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Mechanical and Thermal Buckling Analysis of Rectangular Sandwich Panels Under Different Edge Conditions

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CONTENTS

1. ABSTRACT

The combined load (mechanical or thermal load) buckling equations were established for orthotropic rectangular sandwich panels under four different edge conditions by using the Rayleigh-Ritz method of minimizing the total potential energy of a structural system. Two-dimensional buckling interaction curves and three-dimensional buckling interaction surfaces were constructed for high-temperature honeycomb-core sandwich panels supported under four different edge conditions. The interaction surfaces provide overall comparison of the panel buckling strengths and the domains of symmetrical and antisymmetrical buckling associated with the different edge conditions. In addition, thermal buckling curves of these sandwich panels are presented. The thermal buckling conditions for the cases with and without thermal moments were found to be identical for the small deformation theory.

In sandwich panels, the effect of transverse shear is quite large, and by neglecting the transverse shear effect, the buckling loads could be overpredicted considerably. Clamping the edges could greatly increase buckling strength, more in compression than in shear.

2. NOMENCLATURE

$A_{mn}, A_{k\ell}$	Fourier coefficients of trial function for w, in.
$\bar{A}_{ij}$	extensional stiffnesses of sandwich panel, lb/in.
a	length of sandwich panel, in.
a_o	edge length of square sandwich panel, in.
a^{ij}_{mnkl}	coefficients of characteristic equations
$B_{mn}, B_{k\ell}$	Fourier coefficients of trial function for γ_{xz}, in/in.
b	width of sandwich panel, in.
$C_{mn}, C_{k\ell}$	Fourier coefficients of trial function for γ_{yz}, in/in.
D_{ij}	bending stiffnesses of sandwich panel, in–lb
D_{Qx}, D_{Qy}	transverse shear stiffnesses in xz, yz planes, lb/in.
D^*	flexural stiffness parameters, $\sqrt{D_{11}D_{22}}$, in–lb
dx, dy	differentials of x and y, in.
E_x, E_y	Young's moduli of face sheets, lb/in^2
E_{Cx}, E_{Cy}, E_{Cx}	effective Young's moduli of honeycomb core, lb/in^2
F_{mn}	Fourier coefficients for M_x^T, (in-lb)/in
$G_{Cxy}, G_{Cxz}, G_{Cyz}$	effective shear moduli of honeycomb core, lb/in^2
G_{xy}	shear modulus of face sheets, lb/in^2
H_{mn}	Fourier coefficients for M_y^T, (in-lb)/in.
h	depth of sandwich panel = distance between middle plane of two face sheets, in.
h_c	depth of honeycomb core, $h_c = h - t_s$, in.
I_s	moment of inertia, per unit width, of a face sheet taken with respect to horizontal centroidal axis of the sandwich panel, $I_s = \frac{1}{4}t_s h^2 + \frac{1}{12}t_s^3$, in^4/in.

i	index, 1, 2, 3, $\cdots$
j	index, 1, 2, 3, $\cdots$
k	index, 1, 2, 3, $\cdots$
k_x, k_y	compressive buckling load factors in x- and y-directions, $k_x = \frac{N_x a^2}{\pi^2 D^*}$, $k_y = \frac{N_y a^2}{\pi^2 D^*}$, for a = constant
k_{xy}	shear buckling factor, $k_{xy} = \frac{N_{xy} a^2}{\pi^2 D^*}$, for a = constant
$\bar{k}_x, \bar{k}_y$	modified compressive buckling load factors in x- and y-directions, $\bar{k}_x = \frac{N_x a_o^2}{\pi^2 D^*} = k_x \frac{b}{a}$, $\bar{k}_y = \frac{N_y a_o^2}{\pi^2 D^*} = k_y \frac{b}{a}$, for $ab = a_o^2$ = constant
$\bar{k}_{xy}$	modified shear buckling load factor, $\bar{k}_{xy} = \frac{N_{xy} a_o^2}{\pi^2 D^*} = k_{xy} \frac{b}{a}$, for $ab = a_o^2$ = constant
ℓ	index, 1, 2, 3, $\cdots$
M_x, M_y	bending moment intensities, (in-lb)/in.
M_{xy}	twisting moment intensity, (in-lb)/in.
M_x^T, M_y^T, M_{xy}^T	thermal moments, (in-lb)/in.
m	number of buckle half waves in x-direction
N_x, N_y	normal stress resultants, lb/in.
N_{xy}	shear stress resultant, lb/in.
N_x^T, N_y^T, N_{xy}^T	thermal forces, lb/in.
n	number of buckle half waves in y-direction
Q_x, Q_y	transverse shear force intensities, lb/in.
S_{mn}	Fourier coefficients for M_{xy}^T, (in-lb)/in.
T	temperature, °F
T_a	assumed temperature, °F
T_{cr}	critical buckling temperature, °F
t_s	thickness of sandwich face sheets, in.
V	total potential energy of sandwich panel, in-lb
V_1	strain energy of sandwich panel, in-lb
V_2	work done by external forces, in-lb
ΔV_1	component of V_1 associated with a particular indicial condition, in-lb
ΔV_2	component of V_2 associated with a particular indicial condition, in-lb
ΔV	component of V, $\Delta V = \Delta V_1 + \Delta V_2$, in-lb
u, v, w	displacement components in x-, y-, and z-direction, in.
x, y, z	rectangular Cartesian coordinates
$\alpha_x, \alpha_y, \alpha_{xy}$	coefficients of thermal expansion, in/in-°F
γ_{xz}, γ_{yz}	transverse shear strains in xz- and yz-plane, in/in.
ζ	numerical coefficient of N_y^T in a_{mnkl}^{11}
η	numerical factor in buckling equation, which changes with the edge condition
ξ	numerical coefficient of N_x^T in a_{mnkl}^{11}

ν_{xy}, ν_{yz}	Poisson ratios of face sheets, also used for those of sandwich panel
$\nu_{Cxy}, \nu_{Cyz}, \nu_{Cxz}$	Poisson ratios of honeycomb core
ρ_{Ti}	specific weight of titanium material, lb/in^3
ρ_{Hc}	specific weight of titanium honeycomb core, lb/in^3

3. INTRODUCTION

Structural components of hypersonic flight vehicles (e.g., spacecraft, rockets, reentry vehicles, hypersonic aircraft, etc.) are subjected to hyper-thermal loadings due to hostile aerodynamic heating during ascent and reentry, or due to solar radiation during spaceflights. The structural components of those vehicles have to operate at elevated temperatures and are, therefore, called hot structures. Because of nonuniform heating (which is magnified by the cooler substructural frames which act as heat sinks) and the mechanical structural constraints, severe thermal stresses could build up in those hot structures. Excess thermal loading may induce (1) material degradation, (2) thermal creep, (3) thermal yielding, (4) thermal buckling, (5) thermal crack fracture after cool-down, etc. Any disruption of surface smoothness of the structures (e.g., metallic thermal protection system (ref. 1), hypersonic aircraft engine inlet structures (refs. 2, 3), etc.) caused by the previously mentioned failure modes, especially thermal buckling, could disturb the flow field, creating hot spots which could cause serious consequences on the structures. Thus, the thermal load does play a key factor in the design of the hot structures. Reference 1 discusses various design concepts of hot and cryogenic structural components for the hypersonic flight vehicles. The potential candidates of high-buckling-strength-hot-structural panels (fabricated with super alloys) for hypersonic aircraft applications are tubular panels, beaded panels, truss-core sandwich panels, hat-stiffened panels, honeycomb-core sandwich panels, etc. (refs. 4, 5). The combined-load buckling behavior of the tubular panels was extensively studied by Ko et al. (ref. 4), theoretically and experimentally. The compressive buckling characteristics of the beaded panels were investigated by Siegel (ref. 5).

Recently Ko and Jackson (ref. 6), and Percy and Fields (ref. 7) studied the compressive buckling behavior of the hat-stiffened panel designed for application to a hypersonic aircraft fuselage skin panel. Furthermore, Ko and Jackson conducted simple analysis of thermal behavior (thermal buckling of a face sheet) of the honeycomb-core sandwich panel (ref. 8), and compared the relative combined-load buckling strengths of the truss-core and honeycomb-core sandwich panel (ref. 9). They also investigated the effect of fiber orientation of the metal-matrix face sheet on the combined-load buckling strength of honeycomb-core sandwich panels (refs. 10, 11). Most of the past mechanical buckling analyses of the sandwich panels (refs. 4–7, refs. 9–12) and flat plates (refs. 13, 14) were conducted for simply supported edge conditions because the analysis was mathematically less involved. For the case of clamped edge conditions, one can cite the work by Green and Hearmon (ref. 15), who studied combined loading stability of plywood plates, and Smith (ref. 16), who considered only pure shear buckling of the plywood plates. Kuenzi, Erickson, and Zahn (ref. 17) considered also shear stability of flat panels of sandwich construction. The works cited here ignored the transverse shear effect in their analyses. King (ref. 18) analyzed the stability of clamped rectangular sandwich plates subjected to in-plane combined loadings, taking into account the rotational effect of the sandwich core. He used a less-compact displacement function (which could be reduced to a simpler Green and Hearmon displacement function (ref. 15)), resulting in a complicated expression for the potential energy of the sandwich system. Most of the past thermal buckling analysis was done on single plates (refs. 19–22) or laminated composite plates (refs. 23–27), for which the transverse shear effect may be neglected. In actual application of the hot structural panels, most panel boundary conditions are closer to the clamped edges rather than to the simply supported edges. Therefore, this report will consider the combined-load mechanical and thermal buckling of sandwich panels under different types of edge conditions by taking into account the transverse shear effect. The report also compares the buckling interaction curves and surfaces for different edge conditions.

4. DESCRIPTION OF THE PROBLEM

Figure 1 shows the geometry of a rectangular honeycomb-core sandwich panel having identical face sheets. The extensional and bending stiffnesses of the sandwich panel will be provided by the two face sheets only, and the transverse shear stiffnesses by the honeycomb core only.

This type of sandwich panel, when fabricated with a high-temperature alloy (e.g., titanium), becomes the so-called hot structure, and could be a potential candidate for hypersonic aircraft structural applications (ref. 1). Figure 2 shows the sandwich panel subjected to combined compressive and shear loadings in its middle plane. The conventional Rayleigh-Ritz method of minimizing the panel's total potential energy will be used in the combined-load buckling analysis, accounting for the transverse shear effect (fig. 3). The sandwich panel will be supported under four different edge conditions (fig. 4)

- Case 1: Four edges simply supported (4S edge condition),
- Case 2: Four edges clamped (4C edge condition),
- Case 3: Two sides clamped, two ends simply supported (2C2S edge condition), and
- Case 4: Two sides simply supported, two ends clamped (2S2C edge condition)

where sides and ends are parallel to x and y axes respectively.

The problem is to study the effects of the panel edge condition and the panel aspect ratio on the combined-load buckling behavior of the sandwich panel. Case 1 has already been solved and has been published in reference 9. However, for completeness, some key equations for Case 1 will be repeated in this report.

5. GOVERNING EQUATIONS

5.1 Constitutive Equations

Following the classic orthotropic thick plate theory, which accounts for the transverse shear effect, the membrane force intensities $\{N_x, N_y, N_{xy}\}$, the transverse shear force intensities $\{Q_x, Q_y\}$, and the moment intensities $\{M_x, M_y, M_{xy}\}$ in an orthotropic sandwich panel may be related to the middle surface displacement components $\{u, v, w\}$, the transverse shear strains $\{\gamma_{xz}, \gamma_{yz}\}$, thermal forces $\{N_x^T, N_y^T, N_{xy}^T\}$, and thermal moments $\{M_x^T, M_y^T, M_{xy}^T\}$ through the following constitutive equations (fig. 5)

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} = \begin{bmatrix} \bar{A}_{11} & \bar{A}_{12} & 0 \\ \bar{A}_{21} & \bar{A}_{22} & 0 \\ 0 & 0 & \bar{A}_{66} \end{bmatrix} \begin{bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial u}{\partial y} \\ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \end{bmatrix} - \begin{bmatrix} N_x^T \\ N_y^T \\ N_{xy}^T \end{bmatrix} \tag{1}$$

$$\begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} D_{11} & D_{12} & 0 \\ D_{21} & D_{22} & 0 \\ 0 & 0 & D_{66} \end{bmatrix} \begin{bmatrix} -\frac{\partial}{\partial x}\left(\frac{\partial w}{\partial x} - \gamma_{xz}\right) \\ -\frac{\partial}{\partial y}\left(\frac{\partial w}{\partial y} - \gamma_{yz}\right) \\ -\frac{\partial}{\partial y}\left(\frac{\partial w}{\partial x} - \gamma_{xz}\right) - \frac{\partial}{\partial x}\left(\frac{\partial w}{\partial y} - \gamma_{yz}\right) \end{bmatrix} - \begin{bmatrix} M_x^T \\ M_y^T \\ M_{xy}^T \end{bmatrix} \tag{2}$$

$$\begin{bmatrix} Q_x \\ Q_y \end{bmatrix} = \begin{bmatrix} D_{Qx} & 0 \\ 0 & D_{Qy} \end{bmatrix} \begin{bmatrix} \gamma_{xz} \\ \gamma_{yz} \end{bmatrix} \tag{3}$$

For the sandwich panel whose extensional and bending stiffnesses are provided only by the two identical face sheets, and the transverse shear stiffnesses only by the honeycomb core, the extensional and the bending stiffness $\{\bar{A}_{ij}, D_{ij}\}$ in equations (1) and (2), and the transverse shear stiffnesses $\{D_{Qx}, D_{Qy}\}$ in equation (3) may be written as

$$\begin{bmatrix} \bar{A}_{11} & , & D_{11} \\ \bar{A}_{12} & , & D_{12} \\ \bar{A}_{21} & , & D_{21} \\ \bar{A}_{22} & , & D_{22} \\ \bar{A}_{66} & , & D_{66} \end{bmatrix} = [2t_s,\ 2I_s] \begin{bmatrix} \dfrac{E_x}{1-\nu_{xy}\nu_{yx}} \\ \dfrac{\nu_{yx}E_x}{1-\nu_{xy}\nu_{yx}} \\ \dfrac{\nu_{xy}E_y}{1-\nu_{xy}\nu_{yx}} \\ \dfrac{E_y}{1-\nu_{xy}\nu_{yx}} \\ G_{xy} \end{bmatrix} \tag{4}$$

$$\begin{bmatrix} D_{Qx} \\ D_{Qy} \end{bmatrix} = [h_c] \begin{bmatrix} G_{Cxz} \\ G_{Cyz} \end{bmatrix} \tag{5}$$

In equations (4) and (5), $\{E_x, E_y, G_{xy}, \nu_{xy}, \nu_{yx}\}$ are the elastic constants of the face sheets, $\{G_{Cxz}, G_{Cyz}\}$ are the effective transverse shear moduli of the sandwich core, t_s is the face sheet thickness, h_c is the sandwich core depth, and I_s is the moment of inertia of each face sheet taken with respect to horizontal centroidal axis, given by

$$I_s = \frac{1}{4}t_s h^2 + \frac{1}{12}t_s^3 \tag{6}$$

where h is the depth of the sandwich panel (fig. 1). The 2 in front of $\{t_s, I_s\}$ in equation (4) is associated with two identical face sheets.

The thermal forces $\{N_x^T, N_y^T, N_{xy}^T\}$ and the thermal moments $\{M_x^T, M_y^T, M_{xy}^T\}$ appearing in equations (1) and (2) are defined by

$$\begin{bmatrix} N_x^T & , & M_x^T \\ N_y^T & , & M_y^T \\ N_{xy}^T & , & M_{xy}^T \end{bmatrix} = \sum_{i=1}^{2} \left\{ \left[t_s\, T_i,\ (-1)^i \frac{t_s h}{2}\, T_i \right] \begin{bmatrix} \dfrac{E_x}{1-\nu_{xy}\nu_{yx}} & \dfrac{\nu_{yx}E_x}{1-\nu_{xy}\nu_{yx}} & 0 \\ \dfrac{\nu_{xy}E_y}{1-\nu_{xy}\nu_{yx}} & \dfrac{E_y}{1-\nu_{xy}\nu_{yx}} & 0 \\ 0 & 0 & G_{xy} \end{bmatrix}_i \begin{bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{bmatrix}_i \right\} \tag{7}$$

where $i = 1, 2$ are respectively associated with the lower and the upper face sheets, $\{\alpha_x, \alpha_y, \alpha_{xy}\}$ are the coefficients of thermal expansion of the face sheet material, and $[\ ]_i$ $(i = 1, 2)$ implies that the material properties are associated with temperature T_i $(i = 1, 2)$. The thermal force and thermal moment contributions from the honeycomb core were neglected.

5.2 Energy Equations

Based on the small deformation theory, the strain energy V_1 of the heated sandwich panel may be written as (refs. 23, 24, 26, 27)

$$
\begin{aligned}
V_1 = \int_o^a \int_o^b \Bigg\{ & \frac{A_{11}}{2}\left(\frac{\partial u}{\partial x}\right)^2 + A_{12}\left(\frac{\partial u}{\partial x}\right)\left(\frac{\partial v}{\partial y}\right) + \frac{A_{22}}{2}\left(\frac{\partial v}{\partial y}\right)^2 + \frac{A_{66}}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2 \\
& + \frac{D_{11}}{2}\left[\frac{\partial}{\partial x}\left(\frac{\partial w}{\partial x} - \gamma_{xz}\right)\right]^2 + D_{12}\left[\frac{\partial}{\partial x}\left(\frac{\partial w}{\partial x} - \gamma_{xz}\right)\right]\left[\frac{\partial}{\partial y}\left(\frac{\partial w}{\partial y} - \gamma_{yz}\right)\right] \\
& + \frac{D_{22}}{2}\left[\frac{\partial}{\partial y}\left(\frac{\partial w}{\partial y} - \gamma_{yz}\right)\right]^2 + \frac{D_{66}}{2}\left[\frac{\partial}{\partial y}\left(\frac{\partial w}{\partial x} - \gamma_{xz}\right) + \frac{\partial}{\partial x}\left(\frac{\partial w}{\partial y} - \gamma_{yz}\right)\right]^2 \\
& + \frac{D_{Qx}}{2}\gamma_{xz}^2 + \frac{D_{Qy}}{2}\gamma_{yz}^2 - N_x^T\left(\frac{\partial u}{\partial x}\right) - N_{xy}^T\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) - N_y^T\left(\frac{\partial v}{\partial y}\right) \\
& + M_x^T\left[\frac{\partial}{\partial x}\left(\frac{\partial w}{\partial x} - \gamma_{xz}\right)\right] + M_y^T\left[\frac{\partial}{\partial x}\left(\frac{\partial w}{\partial y} - \gamma_{yz}\right)\right] \\
& + M_{xy}^T\left[\frac{\partial}{\partial y}\left(\frac{\partial w}{\partial x} - \gamma_{xz}\right) + \frac{\partial}{\partial x}\left(\frac{\partial w}{\partial y} - \gamma_{yz}\right)\right]\Bigg\} dxdy
\end{aligned}
\tag{8}
$$

For the buckling problem, the work done V_2 by the in-plane forces to produce transverse deflection is given by

$$
V_2 = \frac{1}{2}\int_o^a \int_o^b \left[N_x\left(\frac{\partial w}{\partial x}\right)^2 + 2N_{xy}\left(\frac{\partial w}{\partial x}\right)\left(\frac{\partial w}{\partial y}\right) + N_y\left(\frac{\partial w}{\partial y}\right)^2\right] dxdy \tag{9}
$$

The total potential energy V of the sandwich panel is then

$$
V = V_1 + V_2 \tag{10}
$$

For pure mechanical buckling problems, the strain energy equation (8) reduces to

$$
\begin{aligned}
V_1 = \int_o^a \int_o^b \Bigg\{ & \frac{D_{11}}{2}\left[\frac{\partial}{\partial x}\left(\frac{\partial w}{\partial x} - \gamma_{xz}\right)\right]^2 + D_{12}\left[\frac{\partial}{\partial x}\left(\frac{\partial w}{\partial x} - \gamma_{xz}\right)\right]\left[\frac{\partial}{\partial y}\left(\frac{\partial w}{\partial y} - \gamma_{yz}\right)\right] \\
& + \frac{D_{22}}{2}\left[\frac{\partial}{\partial y}\left(\frac{\partial w}{\partial y} - \gamma_{yz}\right)\right]^2 + \frac{D_{66}}{2}\left[\frac{\partial}{\partial y}\left(\frac{\partial w}{\partial x} - \gamma_{xz}\right) + \frac{\partial}{\partial x}\left(\frac{\partial w}{\partial y} - \gamma_{yz}\right)\right]^2 \\
& + \frac{D_{Qx}}{2}\gamma_{xz}^2 + \frac{D_{Qy}}{2}\gamma_{yz}^2\Bigg\} dxdy
\end{aligned}
\tag{11}
$$

and the signs of the in-plane forces $\{N_x, N_y, N_{xy}\}$ in equation (9) are to be reversed because the loadings are in the negative direction according to the sign convention shown in fig. 5. Namely,

$$
N_x \rightarrow -N_x, \;\; N_x \rightarrow -N_y, \;\; N_{xy} \rightarrow -N_{xy} \tag{12}
$$

The sign for the shear force N_{xy} is immaterial because, in the plate buckling problems in which shear is present, the eigenvalues (or shear buckling loads) always occur in pairs which are equal in magnitude but opposite in sign.

For thermal buckling problems, if the sandwich panel is under uniform temperature for which $\{M_x^T, M_y^T, M_{xy}^T\} = 0$, (symmetrical temperature distribution in the panel depth direction will also produce zero thermal moments) and if the panel edges are restrained against lateral and in-plane displacements (i.e., $u = v = w = 0$), then the in-plane stress resultants $\{N_x, N_y, N_{xy}\}$ are uniform within the sandwich panel and according to equation (1) can be written as

$$N_x = -N_x^T, \ N_y = -N_y^T, \ N_{xy} = -N_{xy}^T \tag{13}$$

then, as will be discussed later, the thermal buckling problem is equivalent to the mechanical buckling problem if the second-order effect is neglected.

If there exists a temperature difference between the two face sheets, then the thermal moments $\{M_x^T, M_y^T, M_{xy}^T\}$ are no longer zero, and the problem is no longer an eigenvalue problem but a bending problem. As will be seen later, one can solve for $\{w, \gamma_{xz}, \gamma_{yz}\}$ in terms of the thermal moments, and the conditions for unbounded values of $\{w, \gamma_{xz}, \gamma_{yz}\}$ will give the buckling loads.

5.3 Panel Boundary Conditions

The sandwich panel is to be supported at its four edges under the following four cases of boundary (or edge) conditions.

For mechanical buckling:

Case 1. Four edges simply supported (4S edge condition)

$$x = 0, a: \ w = M_x = \gamma_{yz} = 0 \tag{14}$$
$$y = 0, b: \ w = M_y = \gamma_{xz} = 0 \tag{15}$$

Case 2. Four edges clamped (4C edge condition)

$$x = 0, a: \ w = \frac{\partial w}{\partial x} = \gamma_{xz} = \gamma_{yz} = 0 \tag{16}$$
$$y = 0, b: \ w = \frac{\partial w}{\partial y} = \gamma_{xz} = \gamma_{yz} = 0 \tag{17}$$

Case 3. Two sides clamped, two ends simply supported (2C2S edge condition)

$$x = 0, a: \ w = M_x = \gamma_{yz} = 0 \tag{18}$$
$$y = 0, b: \ w = \frac{\partial w}{\partial y} = \gamma_{xz} = \gamma_{yz} = 0 \tag{19}$$

Case 4. Two sides simply supported, two ends clamped (2S2C edge condition)

$$x = 0, a: \ w = \frac{\partial w}{\partial x} = \gamma_{xz} = \gamma_{yz} = 0 \tag{20}$$
$$y = 0, b: \ w = M_y = \gamma_{xz} = 0 \tag{21}$$

For thermal buckling:

In addition to the above boundary conditions, the following edge condition is to be imposed

$$x = 0, a: \ u = v = 0 \tag{22}$$
$$y = 0, b: \ u = v = 0 \tag{23}$$

6. BUCKLING ANALYSIS

The conventional Raleigh-Ritz method of minimization of total potential energy (V) will be used in the buckling analysis. To use this method, one has to assume deformation functions for the sandwich panel in infinite series forms containing unknown coefficients. By minimizing V with respect to each of those unknown coefficients, one will obtain a set of simultaneous homogeneous characteristic equations for eigenvalue solutions (buckling loads).

6.1 Panel Deformation Functions

For an eigenvalue solution via the Rayleigh-Ritz method, the trial functions for the sandwich panel deformation $\{w, \gamma_{xz}, \gamma_{yz}\}$, satisfying the boundary conditions (eqs. (14) through (21)), may be expressed in the following double Fourier series for different edge conditions.

Case 1. Four edges simply supported (4S edge condition) (ref. 9)

$$w(x,y) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} A_{mn} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b} \tag{24}$$

$$\gamma_{xz}(x,y) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} B_{mn} \cos\frac{m\pi x}{a} \sin\frac{n\pi y}{b} \tag{25}$$

$$\gamma_{yz}(x,y) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} C_{mn} \sin\frac{m\pi x}{a} \cos\frac{n\pi y}{b} \tag{26}$$

Case 2. Four edges clamped (4C edge condition) (ref. 15)

$$w(x,y) = \sin\frac{\pi x}{a} \sin\frac{\pi y}{b} \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} A_{mn} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b} \tag{27}$$

$$\begin{aligned} \gamma_{xz}(x,y) = {} & \cos\frac{\pi x}{a} \sin\frac{\pi y}{b} \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} B_{mn} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b} \\ & + \sin\frac{\pi x}{a} \sin\frac{\pi y}{b} \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} mB_{mn} \cos\frac{m\pi x}{a} \sin\frac{n\pi y}{b} \end{aligned} \tag{28}$$

$$\begin{aligned} \gamma_{yz}(x,y) = {} & \sin\frac{\pi x}{a} \cos\frac{\pi y}{b} \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} C_{mn} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b} \\ & + \sin\frac{\pi x}{a} \sin\frac{\pi y}{b} \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} nC_{mn} \sin\frac{m\pi x}{a} \cos\frac{n\pi y}{b} \end{aligned} \tag{29}$$

Case 3. Two sides clamped, two ends simply supported (2C2S edge condition) (ref. 15)

$$w(x,y) = \sin\frac{\pi y}{b} \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} A_{mn} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b} \tag{30}$$

$$\gamma_{xz}(x,y) = \sin\frac{\pi y}{b} \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} B_{mn} \cos\frac{m\pi x}{a} \sin\frac{n\pi y}{b} \tag{31}$$

$$\begin{aligned} \gamma_{yz}(x,y) = {} & \cos\frac{\pi y}{b} \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} C_{mn} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b} \\ & + \sin\frac{\pi y}{b} \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} nC_{mn} \sin\frac{m\pi x}{a} \cos\frac{n\pi y}{b} \end{aligned} \tag{32}$$

Case 4. Two sides simply supported, two ends clamped (2S2C edge condition) (ref. 15)

$$w(x,y) = \sin\frac{\pi x}{a}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} A_{mn}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b} \tag{33}$$

$$\begin{aligned}\gamma_{xz}(x,y) = {} & \cos\frac{\pi x}{a}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} B_{mn}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b} \\ & + \sin\frac{\pi x}{a}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} mB_{mn}\cos\frac{m\pi x}{a}\sin\frac{n\pi y}{b}\end{aligned} \tag{34}$$

$$\gamma_{yz}(x,y) = \sin\frac{\pi x}{a}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} C_{mn}\sin\frac{m\pi x}{a}\cos\frac{n\pi y}{b} \tag{35}$$

In equations (24) through (35), A_{mn}, B_{mn}, and C_{mn} are the undetermined Fourier coefficients of the assumed trial function for w, γ_{xz}, and γ_{yz}, respectively, and m and n are the buckle half wave numbers in the x and y directions.

