DEFINITION OF GROUND TEST FOR VERIFICATION OF LARGE SPACE STRUCTURE CONTROL
FINAL REPORT

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LOGICON
Control Dynamics
600 BOULEVARD SOUTH SUITE 304 • HUNTSVILLE, AL 35802 • TELEPHONE: 205 882-2650
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1.0 INTRODUCTION

This report summarizes the work performed under contract NAS8-35835. This contract was initially awarded to Logicon Control Dynamics (LCD) June 20, 1984 when the company was known as Control Dynamics Company and was a small business. Under the contract, the Large Space Structure Ground Test Verification (LSSGTV) Facility at the George C. Marshall Space Flight Center (MSFC) was developed. Planning in coordination with NASA was finalized and implemented. The contract was modified and extended with several increments of funding to procure additional hardware and to continue support for the LSSGTV facility. Additional tasks were defined for the performance of studies in the dynamics, control and simulation of tethered satellites. When the LSSGTV facility development task was completed, support and enhancement activities were funded through a new competitive contract won by LCD. All work related to LSSGTV performed under NAS8-35835 has been completed and documented (see references 1-7). No further discussion of these activities will appear in this report.

In may of 1985 the contract was modified to incorporate tether dynamics analysis and simulation studies under the simulations task of the contract. The rationale for fitting these activities into this contract was that tethered satellite structures represent the largest structures anticipated for operation in space and have the earliest likelihood of flight. Tether lengths up to 100 km are planned. Tethered satellite dynamics related tasks continued to be funded through NAS8-35835. Additional contract modifications have incorporated more such analysis and support activity. This report summarizes the tether dynamics and control studies performed.

2.0 TETHERED SATELLITE DYNAMICS AND CONTROL STUDIES

Tether dynamics and control studies performed under NAS8-35835 supported Advanced Tether Application Studies and Tethered Satellite System dynamics and control studies.

2.1 ADVANCED TETHER APPLICATION STUDIES

The first area of analysis was the study of advanced concepts for tether applications in space. Several applications have been studied.

The space tether energy storage and retrieval system (TESS) was a concept to store
energy in the rotation of a tethered pair of satellites. The tether would be reeled in to a relatively short length and rockets would be used to increase the rotational velocity, thus storing energy in the motion. This spinup could be done at a leisurely pace so that modest size and efficient rockets could be employed. The energy could then be released quickly by allowing the tether to deploy and turn a generator as it does so. This system might be of value for any application where short, high intensity power bursts were required. The dynamics of this system (see Appendix A) were modeled and studied.

A tethered, artificial gravity satellite, suitable for application to a space station or a deep space, manned flight was studied [8]. This concept also required deployment and retrieval of the tether so that the endbodies could be joined for powered flight when required. After powered flight, the tether would be deployed and rockets would be fired to impart rotation at the appropriate rate and g level.

The Small, Expendable Tether Deployer System (SEDS) was modeled and analyzed to verify feasibility. This is a low cost, non-reusable tether system for deployment of packages from an orbiting carrier vehicle. More recently, this line of study included the development of a Design Reference Mission Document with updates as required for the SEDS-1 mission [9]. SEDS-1 was flown successfully March 30, 1993. The results from the SEDS-1 flight were compared to pre-flight simulations [10].

An analysis of the dynamics of a tether deployment and retrieval system for possible application on the space station was performed [11]. This system was patterned after the Tethered Satellite System.

Each of the above applications required improvements to our tethered satellites simulation (TSSIM) program to account for specific characteristics. Special graphics software was developed to aid in visualization and understanding of the dynamic responses. A new version of the simulation called TSSIM-R was also developed. This version includes rotational dynamics of the end bodies and has become the main analysis tool. These activities were performed in support of MSFC Program Development, the SEDS Project Office and the Science and Engineering Structures and Dynamics Laboratory.

Results from these studies were reported through viewgraph presentations and delivery of simulation and graphics software. In addition, papers were prepared and presented at several conferences. These conferences included the three International Conferences on
Tethers in Space, held in Arlington, VA, Venice, Italy and San Francisco [12, 8, 13], respectively.

