left, the plotter view window is in the upper right, the output box is in the lower right, and the input box is in the lower left.

The main view window shows the geometry, the shock (or expansion), and labels for the hinge and the wedges. On the workstation, these features are color-coded, but are presented here in black and white. The wedge appears as the lower nearly horizontal line with the small semi-circle denoting the hinge location. The upstream flat portion is tagged "0" while the movable wedge is tagged "1". The shock appears as the upper nearly diagonal line originating at the hinge location and flow is from left to right above the wedge surface. Sliders are located to the left and below the main view window to allow repositioning of the flow problem within the window. Fig. 4 shows an example of moving the problem up in the view window.

The plotter view window is located to the right of the main view window and displays the plots generated by the student. The student can select which sets of variables to plot using the input box as described below. The size and orientation of the plot can be modified by the sliders located around the plotter view window. Details of the generation of plots is discussed below.

Above the plotter window are four buttons which invoke various utilities for the simulator. Pushing the "Help" button on the background screen invokes the Help window shown in Fig. 2. Most of the information in the help screens pertain to how the simulator works, however the group marked "Hints" is intended for providing students with information concerning compressible aerodynamics. This help screen will point out various experiences of users, and interesting flow problems to duplicate. Anything in any of the help screens can be easily modified by the owner, since the help screens merely display ASCII data sets which accompany the package.
and which can be edited by the owner. The “Output” button on the background screen will freeze all of the other screens so that graphical output can be made using the “Snapshot” utility of the workstation. There is no printed output from this package. The “Redraw” and “Exit” buttons are self explanatory.

The computed output flow conditions are displayed in the output box below the plotter view window. This data is displayed in two ways; numerically in the row of boxes at the left, and as bar charts to the right. Each bar is a different color corresponding to a different flow variable. The top bar shows the shock angle in degrees, the bar below that shows the total pressure ratio from zero at the left to one on the right. The bottom four bars are grouped somewhat because the scale on these four bars are from zero to ten. Numbers at the bottom of the panel aid the student in evaluating the bars, though the exact value can also be obtained from the numerical boxes. As the flow conditions are changed in the input box, the recalculated numbers are displayed and the bar charts move much like a thermometer. This type of visual output allows the student to immediately sense in what direction the flow variables change and by how much for a given input. The zone choice button at the top has been included for later, more complex flow problems but is always set to one for the single wedge problem.

The input box is located to the left of the output box. It includes some buttons to select a problem for study and four sub-panels to vary conditions in the problem. Provisions have been made in the simulator to eventually solve for the flow past a single wedge, double wedges, and opposed wedges. In this paper we will consider only the single wedge problem, as indicated by the darkened “light” on the button in Fig. 1, and only three of the four sub-panels are required for input. The first sub-panel controls geometry and free stream flow conditions. The student can type in values of the ratio of specific heats, the wedge angle and the upstream Mach number, or can use the sliders to pick wedge angle and Mach number. The choice is indicated by pushing the “Sliders/Enter” buttons between the slider and the input box; Figs. 4-6 show the use of the “Enter” style of input. The lower two sub-panels are used to control the plotting of data. Two sub-panels are used to allow flexibility in the types of plots one can generate for different physics problems.

RESULTS

The simulator can be operated in two modes; interactive mode and plotting mode. In the interactive mode, the student can vary the Mach number and wedge angle using the input box and monitor the changes of the dependent flow variables using the output box. The student can determine the value of all dependent variables for a set of conditions using the output numbers, and get a sense of the variation of the dependent variables from the bar charts as independent conditions are changed. To study the variation of a single flow variable with either a change in Mach number or wedge angle, the simulator should be run in plotting mode. In plotting mode, the student fixes either the Mach number or wedge angle, varies the other parameter and records some flow variable in the plotter window as described below.