6.2 Uniform Temperature (Zero Thermal Moments)

6.2.1 Mechanical and Thermal Bucklings

It is well known that thermal stress is not caused by external loads, but is the consequence of restrained thermal distortion. The intensity of thermal stress changes when the structure is deformed and, therefore, the thermal stress level is the function of strain. In the classical thermal buckling of a structural panel (under uniform temperature rise with constrained edges), the first-order lateral deflections of the panel will cause only second-order small changes in the in-plane strains (thus, thermal stresses) at the onset of thermal buckling (ref. 28). However, in mechanical buckling, the external loads are held constant during buckling. If the second-order effect is neglected, then the in-plane thermal loads may be considered constant during the thermal buckling. Thus the thermal buckling problems would be equivalent to the mechanical buckling problems, and, therefore, the conventional methods of structural stability analysis could be applied to the thermal buckling analysis.

The buckling equations will be developed first for the mechanical buckling under the combined loading condition described in equation (12). The resulting mechanical buckling equation could then be applied directly to the thermal buckling of the sandwich panels with constraint edges under uniform panel temperature, (i.e., $\{N_x = -N_x^T,\ N_y = -N_y^T,\ N_{xy} = -N_{xy}^T\}$ (eq. (13)), $\{M_x^T,\ M_y^T,\ M_{xy}^T\} = 0$, $u = v = w = 0$).

6.2.2 Rayleigh-Ritz Method

After substitutions of the trial deformation functions (eqs. (24) through (35)) into energy equations for V_1 (eq. (11)) and V_2 (eq. (9)) (signs of forcing functions reversed according to eq. (12) or (13)), and after performing the double integrations using the integral relations given in Appendix A, the components of V_1 and V_2 may be calculated for different indicial conditions under different panel edge conditions. The results are presented in Appendix B.

Substituting the expressions of V_1 and V_2 given in Appendix B into equation (10), and minimizing V with respect to each Fourier coefficient A_{mn}, B_{mn}, and C_{mn} according to the Rayleigh-Ritz principle

$$\frac{\partial V}{\partial A_{mn}} = \frac{\partial V}{\partial B_{mn}} = \frac{\partial V}{\partial C_{mn}} = 0 \tag{36}$$

there results three homogeneous simultaneous equations (i.e., characteristic equations) for each indicial set of $\{m, n\}$

$$\sum_{k}^{\infty}\sum_{\ell}^{\infty}\left\{\left[a^{11}_{mnk\ell} + \eta\frac{D^*}{ab}\left(\frac{\pi}{a}\right)^2 k_{xy}\delta_{mnk\ell}\right]A_{k\ell} + a^{12}_{mnk\ell}B_{k\ell} + a^{13}_{mnk\ell}C_{k\ell}\right\} = 0 \tag{37}$$

$$\sum_{k}^{\infty}\sum_{\ell}^{\infty}\left[a^{21}_{mnk\ell}A_{k\ell} + a^{22}_{mnk\ell}B_{k\ell} + a^{23}_{mnk\ell}C_{k\ell}\right] = 0 \tag{38}$$

$$\sum_{k}^{\infty}\sum_{\ell}^{\infty}\left[a^{31}_{mnk\ell}A_{k\ell} + a^{32}_{mnk\ell}B_{k\ell} + a^{33}_{mnk\ell}C_{k\ell}\right] = 0 \tag{39}$$

where the coefficients $a^{ij}_{mnk\ell}$ $(i, j = 1, 2, 3)$ are defined in Appendix C for different edge conditions under particular indicial conditions, ($a^{ij}_{mnk\ell}$ are nonzero only when $\{k, \ell\}$ are related to given $\{m, n\}$ as shown in Appendix C), $\{k_x, k_y\}$ (contained in the coefficient $a^{11}_{mnk\ell}$, Appendix C) and k_{xy} are respectively the compressive and shear buckling load factors defined as

$$k_x = \frac{N_x a^2}{\pi D^*},\quad k_y = \frac{N_y a^2}{\pi D^*},\quad k_{xy} = \frac{N_{xy} a^2}{\pi D^*} \tag{40}$$

and the flexural stiffness parameter D^* is defined as

$$D^* = \frac{\sqrt{E_x E_y}}{1 - \nu_{xy}\nu_{yx}} I_s \tag{41}$$

In equations (37) through (39), the indices $\{k, \ell\}$ for nonzero $a^{ij}_{mnk\ell}$ terms are determined respectively from the given indices $\{m, n\}$ as shown in Appendix C, and in equation (37), η is a numerical parameter, and $\delta_{mnk\ell}$ is a special delta function which is nonzero only under the indicial conditions $m \pm k =$ odd, and $n \pm \ell =$ odd. Both η and $\delta_{mnk\ell}$ respectively change their numerical values and functional forms with the change of panel edge condition as follows.

Case 1. 4S edge condition:

$$\left[\begin{array}{ll} \eta & = 32 \\ \delta_{mnk\ell} & = \dfrac{mnk\ell}{(m^2 - k^2)(n^2 - \ell^2)}\ ;\ m \pm k = odd, n \pm \ell = odd \end{array}\right. \tag{42}$$

Case 2. 4C edge condition:

$$\left[\begin{array}{ll} \eta & = \dfrac{(16)^3}{2} \\ \delta_{mnk\ell} & = \dfrac{mnk\ell[m^2 + k^2 - 2][n^2 + \ell^2 - 2]}{(m^2 - k^2)(n^2 - \ell^2)[(m+k)^2 - 4][(m-k)^2 - 4][(n+\ell)^2 - 4][(n-\ell)^2 - 4]}\ ; \\ & \quad m \pm k = odd, n \pm \ell = odd \end{array}\right. \tag{43}$$

Case 3. 2C2S edge condition:

$$\left[\begin{array}{ll} \eta & = 8^3 \\ \delta_{mnk\ell} & = \dfrac{mnk\ell[2 - (n^2 + \ell^2)]}{(m^2 - k^2)(n^2 - \ell^2)[(n+\ell)^2 - 4][(n-\ell)^2 - 4]}\ ;\ m \pm k = odd, n \pm \ell = odd \end{array}\right. \tag{44}$$

Case 4. 2S2C edge condition:

$$\left[\begin{array}{ll} \eta & = 8^3 \\ \delta_{mnk\ell} & = \dfrac{mnk\ell[2 - (m^2 + k^2)]}{(m^2 - k^2)(n^2 - \ell^2)[(m+k)^2 - 4][(m-k)^2 - 4]}\ ;\ m \pm k = odd, n \pm \ell = odd \end{array}\right. \tag{45}$$

6.2.3 Characteristic Equations in Terms of Load Factors

From equations (38) and (39), for each set of $\{mnk\ell\}$, $B_{k\ell}$ and $C_{k\ell}$ may be solved in terms of $A_{k\ell}$ as

$$B_{k\ell} = \frac{a^{23}_{mnk\ell}a^{31}_{mnk\ell} - a^{21}_{mnk\ell}a^{23}_{mnk\ell}}{a^{22}_{mnk\ell}a^{33}_{mnk\ell} - a^{23}_{mnk\ell}a^{32}_{mnk\ell}} A_{k\ell} \tag{46}$$

$$C_{k\ell} = \frac{a^{21}_{mnk\ell}a^{32}_{mnk\ell} - a^{22}_{mnk\ell}a^{31}_{mnk\ell}}{a^{22}_{mnk\ell}a^{33}_{mnk\ell} - a^{23}_{mnk\ell}a^{32}_{mnk\ell}} A_{k\ell} \tag{47}$$

Substitution of equations (46) and (47) into equation (37) term-by-term yields a homogeneous linear characteristic equation containing only the panel deflection coefficient $A_{k\ell}$

$$\sum_k \sum_\ell \left[\frac{M_{mnk\ell}}{k_{xy}} + \delta_{mnk\ell} \right] A_{k\ell} = 0 \tag{48}$$

for every integral value of $\{m, n\}$. The stiffness–geometry parameter $M_{mnk\ell}$ appearing in equation (48) is defined as

$$M_{mnk\ell} \equiv \frac{1}{\eta}\frac{ab}{D^*}\left(\frac{a}{\pi}\right)^2 \left[\underbrace{a^{11}_{mnk\ell}}_{\substack{\text{classical}\\ \text{thin plate}\\ \text{theory term}}} + \underbrace{\frac{a^{12}_{mnk\ell}\left(a^{23}_{mnk\ell}a^{31}_{mnk\ell} - a^{21}_{mnk\ell}a^{33}_{mnk\ell}\right) + a^{13}_{mnk\ell}\left(a^{21}_{mnk\ell}a^{32}_{mnk\ell} - a^{22}_{mnk\ell}a^{31}_{mnk\ell}\right)}{a^{22}_{mnk\ell}a^{33}_{mnk\ell} - a^{23}_{mnk\ell}a^{32}_{mnk\ell}}}_{\text{transverse shear effect terms}} \right] \tag{49}$$

The characteristic equation (48) is written in terms of the load factors $\{k_x, k_y, k_{xy}\}$, and is suited for combined load mechanical buckling analysis. For the thermal buckling case, equation (48) needs to be rewritten in terms of temperature as shown in the next section.

The characteristic equation (48) forms a system of an infinite number of simultaneous homogeneous equations associated with the whole range of indicial combinations of $\{m, n\}$ (one infinite series equation for each set of $\{m, n\}$ values).

Those simultaneous equations generated by equation (48) have the following interesting characteristics. In equation (48), the nonzero $M_{mnk\ell}$ of the first term requires the indicial restrictions $\{m = k \text{ or } m-k = 2\}$ and $\{n = \ell \text{ or } n - \ell = 2\}$ because of the same indicial restriction for nonzero $a^{ij}_{mnk\ell}$ (eq. (C-49), Appendix C). Thus, if $m \pm n$ is even, then $k \pm \ell$ is also even, and if $m \pm n$ is odd, then $k \pm \ell$ is also odd. In the second term of equation (48), the nonzero $\delta_{mnk\ell}$ requires the indicial conditions: $m \pm k = odd$ and $n \pm \ell = odd$. It follows that $(m \pm k) \pm (n \pm \ell) = (m \pm n) \pm (k \pm \ell) =$ even. Thus, the second term of equation (48) also has the same characteristics as the first term. Namely, if $m \pm n$ is even, then $k \pm \ell$ is also even. Likewise, if $m \pm n$ is odd, then $k \pm \ell$ is also odd. Therefore, there is no coupling between the even case (symmetric buckling) and the odd case (antisymmetric buckling) in the simultaneous homogeneous equations generated from equation (48). Thus, those simultaneous homogeneous equations may be divided into two groups which are independent of each other; one group in which $m \pm n$ is even, and the other group in which $m \pm n$ is odd (refs. 9, 13, 14).

For the deflection coefficients $A_{k\ell}$ to have nontrivial solutions under the assigned values of $\{k_x, k_y, \frac{b}{a}\}$, the determinant of coefficients of unknown $A_{k\ell}$ of the simultaneous homogeneous equations generated from equation (48) must vanish. The largest eigenvalue $1/k_{xy}$ thus obtained will give the lowest shear buckling load factor k_{xy} for given $\{k_x, k_y, \frac{b}{a}\}$. When the transverse shear effect is neglected (eq. (48)), $\{k_x, k_y, k_{xy}\}$ are a function only of $\frac{b}{a}$, and independent of panel size. However, they will become panel-size-dependent if the transverse shear effect is considered.

In the eigenvalue computations, the infinite number of the simultaneous equations and the infinite series summation of each equation may be truncated up to a certain identical finite number if the convergency of the eigenvalue solutions has reached a specified criterion for convergency.

The determinants in terms of the coefficients of the simultaneous equations written out from equation (48) up to order 12 are given in Appendix D for the cases $m \pm n$ = even (symmetric buckling) and $m \pm n$ = odd (antisymmetric buckling) for different edge conditions. The determinants of order 12 were found to give sufficiently accurate eigenvalue solutions (ref. 9). In Appendix D, one notices that for the 4S edge condition only, the $\frac{M_{mnk\ell}}{k_{xy}}$ term (eq. (48)) forms the diagonal terms of the determinant, and the nonzero off-diagonal terms consist of known numerical values. However, for other edge conditions, $\frac{M_{mnk\ell}}{k_{xy}}$ not only forms the diagonal terms of the determinant, but also appears in the off-diagonal terms (mixed with known numerical terms).

6.2.4 Characteristic Equations in Terms of Temperature

For thermal buckling, the main objective is to find the buckling temperature, T_{cr}. Therefore, equation (48) needs to be rewritten in terms of temperature rather than load factors. For the uniform temperature case, the thermal forces have the following forms

$$N_x^T = k_x D^* \left(\frac{\pi}{a}\right)^2 = \left(\bar{A}_{11}\alpha_x + \bar{A}_{12}\alpha_y\right) T \tag{50}$$

$$N_y^T = k_y D^* \left(\frac{\pi}{a}\right)^2 = \left(\bar{A}_{21}\alpha_x + \bar{A}_{22}\alpha_y\right) T \tag{51}$$

$$N_{xy}^T = k_{xy} D^* \left(\frac{\pi}{a}\right)^2 = \bar{A}_{66}\alpha_{xy} T \tag{52}$$

which were obtained from equations (4) and (7) setting $T_1 = T_2 = T$.

The coefficient $a_{mnk\ell}^{11}$ appearing in equation (49) contains thermal forcing terms (Appendix C). Thus, $a_{mnk\ell}^{11}$ may be written in two parts as

$$a_{mnk\ell}^{11} = \bar{a}_{mnk\ell}^{11} + \left[\xi(m,k) N_x^T + \zeta(n,\ell) N_y^T\right] \tag{53}$$

where $\bar{a}_{mnk\ell}^{11}$ is the first part of $a_{mnk\ell}^{11}$ without the thermal forcing terms, $\xi(m,k)$ and $\zeta(n,\ell)$ are respectively the numerical coefficients of N_x^T and N_y^T, and whose values change with the indicial and edge conditions.

In light of equations (50) through (53), equation (48) could be rewritten as

$$\sum_k \sum_\ell \left[\frac{\overline{M}_{mnk\ell}}{T} + P_{mnk\ell} + \delta_{mnk\ell}\right] A_{k\ell} = 0 \tag{54}$$

where

$$\overline{M}_{mnk\ell} \equiv \frac{ab}{\eta A_{66}\alpha_{xy}} \left[\bar{a}_{mnk\ell} \right.$$

$$\left. + \frac{a^{12}_{mnk\ell}\left(a^{23}_{mnk\ell}a^{31}_{mnk\ell} - a^{21}_{mnk\ell}a^{33}_{mnk\ell}\right) + a^{13}_{mnk\ell}\left(a^{21}_{mnk\ell}a^{32}_{mnk\ell} - a^{22}_{mnk\ell}a^{31}_{mnk\ell}\right)}{a^{22}_{mnk\ell}a^{33}_{mnk\ell} - a^{23}_{mnk\ell}a^{32}_{mnk\ell}} \right] \tag{55}$$

$$P_{mnk\ell} \equiv \frac{ab}{\eta A_{66}\alpha_{xy}} \left[\xi(m,k)(\bar{A}_{11}\alpha_x + \bar{A}_{12}\alpha_y) + \zeta(n,\ell)(\bar{A}_{21}\alpha_x + \bar{A}_{22}\alpha_y)\right] \tag{56}$$

In equation (54), $\overline{M}_{mnk\ell}$ and $P_{mnk\ell}$ terms contain material properties which are temperature dependent. Thus, in the eigenvalue solution process using equation (54), one has to assume a temperature T_a and use the corrresponding material properties as inputs to calculate the eigenvalue $1/T_{cr}$ where T_{cr} is the buckling temperature. This iteration process must be continued until the assumed temperature T_a approaches the buckling temperature T_{cr}.

Thus, in thermal buckling, when the buckling temperature is to be calculated instead of the buckling load factors, the eigenvalue solution process requires a temperature iteration process, and therefore, is slightly different from that in mechanical buckling for which only a one-step eigenvalue solution process is needed.

6.3 Different Face Sheets Temperatures (Nonzero Thermal Moments)

When the face sheets temperatures are different (i.e., $T_1 \neq T_2$) the sandwich panel will be subjected not only to thermal forces $\{N_x^T, N_y^T, N_{xy}^T\}$ but also thermal moments $\{M_x^T, M_y^T, M_{xy}^T\}$. The problem will then become a "bending" problem, and no longer an "eigenvalue" problem. One can then calculate the panel deflection w in terms of Fourier coefficient A_{mn}. The buckling condition will correspond to the condition for which the term in series representation of w, (associated with a particular buckling mode shape), becomes unbounded (i.e., $A_{mn} \to \infty$ for a given $\{m, n\}$).

As will be seen later, the buckling conditions for the case with thermal moments turned out to be the buckling equation for the case without thermal moments. Therefore, only the case with 4S edge condition will be analyzed as an example.

6.3.1 Thermal Moments

Let the thermal moments $\{M_x^T, M_y^T, M_{xy}^T\}$ be expressed in double Fourier series in accordance with the deformation functions given in equations (24) to (26) for the 4S edge condition as

$$M_x^T = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} F_{mn} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b} \tag{57}$$

$$M_y^T = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} H_{mn} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b} \tag{58}$$

$$M_{xy}^T = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} S_{mn} \cos\frac{m\pi x}{a} \cos\frac{n\pi y}{b} \tag{59}$$

where the Fourier coefficients F_{mn}, H_{mn}, S_{mn} are given by

$$F_{mn} = \frac{4}{ab}\int_o^a \int_o^b M_x^T \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b} dxdy \tag{60}$$

$$H_{mn} = \frac{4}{ab} \int_o^a \int_o^b M_y^T \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} dx dy \tag{61}$$

$$S_{mn} = \frac{4}{ab} \int_o^a \int_o^b M_{xy}^T \cos \frac{m\pi x}{a} \cos \frac{n\pi y}{b} dx dy \tag{62}$$

In light of the deformation functions (eqs. (24) to (26)) and the thermal moment expressions (eqs. (57) to (59)), the energy equations (8) (setting $u = v = 0$) and (9) may be integrated to yield the forms given in Appendix E. Notice that in equation (E-1), the thermal moment terms turned out to be linear functions of $\{A_{mn}, B_{mn}, C_{mn},\}$ and not quadratic functions of $\{A_{mn}, B_{mn}, C_{mn}\}$.

6.3.2 Nonhomogeneous Equations

After the application of the Rayleigh-Ritz method according to equation (36), one obtains three non-homogeneous simultaneous equations for each indicial set of $\{m, n\}$ shown in the following

$$a^{11}_{mnmn} A_{mn} + 32 \frac{D^*}{ab} \left(\frac{\pi}{a} \right)^2 k_{xy} \sum_k \sum_\ell \delta_{mnk\ell} A_{k\ell} + a^{12}_{mnmn} B_{mn} + a^{13}_{mnmn} C_{mn} = J_{mn} \tag{63}$$

$$a^{21}_{mnmn} A_{mn} + a^{22}_{mnmn} B_{mn} + a^{23}_{mnmn} C_{mn} = K_{mn} \tag{64}$$

$$a^{31}_{mnmn} A_{mn} + a^{32}_{mnmn} B_{mn} + a^{33}_{mnmn} C_{mn} = L_{mn} \tag{65}$$

where the coefficients a^{11}_{mnmn} are defined by equation (C-1), and the nonhomogeneous terms J_{mn}, K_{mn}, and L_{mn} are defined as

$$J_{mn} \equiv F_{mn} \left(\frac{m\pi}{a} \right)^2 + H_{mn} \left(\frac{n\pi}{b} \right)^2 - 2S_{mn} \left(\frac{m\pi}{a} \right) \left(\frac{n\pi}{b} \right) \tag{66}$$

$$K_{mn} \equiv -F_{mn} \left(\frac{m\pi}{a} \right) + S_{mn} \left(\frac{n\pi}{b} \right) \tag{67}$$

$$L_{mn} \equiv -H_{mn} \left(\frac{m\pi}{a} \right) + S_{mn} \left(\frac{n\pi}{b} \right) \tag{68}$$

From equations (64) and (65), B_{mn} and C_{mn} may be solved in terms of $\{A_{mn}, K_{mn}, L_{mn}, a^{ij}_{mnmn}\}$ as

$$B_{mn} = \frac{a^{23}_{mnmn} a^{31}_{mnmn} - a^{21}_{mnmn} a^{33}_{mnmn}}{a^{22}_{mnmn} a^{33}_{mnmn} - a^{23}_{mnmn} a^{32}_{mnmn}} A_{mn} + \frac{K_{mn} a^{33}_{mnmn} - L_{mn} a^{23}_{mnmn}}{a^{22}_{mnmn} a^{33}_{mnmn} - a^{23}_{mnmn} a^{32}_{mnmn}} \tag{69}$$

$$C_{mn} = \frac{a^{21}_{mnmn} a^{32}_{mnmn} - a^{22}_{mnmn} a^{31}_{mnmn}}{a^{22}_{mnmn} a^{33}_{mnmn} - a^{23}_{mnmn} a^{32}_{mnmn}} A_{mn} + \frac{L_{mn} a^{22}_{mnmn} - K_{mn} a^{32}_{mnmn}}{a^{22}_{mnmn} a^{33}_{mnmn} - a^{23}_{mnmn} a^{32}_{mnmn}} \tag{70}$$

Substitution of equations (69) and (70) into equation (63) yields, for each indicial set $\{m, n\}$:

$$\frac{M_{mnmn}}{k_{xy}} A_{mn} + \sum_k \sum_\ell \delta_{mnk\ell} A_{k\ell} = R_{mnmn} \tag{71}$$

where M_{mnmn} is defined in equation (49) under the condition m=k, n=ℓ, η=32, and R_{mnmn} is defined by

$$R_{mnmn} = \frac{1}{32 k_{xy}} \frac{ab}{D^*} \left(\frac{a}{\pi} \right)^2 \left[J_{mn} - \frac{a^{12}_{mnmn} (K_{mn} a^{33}_{mnmn} - L_{mn} a^{23}_{mnmn})}{a^{22}_{mnmn} a^{33}_{mnmn} - a^{23}_{mnmn} a^{32}_{mnmn}} \right.$$

$$\left. + \frac{a^{13}_{mnmn} (K_{mn} a^{32}_{mnmn} - L_{mn} a^{22}_{mnmn})}{a^{22}_{mnmn} a^{33}_{mnmn} - a^{23}_{mnmn} a^{32}_{mnmn}} \right] \tag{72}$$

Equation (71) forms an infinite number of nonhomogeneous simultaneous equations each of which is associated with a set of $\{m, n\}$ values (or mode shape) for the calculation of infinite number of Fourier (or Ritz) coefficients A_{mn} in the series representation of panel deflection $w(x, y)$ equation (24).

6.3.3 Buckling Condition

The calculated Ritz coefficients A_{mn} have the following form

$$A_{mn} = \frac{[\quad]_{mn}}{\Delta} \tag{73}$$

where the numerator $[\quad]_{mn}$ contains $\{M_{mnmn}, R_{mnmn}, \delta_{mnk\ell}\}$, and the denominator Δ is the determinant of the coefficients of $A_{k\ell}$ of the nonhomogeneous simultaneous equations written out from equation (71). The determinant Δ of order 12 is shown in Appendix D for either symmetrical (eq. (D-1)) or antisymmetrical buckling (eq. (D-2)).

The mathematical meaning of the buckling state in light of equation (73) is that the Ritz coefficient A_{mn} becomes unbounded (i.e., infinite panel deflection, or $\Delta \to 0$). Namely, when the buckling state is reached, the term in the series (eq. (24)) which corresponds to the particular deformation mode shape becomes the most important term.

From the above analysis, one sees that the buckling conditions for the cases with and without the thermal moments are identical (i.e., $\Delta = 0$) under the classical small deformation theory. Because of this finding, similar bending analyses for nonzero thermal moments for other edge conditions were not carried out.

7. NUMERICAL EXAMPLES

7.1 Physical Properties of Sandwich Panels

The sandwich panel is assumed to be fabricated with titanium face sheets and titanium honeycomb core, having the following geometrical and material properties.