2.2 TETHERED SATELLITE SYSTEM STUDIES

2.2.1 Dynamics And Control Review Panel

The second area of analysis was the joint USA/Italy Tethered Satellite System (TSS). The first phase of activity performed in this area was the formation of the TSS Dynamics and Control Review (DACR) Panel. This panel was formed at the request of NASA to perform an independent review of the dynamics and control aspects of TSS and to report the findings to TSS Program management. LCD hosted the panel meetings and provided the Chairman. Three panel meetings in all were held. The meetings were in March and June of 1987 and April of 1990. Reports of findings were delivered in several presentations and written reports.

2.2.2 TSS Dynamics And Control Analyses

The second phase of TSS related tethered satellite dynamics studies focused on the development of improvements to the detailed tethered satellite dynamics simulation (TSSIM) and participation in various TSS Project Reviews. LCD also participated in major TSS reviews and was a test observer during testing of the deployer hardware. LCD participated in the activities of the TSS Dynamics and Control Working Group (DWG) and was on the TSS Mission Control Team at the Johnson Space Center during the TSS-1 mission. After TSS-1 with its much shortened tether deployment and anomalies, LCD conducted a study of the flight telemetry data pertaining to the deployer and the satellite as well as the Orbiter radar data. The results are documented in data evaluation reports [14, 15]. Finally, LCD participated in the TSS-1 Reflight Study during the first half of 1993. This work was performed in support of the TSS Project Office at MSFC. The activities of the first two phases are documented in reference 9.

2.2.3 TSS Hardware Testing

The third phase of TSS dynamics and control study activities was devoted to participation in test activities qualifying the TSS Deployer hardware for flight. The first test was the Formal Qualification Test (FQT) which Occurred in November of 1989. This qualified the
deployer software and demonstrated that representative TSS hardware could be controlled through a complete mission by simulated uplink commands. The second test was the integrated Hardware/Software Integration Test (HSIT) and demonstrated the ability of the system to perform all phases of a TSS mission with nominal and contingency components. It also served to calibrate various encoders. The third phase activities are described in reference [17].

The fourth phase of TSS related studies under this contract consisted of several tasks. The first task was to observe TSS deployer tests performed at the Kennedy Space Center (KSC) in August of 1991. These tests were performed in the O&C building at KSC (see Appendix B).

2.2.4 DWG Support Activities

The second task was to participate in activities of the TSS Dynamics Working Group (DWG) and to support the final development and implementation of the frequency domain skiprope observer (FDSO).

The Dynamics and Control Working Group was the focal organization within NASA for the resolutions of problems in the dynamics and control of TSS hardware in orbital operations. The most difficult problem and consequently, the most frequently addressed issue was tether dynamics and the behavior of the deployer system hardware. Many meetings at MSFC, Martin Marietta, JSC and KSC were devoted to developing an understanding of such matters as in plane and out of plane libration, satellite attitude control and tether string dynamics including the skiprope oscillations.

Skiprope, particularly, generated much activity because of its ability to prevent retrieval of the satellite if amplitudes were allowed to grow to significant levels. Skiprope was not initially recognized as a problem by the DWG. Simulations done by Martin Marietta and by Johnson Space Center indicated that the effect was adequately damped. Since two apparently independent simulations discounted the effect, it was not considered a problem. Dave Arnold, Smithsonian Astrophysical Observatory (SAO) in Cambridge, MA persisted in his warning about skiprope. His arguments were based on certain fundamental physical principles such as conservation of angular momentum and minimal damping. He also had done simulations with the SAO tether dynamics programs. These considerations implied that skiprope is minimally damped and that the amplitude of skiprope oscillations
varies inversely with tether length raised to the 1/4 power. Since the SAO simulations did not contain details of the TSS deployer and end body control systems, it was assumed that the difference of results were explainable by this. However, runs made at JSC showed that removal of these effects did not significantly affect the damping of skiprope seen in the JSC tether simulation. After the puzzling JSC results and after taking a closer look at the physical arguments regarding skiprope, John Glaese at LCD confirmed the SAO calculations and duplicated the simulation results. With this agreement between SAO and LCD results, the other simulations were reviewed. Through cooperative efforts between Dave Lang, Lang Associates (LA) who formulated the JSC tether dynamics simulations (called GTOSS), Dave Arnold and John Glaese, it was discovered that GTOSS did not include so-called inductive derivative terms in its tether dynamics formulation. These terms are required to model tether deployment/retrieval because of the mass flow effects through the discretized tether elements (usually called beads or nodes). A similar review of the Martin Marietta Denver Aerospace (MMDA) tether dynamics simulation showed that the same terms had also been omitted from it, even though the formulations were otherwise independent. When these terms were properly included in the JSC and Martin simulations, the skiprope oscillations were no longer damped and the problem was manifest. The simulations were now in basic agreement and skiprope was perceived to be a significant problem.