Figures 1-6 can be used again to demonstrate how a student might use the interactive mode of the simulator. As the input is varied the simulator recalculates the flow conditions and instantly changes the geometry and shock orientation to reflect the new conditions. In Fig. 1, Mach 3.0 flow past a 10.0 degree wedge has been calculated. The results presented in the output box have been checked against the NACA-1135 curves to ensure proper coding. Between Fig.1 and Fig.3 the wedge angle has been increased to 20.96 degrees. The output box reflects the increase in shock angle, static pressure, temperature and density ratio and the decrease in total pressure ratio and downstream Mach number associated with the stronger shock. In Fig. 4 a negative wedge angle has been chosen using the input box. An expansion fan is then displayed in the main view window; to the right of the hinge the lower line is the wedge surface, the top line is the beginning and the middle line the end of the expansion fan. On the actual simulator these lines are color-coded for easier recognition. The output box indicates that the flow has accelerated from Mach 2.0 to Mach 3.37, the total pressure ratio is 1.0 because the flow is isentropic, and the pressure, density, and temperature ratios are less than 1.0 because the flow is expanded. The “Shock” value is equal to the Mach angle for the incoming flow. In Fig. 5, the wedge angle exceeds the detachment angle for Mach = 2.0 and the
normal shock is drawn in the main view window as a vertical line. In reality, the normal shock would be perpendicular to the incoming flow near the wall, displaced some distance upstream of the hinge, and then curve back far from the hinge. In the simulator, the vertical line is colored different than an oblique shock to alert the student. The output box also shows the shock angle at 90.00 degrees, and the downstream flow to be subsonic. Because the simulator solves the normal shock equations for detached shocks, a large wedge angle can be used to study the variation of flow properties with Mach number for normal shocks. In Fig. 3, the Mach number has been reduced to 1.6 while the wedge angle remains large. Compared to Fig. 5, the total pressure ratio and downstream Mach number have increased while the static pressure, density, and temperature ratios have decreased. The curve in the plotter view window of Fig. 6 shows the variation in total pressure ratio with Mach number for a normal shock.

In plotting mode, the simulator behaves like a desk top wind tunnel. The plotter is invoked by pushing the "ON" button in the left sub-panel of the input box. The user then pushes the "New Plot" button which enables the choice buttons in the right sub-panel marked "Variables". The student chooses a dependent variable from the upper group of variables and an independent variable from the lower group by pushing the appropriate buttons. As the choices are made, the axes in the plotter view window are automatically labeled and scaled. In Fig. 3, the user has chosen to plot static pressure ratio versus wedge angle as indicated by the lights on the buttons. The Mach number then becomes a parameter for generating curves. The student sets a value of Mach number and pushes the "Begin Trace" button which freezes the value of the Mach number parameter. The student then varies the wedge angle to any desired value and presses the "Take Data" button. At this point a "*" appears on the graph corresponding to the chosen value of independent variable (wedge angle) and calculated value of dependent variable (pressure ratio). The student then uses the input sub-panel to select a new wedge angle and again "Take Data". A new point appears on the plot and the procedure is repeated to a maximum of twenty five data points. The data can be taken in any order, so the student can fill in interesting portions of the curve. In Fig. 3, six data points have been taken. When the student has completed a trace, the "End Trace" button is pushed which frees the Mach number parameter, draws a solid color coded line through the data, and affixes a colored label with the value of the parameter. The student can then choose a new value of the parameter and begin a new trace as before. Fig. 4 shows the screen dump after the student has begun a new trace at Mach number equal to 2.0. In this example, ten data points have been taken along the new trace, and the old trace has been labeled in the upper right corner of the plotter window. The student can put up to five traces on the plotter. To begin a new plot, the student pushes the "New Plot" button, as shown in Fig. 5, the old plots are erased, the count boxes for traces and data are reset to zero, and the "Variables" buttons can be used to pick new independent and dependent variables. In Fig. 5, the student has moved on to total pressure ratio versus Mach number.