Geometry:

$$h = 1.2\,\text{in.}$$
$$t_s = 0.032\,\text{in.}$$
$$a = a_o = 24\,\text{in. (for varying } b\text{), or } ab = a_o^2 \text{ (for constant panel area)}$$

Material properties:
Face sheets

	70 °F	1000 °F
$E_x = E_y$, lb/in^2	16×10^6	10.5×10^6
G_{xy}, lb/in^2	6.2×10^6	4.7×10^6
$\nu_{xy} = \nu_{yx}$	0.31	0.31
$\alpha_x = \alpha_y$, in/in-°F	4.85×10^{-6}	5.6×10^{-6}
α_{xy} in/in-°F	0	0
ρ_{Ti} lb/in^3	0.16	0.16

Honeycomb core (properties at 600 °F)

$$
\begin{array}{lcll}
E_{Cx} & = & 2.7778 \times 10^4 & \mathrm{lb/in^2} \\
E_{Cy} & = & 2.7778 \times 10^4 & \mathrm{lb/in^2} \\
E_{Cz} & = & 2.7778 \times 10^5 & \mathrm{lb/in^2} \\
G_{Cxy} & = & 0.00613 & \mathrm{lb/in^2} \\
G_{Cyz} & = & 0.81967 \times 10^5 & \mathrm{lb/in^2} \\
G_{Czx} & = & 1.81 \times 10^5 & \mathrm{lb/in^2} \\
\nu_{Cxy} & = & 0.658 \times 10^{-2} & \\
\nu_{Cyz} & = & 0.643 \times 10^{-6} & \\
\nu_{Cxz} & = & 0.643 \times 10^{-6} & \\
\alpha_x = \alpha_y & = & 5.37 \times 10^{-6} & \mathrm{in/in\text{-}^\circ F} \\
\alpha_{xy} & = & 0 & \mathrm{in/in\text{-}^\circ F} \\
\rho_{Hc} & = & 3.674 \times 10^{-3} & \mathrm{lb/in^3}
\end{array}
$$

7.2 Buckling Interaction Curves

In generating the data for plotting the buckling interaction curves, k_y was set to zero. For a given aspect ratio $\frac{b}{a}$, different values of k_x were assigned; then the corresponding eigenvalues $1/k_{xy}$ were calculated from equation (48). The buckling interaction curves plotted in k_x-k_{xy} space for different aspect ratios (a = constant) are shown in figure 6. For the case of the square ($\frac{b}{a} = 1$) panel, the additional set of buckling interaction curves shown in broken curves is for the case when the effect of transverse shear is neglected. For the square panel, the buckling interaction curves for 4S, 2S2C, and 4C cases are continuous curves of symmetric buckling. However, for the 2C2S case, the buckling interaction curve is a composite curve, partly for symmetric buckling and partly for antisymmetric buckling. Notice that the effect of the transverse shear is quite large for the sandwich panel. Without the consideration of the transverse shear effect, the sandwich panel buckling strength could be overpredicted considerably. The 4S case has the lowest buckling strength. Clamping the two opposite edges (i.e., from 4S case to 2C2S and 2S2C cases), could enhance the buckling strength considerably. By additional clamping of the other two opposite edges (i.e., from 2C2S and 2S2C cases to 4C case), further improvement of the buckling strength could be achieved. However, the improvement is not as large as that for the previous case (i.e., from 4S case to 2C2S and 2S2C cases). With or without the consideration of the transverse shear effect, the improvement of buckling strength through edge clampings is larger in pure compression than in pure shear.

At aspect ratio $\frac{b}{a} = 0.7$ (a = constant), only the buckling interaction curve for the 2S2C case is continuous, and for symmetric buckling only. For the remaining three cases (i.e., 4S, 2C2S, and 4C cases), the buckling interaction curves are composite curves, each consisting of two segments: one segment for symmetric buckling, the other segment for antisymmetric buckling.

At higher aspect ratios (i.e., $\frac{b}{a} = 2, 3, 4$), most buckling interaction curves are composite curves. Some interaction curves contain more than two segments. A maximum of four segments could be found for the cases of 4C and 2S2C at $\frac{b}{a} = 4$. At high aspect ratio (i.e., $\frac{b}{a} = 4$) the buckling strengths of the 4S and 2C2S cases are relatively close, and also those for the 4C and 2S2C cases.

7.3 Buckling Curves for Pure Compression and Pure Shear

Figures 7 and 8, respectively, show k_x (pure compression) and k_{xy} (pure shear) plotted as functions of aspect ratio $\frac{b}{a}$. In changing $\frac{b}{a}$, the length of the panel (a) was kept unchanged (i.e., a = 24 in.). Notice that the rate of reductions of k_x and k_{xy} values with the increase of $\frac{b}{a}$ are the most severe in the region $\frac{b}{a} < 1$, and gradually decrease as $\frac{b}{a}$ increases. At higher aspect ratios ($\frac{b}{a} > 2$), the buckling curves of k_x (fig. 7) and k_{xy} (fig. 8) for the 2C2S and 2S2C cases, respectively, converge toward those for the 4S and 4C cases because the edge effects from the two sides (i.e., two vertical edges in figs. 7 and 8) of the panel diminish as $\frac{b}{a}$ increases.

Figures 7 and 8 are the conventional plots of buckling curves. They may not serve as ideal design curves for aerospace structural panels because when $\frac{b}{a}$ is changed (under a = constant), the panel weight (i.e., panel area ab) is also changed accordingly. In the aerospace structural designs, the main objective is the structural optimization. Namely, for a given panel weight, the objective is to search for a panel with optimum buckling strengths (or stiffnesses). For this reason, modified buckling load factors $\bar{k}_x$ and $\bar{k}_{xy}$ ($\bar{k}_y = 0$) were recalculated as functions of $\frac{b}{a}$ under the condition $ab = a_o^2$ = constant (instead of a = constant). Figures 9 and 10, respectively, show the alternative plots of $\bar{k}_x$ vs $\frac{b}{a}$ and $\bar{k}_{xy}$ vs $\frac{b}{a}$ when the panel area ab was kept unchanged. In practical applications, the panel has to be supported by an edge frame (cross section assumed constant), therefore, the edge frame weight (or edge frame length, $(a+b)/2a_o$) was also plotted in figures 9 and 10 as a function of $\frac{b}{a}$. The square panel ($\frac{b}{a} = 1$) has the minimum edge frame weight; however, it does not have the optimum buckling strengths in compression and shear. The compressive buckling strengths (fig. 9) reached minimum at $\frac{b}{a}$ = 1.6, 1.4, 2.2, 1.0, respectively, for the 4S, 4C, 2C2S, and 2S2C cases. The lowest shear buckling strengths (fig. 10) occur at $\frac{b}{a}$ = 0.9, 0.9, 1.2, 0.7, respectively, for the 4S, 4C, 2C2S, and 2S2C cases. Figures 9 and 10 serve as design curves for selecting the desired sandwich panel geometry (i.e., $\frac{b}{a}$ value). To boost the panel buckling strengths in compression and shear, some weight penalty due to edge frame is inevitable. The desirable high-stiffness-to-weight ratio panel shapes will be slightly slender ($\frac{b}{a} < 1$).

7.4 Buckling Interaction Surfaces

Figure 11 shows three-dimensional buckling surfaces which are plotted in $\{k_x, k_{xy}, \frac{b}{a}\}$ space for different edge conditions. In the figure, the domains of symmetric and antisymmetric buckling (lowest buckling modes) are also shown. Figure 11 gives better visualization of the buckling behavior of the sandwich panel than the traditional buckling plots shown in figures 6 to 8. For slender rectangular panels (i.e., $\frac{b}{a} < 1$), antisymmetric bucklings occur mostly in the compression-dominated regions. For wider panels (i.e., $\frac{b}{a} > 1$), the antisymmetric bucklings take place in the shear-dominated regions. In the neighborhood of $\frac{b}{a} = 1$, the lowest buckling modes are all symmetric (i.e., $m = 1$, $n = 1$, fig. 4) for the 4S, 4C, and 2S2C cases. Only for the 2C2S case is the lowest buckling mode in the compression-dominated region antisymmetric (i.e., $m = 2$, $n = 1$, fig. 4). Such buckling behavior also occurs in the flat rectangular plates of $\frac{b}{a} \approx 1$.

7.5 Thermal Buckling Curves

For most of the practical materials, the coupling coefficient of thermal expansion α_{xy} is zero. Therefore, in generating the data for thermal buckling curves, the following conditions were imposed.

$$\alpha_x = \alpha_y, \ \alpha_{xy} = 0 \ \ (i.e., N_{xy}^T = M_{xy}^T = 0) \tag{74}$$

$$M_x^T = M_y^T = 0 \tag{75}$$

The above conditions will induce in-plane biaxial thermal compression without shear and bending. Figure 12 shows the critical buckling temperature (T_{cr}) plotted as a function of $\frac{b}{a}$, with the panel length a

being kept constant. Those thermal (biaxial compression) buckling curves somewhat resemble the uniaxial compressive buckling (mechanical buckling) curves shown in figure 7. For buckling temperatures higher than 1000 °F, the face sheet material property data at 1000 °F were used as inputs to equation (54) for T_{cr} calculations because of the lack of material property data at high temperatures. For the honeycomb core, the only available material property data at 600 °F had to be used as inputs for T_{cr} calculations. It was found that T_{cr} was relatively insensitive to the material property change with temperature.

For the present particular panel (i.e., dimensions chosen) the thermal buckling temperatures T_{cr} exceed the titanium melting point (3074 °F) at low $\frac{b}{a}$, and gradually decrease with the increase of $\frac{b}{a}$. At high aspect ratios, T_{cr} for the 4S and 2C2S cases level off at about 1000 °F (below superplastic temperature, 1650 °F), and for 4C and 2S2C cases, at temperatures slightly below the melting point.

Figure 13 shows the alternative plots of T_{cr} as a function of $\frac{b}{a}$ for constant-area panels (i.e., ab = constant) the lowest buckling temperatures for 4S, 4C, 2C2S, and 2S2C cases are, respectively, 1297 °F, 3702 °F (above melting point), 2194 °F (above superplastic temperature), and 2205 °F (above superplastic temperature), and which occur respectively at $\frac{b}{a} = 1.0, 0.975, 1.8$, and 0.5.

For the present sandwich panel, the actual thermal buckling will take place only for the 4S case in the region $1.5 < \frac{b}{a} < 1.8$. Outside this region for the 4S case and for all the whole range of $\frac{b}{a}$ for the other three edge conditions, no "real" thermal buckling could take place because the sandwich panel will first undergo superplastic creep or melting.

8. CONCLUDING REMARKS

Through the use of the Rayleigh-Ritz method of minimizing the total potential energy of a structural system, the combined load (mechanical or thermal) buckling equations were established for orthotropic rectangular sandwich panels under four types of edge conditions. Two-dimensional buckling interaction curves, and three-dimensional buckling interaction surfaces were constructed for high-temperature honeycomb-core sandwich panels supported under four different edge conditions. The buckling interaction surfaces provide easy visualization of the variation of the panel buckling strengths, and the domains of buckling modes (symmetric and antisymmetric) with the edge condition. Furthermore, the buckling temperature curves for the sandwich panels were presented.

The effect of transverse shear on the buckling strength is quite large for sandwich panels, and by neglecting the transverse shear effect, the buckling strengths could be overpredicted considerably. With the inclusion of the transverse shear effect, the buckling load factors became panel-size-dependent in addition to panel-aspect-ratio dependent. Clamping the edges could enhance the buckling strength greatly more in compression than in shear. Thermal buckling conditions for the cases with and without thermal moments were found to be identical for the small deformation theory.

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National Aeronautics and Space Administration
Edwards, California, June 30, 1993

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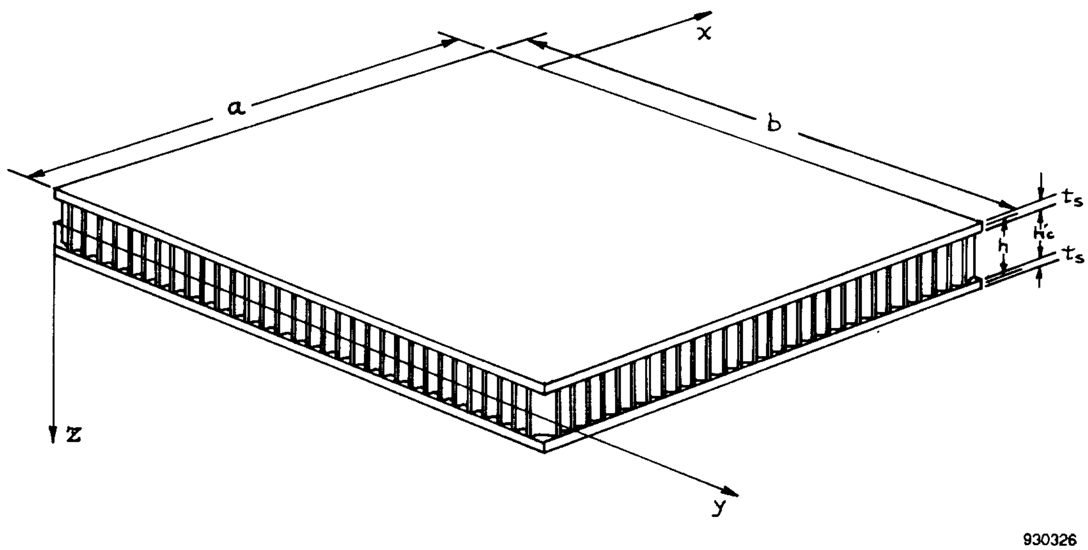


Figure 1. Honeycomb-core sandwich panel.

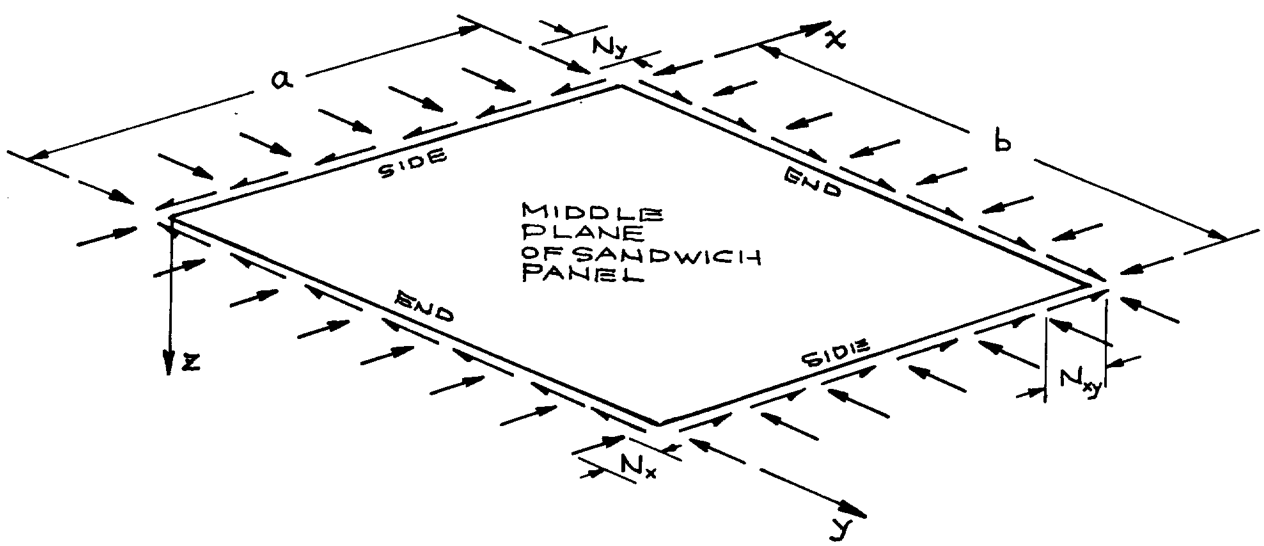


Figure 2. Combined compressive and shear loadings of a rectangular sandwich panel.

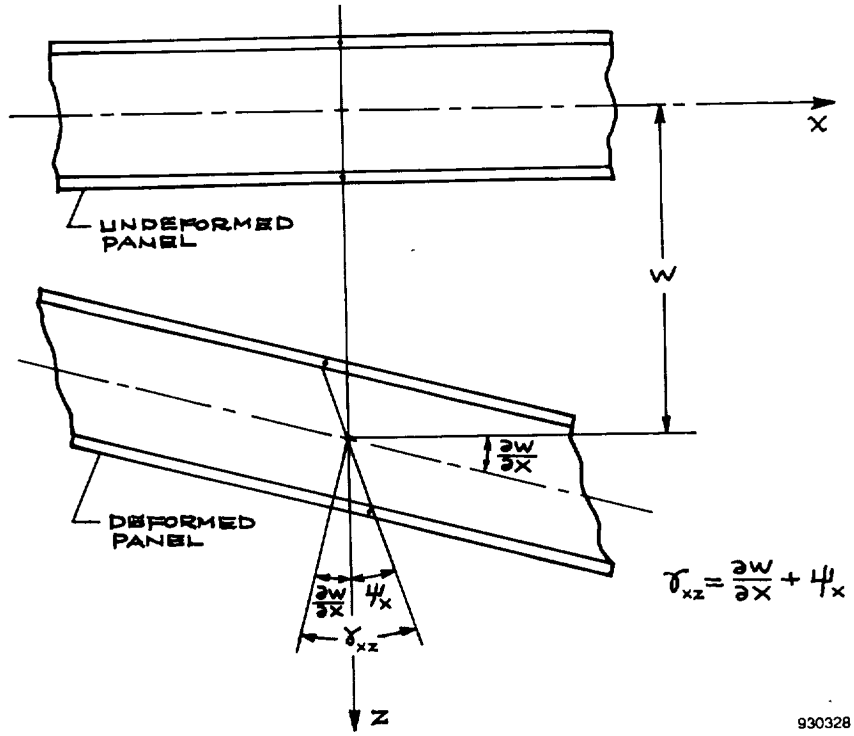


Figure 3. Deformation of a sandwich panel in the x-z plane.

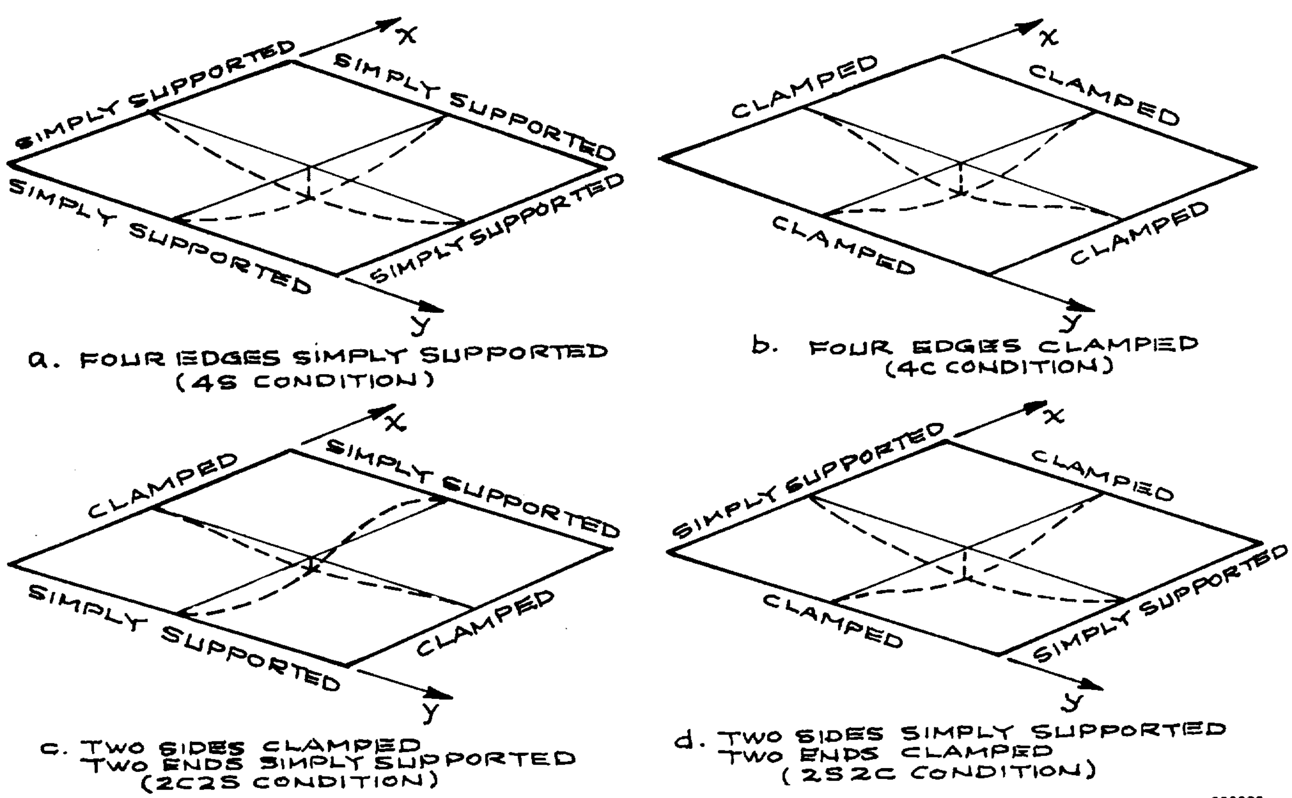


Figure 4. Four types of edge conditions.

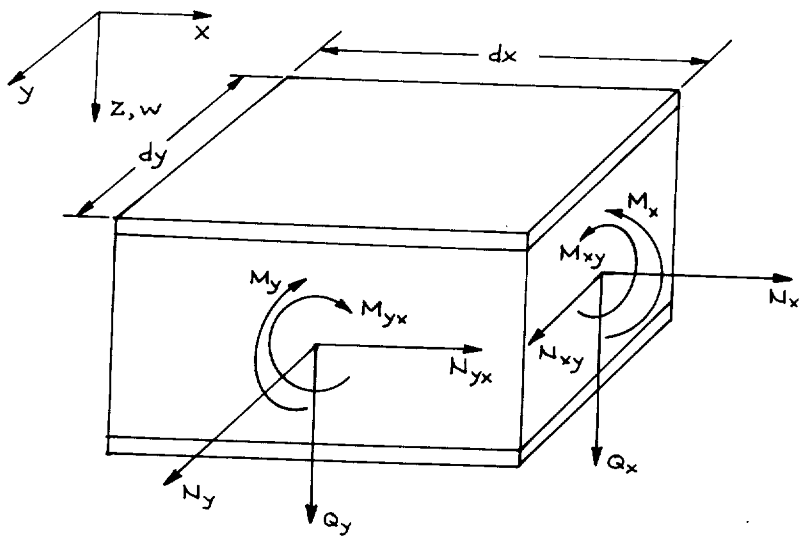
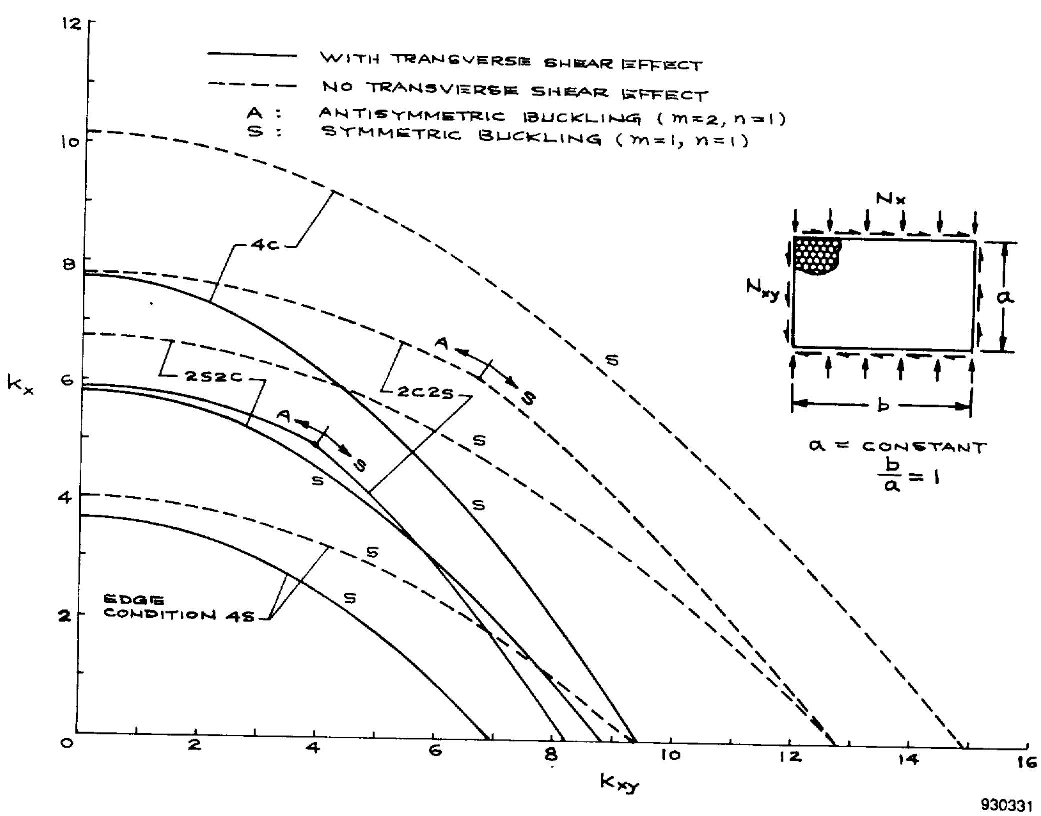
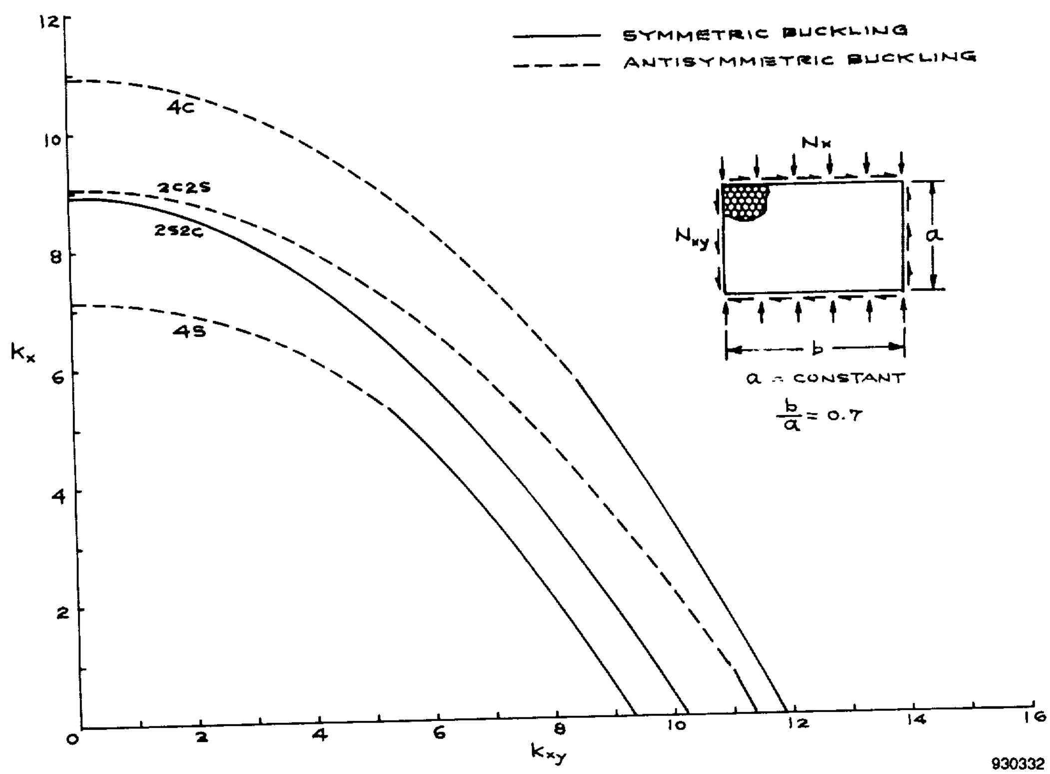


Figure 5. Forces and moments acting on the differential element of a sandwich panel.