Skiprope is induced primarily by current flowing in the tether and, to a much lesser degree, by dynamic events such as orbiter attitude maneuvers and thruster firings. Current induced skiprope growth is greatest for the fully deployed tether. Methods for determining skiprope amplitude, phase and frequency were quickly sought. It was realized that satellite attitude motion was a good source from which skiprope information could be developed. A time domain skiprope observer (TDSO) was developed at Martin Marietta and a frequency domain skiprope observer (FDSO) was developed at MSFC. LCD developed an enhanced form of the FDSO which significantly improved its capability. This was called the Complex frequency domain skiprope observer (CFDSO). The term complex refers to the fact a complex number form of the satellite x and y gyro rates are used in the CFDSO calculations. This development was originally for post flight data evaluation and came too close to the flight to be implemented in ground support software. When the utility of the modified observer became apparent, it was made available for flight support on a laptop computer. Since deployed tether length never exceeded 256 m, skiprope amplitudes in flight stayed small, never exceeding 1 m. Thus, the observers were never really tested in flight. Appendix C contains a memo documenting the
2.2.5 TSS-1 Flight Support

The third task was to prepare for support of the flight of TSS-1. This preparation consisted of participation in three training sessions at the Mission Control Facility at the Johnson Space Center. The fourth task was the support of the TSS-1 mission operations (see Appendix D). The fifth task was to assist in analysis of the TSS-1 telemetry data. The sixth task was to assist in the planning and activities of the TSS-1 reflight studies by serving on the Dynamics Discipline Team and the Single Point Failure Study Team. The final task was to provide support to the Dynamics Discipline Team and the Dynamics Working Group in the planning and analysis activity for the Reflight of TSS-1.

3.0 CONCLUSIONS

A wide variety of activities related to the development of a Definition of Ground Test for Verification of Large Space Structure Control have been performed under contract NAS8-35835. This work included analysis and simulation of large space structure test specimens and configurations which became a part of the LSSGTV facility. In addition, studies of tethered satellite configurations and systems were performed. These studies contributed to two flight programs for two different tethered satellite configurations. As a result of these activities a significant analysis capability has been developed at LCD and a sophisticated tool has been added to the repertoire available to analyze tethered configurations. TSSIM-R is probably the most versatile tool for the simulation of tethers dynamics.
REFERENCES:

1. 1-Cat: A MIMO Design Methodology, Control Dynamics Report, April 1986.


7. Air Bearing Tests,


APPENDIX A

SPINNING TETHER DEPLOYMENT
(TESS)
TETHER DYNAMICS STUDY

(presented at NASA HQ, October 1986)
BACKGROUND

ENERGY STORED IN ROTATION

\[ E_0 = \frac{1}{2} (m + \frac{\mu L_0}{3}) L_0^2 \omega_0^2 \]

ANGULAR MOMENTUM

\[ H = (m + \frac{\mu L_0}{3}) L_0^2 \omega_0 \]

CONSTANT LENGTH \( L_0 \)

ENERGY RELEASED IN DEPLOYMENT

\[ P = TL' \quad \text{(POWER)} \]
\[ L > 0 ; \quad L_0 < L < L_1 \]

ENERGY RELEASED

\[ \Delta E = \frac{1}{2} E_0 \left( 1 - \frac{m + \frac{\mu L_0}{3}}{m + \frac{\mu L_1}{3}} \cdot \frac{L_0^2}{L_1^2} \right) \]

\[ E = \frac{1}{2} \frac{[(m + \frac{\mu L_0}{3}) L_0^2 \omega_0]^2}{(m + \frac{\mu L_1}{3}) L_1^2} \quad \text{RESIDUAL ENERGY} \]
ENERGY STORAGE EQUATIONS

ENERGY OF STRAIGHT, INEXTENSIBLE TETHER DEPLOYING AT RATE $L$:

$$E = \frac{1}{2} (m + \mu L) L^2 + \frac{1}{2} (m + \frac{\mu L}{3}) L^2 \omega^2$$