No educational tool would be complete without an examination; a question and answer box has been added at the lower right corner as shown in Fig. 5. The questions and answers are stored in a data set which the simulator accesses. Teachers (or for that matter students!) can edit this data set and add, modify, or delete questions and answers as required. The questions can appear in the window either sequentially or randomly as chosen by the student with the appropriate buttons. The current sequential set of questions and answers have been chosen to promote an interaction of the student with the simulator and to bring out several important topics regarding compressible aerodynamics. To use this feature the student pushes the "Question" button for a question, then uses the simulator to obtain an answer, then presses the "Answer" button to check the answer. As currently configured, the simulator runs through the same questions every time the simulator is invoked. This portion of tool could be modified to present questions from different data sets or even questions with a timer but without the provided answers - a true examination tool. The particular example in Fig. 5 shows a flow condition requested in the question box and the answer appears at the bottom of Fig. 6.

SUMMARY

A workstation-based, highly interactive educa-
tional tool for compressible aerodynamics has been developed. The underlying assumptions and equations which form the basis for this tool have been presented in this paper as well as several examples of the results from the simulator. The tool employs two basic ideas to help undergraduates better understand fluid mechanics. First, the student is required to interact with the tool to produce results. The student is in control of the parameters defining the problem and must perform several steps to produce the plotted output. The question and answer box also requires the student to act and not merely observe. Second, the output results are presented to the student in a variety of forms. For a given flow problem the student can see a schematic of the geometry, the shock location and motions, tabulated numbers of the dependent flow variables, moving bar charts of the flow variables, and plotted data. During the development of the tool, some new ideas have been explored, particularly the desk top wind tunnel. The tool is currently being extended to solve double wedge and opposed wedge compressible flow problems. Further extensions of this technology into other flow regimes and other physics problems are planned.

ACKNOWLEDGEMENTS

The current simulator uses the FORMS library of GUI developed by Mark H. Overmars, Department of Computer Science, Utrecht University, The Netherlands for Silicon Graphics workstations. Some modifications to this library have been made to allow the package to also be used on IBM Risc 6000 machines. All of this software is public domain and may be copied and used for non-commercial products, but not resold. Copies of the source are available from the author at the NASA Lewis Research Center.

REFERENCES

**Fig. 1** Simulator screen dump.
This simulator calculates the inviscid, supersonic flow along a compression or expansion surface using the equations found in NACA 1135.

It makes extensive use of the FORMS GUI Library Version 2.1, developed by Mark Overmars, Utrecht University, the Netherlands.

Any suggestions, corrections, bugs, experiences (good or bad), should be sent to:

Tom Benson
NASA Lewis Research Center, MS 5-11
Cleveland, Ohio 44135

Fig. 2 Help screen
Fig. 3 Acquiring data using the plotter.
Fig. 4 Plot completed.
**Fig. 5** Question and answer box.
Fig. 6 Normal shock relations.

A 3: 90.00 degrees. Oblique shock has detached and became a normal shock.
**4. TITLE AND SUBTITLE**

An Interactive Educational Tool for Compressible Aerodynamics

**6. AUTHOR(S)**

Thomas J. Benson

**7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)**

National Aeronautics and Space Administration  
Lewis Research Center  
Cleveland, Ohio 44135–3191

**9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)**

National Aeronautics and Space Administration  
Washington, D.C. 20546–0001

**11. SUPPLEMENTARY NOTES**

Responsible person, Thomas J. Benson, organization code 2670, (216) 433–5920.

**13. ABSTRACT (Maximum 200 words)**

A workstation-based interactive educational tool has been developed to aid in the teaching of undergraduate compressible aerodynamics. The tool solves for the supersonic flow past a wedge using the equations found in NACA 1135. The student varies the geometry or flow conditions through a graphical user interface and the new conditions are calculated immediately. Various graphical formats present the variation of flow results to the student. One such format leads the student to the generation of some of the graphs found in NACA-1135. The tool includes interactive questions and answers to aid in both the use of the tool and to develop an understanding of some of the complexities of compressible aerodynamics. A series of help screens make the simulator easy to learn and use. This paper will detail the numerical methods used in the tool and describe how it can be used and modified.

**14. SUBJECT TERMS**

Educational; Software; Compressible Fluids