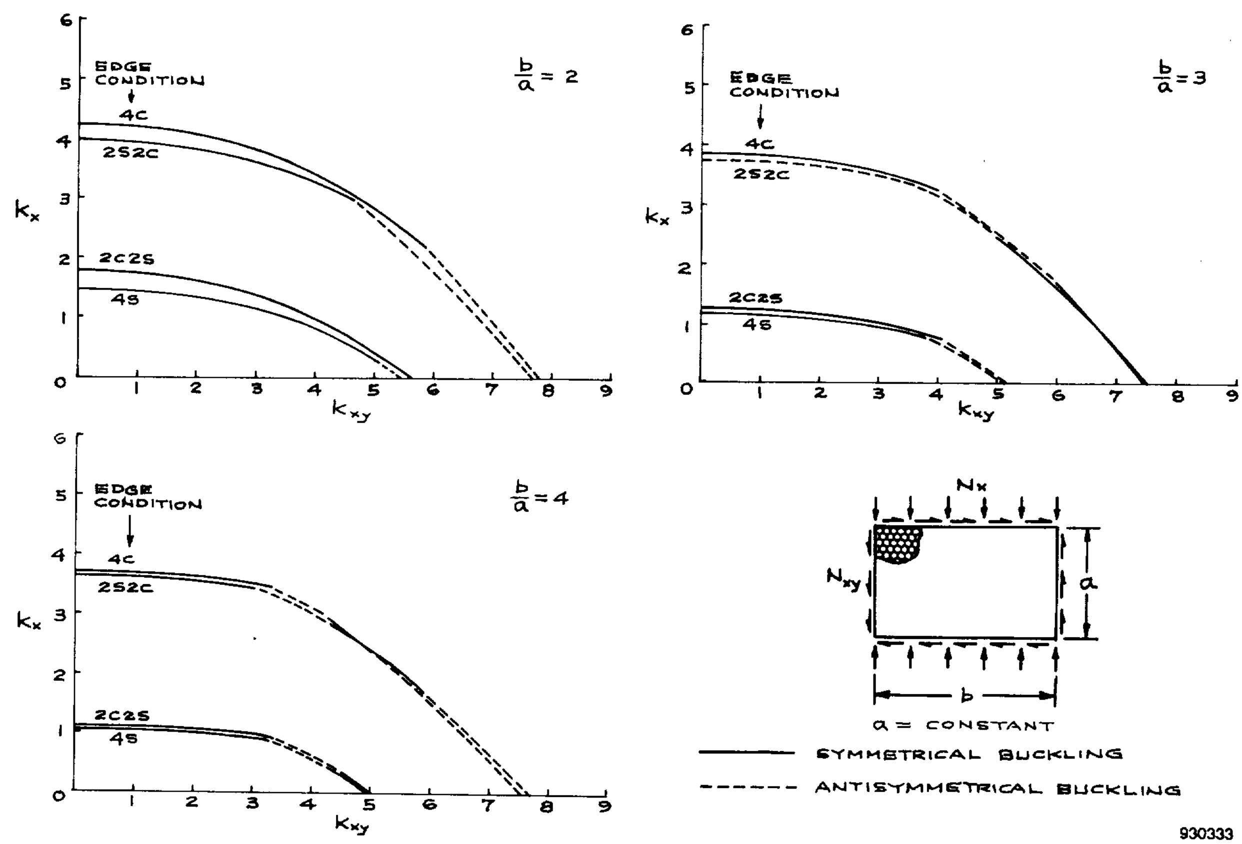
(a) $\frac{b}{a} = 1$.

Figure 6. Combined load buckling interaction plots for honeycomb-core sandwich panel under different edge conditions.



(b) $\frac{b}{a} = 0.7$.

Figure 6. Continued.



(c) $\frac{b}{a} = 2$, $\frac{b}{a} = 3$, $\frac{b}{a} = 4$ respectively.

Figure 6. Concluded.

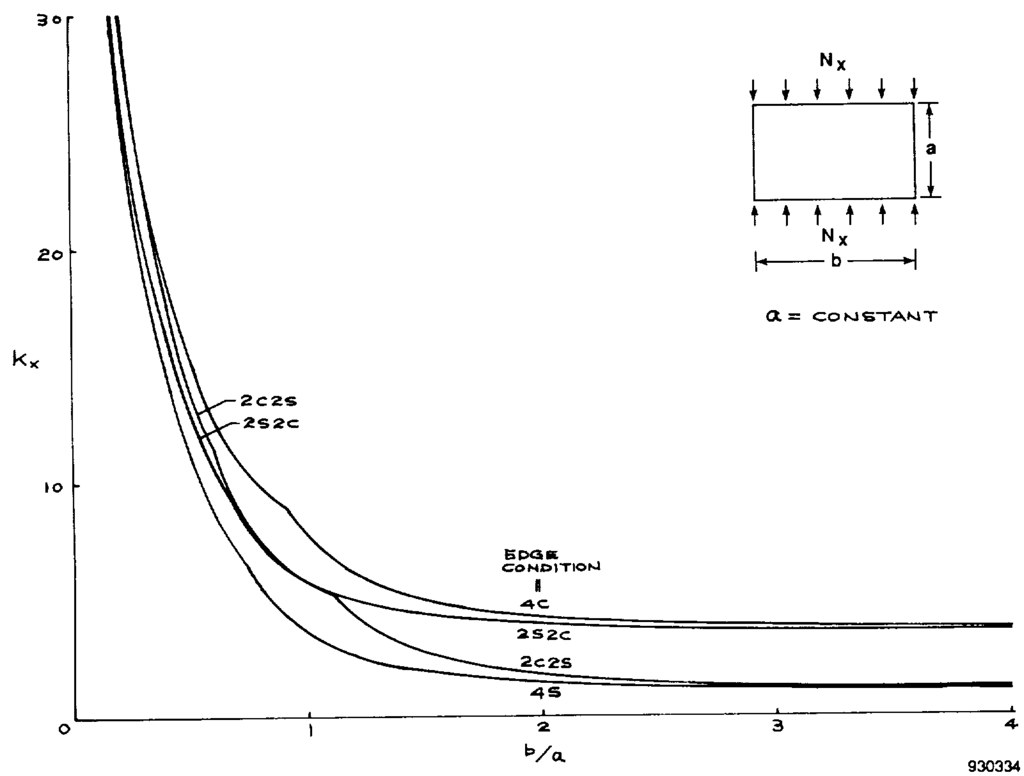


Figure 7. Comparison of compressive buckling strengths of honeycomb-core sandwich panels under different edge conditions. a = constant.

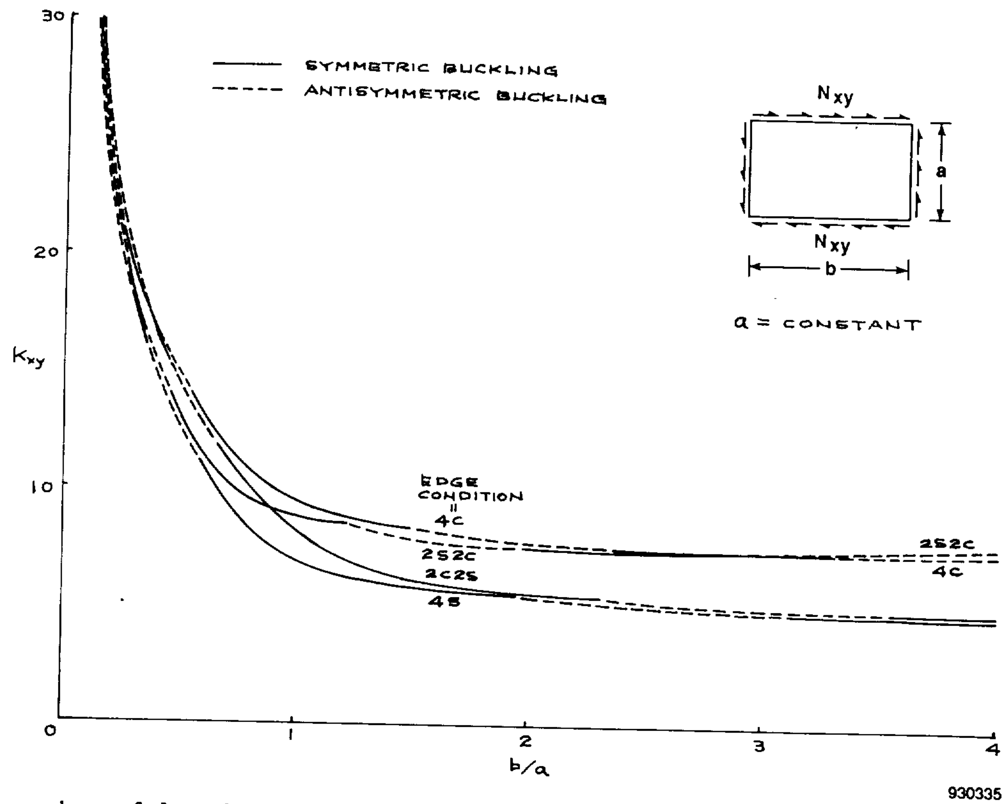


Figure 8. Comparison of shear buckling strengths of honeycomb-core sandwich panels under different edge conditions. a = constant.

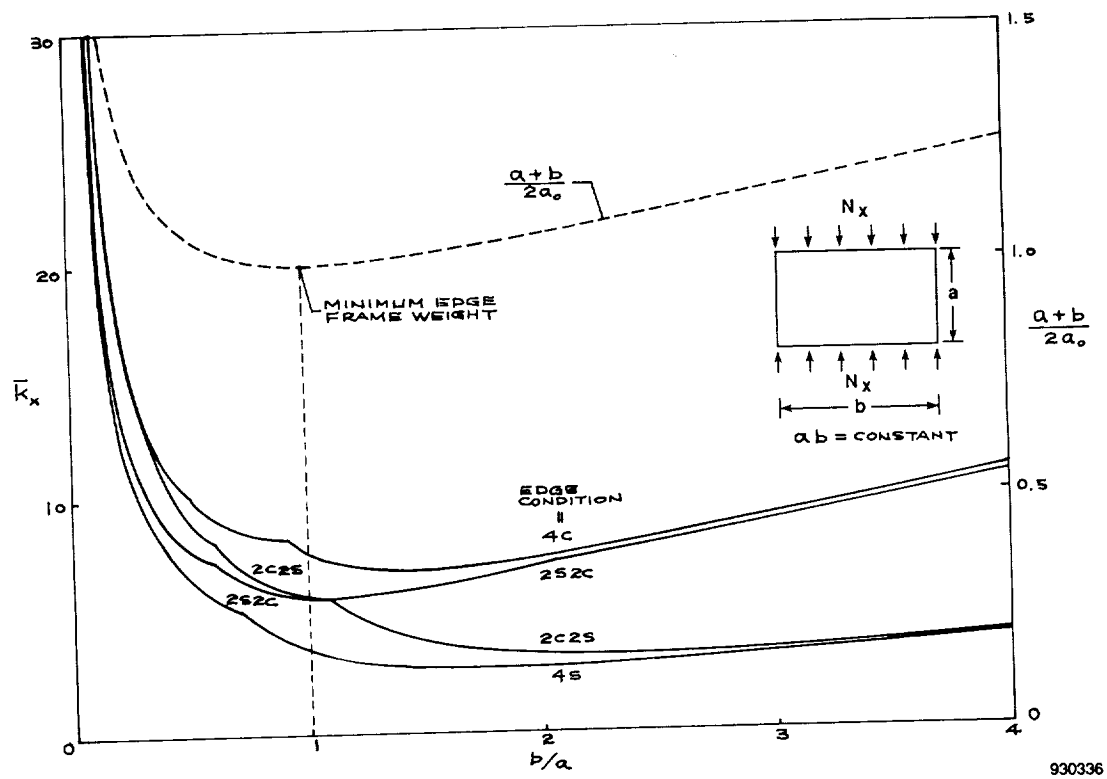


Figure 9. Comparison of compressive buckling strengths of honeycomb-core sandwich panels under different edge conditions. Constant panel areas.

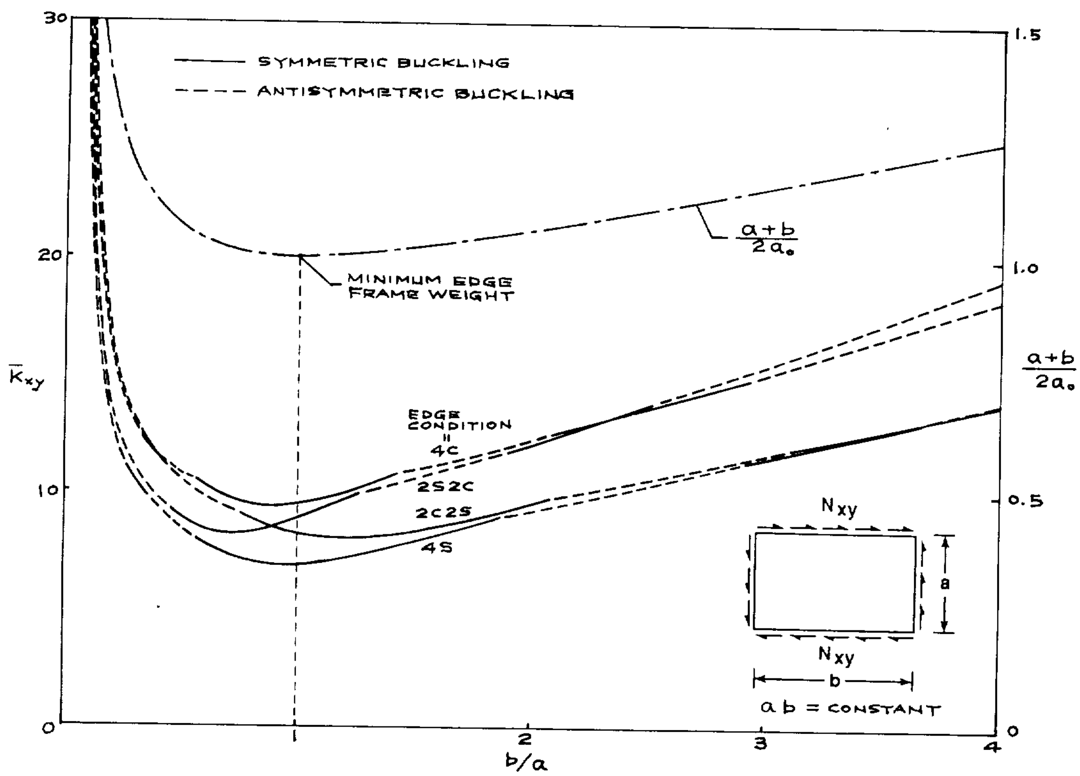
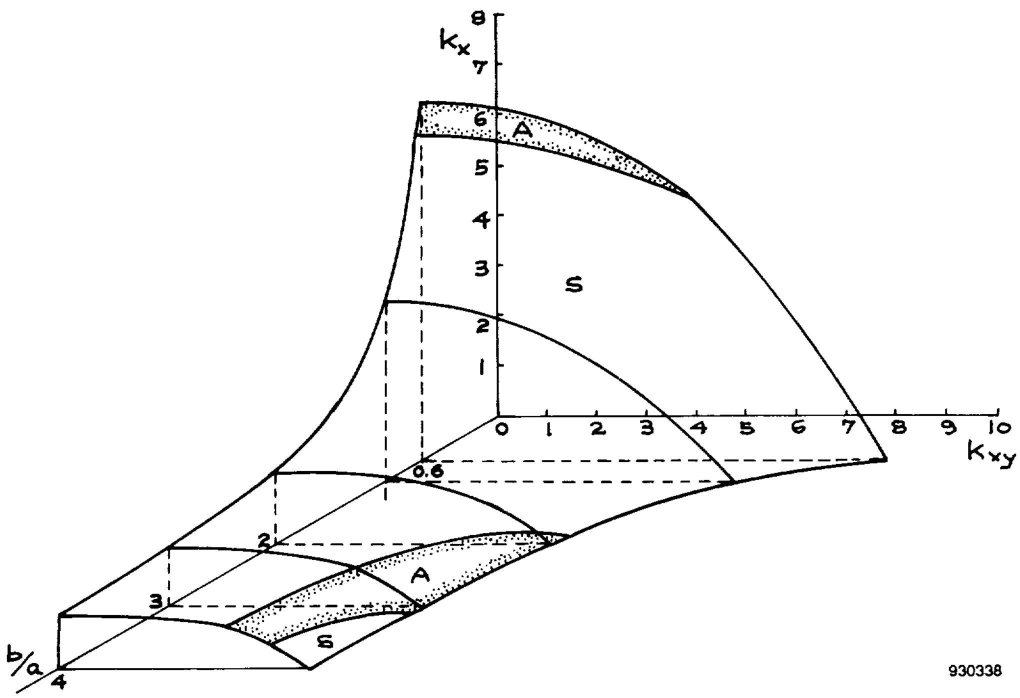
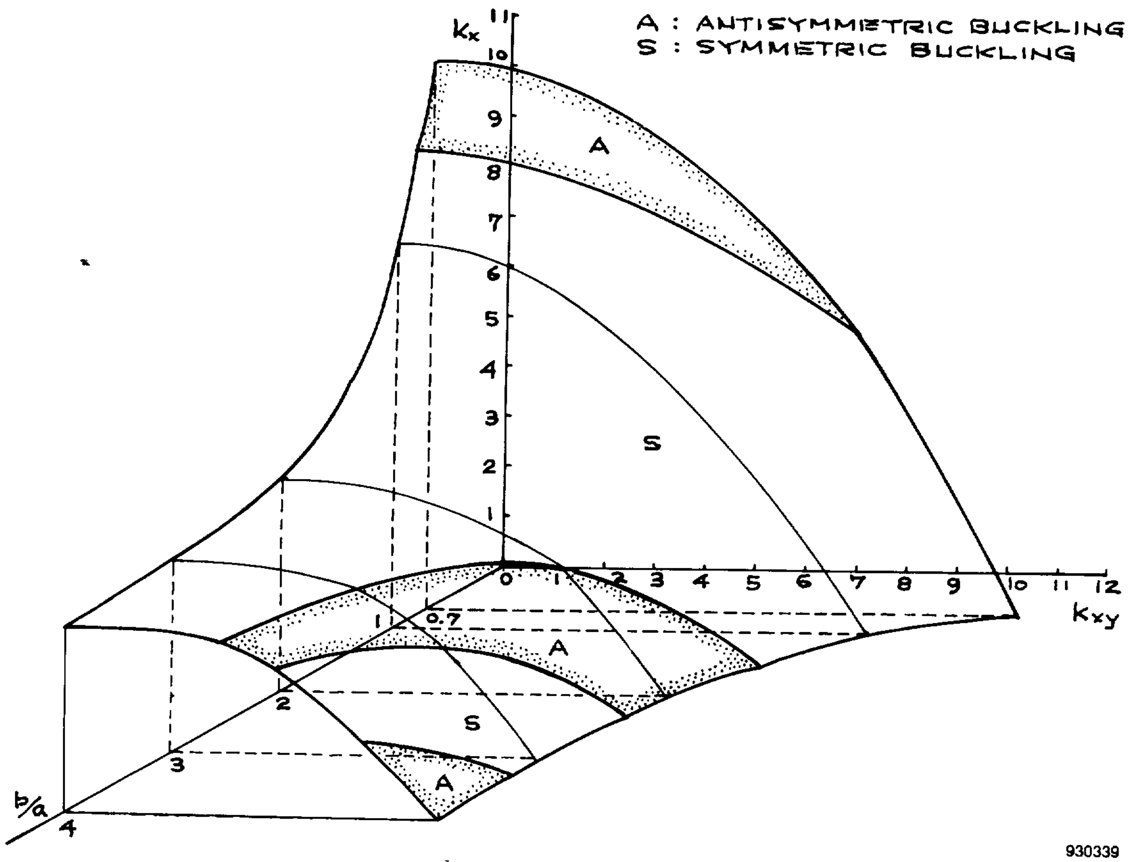


Figure 10. Comparison of shear buckling strengths of honeycomb-core sandwich panels under different edge conditions. Constant panel areas.



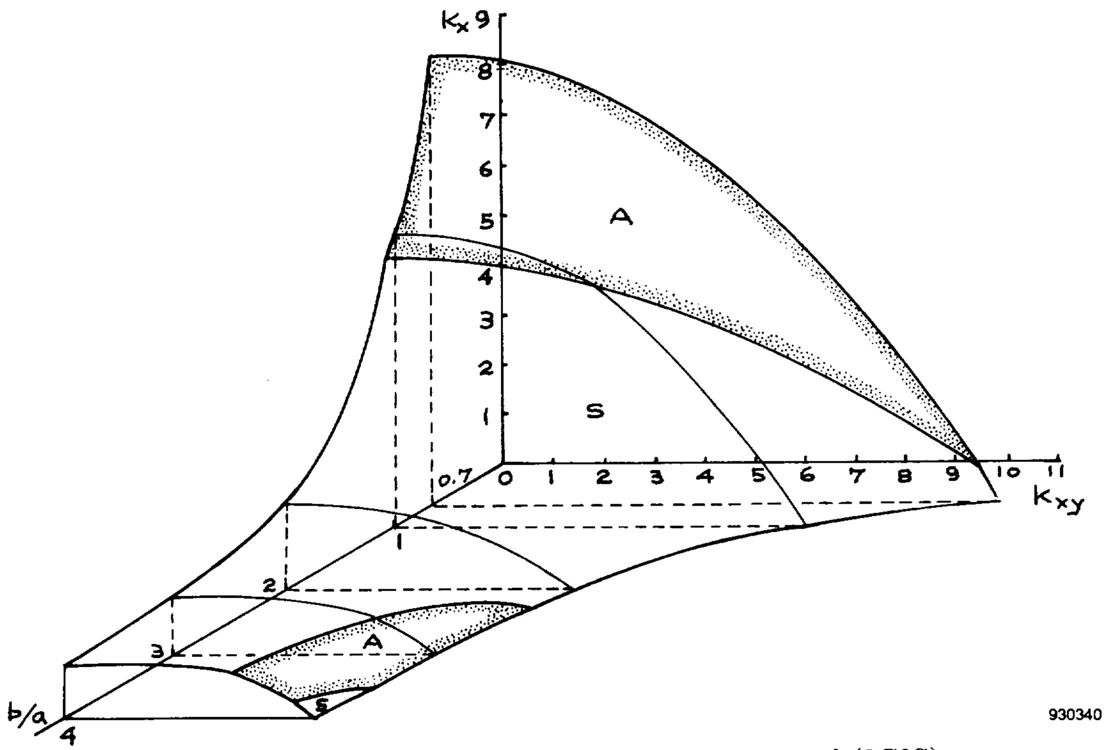
(a) Four edges simply supported (4S).

Figure 11. Buckling interaction surfaces for honeycomb-core sandwich panels under different edge conditions. a = constant.



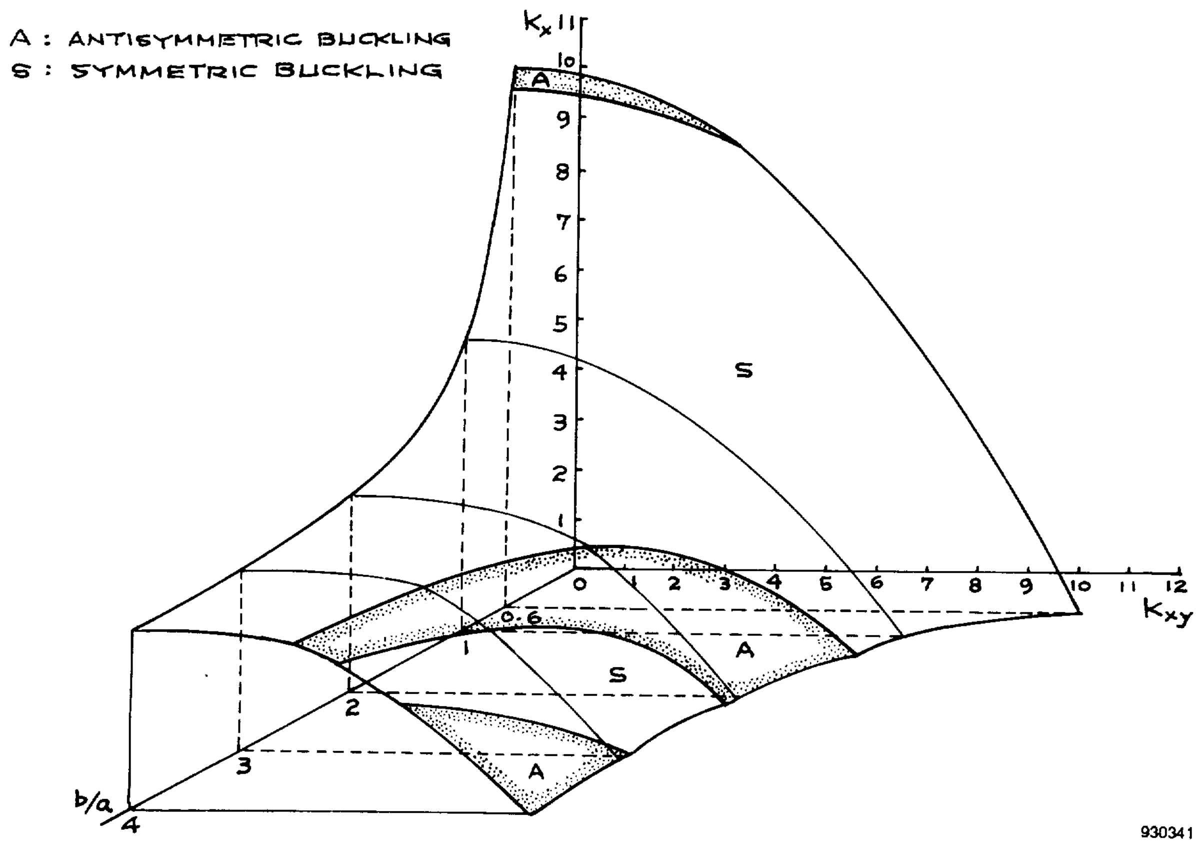
(b) Four edges clamped (4C).

Figure 11. Continued.



(c) Two sides clamped, two sides simply supported (2C2S).

Figure 11. Continued.



(d) Two sides simply supported, two ends clamped (2S2C).

Figure 11. Concluded.

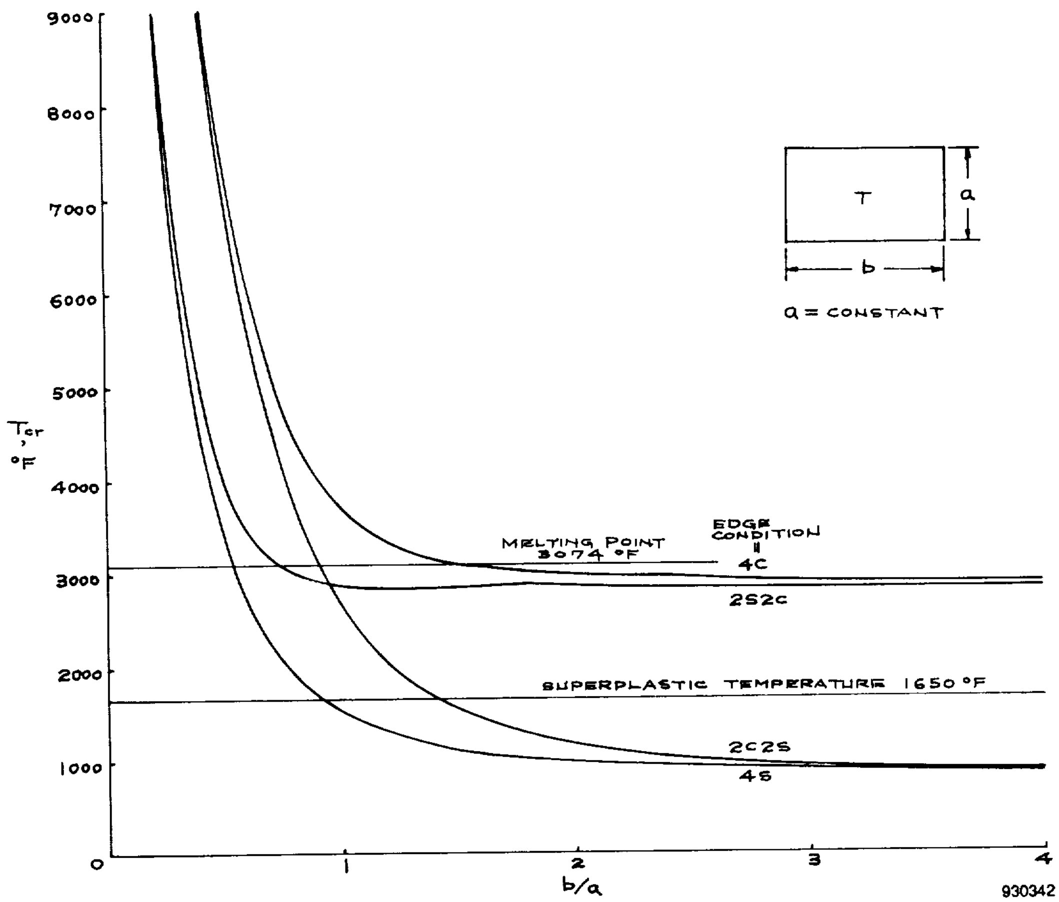


Figure 12. Thermal buckling temperatures for honeycomb-core sandwich panels under different edge conditions. a = constant.

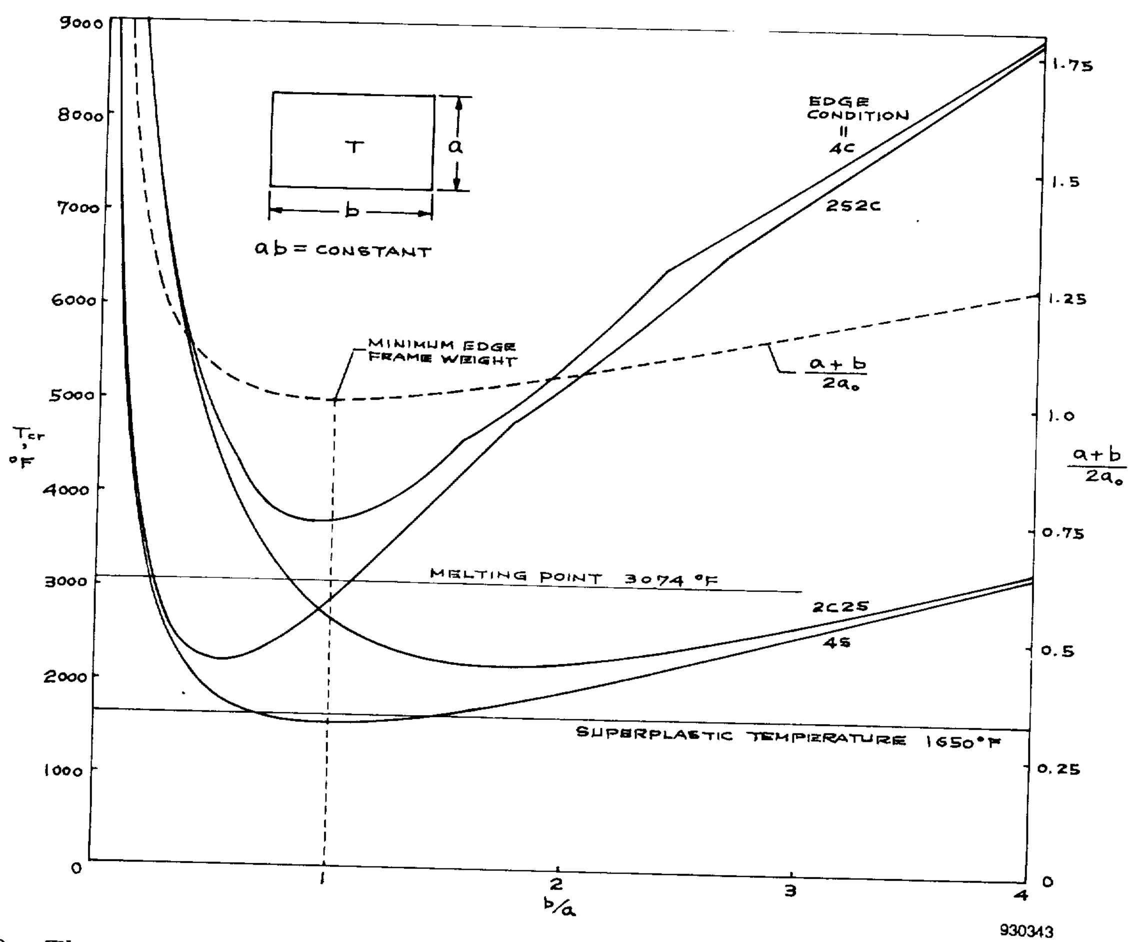


Figure 13. Thermal buckling temperatures for honeycomb-core sandwich panels under different edge conditions. Constant panel areas.

APPENDIX A
INTEGRAL RELATIONS

In the integrations of the potential energies V_1 and V_2, the following integral relations were used.

For $m = k = 1$:

$$\int_o^a \sin^2 \frac{\pi x}{a} \sin \frac{\pi x}{a} \sin \frac{\pi x}{a}\, dx = \frac{3}{8}a$$
$$\int_o^a \cos^2 \frac{\pi x}{a} \cos \frac{\pi x}{a} \cos \frac{\pi x}{a}\, dx = \frac{3}{8}a$$
$$\int_o^a \cos^2 \frac{\pi x}{a} \sin \frac{\pi x}{a} \sin \frac{\pi x}{a}\, dx = \frac{a}{8}$$
$$\int_o^a \sin \frac{2\pi x}{a} \sin \frac{\pi x}{a} \cos \frac{\pi x}{a}\, dx = \frac{a}{4}$$
$$\int_o^a \sin \frac{2\pi x}{a} \sin \frac{\pi x}{a} \sin \frac{\pi x}{a}\, dx = 0 \tag{A-1}$$

For $m - k = 2$:

$$\int_o^a \sin^2 \frac{\pi x}{a} \sin \frac{m\pi x}{a} \sin \frac{k\pi x}{a}\, dx = -\frac{a}{8}$$
$$\int_o^a \sin^2 \frac{\pi x}{a} \cos \frac{m\pi x}{a} \cos \frac{k\pi x}{a}\, dx = -\frac{a}{8}$$
$$\int_o^a \cos^2 \frac{\pi x}{a} \sin \frac{m\pi x}{a} \sin \frac{k\pi x}{a}\, dx = \frac{a}{8}$$
$$\int_o^a \cos^2 \frac{\pi x}{a} \cos \frac{m\pi x}{a} \cos \frac{k\pi x}{a}\, dx = \frac{a}{8}$$
$$\int_o^a \sin \frac{2\pi x}{a} \sin \frac{m\pi x}{a} \cos \frac{k\pi x}{a}\, dx = \frac{a}{4}$$
$$\int_o^a \sin \frac{2\pi x}{a} \cos \frac{m\pi x}{a} \sin \frac{k\pi x}{a}\, dx = -\frac{a}{4}$$
$$\int_o^a \sin^2 \frac{\pi x}{a} \cos \frac{m\pi x}{a} \sin \frac{k\pi x}{a}\, dx = 0$$
$$\int_o^a \sin \frac{2\pi x}{a} \sin \frac{m\pi x}{a} \sin \frac{k\pi x}{a}\, dx = 0 \tag{A-2}$$

For $m = k \neq 1$:

$$\begin{aligned}
\int_o^a \sin^2 \frac{\pi x}{a} \sin \frac{m\pi x}{a} \sin \frac{k\pi x}{a}\, dx &= \frac{a}{4} \\
\int_o^a \sin^2 \frac{\pi x}{a} \cos \frac{m\pi x}{a} \cos \frac{k\pi x}{a}\, dx &= \frac{a}{4} \\
\int_o^a \cos^2 \frac{\pi x}{a} \sin \frac{m\pi x}{a} \sin \frac{k\pi x}{a}\, dx &= \frac{a}{4} \\
\int_o^a \cos^2 \frac{\pi x}{a} \cos \frac{m\pi x}{a} \cos \frac{k\pi x}{a}\, dx &= \frac{a}{4} \\
\int_o^a \sin \frac{2\pi x}{a} \sin \frac{m\pi x}{a} \cos \frac{k\pi x}{a}\, dx &= 0
\end{aligned} \tag{A-3}$$

For $m = k$ or $m \neq k$:

$$\int_o^a \sin \frac{m\pi x}{a} \sin \frac{k\pi x}{a}\, dx = \begin{cases} \frac{a}{2} & ; \quad m = k \\ 0 & ; \quad m \neq k \end{cases} \tag{A-4}$$

For $m \pm k =$ odd or even:

$$\int_o^a \sin \frac{m\pi x}{a} \cos \frac{k\pi x}{a}\, dx = \begin{cases} \frac{2a}{\pi} \frac{m}{m^2 - k^2} & ; \quad m \pm k = \text{odd} \\ 0 & ; \quad m \pm k = \text{even} \end{cases}$$

$$\int_o^a \sin \frac{2\pi x}{a} \sin \frac{m\pi x}{a} \sin \frac{k\pi x}{a}\, dx = \begin{cases} -\frac{8a}{\pi} \frac{mk}{[(m+k)^2 - 4][(m-k)^2 - 4]} & ; \quad m \pm k = \text{odd} \\ 0 & ; \quad m \pm k = \text{even} \end{cases}$$

$$\int_o^a \sin^2 \frac{\pi x}{a} \sin \frac{m\pi x}{a} \cos \frac{k\pi x}{a}\, dx = \begin{cases} \frac{am}{\pi} \left\{ \frac{1}{m^2 - k^2} - \frac{(m^2 - k^2) - 4}{[(m+k)^2 - 4][(m-k)^2 - 4]} \right\} & ; \quad m \pm k = \text{odd} \\ 0 & ; \quad m \pm k = \text{even} \end{cases} \tag{A-5}$$

APPENDIX B
ENERGY EQUATIONS

After substitutions of the trial deformation functions (eqs. (24) through (35)) into the energy equations for V_1 (eq. (11)) and V_2 (eq. (9)) (signs of forcing functions reversed according to eq. (12) or (13)), and carrying out double integrations using the special integral relations given in Appendix A, the energy components ΔV_1 and ΔV_2 respectively for V_1 and V_2 have the following expressions for different edge conditions under particular indicial conditions.

Case 1. Four edges simply supported (4S edge condition).

(1) $\underline{m = k,\ n = \ell}$

$$\begin{aligned}\Delta V_1 &= \frac{ab}{4}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left\{\frac{1}{2}\left[D_{11}\left(\frac{m\pi}{a}\right)^4 + 2(D_{12}+2D_{66})\left(\frac{m\pi}{a}\right)^2\left(\frac{n\pi}{b}\right)^2 + D_{22}\left(\frac{n\pi}{b}\right)^2\right] A_{mn}^2\right.\\
&\quad - \left[D_{11}\left(\frac{m\pi}{a}\right)^3 + (D_{12}+2D_{66})\left(\frac{m\pi}{a}\right)\left(\frac{n\pi}{b}\right)^2\right] A_{mn}\,B_{mn}\\
&\quad - \left[D_{22}\left(\frac{n\pi}{b}\right)^3 + (D_{12}+2D_{66})\left(\frac{m\pi}{a}\right)^2\left(\frac{n\pi}{b}\right)\right] A_{mn}\,C_{mn}\\
&\quad + \frac{1}{2}\left[D_{11}\left(\frac{m\pi}{a}\right)^2 + D_{66}\left(\frac{n\pi}{b}\right)^2 + D_{Qx}\right] B_{mn}^2\\
&\quad + \left[(D_{12}+D_{66})\left(\frac{m\pi}{a}\right)\left(\frac{n\pi}{b}\right)\right] B_{mn}C_{mn}\\
&\quad \left. + \frac{1}{2}\left[D_{22}\left(\frac{n\pi}{b}\right)^2 + D_{66}\left(\frac{m\pi}{a}\right)^2 + D_{Qy}\right] C_{mn}^2\right\}\end{aligned} \tag{B-1}$$

$$\Delta V_2 = \frac{ab}{4}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left\{-\frac{1}{2}\left[N_x\left(\frac{m\pi}{a}\right)^2 + N_y\left(\frac{n\pi}{b}\right)^2\right] A_{mn}^2\right\} \tag{B-2}$$

(2) $\underline{m \pm k = \text{odd},\ n \pm \ell = \text{odd}}$

$$\Delta V_1 = 0 \tag{B-3}$$

$$\Delta V_2 = 4N_{xy}\sum_{m}\sum_{n}\sum_{k}\sum_{\ell}\frac{mnk\ell}{(m^2-k^2)(n^2-\ell^2)}A_{mn}A_{k\ell} \tag{B-4}$$

Case 2. Four edges clamped (4C edge condition).

(1) $\underline{m = k \neq 1,\ n = \ell = 1}$

$$\begin{aligned}\Delta V_1 = \frac{ab}{32}\Bigg\{&\left[12D_{11}\left(\frac{\pi}{a}\right)^2 + 4D_{66}\left(\frac{\pi}{b}\right)^2\right]\left(\frac{\pi}{a}A_{11} - B_{11}\right)^2 \\ &+ \left[8(D_{12} + D_{66})\left(\frac{\pi}{a}\right)\left(\frac{\pi}{b}\right)\right]\left(\frac{\pi}{a}A_{11} - B_{11}\right)\left(\frac{\pi}{b}A_{11} - C_{11}\right) \\ &+ \left[12D_{22}\left(\frac{\pi}{b}\right)^2 + 4D_{66}\left(\frac{\pi}{a}\right)^2\right]\left(\frac{\pi}{b}A_{11} - C_{11}\right)^2 \\ &+ 3D_{Qx}B_{11}^2 + 3D_{Qy}C_{11}^2\Bigg\}\end{aligned} \tag{B-5}$$

$$\Delta V_2 = \frac{ab}{32}\left\{-3\left[N_x\left(\frac{\pi}{a}\right)^2 + N_y\left(\frac{\pi}{b}\right)^2\right]A_{11}^2\right\} \tag{B-6}$$

(2) $\underline{m = k = 1,\ n = \ell \neq 1}$

$$\begin{aligned}\Delta V_1 = \frac{ab}{32}\sum_{n=2}^{\infty}\Bigg\{&\left[8D_{11}\left(\frac{\pi}{a}\right)^2 + 2D_{66}\left(\frac{\pi}{b}\right)^2(1+n^2)\right]\left(\frac{\pi}{a}A_{1n} - B_{1n}\right)^2 \\ &+ \left[4(D_{12} + D_{66})\left(\frac{\pi}{a}\right)\left(\frac{\pi}{b}\right)(1+n^2)\right]\left(\frac{\pi}{a}A_{1n} - B_{1n}\right)\left(\frac{\pi}{b}A_{1n} - C_{1n}\right) \\ &+ \left[\frac{3}{2}D_{22}\left(\frac{\pi}{b}\right)^2[(1+n^2)^2 + 4n^2] + 2D_{66}\left(\frac{\pi}{a}\right)^2(1+n^2)\right]\left(\frac{\pi}{b}A_{1n} - C_{1n}\right)^2 \\ &+ 2D_{Qx}B_{in}^2 + \frac{3}{2}D_{Qy}(1+n^2)C_{1n}^2\Bigg\}\end{aligned} \tag{B-7}$$

$$\Delta V_2 = \frac{ab}{32}\sum_{n=2}^{\infty}\left\{-\left[2N_x\left(\frac{\pi}{a}\right)^2 + \frac{3}{2}N_y\left(\frac{\pi}{b}\right)^2(1+n^2)\right]A_{1n}^2\right\} \tag{B-8}$$

(3) $\underline{m = k \neq 1,\ n = \ell = 1}$

$$\begin{aligned}\Delta V_1 = \frac{ab}{32}\sum_{m=2}^{\infty}\Bigg\{&\left[\frac{3}{2}D_{11}\left(\frac{\pi}{a}\right)^2[(1+m^2)^2 + 4m^2] + 2D_{66}\left(\frac{\pi}{b}\right)^2(1+m^2)\right]\left(\frac{\pi}{a}A_{m1} - B_{m1}\right)^2 \\ &+ \left[4(D_{12} + D_{66})\left(\frac{\pi}{a}\right)\left(\frac{\pi}{b}\right)(1+m^2)\right]\left(\frac{\pi}{a}A_{m1} - B_{m1}\right)\left(\frac{\pi}{b}A_{m1} - C_{m1}\right) \\ &+ \left[8D_{22}\left(\frac{\pi}{b}\right)^2 + 2D_{66}\left(\frac{\pi}{a}\right)^2(1+m^2)\right]\left(\frac{\pi}{b}A_{m1} - C_{m1}\right)^2 \\ &+ \frac{3}{2}D_{Qx}(1+m^2)B_{m1}^2 + 2D_{Qy}C_{m1}^2\Bigg\}\end{aligned} \tag{B-9}$$

$$\Delta V_2 = \frac{ab}{32}\sum_{m=2}^{\infty}\left\{-\left[\frac{3}{2}N_x\left(\frac{\pi}{a}\right)^2(1+m^2) + 2N_y\left(\frac{\pi}{b}\right)^2\right]A_{m1}^2\right\} \tag{B-10}$$

(4) $\underline{m = k \neq 1,\ n = \ell \neq 1}$

$$\Delta V_1 = \frac{ab}{32}\sum_{m=2}^{\infty}\sum_{n=2}^{\infty}\left\{\left[D_{11}\left(\frac{\pi}{a}\right)^2[(1+m^2)^2+4m^2]+D_{66}\left(\frac{\pi}{b}\right)^2(1+m^2)(1+n^2)\right]\left(\frac{\pi}{a}A_{mn}-B_{mn}\right)^2\right.$$
$$+\left[2(D_{12}+D_{66})\left(\frac{\pi}{a}\right)\left(\frac{\pi}{b}\right)(1+m^2)(1+n^2)\right]\left(\frac{\pi}{a}A_{mn}-B_{mn}\right)\left(\frac{\pi}{b}A_{mn}-C_{mn}\right)$$
$$+\left[D_{22}\left(\frac{\pi}{b}\right)^2[(1+n^2)^2+4n^2]+D_{66}\left(\frac{\pi}{a}\right)^2(1+m^2)(1+n^2)\right]\left(\frac{\pi}{b}A_{mn}-C_{mn}\right)^2$$
$$\left.+D_{Qx}(1+m^2)\,B_{mn}^2+D_{Qy}(1+n^2)C_{mn}^2\right\} \tag{B-11}$$

$$\Delta V_2 = \frac{ab}{32}\sum_{m=2}^{\infty}\sum_{n=2}^{\infty}\left\{-\left[N_x\left(\frac{\pi}{a}\right)^2(1+m^2)+N_y\left(\frac{\pi}{b}\right)^2(1+n^2)\right]A_{mn}^2\right\} \tag{B-12}$$

(5) $\underline{m = k = 1,\ n - \ell = 2}$

$$\Delta V_1 = \frac{ab}{32}\sum_{n}\sum_{\ell}\left\{-\left[4D_{11}\left(\frac{\pi}{a}\right)^2+D_{66}\left(\frac{\pi}{b}\right)^2(1+\ell)^2\right]\left(\frac{\pi}{a}A_{1n}-B_{1n}\right)\left(\frac{\pi}{a}A_{1\ell}-B_{1\ell}\right)\right.$$
$$-\left[2(D_{12}+D_{66})\left(\frac{\pi}{a}\right)\left(\frac{\pi}{b}\right)(1+\ell)^2\right]\left(\frac{\pi}{a}A_{1n}-B_{1n}\right)\left(\frac{\pi}{b}A_{1\ell}-C_{1\ell}\right)$$
$$-\left[\frac{3}{4}D_{22}\left(\frac{\pi}{b}\right)^2(1+\ell)^4+D_{66}\left(\frac{\pi}{a}\right)^2(1+\ell)^2\right]\left(\frac{\pi}{b}A_{1n}-C_{1n}\right)\left(\frac{\pi}{b}A_{1\ell}-C_{1\ell}\right)$$
$$\left.-D_{Qx}B_{1n}B_{1\ell}-\frac{3}{4}D_{Qy}(1+\ell)^2\,C_{1n}C_{1\ell}\right\} \tag{B-13}$$

$$\Delta V_2 = \frac{ab}{32}\sum_{n}\sum_{\ell}\left\{\left[N_x\left(\frac{\pi}{a}\right)^2+\frac{3}{4}N_y\left(\frac{\pi}{b}\right)^2(1+\ell)^2\right]A_{1n}A_{1\ell}\right\} \tag{B-14}$$

(6) $\underline{m = k \neq 1,\ n - \ell = 2}$

$$\Delta V_1 = \frac{ab}{32}\sum_{m=2}^{\infty}\sum_{n}\sum_{\ell}\left\{-\frac{1}{2}\left[D_{11}\left(\frac{\pi}{a}\right)^2[(1+m^2)^2+4m^2]+D_{66}\left(\frac{\pi}{b}\right)^2(1+m^2)(1+\ell)^2\right]\right.$$
$$\times\left(\frac{\pi}{a}A_{mn}-B_{mn}\right)\left(\frac{\pi}{a}A_{m\ell}-B_{m\ell}\right)$$
$$-\left[(D_{12}+D_{66})\left(\frac{\pi}{a}\right)\left(\frac{\pi}{b}\right)(1+m^2)(1+\ell)^2\right]\left(\frac{\pi}{a}A_{mn}-B_{mn}\right)\left(\frac{\pi}{b}A_{m\ell}-C_{m\ell}\right)$$
$$-\frac{1}{2}\left[D_{22}\left(\frac{\pi}{b}\right)^2(1+\ell)^4+D_{66}\left(\frac{\pi}{a}\right)^2(1+m^2)(1+\ell)^2\right]\left(\frac{\pi}{b}A_{mn}-C_{mn}\right)$$
$$\times\left(\frac{\pi}{b}A_{m\ell}-C_{m\ell}\right)$$
$$\left.-\frac{D_{Qx}}{2}(1+m^2)B_{mn}B_{m\ell}-\frac{D_{Qy}}{2}(1+\ell)^2\,C_{mn}C_{m\ell}\right\} \tag{B-15}$$

$$\Delta V_2 = \frac{ab}{32}\sum_{m=2}^{\infty}\sum_{n}\sum_{\ell}\left\{\frac{1}{2}\left[N_x\left(\frac{\pi}{a}\right)^2(1+m^2)+N_y\left(\frac{\pi}{b}\right)^2(1+\ell)^2\right]A_{mn}A_{m\ell}\right\} \tag{B-16}$$

(7) $\underline{m - k = 2, n = \ell = 1}$

$$\Delta V_1 = \frac{ab}{32}\sum_m\sum_k\left\{-\left[\frac{3}{4}D_{11}\left(\frac{\pi}{a}\right)^2(1+k)^4 + D_{66}\left(\frac{\pi}{b}\right)^2(1+k)^2\right]\left(\frac{\pi}{a}A_{m1} - B_{m1}\right)\left(\frac{\pi}{a}A_{k1} - B_{k1}\right)\right.$$
$$- \left[2(D_{12}+D_{66})\left(\frac{\pi}{a}\right)\left(\frac{\pi}{b}\right)(1+k)^2\right]\left(\frac{\pi}{a}A_{m1} - B_{m1}\right)\left(\frac{\pi}{b}A_{k1} - C_{k1}\right)$$
$$- \left[4D_{22}\left(\frac{\pi}{b}\right)^2 + D_{66}\left(\frac{\pi}{a}\right)^2(1+k)^2\right]\left(\frac{\pi}{b}A_{m1} - C_{m1}\right)\left(\frac{\pi}{b}A_{k1} - C_{k1}\right)$$
$$\left. - \frac{3}{4}D_{Qx}(1+k)^2 B_{m1}B_{k1} - D_{Qy}C_{m1}C_{k1}\right\} \tag{B-17}$$

$$\Delta V_2 = \frac{ab}{32}\sum_m\sum_k\left\{\left[\frac{3}{4}N_x\left(\frac{\pi}{a}\right)^2(1+k)^2 + N_y\left(\frac{\pi}{b}\right)^2\right]A_{m1}A_{k1}\right\} \tag{B-18}$$

(8) $\underline{m - k = 2\ ,\ n = \ell \neq 1}$

$$\Delta V_1 = \frac{ab}{32}\sum_m\sum_{n=2}^{\infty}\sum_k\left\{-\frac{1}{2}\left[D_{11}\left(\frac{\pi}{a}\right)^2(1+k)^4 + D_{66}\left(\frac{\pi}{b}\right)^2(1+k)^2(1+n^2)\right]\right.$$
$$\times\left(\frac{\pi}{a}A_{mn} - B_{mn}\right)\left(\frac{\pi}{a}A_{kn} - B_{kn}\right)$$
$$- \left[(D_{12}+D_{66})\left(\frac{\pi}{a}\right)\left(\frac{\pi}{b}\right)(1+k)^2(1+n^2)\right]\left(\frac{\pi}{a}A_{mn} - B_{mn}\right)\left(\frac{\pi}{b}A_{kn} - C_{kn}\right)$$
$$- \frac{1}{2}\left[D_{22}\left(\frac{\pi}{b}\right)^2[(1+n^2)^2 + 4n^2] + D_{66}\left(\frac{\pi}{a}\right)^2(1+k)^2(1+n^2)\right]\left(\frac{\pi}{b}A_{mn} - C_{mn}\right)$$
$$\times\left(\frac{\pi}{b}A_{kn} - C_{kn}\right)$$
$$\left. - \frac{D_{Qx}}{2}(1+k)^2 B_{mn}B_{kn} - \frac{D_{Qy}}{2}(1+n^2)\,C_{mn}C_{kn}\right\} \tag{B-19}$$

$$\Delta V_2 = \frac{ab}{32}\sum_m\sum_{n=2}^{\infty}\sum_k\left\{\frac{1}{2}\left[N_x\left(\frac{\pi}{a}\right)^2(1+k)^2 + N_y\left(\frac{\pi}{b}\right)^2(1+n^2)\right]A_{mn}A_{kn}\right\} \tag{B-20}$$

(9) $\underline{m - k = 2 \,,\, n - \ell = 2}$

$$\begin{aligned}\Delta V_1 = \frac{ab}{32}\sum_m\sum_n\sum_k\sum_\ell \Bigg\{ & \frac{1}{4}\left[D_{11}\left(\frac{\pi}{a}\right)^2(1+k)^4 + D_{66}\left(\frac{\pi}{b}\right)^2(1+k)^2(1+\ell)^2\right] \\ & \times \left(\frac{\pi}{a}A_{mn} - B_{mn}\right)\left(\frac{\pi}{a}A_{k\ell} - B_{k\ell}\right) \\ & + \frac{1}{2}\left[(D_{12}+D_{66})\left(\frac{\pi}{a}\right)\left(\frac{\pi}{b}\right)(1+k)^2(1+\ell)^2\right]\left(\frac{\pi}{a}A_{mn} - B_{mn}\right)\left(\frac{\pi}{b}A_{k\ell} - C_{k\ell}\right) \\ & + \frac{1}{4}\left[D_{22}\left(\frac{\pi}{b}\right)^2(1+\ell)^4 + D_{66}\left(\frac{\pi}{a}\right)^2(1+k)^2(1+\ell)^2\right]\left(\frac{\pi}{a}A_{mn} - C_{mn}\right) \\ & \times \left(\frac{\pi}{b}A_{k\ell} - C_{k\ell}\right) \\ & + \frac{D_{Qx}}{4}(1+k)^2 B_{mn}\, B_{k\ell} + \frac{D_{Qy}}{4}(1+\ell)^2 C_{mn}C_{k\ell}\Bigg\} \end{aligned} \tag{B-21}$$

$$\Delta V_2 = \frac{ab}{32}\sum_m\sum_n\sum_k\sum_\ell\left\{-\frac{1}{4}\left[N_x\left(\frac{\pi}{a}\right)^2(1+k)^2 + N_y\left(\frac{\pi}{b}\right)^2(1+\ell)^2\right]A_{mn}A_{k\ell}\right\} \tag{B-22}$$

(10) $\underline{m \pm k = \text{odd},\ n \pm \ell = \text{odd}}$

$$\Delta V_1 = 0 \tag{B-23}$$

$$\Delta V_2 = 64N_{xy}\sum_m\sum_n\sum_k\sum_\ell \frac{mnk\ell[m^2+k^2-2][n^2+\ell^2-2]A_{mn}A_{k\ell}}{(m^2-k^2)(n^2-\ell^2)[(m+k)^2-4][(m-k)^2-4][(n+\ell)^2-4][(n-\ell)^2-4]} \tag{B-24}$$

Case 3. Two sides clamped, two ends simply supported (2C2S edge condition).

(1) $\underline{m = k,\ n = \ell = 1}$

$$\begin{aligned}\Delta V_1 = \frac{ab}{32}\sum_{m=1}^{\infty}\Bigg\{ & \left[3D_{11}\left(\frac{m\pi}{a}\right)^2 m^2 + 4D_{66}\left(\frac{\pi}{b}\right)^2 m^2\right]\left(\frac{\pi}{a}A_{m1} - \frac{B_{m1}}{m}\right)^2 \\ & + \left[8(D_{12}+D_{66})\left(\frac{\pi}{a}\right)\left(\frac{\pi}{b}\right)m^2\right]\left(\frac{\pi}{a}A_{m1} - \frac{B_{m1}}{m}\right)\left(\frac{\pi}{b}A_{m1} - C_{m1}\right) \\ & + \left[16D_{22}\left(\frac{\pi}{b}\right)^2 + 4D_{66}\left(\frac{m\pi}{a}\right)^2\right]\left(\frac{\pi}{b}A_{m1} - C_{m1}\right)^2 \\ & + 3D_{Qx}B_{m1}^2 + 4D_{Qy}C_{m1}^2\Bigg\} \end{aligned} \tag{B-25}$$

$$\Delta V_2 = \frac{ab}{32}\sum_{m=1}^{\infty}\left\{-\left[3N_x\left(\frac{m\pi}{a}\right)^2 + 4N_y\left(\frac{\pi}{b}\right)^2\right]A_{m1}^2\right\} \tag{B-26}$$

(2) $m = k,\ n = \ell \neq 1$

$$\Delta V_1 = \frac{ab}{32}\sum_{m=1}^{\infty}\sum_{n=2}^{\infty}\left\{\left[2D_{11}\left(\frac{m\pi}{a}\right)^2 m^2 + 2D_{66}\left(\frac{\pi}{b}\right)^2 m^2(1+n^2)\right]\left(\frac{\pi}{a}A_{mn} - \frac{B_{mn}}{m}\right)^2\right.$$
$$+\left[4(D_{12}+D_{66})\left(\frac{\pi}{a}\right)\left(\frac{\pi}{b}\right)m^2(1+n^2)\right]\left(\frac{\pi}{a}A_{mn} - \frac{B_{mn}}{m}\right)\left(\frac{\pi}{b}A_{mn} - C_{mn}\right)$$
$$+\left[2D_{22}\left(\frac{\pi}{b}\right)^2[(1+n^2)^2+4n^2] + 2D_{66}\left(\frac{m\pi}{a}\right)^2(1+n^2)\right]\left(\frac{\pi}{b}A_{mn} - C_{mn}\right)^2$$
$$\left.+\,2D_{Qx}B_{mn}^2 + 2D_{Qy}(1+n^2)C_{mn}^2\right\} \tag{B-27}$$

$$\Delta V_2 = \frac{ab}{32}\sum_{m=1}^{\infty}\sum_{n=2}^{\infty}\left\{-2\left[N_x\left(\frac{m\pi}{a}\right)^2 + N_y\left(\frac{\pi}{b}\right)^2(1+n^2)\right]A_{mn}^2\right\} \tag{B-28}$$

(3) $m = k,\ n - \ell = 2$

$$\Delta V_1 = \frac{ab}{32}\sum_{m=1}^{\infty}\sum_{n}\sum_{\ell}\left\{-\left[D_{11}\left(\frac{m\pi}{a}\right)^2 m^2 + D_{66}\left(\frac{\pi}{b}\right)^2 m^2(1+\ell)^2\right]\right.$$
$$\times\left(\frac{\pi}{a}A_{mn} - \frac{B_{mn}}{m}\right)\left(\frac{\pi}{a}A_{m\ell} - \frac{B_{m\ell}}{m}\right)$$
$$-\left[2(D_{12}+D_{66})\left(\frac{\pi}{a}\right)\left(\frac{\pi}{b}\right)m^2(1+\ell)^2\right]\left(\frac{\pi}{a}A_{mn} - \frac{B_{mn}}{m}\right)\left(\frac{\pi}{b}A_{m\ell} - C_{m\ell}\right)$$
$$-\left[D_{22}\left(\frac{\pi}{b}\right)^2(1+\ell)^4 + D_{66}\left(\frac{m\pi}{a}\right)^2(1+\ell)^2\right]\left(\frac{\pi}{b}A_{mn} - C_{mn}\right)\left(\frac{\pi}{b}A_{m\ell} - C_{m\ell}\right)$$
$$\left.-\,D_{Qx}B_{mn}B_{m\ell} - D_{Qy}(1+\ell)^2 C_{mn}C_{m\ell}\right\} \tag{B-29}$$

$$\Delta V_2 = \frac{ab}{32}\sum_{m=1}^{\infty}\sum_{n}\sum_{\ell}\left\{\left[N_x\left(\frac{m\pi}{a}\right)^2 + N_y\left(\frac{\pi}{b}\right)^2(1+\ell)^2\right]A_{mn}A_{m\ell}\right\} \tag{B-30}$$

(4) $m \pm k = \text{odd},\ n \pm \ell = \text{odd}$

$$\Delta V_1 = 0 \tag{B-31}$$

$$\Delta V_2 = 16N_{xy}\sum_{m}\sum_{n}\sum_{k}\sum_{\ell}\frac{mnk\ell\left[2-(n^2+\ell^2)\right]A_{mn}A_{k\ell}}{(m^2-k^2)(n^2-\ell^2)\left[(n+\ell)^2-4\right]\left[(n-\ell)^2-4\right]} \tag{B-32}$$

Case 4. Two sides simply supported, two ends clamped (2S2C edge condition).

(1) $\underline{m = k = 1,\ n = \ell}$

$$\begin{aligned}\Delta V_1 = \frac{ab}{32}\sum_{n=1}^{\infty}\Bigg\{&\left[16D_{11}\left(\frac{\pi}{a}\right)^2 + 4D_{66}\left(\frac{n\pi}{b}\right)^2\right]\left(\frac{\pi}{a}A_{1n} - B_{1n}\right)^2 \\ &+ \left[8(D_{12}+D_{66})\left(\frac{\pi}{a}\right)\left(\frac{\pi}{b}\right)n^2\right]\left(\frac{\pi}{a}A_{1n} - B_{1n}\right)\left(\frac{\pi}{b}A_{1n} - \frac{C_{1n}}{n}\right) \\ &+ \left[3D_{22}\left(\frac{n\pi}{b}\right)^2 n^2 + 4D_{66}\left(\frac{\pi}{a}\right)^2 n^2\right]\left(\frac{\pi}{b}A_{1n} - \frac{C_{1n}}{n}\right)^2 \\ &+ 4D_{Qx}B_{1n}^2 + 3D_{Qy}C_{1n}^2\Bigg\}\end{aligned} \tag{B-33}$$

$$\Delta V_2 = \frac{ab}{32}\sum_{n=1}^{\infty}\left\{-\left[4N_x\left(\frac{\pi}{a}\right)^2 + 3N_y\left(\frac{n\pi}{b}\right)^2\right]A_{1n}^2\right\} \tag{B-34}$$

(2) $\underline{m = k \neq 1,\ n = \ell}$

$$\begin{aligned}\Delta V_1 = \frac{ab}{32}\sum_{m=2}^{\infty}\sum_{n=1}^{\infty}\Bigg\{&\left[2D_{11}\left(\frac{\pi}{a}\right)^2\left[(1+m^2)^2 + 4m^2\right] + 2D_{66}\left(\frac{n\pi}{b}\right)^2(1+m^2)\right]\left(\frac{\pi}{a}A_{mn} - B_{mn}\right)^2 \\ &+ \left[4(D_{12}+D_{66})\left(\frac{\pi}{a}\right)\left(\frac{\pi}{b}\right)(1+m^2)n^2\right]\left(\frac{\pi}{a}A_{mn} - B_{mn}\right)\left(\frac{\pi}{b}A_{mn} - \frac{C_{mn}}{n}\right) \\ &+ \left[2D_{22}\left(\frac{n\pi}{b}\right)^2 n^2 + 2D_{66}\left(\frac{\pi}{a}\right)^2(1+m^2)n^2\right]\left(\frac{\pi}{b}A_{mn} - \frac{C_{mn}}{n}\right)^2 \\ &+ 2D_{Qx}\left(1+m^2\right)B_{mn}^2 + 2D_{Qy}C_{mn}^2\Bigg\}\end{aligned} \tag{B-35}$$

$$\Delta V_2 = \frac{ab}{32}\sum_{m=2}^{\infty}\sum_{n=1}^{\infty}\left\{-2\left[N_x\left(\frac{\pi}{a}\right)^2(1+m^2) + N_y\left(\frac{n\pi}{b}\right)^2\right]A_{mn}^2\right\} \tag{B-36}$$

(3) $\underline{m - k = 2,\ n = \ell}$

$$\begin{aligned}\Delta V_1 = \frac{ab}{32}\sum_{m}\sum_{n=1}^{\infty}\sum_{k}\Bigg\{&-\left[D_{11}\left(\frac{\pi}{a}\right)^2(1+k)^4 + D_{66}\left(\frac{n\pi}{b}\right)^2(1+k)^2\right]\left(\frac{\pi}{a}A_{mn} - B_{mn}\right)\left(\frac{\pi}{a}A_{kn} - B_{kn}\right) \\ &- \left[2(D_{12}+D_{66})\left(\frac{\pi}{a}\right)\left(\frac{\pi}{b}\right)(1+k)^2n^2\right]\left(\frac{\pi}{a}A_{mn} - B_{mn}\right)\left(\frac{\pi}{b}A_{kn} - C_{kn}\right) \\ &- \left[D_{22}\left(\frac{n\pi}{b}\right)^2 n^2 + D_{66}\left(\frac{\pi}{a}\right)^2(1+k)^2n^2\right]\left(\frac{\pi}{b}A_{mn} - \frac{C_{mn}}{n}\right)\left(\frac{\pi}{b}A_{kn} - \frac{C_{kn}}{n}\right) \\ &- D_{Qx}\left(1+k\right)^2 B_{mn}B_{kn} - D_{Qy}C_{mn}C_{kn}\Bigg\}\end{aligned} \tag{B-37}$$

$$\Delta V_2 = \frac{ab}{32}\sum_{m}\sum_{n=1}^{\infty}\sum_{k}\left\{\left[N_x\left(\frac{\pi}{a}\right)^2(1+k)^2 + N_y\left(\frac{n\pi}{b}\right)^2\right]A_{mn}A_{kn}\right\} \tag{B-38}$$

(4) $\underline{m \pm k = \text{odd},\ n \pm \ell = \text{odd}}$

$$\Delta V_1 \ = 0 \tag{B-39}$$

$$\Delta V_2 \ = 16 N_{xy} \sum_m \sum_n \sum_k \sum_\ell \frac{mnk\ell\,[2-(m^2+k^2)]\,A_{mn}A_{k\ell}}{(m^2-k^2)(n^2-\ell^2)\,[(m+k)^2-4]\,[(m-k)^2-4]} \tag{B-40}$$

APPENDIX C
COEFFICIENTS OF CHARACTERISTIC EQUATIONS

The coefficients $a^{ij}_{mnk\ell}$ of the simultaneous characteristic equations (37) through (39) are defined in the following for different edge conditions under different indicial conditions.

Case 1. Four edges simply supported (4S edge condition).

(1) $\underline{m = k,\ n = \ell}$

$$
\begin{aligned}
a^{11}_{mnmn} &= D_{11}\left(\frac{m\pi}{a}\right)^4 + 2(D_{12}+2D_{66})\left(\frac{m\pi}{a}\right)^2\left(\frac{n\pi}{b}\right)^2 + D_{22}\left(\frac{n\pi}{b}\right)^4 \\
&\quad - D^*\left(\frac{\pi}{a}\right)^2\left[k_x\left(\frac{m\pi}{a}\right)^2 + k_y\left(\frac{n\pi}{b}\right)^2\right] \\
a^{12}_{mnmn} &= a^{21}_{mnmn} = -\left[D_{11}\left(\frac{m\pi}{a}\right)^3 + (D_{12}+2D_{66})\left(\frac{m\pi}{a}\right)\left(\frac{n\pi}{b}\right)^2\right] \\
a^{13}_{mnmn} &= a^{31}_{mnmn} = -\left[D_{22}\left(\frac{n\pi}{b}\right)^3 + (D_{12}+2D_{66})\left(\frac{m\pi}{a}\right)^2\left(\frac{n\pi}{b}\right)\right] \\
a^{22}_{mnmn} &= D_{11}\left(\frac{m\pi}{a}\right)^2 + D_{66}\left(\frac{n\pi}{b}\right)^2 + D_{Qx} \\
a^{23}_{mnmn} &= a^{32}_{mnmn} = (D_{12}+D_{66})\left(\frac{m\pi}{a}\right)\left(\frac{n\pi}{b}\right) \\
a^{33}_{mnmn} &= D_{22}\left(\frac{n\pi}{b}\right)^2 + D_{66}\left(\frac{m\pi}{a}\right)^2 + D_{Qy}
\end{aligned}
\qquad \text{(C-1)}
$$

(2) $\underline{m \neq k,\ n \neq \ell}$

$$a^{ij}_{mnk\ell} = 0 \qquad \text{(C-2)}$$

Case 2. Four edges clamped (4C edge condition).

(1) $\underline{m = n = k = \ell = 1}$

$$\begin{aligned}
a_{1111}^{11} &= 12D_{11}\left(\frac{\pi}{a}\right)^4 + 8(D_{12}+2D_{66})\left(\frac{\pi}{a}\right)^2\left(\frac{\pi}{b}\right)^2 + 12D_{22}\left(\frac{\pi}{b}\right)^4 \\
&\quad - 3D^*\left(\frac{\pi}{a}\right)^2\left[k_x\left(\frac{\pi}{a}\right)^2 + k_y\left(\frac{\pi}{b}\right)^2\right] \\
a_{1111}^{12} &= a_{1111}^{21} = -\left[12D_{11}\left(\frac{\pi}{a}\right)^3 + 4(D_{12}+2D_{66})\left(\frac{\pi}{a}\right)\left(\frac{\pi}{b}\right)^2\right] \\
a_{1111}^{13} &= a_{1111}^{31} = -\left[12D_{22}\left(\frac{\pi}{b}\right)^3 + 4(D_{12}+2D_{66})\left(\frac{\pi}{a}\right)^2\left(\frac{\pi}{b}\right)\right] \\
a_{1111}^{22} &= 12D_{11}\left(\frac{\pi}{a}\right)^2 + 4D_{66}\left(\frac{\pi}{b}\right)^2 + 3D_{Qx} \\
a_{1111}^{23} &= a_{1111}^{32} = 4(D_{12}+D_{66})\left(\frac{\pi}{a}\right)\left(\frac{\pi}{b}\right) \\
a_{1111}^{33} &= 12D_{22}\left(\frac{\pi}{b}\right)^2 + 4D_{66}\left(\frac{\pi}{a}\right)^2 + 3D_{Qy}
\end{aligned} \tag{C-3}$$

(2) $\underline{m = k = 1,\ n = \ell \neq 1}$

$$\begin{aligned}
a_{1n1n}^{11} &= 8D_{11}\left(\frac{\pi}{a}\right)^4 + 4(D_{12}+2D_{66})\left(\frac{\pi}{a}\right)^2\left(\frac{\pi}{b}\right)^2(1+n^2) + \frac{3}{2}D_{22}\left(\frac{\pi}{b}\right)^4\left[(1+n^2)^2+4n^2\right] \\
&\quad - D^*\left(\frac{\pi}{a}\right)^2\left[2k_x\left(\frac{\pi}{a}\right)^2 + \frac{3}{2}k_y\left(\frac{\pi}{b}\right)^2(1+n^2)\right] \\
a_{1n1n}^{12} &= a_{1n1n}^{21} = -\left[8D_{11}\left(\frac{\pi}{a}\right)^3 + 2(D_{12}+2D_{66})\left(\frac{\pi}{a}\right)\left(\frac{\pi}{b}\right)^2(1+n^2)\right] \\
a_{1n1n}^{13} &= a_{1n1n}^{31} = -\left\{\frac{3}{2}D_{22}\left(\frac{\pi}{b}\right)^3\left[(1+n^2)^2+4n^2\right] + 2(D_{12}+2D_{66})\left(\frac{\pi}{a}\right)^2\left(\frac{\pi}{b}\right)(1+n^2)\right\} \\
a_{1n1n}^{22} &= 8D_{11}\left(\frac{\pi}{a}\right)^2 + 2D_{66}\left(\frac{\pi}{b}\right)^2(1+n^2) + 2D_{Qx} \\
a_{1n1n}^{23} &= a_{1n1n}^{32} = 2(D_{12}+D_{66})\left(\frac{\pi}{a}\right)\left(\frac{\pi}{b}\right)(1+n^2) \\
a_{1n1n}^{33} &= \frac{3}{2}D_{22}\left(\frac{\pi}{b}\right)^2\left[(1+n^2)^2+4n^2\right] + 2D_{66}\left(\frac{\pi}{a}\right)^2(1+n^2) + \frac{3}{2}D_{Qy}(1+n^2)
\end{aligned} \tag{C-4}$$

(3) $\underline{m = k \neq 1, n = \ell = 1}$

$$a^{11}_{m1m1} = \frac{3}{2}D_{11}\left(\frac{\pi}{a}\right)^4\left[(1+m^2)^2+4m^2\right] + 4(D_{12}+2D_{66})\left(\frac{\pi}{a}\right)^2\left(\frac{\pi}{b}\right)^2(1+m^2) + 8D_{22}\left(\frac{\pi}{b}\right)^4 - D^*\left(\frac{\pi}{a}\right)^2\left[\frac{3}{2}k_x\left(\frac{\pi}{a}\right)^2(1+m^2) + 2k_y\left(\frac{\pi}{b}\right)^2\right]$$

$$a^{12}_{m1m1} = a^{21}_{m1m1} = -\left\{\frac{3}{2}D_{11}\left(\frac{\pi}{a}\right)^3\left[(1+m^2)^2+4m^2\right] + 2(D_{12}+2D_{66})\left(\frac{\pi}{a}\right)\left(\frac{\pi}{b}\right)^2(1+m^2)\right\}$$

$$a^{13}_{m1m1} = a^{31}_{m1m1} = -\left[8D_{22}\left(\frac{\pi}{b}\right)^3 + 2(D_{12}+2D_{66})\left(\frac{\pi}{a}\right)^2\left(\frac{\pi}{b}\right)(1+m^2)\right]$$

$$a^{22}_{m1m1} = \frac{3}{2}D_{11}\left(\frac{\pi}{a}\right)^2\left[(1+m^2)^2+4m^2\right] + 2D_{66}\left(\frac{\pi}{b}\right)^2(1+m^2) + \frac{3}{2}D_{Qx}(1+m^2)$$

$$a^{23}_{m1m1} = a^{32}_{m1m1} = 2(D_{12}+D_{66})\left(\frac{\pi}{a}\right)\left(\frac{\pi}{b}\right)(1+m^2)$$

$$a^{33}_{m1m1} = 8D_{22}\left(\frac{\pi}{b}\right)^2 + 2D_{66}\left(\frac{\pi}{a}\right)^2(1+m^2) + 2D_{Qy} \tag{C-5}$$

(4) $\underline{m = k \neq 1, n = \ell \neq 1}$

$$a^{11}_{mnmn} = D_{11}\left(\frac{\pi}{a}\right)^4\left[(1+m^2)^2+4m^2\right] + 2(D_{12}+2D_{66})\left(\frac{\pi}{a}\right)^2\left(\frac{\pi}{b}\right)^2(1+m^2)(1+n^2) + D_{22}\left(\frac{\pi}{b}\right)^4\left[(1+n^2)^2+4n^2\right] - D^*\left(\frac{\pi}{a}\right)^2\left[k_x\left(\frac{\pi}{a}\right)^2(1+m^2) + k_y\left(\frac{\pi}{b}\right)^2(1+n^2)\right]$$

$$a^{12}_{mnmn} = a^{21}_{mnmn} = -\left\{D_{11}\left(\frac{\pi}{a}\right)^3\left[(1+m^2)^2+4m^2\right] + (D_{12}+2D_{66})\left(\frac{\pi}{a}\right)\left(\frac{\pi}{b}\right)^2(1+m^2)(1+n^2)\right\}$$

$$a^{13}_{mnmn} = a^{31}_{mnmn} = -\left\{D_{22}\left(\frac{\pi}{b}\right)^3\left[(1+n^2)^2+4n^2\right] + (D_{12}+2D_{66})\left(\frac{\pi}{a}\right)^2\left(\frac{\pi}{b}\right)(1+m^2)(1+n^2)\right\}$$

$$a^{22}_{mnmn} = D_{11}\left(\frac{\pi}{a}\right)^2\left[(1+m^2)^2+4m^2\right] + D_{66}\left(\frac{\pi}{b}\right)^2(1+m^2)(1+n^2) + D_{Qx}(1+m^2)$$

$$a^{23}_{mnmn} = a^{32}_{mnmn} = (D_{12}+D_{66})\left(\frac{\pi}{a}\right)\left(\frac{\pi}{b}\right)(1+m^2)(1+n^2)$$

$$a^{33}_{mnmn} = D_{22}\left(\frac{\pi}{b}\right)^2\left[(1+n^2)^2+4n^2\right] + 2D_{66}\left(\frac{\pi}{a}\right)^2(1+m^2)(1+n^2) + D_{Qy}(1+n^2) \tag{C-6}$$

(5) $\underline{m = k = 1,\ n - \ell = 2}$

$$a^{11}_{1n1\ell} = -\left[4D_{11}\left(\frac{\pi}{a}\right)^4 + 2(D_{12}+2D_{66})\left(\frac{\pi}{a}\right)^2\left(\frac{\pi}{b}\right)^2(1+\ell)^2 + \frac{3}{4}D_{22}\left(\frac{\pi}{b}\right)^4(1+\ell)^4\right]$$
$$+ D^*\left(\frac{\pi}{a}\right)^2\left[k_x\left(\frac{\pi}{a}\right)^2 + \frac{3}{4}k_y\left(\frac{\pi}{b}\right)^2(1+\ell)^2\right]$$
$$a^{12}_{1n1\ell} = a^{21}_{1n1\ell} = 4D_{11}\left(\frac{\pi}{a}\right)^3 + (D_{12}+2D_{66})\left(\frac{\pi}{a}\right)\left(\frac{\pi}{b}\right)^2(1+\ell)^2$$
$$a^{13}_{1n1\ell} = a^{31}_{1n1\ell} = \frac{3}{4}D_{22}\left(\frac{\pi}{b}\right)^3(1+\ell)^4 + (D_{12}+2D_{66})\left(\frac{\pi}{a}\right)^2\left(\frac{\pi}{b}\right)(1+\ell)^2$$
$$a^{22}_{1n1\ell} = -\left[4D_{11}\left(\frac{\pi}{a}\right)^2 + D_{66}\left(\frac{\pi}{b}\right)^2(1+\ell)^2 + D_{Qx}\right]$$
$$a^{23}_{1n1\ell} = a^{32}_{1n1\ell} = -(D_{12}+D_{66})\left(\frac{\pi}{a}\right)\left(\frac{\pi}{b}\right)(1+\ell)^2$$
$$a^{33}_{1n1\ell} = -\left[\frac{3}{4}D_{22}\left(\frac{\pi}{b}\right)^2(1+\ell)^4 + D_{66}\left(\frac{\pi}{a}\right)^2(1+\ell)^2 + \frac{3}{4}D_{Qy}(1+\ell)^2\right] \tag{C-7}$$

(6) $\underline{m = k \neq 1,\ n - \ell = 2}$

$$a^{11}_{mnm\ell} = -\frac{1}{2}\left\{D_{11}\left(\frac{\pi}{a}\right)^4\left[(1+m^2)^2+4m^2\right] + 2(D_{12}+2D_{66}\left(\frac{\pi}{a}\right)^2\left(\frac{\pi}{b}\right)^2(1+m^2)(1+\ell)^2\right.$$
$$\left. + D_{22}\left(\frac{\pi}{b}\right)^4(1+\ell)^4\right\} + \frac{D^*}{2}\left(\frac{\pi}{a}\right)^2\left[k_x\left(\frac{\pi}{a}\right)^2(1+m^2) + k_y\left(\frac{\pi}{b}\right)^2(1+\ell)^2\right]$$
$$a^{12}_{mnm\ell} = a^{21}_{mnm\ell} = \frac{1}{2}\left\{D_{11}\left(\frac{\pi}{a}\right)^3\left[(1+m^2)^2+4m^2\right] + (D_{12}+2D_{66})\left(\frac{\pi}{a}\right)\left(\frac{\pi}{b}\right)^2(1+m^2)(1+\ell)^2\right\}$$
$$a^{13}_{mnm\ell} = a^{31}_{mnm\ell} = \frac{1}{2}\left\{D_{22}\left(\frac{\pi}{b}\right)^3(1+\ell)^4 + (D_{12}+2D_{66})\left(\frac{\pi}{a}\right)^2\left(\frac{\pi}{b}\right)(1+m^2)(1+\ell)^2\right\}$$
$$a^{22}_{mnm\ell} = -\frac{1}{2}\left\{D_{11}\left(\frac{\pi}{a}\right)^2\left[(1+m^2)^2+4m^2\right] + D_{66}\left(\frac{\pi}{b}\right)^2(1+m^2)(1+\ell)^2 + D_{Qx}(1+m^2)\right\}$$
$$a^{23}_{mnm\ell} = a^{32}_{mnm\ell} = -\frac{1}{2}(D_{12}+D_{66})\left(\frac{\pi}{a}\right)\left(\frac{\pi}{b}\right)(1+m^2)(1+\ell)^2$$
$$a^{33}_{mnm\ell} = -\frac{1}{2}\left[D_{22}\left(\frac{\pi}{b}\right)^2(1+\ell)^4 + D_{66}\left(\frac{\pi}{a}\right)^2(1+m^2)(1+\ell)^2 + D_{Qy}(1+\ell)^2\right] \tag{C-8}$$

(7) $\underline{m - k = 2,\ n = \ell = 1}$

$$a^{11}_{m1k1} = -\left[\frac{3}{4}D_{11}\left(\frac{\pi}{a}\right)^4(1+k)^4 + 2(D_{12}+2D_{66})\left(\frac{\pi}{a}\right)^2\left(\frac{\pi}{b}\right)^2(1+k)^2 + 4D_{22}\left(\frac{\pi}{b}\right)^4\right] + D^*\left(\frac{\pi}{a}\right)^2\left[\frac{3}{4}k_x\left(\frac{\pi}{a}\right)^2(1+k)^2 + k_y\left(\frac{\pi}{b}\right)^2\right]$$

$$a^{12}_{m1k1} = a^{21}_{m1k1} = \frac{3}{4}D_{11}\left(\frac{\pi}{a}\right)^3(1+k)^4 + (D_{12}+2D_{66})\left(\frac{\pi}{a}\right)\left(\frac{\pi}{b}\right)^2(1+k)^2$$

$$a^{13}_{m1k1} = a^{31}_{m1k1} = 4D_{22}\left(\frac{\pi}{b}\right)^3 + (D_{12}+2D_{66})\left(\frac{\pi}{a}\right)^2\left(\frac{\pi}{b}\right)(1+k)^2$$

$$a^{22}_{m1k1} = -\left[\frac{3}{4}D_{11}\left(\frac{\pi}{a}\right)^2(1+k)^4 + D_{66}\left(\frac{\pi}{b}\right)^2(1+k)^2 + \frac{3}{4}D_{Qx}(1+k)^2\right]$$

$$a^{23}_{m1k1} = a^{32}_{m1k1} = -(D_{12}+D_{66})\left(\frac{\pi}{a}\right)\left(\frac{\pi}{b}\right)(1+k)^2$$

$$a^{33}_{m1k1} = -\left[4D_{22}\left(\frac{\pi}{b}\right)^2 + D_{66}\left(\frac{\pi}{a}\right)^2(1+k)^2 + D_{Qy}\right] \tag{C-9}$$

(8) $\underline{m - k = 2,\ n = \ell \neq 1}$

$$a^{11}_{mnkn} = -\frac{1}{2}\left\{D_{11}\left(\frac{\pi}{a}\right)^4(1+k)^4 + 2(D_{12}+2D_{66})\left(\frac{\pi}{a}\right)^2\left(\frac{\pi}{b}\right)^2(1+k)^2(1+n^2) + D_{22}\left(\frac{\pi}{b}\right)^4\left[(1+n^2)^2+4n^2\right]\right\} + \frac{D^*}{2}\left(\frac{\pi}{a}\right)^2\left[k_x\left(\frac{\pi}{a}\right)^2(1+k)^2 + k_y\left(\frac{\pi}{b}\right)^2(1+n^2)\right]$$

$$a^{12}_{mnkn} = a^{21}_{mnkn} = \frac{1}{2}\left[D_{11}\left(\frac{\pi}{a}\right)^3(1+k)^4 + (D_{12}+2D_{66})\left(\frac{\pi}{a}\right)\left(\frac{\pi}{b}\right)^2(1+k)^2(1+n^2)\right]$$

$$a^{13}_{mnkn} = a^{31}_{mnkn} = \frac{1}{2}\left[D_{22}\left(\frac{\pi}{b}\right)^3\left[(1+n^2)^2+4n^2\right] + (D_{12}+2D_{66})\left(\frac{\pi}{a}\right)^2\left(\frac{\pi}{b}\right)(1+k)^2(1+n^2)\right]$$

$$a^{22}_{mnkn} = -\frac{1}{2}\left[D_{11}\left(\frac{\pi}{a}\right)^2(1+k)^4 + D_{66}\left(\frac{\pi}{b}\right)^2(1+k)^2(1+n^2) + D_{Qx}(1+k)^2\right]$$

$$a^{23}_{mnkn} = a^{32}_{mnkn} = -\frac{1}{2}(D_{12}+D_{66})\left(\frac{\pi}{a}\right)\left(\frac{\pi}{b}\right)(1+k)^2(1+n^2)$$

$$a^{33}_{mnkn} = -\frac{1}{2}\left\{D_{22}\left(\frac{\pi}{b}\right)^2\left[(1+n^2)^2+4n^2\right] + D_{66}\left(\frac{\pi}{a}\right)^2(1+k)^2(1+n^2) + D_{Qy}(1+n^2)\right\} \tag{C-10}$$

(9) $\underline{m - k = 2,\ n - \ell = 2}$

$$a^{11}_{mnk\ell} = \frac{1}{4}\left[D_{11}\left(\frac{\pi}{a}\right)^4(1+k)^4 + 2(D_{12}+2D_{66})\left(\frac{\pi}{a}\right)^2\left(\frac{\pi}{b}\right)^2(1+k)^2(1+\ell)^2 + D_{22}\left(\frac{\pi}{b}\right)^4(1+\ell)^4\right] - \frac{D^*}{4}\left(\frac{\pi}{a}\right)^2\left[k_x\left(\frac{\pi}{a}\right)^2(1+k)^2 + k_y\left(\frac{\pi}{b}\right)^2(1+\ell)^2\right]$$

$$a^{12}_{mnk\ell} = a^{21}_{mnk\ell} = -\frac{1}{4}\left[D_{11}\left(\frac{\pi}{a}\right)^3(1+k)^4 + (D_{12}+2D_{66})\left(\frac{\pi}{a}\right)\left(\frac{\pi}{b}\right)^2(1+k)^2(1+\ell)^2\right]$$

$$a^{13}_{mnk\ell} = a^{31}_{mnk\ell} = -\frac{1}{4}\left[D_{22}\left(\frac{\pi}{b}\right)^3(1+\ell)^4 + (D_{12}+2D_{66})\left(\frac{\pi}{a}\right)^2\left(\frac{\pi}{b}\right)(1+k)^2(1+\ell)^2\right]$$

$$a^{22}_{mnk\ell} = \frac{1}{4}\left[D_{11}\left(\frac{\pi}{a}\right)^2(1+k)^4 + D_{66}\left(\frac{\pi}{b}\right)^2(1+k)^2(1+\ell)^2 + D_{Qx}(1+k)^2\right]$$

$$a^{23}_{mnk\ell} = a^{32}_{mnk\ell} = \frac{1}{4}(D_{12}+D_{66})\left(\frac{\pi}{a}\right)\left(\frac{\pi}{b}\right)(1+k)^2(1+\ell)^2$$

$$a^{33}_{mnk\ell} = \frac{1}{4}\left[D_{22}\left(\frac{\pi}{b}\right)^2(1+\ell)^4 + D_{66}\left(\frac{\pi}{a}\right)^2(1+k)^2(1+\ell)^2 + D_{Qy}(1+\ell)^2\right] \qquad \text{(C-11)}$$

Case 3. Two sides clamped, two ends simply supported (2C2S edge condition).

(1) $\underline{m = k,\ n = \ell = 1}$

$$a^{11}_{m1m1} = 3D_{11}\left(\frac{m\pi}{a}\right)^4 + 8(D_{12}+2D_{66})\left(\frac{m\pi}{a}\right)^2\left(\frac{\pi}{b}\right)^2 + 16D_{22}\left(\frac{\pi}{b}\right)^4 - D^*\left(\frac{\pi}{a}\right)^2\left[3k_x\left(\frac{m\pi}{a}\right)^2 + 4k_y\left(\frac{\pi}{b}\right)^2\right]$$

$$a^{12}_{m1m1} = a^{21}_{m1m1} = -\left[3D_{11}\left(\frac{m\pi}{a}\right)^3 + 4(D_{12}+2D_{66})\left(\frac{m\pi}{a}\right)\left(\frac{\pi}{b}\right)^2\right]$$

$$a^{13}_{m1m1} = a^{31}_{m1m1} = -\left[16D_{22}\left(\frac{\pi}{b}\right)^3 + 4(D_{12}+2D_{66})\left(\frac{m\pi}{a}\right)^2\left(\frac{\pi}{b}\right)\right]$$

$$a^{22}_{m1m1} = 3D_{11}\left(\frac{m\pi}{a}\right)^2 + 4D_{66}\left(\frac{\pi}{b}\right)^2 + 3D_{Qx}$$

$$a^{23}_{m1m1} = a^{32}_{m1m1} = 4(D_{12}+D_{66})\left(\frac{m\pi}{a}\right)\left(\frac{\pi}{b}\right)$$

$$a^{33}_{m1m1} = 16D_{22}\left(\frac{\pi}{b}\right)^2 + 4D_{66}\left(\frac{m\pi}{a}\right)^2 + 4D_{Qx} \qquad \text{(C-12)}$$

(2) $\underline{m = k,\ n = \ell \neq 1}$

$$a^{11}_{mnmn} = 2D_{11}\left(\frac{m\pi}{a}\right)^4 + 4(D_{12}+2D_{66})\left(\frac{m\pi}{a}\right)^2\left(\frac{\pi}{b}\right)^2(1+n^2) + 2D_{22}\left(\frac{\pi}{b}\right)^4\left[(1+n^2)^2+4n^2\right]$$
$$- 2D^*\left(\frac{\pi}{a}\right)^2\left[k_x\left(\frac{m\pi}{a}\right)^2 + k_y\left(\frac{\pi}{b}\right)^2(1+n^2)\right]$$
$$a^{12}_{mnmn} = a^{21}_{mnmn} = -\left[2D_{11}\left(\frac{m\pi}{a}\right)^3 + 2(D_{12}+2D_{66})\left(\frac{m\pi}{a}\right)\left(\frac{\pi}{b}\right)^2(1+n^2)\right]$$
$$a^{13}_{mnmn} = a^{31}_{mnmn} = -\left\{2D_{22}\left(\frac{\pi}{b}\right)^3\left[(1+n^2)^2+4n^2\right] + 2(D_{12}+2D_{66})\left(\frac{m\pi}{a}\right)^2\left(\frac{\pi}{b}\right)(1+n^2)\right\}$$
$$a^{22}_{mnmn} = 2D_{11}\left(\frac{m\pi}{a}\right)^2 + 2D_{66}\left(\frac{\pi}{b}\right)^2(1+n^2) + 2D_{Qx}$$
$$a^{23}_{mnmn} = a^{32}_{mnmn} = 2(D_{12}+D_{66})\left(\frac{m\pi}{a}\right)\left(\frac{\pi}{b}\right)(1+n^2)$$
$$a^{33}_{mnmn} = 2D_{22}\left(\frac{\pi}{b}\right)^2\left[(1+n^2)^2+4n^2\right] + 2D_{66}\left(\frac{m\pi}{a}\right)^2(1+n^2) + 2D_{Qy}(1+n^2) \qquad \text{(C-13)}$$

(3) $\underline{m = k,\ n - \ell = 2}$

$$a^{11}_{mnm\ell} = -\left[D_{11}\left(\frac{m\pi}{a}\right)^4 + 2(D_{12}+2D_{66})\left(\frac{m\pi}{a}\right)^2\left(\frac{\pi}{b}\right)^2(1+\ell)^2 + D_{22}\left(\frac{\pi}{b}\right)^4(1+\ell)^4\right]$$
$$+ D^*\left(\frac{\pi}{a}\right)^2\left[k_x\left(\frac{m\pi}{a}\right)^2 + k_y\left(\frac{\pi}{b}\right)^2(1+\ell)^2\right]$$
$$a^{12}_{mnm\ell} = a^{21}_{mnm\ell} = D_{11}\left(\frac{m\pi}{a}\right)^3 + (D_{12}+2D_{66})\left(\frac{m\pi}{a}\right)\left(\frac{\pi}{b}\right)^2(1+\ell)^2$$
$$a^{13}_{mnm\ell} = a^{31}_{mnm\ell} = D_{22}\left(\frac{\pi}{b}\right)^3(1+\ell)^4 + (D_{12}+2D_{66})\left(\frac{m\pi}{a}\right)^2\left(\frac{\pi}{b}\right)(1+\ell)^2$$
$$a^{22}_{mnm\ell} = -\left[D_{11}\left(\frac{m\pi}{a}\right)^2 + D_{66}\left(\frac{\pi}{b}\right)^2(1+\ell)^2 + D_{Qx}\right]$$
$$a^{23}_{mnm\ell} = a^{32}_{mnm\ell} = -(D_{12}+D_{66})\left(\frac{m\pi}{a}\right)\left(\frac{\pi}{b}\right)(1+\ell)^2$$
$$a^{33}_{mnm\ell} = -\left[D_{22}\left(\frac{\pi}{b}\right)^2(1+\ell)^4 + D_{66}\left(\frac{m\pi}{a}\right)^2(1+\ell)^2 + D_{Qy}(1+\ell)^2\right] \qquad \text{(C-14)}$$

Case 4. Two sides simply supported, two ends clamped (4S4C edge condition).

(1) $\underline{m = k = 1,\ n = \ell}$

$$a^{11}_{1n1n} = 16D_{11}\left(\frac{\pi}{a}\right)^4 + 8(D_{12}+2D_{66})\left(\frac{\pi}{a}\right)^2\left(\frac{n\pi}{b}\right)^2 + 3D_{22}\left(\frac{n\pi}{b}\right)^4$$
$$- D^*\left(\frac{\pi}{a}\right)^2\left[4k_x\left(\frac{\pi}{a}\right)^2 + 3k_y\left(\frac{n\pi}{b}\right)^2\right]$$
$$a^{12}_{1n1n} = a^{21}_{1n1n} = -\left[16D_{11}\left(\frac{\pi}{a}\right)^3 + 4(D_{12}+2D_{66})\left(\frac{\pi}{a}\right)\left(\frac{n\pi}{b}\right)^2\right]$$
$$a^{13}_{1n1n} = a^{31}_{1n1n} = -\left[3D_{22}\left(\frac{n\pi}{b}\right)^3 + 4(D_{12}+2D_{66})\left(\frac{\pi}{a}\right)^2\left(\frac{n\pi}{b}\right)\right]$$
$$a^{22}_{1n1n} = 16D_{11}\left(\frac{\pi}{a}\right)^2 + 4D_{66}\left(\frac{n\pi}{b}\right)^2 + 4D_{Qx}$$
$$a^{23}_{1n1n} = a^{32}_{1n1n} = 4(D_{12}+D_{66})\left(\frac{\pi}{a}\right)\left(\frac{n\pi}{b}\right)$$
$$a^{33}_{1n1n} = 3D_{22}\left(\frac{n\pi}{b}\right)^2 + 4D_{66}\left(\frac{\pi}{a}\right)^2 + 3D_{Qx} \qquad \text{(C-15)}$$

(2) $\underline{m = k \neq 1,\ n = \ell}$

$$a^{11}_{mnmn} = 2D_{11}\left(\frac{\pi}{a}\right)^4\left[(1+m^2)^2+4m^2\right] + 4(D_{12}+2D_{66})\left(\frac{\pi}{a}\right)^2\left(\frac{n\pi}{b}\right)^2(1+m^2) + 2D_{22}\left(\frac{n\pi}{b}\right)^4$$
$$- 2D^*\left(\frac{\pi}{a}\right)^2\left[k_x\left(\frac{\pi}{a}\right)^2(1+m^2) + k_y\left(\frac{n\pi}{b}\right)^2\right]$$
$$a^{12}_{mnmn} = a^{21}_{mnmn} = -\left\{2D_{11}\left(\frac{\pi}{a}\right)^3\left[(1+m^2)^2+4m^2\right] + 2(D_{12}+2D_{66})\left(\frac{\pi}{a}\right)\left(\frac{n\pi}{b}\right)^2(1+m^2)\right\}$$
$$a^{13}_{mnmn} = a^{31}_{mnmn} = -\left[2D_{22}\left(\frac{n\pi}{b}\right)^3 + 2(D_{12}+2D_{66})(1+m^2)\left(\frac{\pi}{a}\right)^2\left(\frac{n\pi}{b}\right)\right]$$
$$a^{22}_{mnmn} = 2D_{11}\left(\frac{\pi}{a}\right)^2\left[(1+m^2)^2+4m^2\right] + 2D_{66}\left(\frac{n\pi}{b}\right)^2(1+m^2) + 2D_{Qy}(1+m^2)$$
$$a^{23}_{mnmn} = a^{32}_{mnmn} = 2(D_{12}+D_{66})\left(\frac{\pi}{a}\right)\left(\frac{n\pi}{b}\right)(1+m^2)$$
$$a^{33}_{mnmn} = 2D_{22}\left(\frac{n\pi}{b}\right)^2 + 2D_{66}\left(\frac{\pi}{a}\right)^2(1+m^2) + 2D_{Qy} \qquad \text{(C-16)}$$

(3) $\underline{m - k = 2, n = \ell}$

$$a^{11}_{mnkn} = -\left[D_{11}\left(\frac{\pi}{a}\right)^4 (1+k)^4 + 2(D_{12}+2D_{66})\left(\frac{\pi}{a}\right)^2\left(\frac{n\pi}{b}\right)^2(1+k)^2 + D_{22}\left(\frac{n\pi}{b}\right)^4\right]$$
$$+ D^*\left(\frac{\pi}{a}\right)^2\left[k_x\left(\frac{\pi}{a}\right)^2(1+k)^2 + k_y\left(\frac{n\pi}{b}\right)^2\right]$$
$$a^{12}_{mnkn} = a^{21}_{mnkn} = D_{11}\left(\frac{\pi}{a}\right)^3(1+k)^4 + (D_{12}+2D_{66})\left(\frac{\pi}{a}\right)\left(\frac{n\pi}{b}\right)^2(1+k)^2$$
$$a^{13}_{mnkn} = a^{31}_{mnkn} = D_{22}\left(\frac{n\pi}{b}\right)^3 + (D_{12}+2D_{66})\left(\frac{\pi}{a}\right)^2\left(\frac{n\pi}{b}\right)(1+k)^2$$
$$a^{22}_{mnkn} = -\left[D_{11}\left(\frac{\pi}{a}\right)^2(1+k)^4 + D_{66}\left(\frac{n\pi}{b}\right)^2(1+k)^2 + D_{Qx}(1+k)^2\right]$$
$$a^{23}_{mnkn} = a^{32}_{mnkn} = -(D_{12}+D_{66})\left(\frac{\pi}{a}\right)\left(\frac{n\pi}{b}\right)(1+k)^2$$
$$a^{33}_{mnkn} = -\left[D_{22}\left(\frac{n\pi}{b}\right)^2 + D_{66}\left(\frac{\pi}{a}\right)^2(1+k)^2 + D_{Qy}\right] \tag{C-17}$$

APPENDIX D
BUCKLING EQUATIONS

The buckling equations (eigenvalue solution equations) written out from equation (48) up to order 12 (i.e., 12 × 12 matrices) for the case $m \pm n$ = even (symmetric buckling) and $m \pm n$ = odd (antisymmetric buckling) for different edge conditions are given below.

Case 1. Four edges simply supported (4S edge condition) (ref. 9).
$m \pm n$ = even (symmetric buckling) (4S)

$$
\begin{array}{r|cccccccccccc|l}
A_{k\ell} \rightarrow & A_{11} & A_{13} & A_{22} & A_{31} & A_{15} & A_{24} & A_{33} & A_{42} & A_{51} & A_{35} & A_{44} & A_{53} & \\
m{=}1,n{=}1 & \frac{M_{1111}}{k_{xy}} & 0 & \frac{4}{9} & 0 & 0 & \frac{8}{45} & 0 & \frac{8}{45} & 0 & 0 & \frac{16}{225} & 0 & \\
m{=}1,n{=}3 & & \frac{M_{1313}}{k_{xy}} & -\frac{4}{5} & 0 & 0 & \frac{8}{7} & 0 & -\frac{8}{25} & 0 & 0 & \frac{16}{35} & 0 & \\
m{=}2,n{=}2 & & & \frac{M_{2222}}{k_{xy}} & -\frac{4}{5} & -\frac{20}{63} & 0 & \frac{36}{25} & 0 & -\frac{20}{63} & \frac{4}{7} & 0 & \frac{4}{7} & \\
m{=}3,n{=}1 & & & & \frac{M_{3131}}{k_{xy}} & 0 & -\frac{8}{25} & 0 & \frac{8}{7} & 0 & 0 & \frac{16}{35} & 0 & \\
m{=}1,n{=}5 & & & & & \frac{M_{1515}}{k_{xy}} & -\frac{40}{27} & 0 & -\frac{8}{63} & 0 & 0 & -\frac{16}{27} & 0 & \\
m{=}2,n{=}4 & & & & & & \frac{M_{2424}}{k_{xy}} & -\frac{72}{35} & 0 & -\frac{8}{63} & \frac{8}{3} & 0 & -\frac{120}{147} & \\
m{=}3,n{=}3 & & & \text{Symmetry} & & & & \frac{M_{3333}}{k_{xy}} & -\frac{72}{35} & 0 & 0 & \frac{144}{49} & 0 & =0 \\
m{=}4,n{=}2 & & & & & & & & \frac{M_{4242}}{k_{xy}} & -\frac{40}{27} & -\frac{120}{147} & 0 & \frac{8}{3} & \\
m{=}5,n{=}1 & & & & & & & & & \frac{M_{5151}}{k_{xy}} & 0 & -\frac{16}{27} & 0 & \\
m{=}3,n{=}5 & & & & & & & & & & \frac{M_{3535}}{k_{xy}} & -\frac{80}{21} & 0 & \\
m{=}4,n{=}4 & & & & & & & & & & & \frac{M_{4444}}{k_{xy}} & -\frac{80}{21} & \\
m{=}5,n{=}3 & & & & & & & & & & & & \frac{M_{5353}}{k_{xy}} &
\end{array}
\qquad \text{(D-1)}
$$

$m \pm n = $ odd (antisymmetric buckling) (4S) (ref. 9)

$$
\begin{array}{c|cccccccccccc|}
A_{k\ell} \rightarrow & A_{12} & A_{21} & A_{14} & A_{23} & A_{32} & A_{41} & A_{16} & A_{25} & A_{34} & A_{43} & A_{52} & A_{61} \\
m{=}1,n{=}2 & \frac{M_{1212}}{k_{xy}} & -\frac{4}{9} & 0 & \frac{4}{5} & 0 & -\frac{8}{45} & 0 & \frac{20}{63} & 0 & \frac{8}{25} & 0 & -\frac{4}{35} \\
m{=}2,n{=}1 & & \frac{M_{2121}}{k_{xy}} & -\frac{8}{45} & 0 & \frac{4}{5} & 0 & -\frac{4}{35} & 0 & \frac{8}{25} & 0 & \frac{20}{63} & 0 \\
m{=}1,n{=}4 & & & \frac{M_{1414}}{k_{xy}} & -\frac{8}{7} & 0 & -\frac{16}{225} & 0 & \frac{40}{27} & 0 & -\frac{16}{35} & 0 & -\frac{8}{175} \\
m{=}2,n{=}3 & & & & \frac{M_{2323}}{k_{xy}} & -\frac{36}{25} & 0 & -\frac{4}{9} & 0 & \frac{72}{35} & 0 & -\frac{4}{7} & 0 \\
m{=}3,n{=}2 & & & & & \frac{M_{3232}}{k_{xy}} & -\frac{8}{7} & 0 & -\frac{4}{7} & 0 & \frac{72}{35} & 0 & -\frac{4}{9} \\
m{=}4,n{=}1 & & & & & & \frac{M_{4141}}{k_{xy}} & -\frac{8}{175} & 0 & -\frac{16}{35} & 0 & \frac{40}{27} & 0 \\
m{=}1,n{=}6 & & & & \text{Symmetry} & & & \frac{M_{1616}}{k_{xy}} & -\frac{20}{11} & 0 & -\frac{8}{45} & 0 & -\frac{36}{1225} \\
m{=}2,n{=}5 & & & & & & & & \frac{M_{2525}}{k_{xy}} & -\frac{8}{3} & 0 & -\frac{100}{441} & 0 \\
m{=}3,n{=}4 & & & & & & & & & \frac{M_{3434}}{k_{xy}} & -\frac{144}{49} & 0 & -\frac{8}{45} \\
m{=}4,n{=}3 & & & & & & & & & & \frac{M_{4343}}{k_{xy}} & -\frac{8}{3} & 0 \\
m{=}5,n{=}2 & & & & & & & & & & & \frac{M_{5252}}{k_{xy}} & -\frac{20}{11} \\
m{=}6,n{=}1 & & & & & & & & & & & & \frac{M_{6161}}{k_{xy}}
\end{array} = 0 \qquad \text{(D-2)}
$$

Case 2. Four edges clamped (4C edge condition).
$m \pm n =$ even (symmetric buckling) (4C)

$$
\begin{array}{c}
A_{k\ell} \rightarrow \\
m=1,n=1 \\ m=1,n=3 \\ m=2,n=2 \\ m=3,n=1 \\ m=1,n=5 \\ m=2,n=4 \\ m=3,n=3 \\ m=4,n=2 \\ m=5,n=1 \\ m=3,n=5 \\ m=4,n=4 \\ m=5,n=3
\end{array}
\begin{array}{c}
\begin{array}{cccccccccccc}
A_{11} & A_{13} & A_{22} & A_{31} & A_{15} & A_{24} & A_{33} & A_{42} & A_{51} & A_{35} & A_{44} & A_{53}
\end{array} \\
\begin{vmatrix}
\frac{M_{1111}}{k_{xy}} & \frac{M_{1113}}{k_{xy}} & \frac{4}{225} & \frac{M_{1131}}{k_{xy}} & 0 & -\frac{8}{1575} & \frac{M_{1133}}{k_{xy}} & -\frac{8}{1575} & 0 & 0 & \frac{16}{11025} & 0 \\
 & \frac{M_{1313}}{k_{xy}} & -\frac{44}{1575} & \frac{M_{1331}}{k_{xy}} & \frac{M_{1315}}{k_{xy}} & \frac{184}{4725} & \frac{M_{1333}}{k_{xy}} & \frac{88}{11025} & 0 & \frac{M_{1335}}{k_{xy}} & -\frac{368}{33075} & 0 \\
 & & \frac{M_{2222}}{k_{xy}} & -\frac{44}{1575} & \frac{4}{525} & \frac{M_{2224}}{k_{xy}} & \frac{484}{11025} & \frac{M_{2242}}{k_{xy}} & \frac{4}{525} & -\frac{44}{3675} & \frac{M_{2244}}{k_{xy}} & -\frac{44}{3675} \\
 & & & \frac{M_{3131}}{k_{xy}} & 0 & \frac{88}{11025} & \frac{M_{3133}}{k_{xy}} & \frac{184}{4725} & \frac{M_{3151}}{k_{xy}} & 0 & -\frac{368}{33075} & \frac{M_{3153}}{k_{xy}} \\
 & & & & \frac{M_{1515}}{k_{xy}} & -\frac{104}{2079} & \frac{M_{1533}}{k_{xy}} & -\frac{8}{3675} & 0 & \frac{M_{1535}}{k_{xy}} & \frac{208}{14553} & 0 \\
 & & & & & \frac{M_{2424}}{k_{xy}} & -\frac{2024}{33075} & \frac{M_{2442}}{k_{xy}} & -\frac{8}{3675} & \frac{104}{1323} & \frac{M_{2444}}{k_{xy}} & \frac{184}{11025} \\
 & & \text{Symmetry} & & & & \frac{M_{3333}}{k_{xy}} & -\frac{2024}{33075} & \frac{M_{3351}}{k_{xy}} & \frac{M_{3335}}{k_{xy}} & \frac{8464}{99225} & \frac{M_{3353}}{k_{xy}} \\
 & & & & & & & \frac{M_{4242}}{k_{xy}} & -\frac{104}{2079} & \frac{184}{11025} & \frac{M_{4244}}{k_{xy}} & \frac{104}{1323} \\
 & & & & & & & & \frac{M_{5151}}{k_{xy}} & 0 & \frac{208}{14553} & \frac{M_{5153}}{k_{xy}} \\
 & & & & & & & & & \frac{M_{3535}}{k_{xy}} & -\frac{4784}{43659} & \frac{M_{3553}}{k_{xy}} \\
 & & & & & & & & & & \frac{M_{4444}}{k_{xy}} & -\frac{4784}{43659} \\
 & & & & & & & & & & & \frac{M_{5353}}{k_{xy}}
\end{vmatrix}
\end{array}
= 0 \qquad \text{(D-3)}
$$

$m \pm n = $ odd (antisymmetric buckling) (4C)

$$\begin{array}{c|cccccccccccc|l}
A_{k\ell} \rightarrow & A_{12} & A_{21} & A_{14} & A_{23} & A_{32} & A_{41} & A_{16} & A_{25} & A_{34} & A_{43} & A_{52} & A_{61} & \\
m{=}1,n{=}2 & \frac{M_{1212}}{k_{xy}} & -\frac{4}{225} & \frac{M_{1214}}{k_{xy}} & \frac{44}{1575} & \frac{M_{1232}}{k_{xy}} & \frac{8}{1575} & 0 & -\frac{4}{525} & \frac{M_{1234}}{k_{xy}} & -\frac{88}{11025} & 0 & \frac{4}{4725} & \\
m{=}2,n{=}1 & & \frac{M_{2121}}{k_{xy}} & \frac{8}{1575} & \frac{M_{2123}}{k_{xy}} & \frac{44}{1575} & \frac{M_{2141}}{k_{xy}} & \frac{4}{4725} & 0 & -\frac{88}{11025} & \frac{M_{2143}}{k_{xy}} & -\frac{4}{525} & 0 & \\
m{=}1,n{=}4 & & & \frac{M_{1414}}{k_{xy}} & -\frac{184}{4725} & \frac{M_{1432}}{k_{xy}} & -\frac{16}{11025} & \frac{M_{1416}}{k_{xy}} & \frac{104}{2079} & \frac{M_{1434}}{k_{xy}} & \frac{368}{33075} & 0 & -\frac{8}{33075} & \\
m{=}2,n{=}3 & & & & \frac{M_{2323}}{k_{xy}} & -\frac{484}{11025} & \frac{M_{2341}}{k_{xy}} & \frac{172}{17325} & \frac{M_{2325}}{k_{xy}} & \frac{2024}{33075} & \frac{M_{2343}}{k_{xy}} & \frac{44}{3675} & 0 & \\
m{=}3,n{=}2 & & & & & \frac{M_{3232}}{k_{xy}} & -\frac{184}{4725} & 0 & \frac{44}{3675} & \frac{M_{3234}}{k_{xy}} & \frac{2024}{33075} & \frac{M_{3252}}{k_{xy}} & \frac{172}{17325} & \\
m{=}4,n{=}1 & & & & & & \frac{M_{4141}}{k_{xy}} & -\frac{8}{33075} & 0 & \frac{368}{33075} & \frac{M_{4143}}{k_{xy}} & \frac{104}{2079} & \frac{M_{4161}}{k_{xy}} & \\
m{=}1,n{=}6 & & & \text{Symmetry} & & & & \frac{M_{1616}}{k_{xy}} & -\frac{236}{3861} & \frac{M_{1634}}{k_{xy}} & -\frac{344}{121275} & 0 & -\frac{4}{99225} & = 0 \\
m{=}2,n{=}5 & & & & & & & & \frac{M_{2525}}{k_{xy}} & -\frac{104}{1323} & \frac{M_{2543}}{k_{xy}} & -\frac{4}{1225} & 0 & \\
m{=}3,n{=}4 & & & & & & & & & \frac{M_{3434}}{k_{xy}} & -\frac{8464}{99225} & \frac{M_{3452}}{k_{xy}} & -\frac{344}{121275} & \\
m{=}4,n{=}3 & & & & & & & & & & \frac{M_{4343}}{k_{xy}} & -\frac{104}{1323} & \frac{M_{4361}}{k_{xy}} & \\
m{=}5,n{=}2 & & & & & & & & & & & \frac{M_{5252}}{k_{xy}} & -\frac{236}{3861} & \\
m{=}6,n{=}1 & & & & & & & & & & & & \frac{M_{6161}}{k_{xy}} &
\end{array} \qquad \text{(D-4)}$$

Case 3. Two sides clamped, two ends simply supported (2C2S edge condition).

$m \pm n =$ even (symmetric buckling) (2C2S)

$$
\begin{array}{r|cccccccccccc|l}
A_{k\ell} \rightarrow & A_{11} & A_{13} & A_{22} & A_{31} & A_{15} & A_{24} & A_{33} & A_{42} & A_{51} & A_{35} & A_{44} & A_{53} & \\
m=1,n=1 & \frac{M_{1111}}{k_{xy}} & \frac{M_{1113}}{k_{xy}} & \frac{4}{45} & 0 & 0 & -\frac{8}{315} & 0 & \frac{8}{225} & 0 & 0 & -\frac{16}{1575} & 0 & \\
m=1,n=3 & & \frac{M_{1313}}{k_{xy}} & -\frac{44}{315} & 0 & \frac{M_{1315}}{k_{xy}} & \frac{184}{945} & 0 & -\frac{88}{1575} & 0 & 0 & \frac{368}{4725} & 0 & \\
m=2,n=2 & & & \frac{M_{2222}}{k_{xy}} & -\frac{4}{25} & \frac{4}{105} & \frac{M_{2224}}{k_{xy}} & \frac{44}{175} & 0 & -\frac{4}{63} & -\frac{12}{175} & 0 & \frac{44}{441} & \\
m=3,n=1 & & & & \frac{M_{3131}}{k_{xy}} & 0 & \frac{8}{175} & \frac{M_{3133}}{k_{xy}} & \frac{8}{35} & 0 & 0 & -\frac{16}{245} & 0 & \\
m=1,n=5 & & & & & \frac{M_{1515}}{k_{xy}} & -\frac{520}{2079} & 0 & \frac{8}{525} & 0 & 0 & -\frac{208}{2079} & 0 & \\
m=2,n=4 & & & & & & \frac{M_{2424}}{k_{xy}} & -\frac{184}{525} & 0 & \frac{8}{441} & \frac{104}{231} & 0 & -\frac{184}{1323} & = 0 \\
m=3,n=3 & & & \text{Symmetry} & & & & \frac{M_{3333}}{k_{xy}} & -\frac{88}{245} & 0 & \frac{M_{3335}}{k_{xy}} & \frac{368}{735} & 0 & \\
m=4,n=2 & & & & & & & & \frac{M_{4242}}{k_{xy}} & -\frac{8}{27} & \frac{24}{245} & \frac{M_{4244}}{k_{xy}} & \frac{88}{189} & \\
m=5,n=1 & & & & & & & & & \frac{M_{5151}}{k_{xy}} & 0 & \frac{16}{189} & \frac{M_{5153}}{k_{xy}} & \\
m=3,n=5 & & & & & & & & & & \frac{M_{3535}}{k_{xy}} & -\frac{1040}{1617} & 0 & \\
m=4,n=4 & & & & & & & & & & & \frac{M_{4444}}{k_{xy}} & -\frac{368}{567} & \\
m=5,n=3 & & & & & & & & & & & & \frac{M_{5353}}{k_{xy}} &
\end{array}
$$

(D-5)

$m \pm n = $ odd (antisymmetric buckling) (2C2S)

$$
\begin{array}{r|cccccccccccc|l}
A_{k\ell} \rightarrow & A_{12} & A_{21} & A_{14} & A_{23} & A_{32} & A_{41} & A_{16} & A_{25} & A_{34} & A_{43} & A_{52} & A_{61} & \\
m=1,n=2 & \frac{M_{1212}}{k_{xy}} & -\frac{4}{25} & \frac{M_{1214}}{k_{xy}} & \frac{44}{315} & 0 & -\frac{8}{225} & 0 & -\frac{4}{105} & 0 & \frac{88}{1575} & 0 & -\frac{4}{175} & \\
m=2,n=1 & & \frac{M_{2121}}{k_{xy}} & \frac{8}{315} & \frac{M_{2123}}{k_{xy}} & \frac{4}{25} & 0 & \frac{4}{925} & 0 & -\frac{8}{175} & 0 & \frac{4}{63} & 0 & \\
m=1,n=4 & & & \frac{M_{1414}}{k_{xy}} & -\frac{184}{945} & 0 & \frac{16}{1575} & \frac{M_{1416}}{k_{xy}} & \frac{520}{2079} & 0 & -\frac{368}{4725} & 0 & \frac{8}{1225} & \\
m=2,n=3 & & & & \frac{M_{2323}}{k_{xy}} & -\frac{44}{175} & 0 & \frac{172}{3465} & \frac{M_{2325}}{k_{xy}} & \frac{184}{525} & 0 & -\frac{44}{441} & 0 & \\
m=3,n=2 & & & & & \frac{M_{3232}}{k_{xy}} & -\frac{8}{35} & 0 & \frac{12}{175} & \frac{M_{3234}}{k_{xy}} & \frac{88}{245} & 0 & -\frac{4}{45} & \\
m=4,n=1 & & & & & & \frac{M_{4141}}{k_{xy}} & \frac{8}{4725} & 0 & \frac{16}{245} & \frac{M_{4143}}{k_{xy}} & \frac{8}{27} & 0 & \\
m=1,n=6 & & & \text{Symmetry} & & & & \frac{M_{1616}}{k_{xy}} & -\frac{1180}{3861} & 0 & \frac{344}{17325} & 0 & \frac{4}{3675} & = 0 \\
m=2,n=5 & & & & & & & & \frac{M_{2525}}{k_{xy}} & -\frac{104}{231} & 0 & \frac{4}{147} & 0 & \\
m=3,n=4 & & & & & & & & & \frac{M_{3434}}{k_{xy}} & -\frac{368}{735} & 0 & \frac{8}{315} & \\
m=4,n=3 & & & & & & & & & & \frac{M_{4343}}{k_{xy}} & -\frac{88}{189} & 0 & \\
m=5,n=2 & & & & & & & & & & & \frac{M_{5252}}{k_{xy}} & -\frac{4}{11} & \\
m=6,n=1 & & & & & & & & & & & & \frac{M_{6161}}{k_{xy}} &
\end{array}
\qquad \text{(D-6)}
$$

Case 4. Two sides simply supported, two ends clamped (2S2C edge condition).

$m \pm n =$ even (symmetric buckling) (2S2C)

$$
\begin{array}{c|cccccccccccc|l}
A_{k\ell} \rightarrow & A_{11} & A_{13} & A_{22} & A_{31} & A_{15} & A_{24} & A_{33} & A_{42} & A_{51} & A_{35} & A_{44} & A_{53} & \\
m{=}1,n{=}1 & \frac{M_{1111}}{k_{xy}} & 0 & \frac{4}{45} & \frac{M_{1131}}{k_{xy}} & 0 & \frac{8}{225} & 0 & -\frac{8}{315} & 0 & 0 & -\frac{16}{1575} & 0 & \\
m{=}1,n{=}3 & & \frac{M_{1313}}{k_{xy}} & -\frac{4}{25} & 0 & 0 & \frac{8}{35} & \frac{M_{1333}}{k_{xy}} & \frac{8}{175} & 0 & 0 & -\frac{16}{245} & 0 & \\
m{=}2,n{=}2 & & & \frac{M_{2222}}{k_{xy}} & -\frac{44}{315} & -\frac{4}{63} & 0 & \frac{44}{175} & \frac{M_{2242}}{k_{xy}} & \frac{4}{105} & \frac{44}{441} & 0 & -\frac{12}{175} & \\
m{=}3,n{=}1 & & & & \frac{M_{3131}}{k_{xy}} & 0 & -\frac{88}{1575} & 0 & \frac{184}{945} & \frac{M_{3151}}{k_{xy}} & 0 & \frac{368}{4725} & 0 & \\
m{=}1,n{=}5 & & & & & \frac{M_{1515}}{k_{xy}} & -\frac{8}{27} & 0 & \frac{8}{441} & 0 & \frac{M_{1535}}{k_{xy}} & \frac{16}{189} & 0 & \\
m{=}2,n{=}4 & & & & & & \frac{M_{2424}}{k_{xy}} & -\frac{88}{245} & 0 & \frac{8}{525} & \frac{88}{189} & \frac{M_{2444}}{k_{xy}} & \frac{24}{245} & = 0 \\
m{=}3,n{=}3 & & & \text{Symmetry} & & & & \frac{M_{3333}}{k_{xy}} & -\frac{184}{525} & 0 & 0 & \frac{368}{735} & \frac{M_{3353}}{k_{xy}} & \\
m{=}4,n{=}2 & & & & & & & & \frac{M_{4242}}{k_{xy}} & -\frac{520}{2079} & -\frac{184}{1323} & 0 & \frac{104}{231} & \\
m{=}5,n{=}1 & & & & & & & & & \frac{M_{5151}}{k_{xy}} & 0 & -\frac{208}{2079} & 0 & \\
m{=}3,n{=}5 & & & & & & & & & & \frac{M_{3535}}{k_{xy}} & -\frac{368}{567} & 0 & \\
m{=}4,n{=}4 & & & & & & & & & & & \frac{M_{4444}}{k_{xy}} & -\frac{1040}{1617} & \\
m{=}5,n{=}3 & & & & & & & & & & & & \frac{M_{5353}}{k_{xy}} &
\end{array}
\qquad \text{(D-7)}
$$

$m \pm n$ = odd (antisymmetric buckling) (2S2C)

$$
\begin{array}{r|cccccccccccc|l}
A_{k\ell} \rightarrow & A_{12} & A_{21} & A_{14} & A_{23} & A_{32} & A_{41} & A_{16} & A_{25} & A_{34} & A_{43} & A_{52} & A_{61} & \\
m{=}1,n{=}2 & \frac{M_{1212}}{k_{xy}} & -\frac{4}{45} & 0 & \frac{4}{25} & \frac{M_{1232}}{k_{xy}} & \frac{8}{315} & 0 & \frac{4}{63} & 0 & -\frac{8}{175} & 0 & \frac{4}{945} & \\
m{=}2,n{=}1 & & \frac{M_{2121}}{k_{xy}} & -\frac{8}{225} & 0 & \frac{44}{315} & \frac{M_{2141}}{k_{xy}} & -\frac{4}{175} & 0 & \frac{88}{1575} & 0 & -\frac{4}{105} & 0 & \\
m{=}1,n{=}4 & & & \frac{M_{1414}}{k_{xy}} & -\frac{8}{35} & 0 & \frac{16}{1575} & 0 & \frac{8}{27} & \frac{M_{1434}}{k_{xy}} & \frac{16}{245} & 0 & \frac{8}{4725} & \\
m{=}2,n{=}3 & & & & \frac{M_{2323}}{k_{xy}} & -\frac{44}{175} & 0 & -\frac{4}{45} & 0 & \frac{88}{245} & \frac{M_{2343}}{k_{xy}} & \frac{12}{175} & 0 & \\
m{=}3,n{=}2 & & & & & \frac{M_{3232}}{k_{xy}} & -\frac{184}{945} & 0 & -\frac{44}{441} & 0 & \frac{184}{525} & \frac{M_{3252}}{k_{xy}} & \frac{172}{3465} & \\
m{=}4,n{=}1 & & & & & & \frac{M_{4141}}{k_{xy}} & \frac{8}{1225} & 0 & -\frac{368}{4725} & 0 & \frac{520}{2079} & \frac{M_{4161}}{k_{xy}} & \\
m{=}1,n{=}6 & & & \multicolumn{2}{c}{\text{Symmetry}} & & & \frac{M_{1616}}{k_{xy}} & -\frac{4}{11} & 0 & \frac{8}{315} & 0 & \frac{4}{3675} & = 0 \\
m{=}2,n{=}5 & & & & & & & & \frac{M_{2525}}{k_{xy}} & -\frac{88}{189} & 0 & \frac{4}{147} & 0 & \\
m{=}3,n{=}4 & & & & & & & & & \frac{M_{3434}}{k_{xy}} & -\frac{368}{735} & 0 & \frac{344}{17325} & \\
m{=}4,n{=}3 & & & & & & & & & & \frac{M_{4343}}{k_{xy}} & -\frac{104}{231} & 0 & \\
m{=}5,n{=}2 & & & & & & & & & & & \frac{M_{5252}}{k_{xy}} & -\frac{1180}{3861} & \\
m{=}6,n{=}1 & & & & & & & & & & & & \frac{M_{6161}}{k_{xy}} &
\end{array}
\tag{D-8}
$$

APPENDIX E
ENERGY EQUATIONS WITH NONZERO THERMAL MOMENTS

Using a similar process mentioned in Appendix B, the energy equations with nonzero thermal moments will be presented in the following for the 4S edge condition as an example.

$$
\begin{aligned}
V_1 = \frac{ab}{4} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \Bigg\{ & \frac{1}{2} \left[D_{11} \left(\frac{m\pi}{a}\right)^4 + 2(D_{12} + 2D_{66}) \left(\frac{m\pi}{a}\right)^2 \left(\frac{n\pi}{b}\right)^2 + D_{22} \left(\frac{n\pi}{b}\right)^4 \right] A_{mn}^2 \\
& - \left[D_{11} \left(\frac{m\pi}{a}\right)^3 + (D_{12} + 2D_{66}) \left(\frac{m\pi}{a}\right) \left(\frac{n\pi}{b}\right)^2 \right] A_{mn} B_{mn} \\
& - \left[D_{22} \left(\frac{n\pi}{b}\right)^3 + (D_{12} + 2D_{66}) \left(\frac{m\pi}{a}\right)^2 \left(\frac{n\pi}{b}\right) \right] A_{mn} C_{mn} \\
& + \frac{1}{2} \left[D_{11} \left(\frac{m\pi}{a}\right)^2 + D_{66} \left(\frac{n\pi}{b}\right)^2 + D_{Qx} \right] B_{mn}^2 \\
& + \left[(D_{12} + D_{66}) \left(\frac{m\pi}{a}\right) \left(\frac{n\pi}{b}\right) \right] B_{mn} C_{mn} \\
& + \frac{1}{2} \left[D_{22} \left(\frac{n\pi}{b}\right)^2 + D_{66} \left(\frac{m\pi}{a}\right)^2 + D_{Qy} \right] C_{mn}^2 \\
\text{thermal moment terms} \quad & - \left[F_{mn} \left(\frac{m\pi}{a}\right)^2 + H_{mn} \left(\frac{n\pi}{b}\right)^2 - 2S_{mn} \left(\frac{m\pi}{a}\right) \left(\frac{n\pi}{b}\right) \right] A_{mn} \\
& + \left[F_{mn} \left(\frac{m\pi}{a}\right) - S_{mn} \left(\frac{n\pi}{b}\right) \right] B_{mn} + \left[H_{mn} \left(\frac{n\pi}{b}\right) - S_{mn} \left(\frac{m\pi}{a}\right) \right] C_{mn} \Bigg\}
\end{aligned}
\tag{E-1}
$$

$$
\begin{aligned}
V_2 = \frac{ab}{4} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \Bigg\{ & -\frac{1}{2} \left[N_x^T \left(\frac{m\pi}{a}\right)^2 + N_y^T \left(\frac{n\pi}{b}\right)^2 \right] A_{mn}^2 \\
& + 16 \sum_{k} \sum_{\ell} N_{xy}^T \frac{mnk\ell}{(m^2 - k^2)(n^2 - \ell^2)} A_{mn} A_{k\ell} \Bigg\}
\end{aligned}
\tag{E-2}
$$

where the last term of equation (E-2) is nonzero only under the indicial conditions: m $\pm$ k = odd and n $\pm$ l = odd.

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The combined load (mechanical or thermal load) buckling equations were established for orthotropic rectangular sandwich panels under four different edge conditions by using the Rayleigh-Ritz method of minimizing the total potential energy of a structural system. Two-dimensional buckling interaction curves and three-dimensional buckling interaction surfaces were constructed for high-temperature honeycomb-core sandwich panels supported under four different edge conditions. The interaction surfaces provide overall comparison of the panel buckling strengths and the domains of symmetrical and antisymmetrical buckling associated with the different edge conditions. In addition, thermal buckling curves of these sandwich panels are presented. The thermal buckling conditions for the cases with and without thermal moments were found to be identical for the small deformation theory.

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