DEPLOYMENT EQUATION:

$$\frac{d}{dt} (m + \mu L) \dot{L} = -\frac{\mu L^2}{2} + \frac{H^2}{2} - \frac{1}{(m + \frac{\mu L}{3})^3}$$
DEPLOYMENT SCENARIO

STEADY STATE TENSION:

\[ T = (m + \frac{\mu l}{3})L\omega^2 \]

"CONSTANT POWER" DEPLOYMENT (ASSUMES \( \mu = 0 \) (small))

\[ L = L_0 \left(1 - \frac{t - t_0}{\tau}\right)^{-\frac{1}{2}} \]

\[ \ddot{\tau} = \frac{E_0}{P} \]

\[ \ddot{L} = \frac{3L_0}{4\tau^2} \left(1 - \frac{t - t_0}{\tau}\right)^{-\frac{5}{2}} \]

\[ \dot{L} = \frac{L_0}{2\tau} \left(1 - \frac{t - t_0}{\tau}\right)^{-\frac{3}{2}} + \dot{L}_0 - \frac{L_0}{2\tau} \]

\[ L = L_0 \left(1 - \frac{t - t_0}{\tau}\right)^{\frac{1}{2}} + (\dot{L}_0 - \frac{L_0}{2\tau})(t - t_0) \]
SIMULATION DESCRIPTION

- SIMULATES NONLINEAR TETHER DYNAMICS
- ASSUMES POINT END MASSES
- INCLUDES NONSPHERICAL EARTH GRAVITATIONAL FIELD EFFECTS
- INCLUDES AERODYNAMIC FORCES
- ASSUMES UNIFORM CROSS SECTION TETHER
CASES STUDIED

- SIMULATION DATA BASE

TETHER RADIUS = 2.6 mm

TETHER DENSITY \( \mu = 31 \text{ kg/km} \)

MATERIAL STIFFNESS \( \beta = 1.79 \times 10^6 \text{ N} \)

MATERIAL DAMPING \( \gamma = 1.91 \times 10^5 \text{ Ns} \)

END MASS = 100 kg

\[ \ddot{L} = 0.05 \text{ km/sec} \quad (t_0, t_1, t_2) = (4, 26, 55) \text{ sec} \]

\[ \ddot{L} = 0.01 \text{ km/sec} \quad \text{"} = (17, 32, 38) \text{ "} \]

\[ \ddot{L} = 0.03 \text{ km/sec} \quad \text{"} = (4, 30, 37) \text{ "} \]

\[ \ddot{L} = 0.03 \text{ km/sec} \quad \text{"} = (4, 26, 46) \text{ "} \]
SAMPLE
RESULTS
Case 1

$L" = 0.05 \hspace{1cm} 0-4$

$CP \hspace{1cm} 4-26$

$L" = -0.01007 \hspace{1cm} 26-55$

Deployment Profile
Case 1 Results

- Tether in-plane projection
- Power vs time
- Max & Min Tension
Case 2
Deployment Profile

L = .01  0-17
CP = 17-32
L = -.1428  32-39
Case 2
° Tether in-plane projection
° Power vs time
° Tension vs time
Case 3

\[ L = 0.03 \text{ km/sec}^2 \quad 0-4 \]
\[ CP \quad 4-30 \]
\[ L = -0.1087 \quad 30-37 \]
Case 3
- In-plane projection
- Power vs time
- Tension vs time
Case 4

\[ L = 0.03 \quad 0-4 \]
\[ CP = 4-26 \]
\[ L = -0.0233 \quad 26-46 \]
Case 4
°In-plane projection
°Power vs time
°Tension vs time

ORIGINAL PAGE IS OF POOR QUALITY
DISCUSSION OF RESULTS

- NOTE LONGITUDINAL OSCILLATIONS AT TETHER FUNDAMENTAL

- TETHER DEFLECTS LATERALLY BECAUSE OF CORIOLIS FORCES

\[
\mu \, dl \, \mathbf{r}(l) \, \omega^2 \quad \text{(centrifugal force)}
\]

\[
2 \mu \, dl \, \dot{\mathbf{r}}(l) \, \omega \quad \text{(Coriolis force)}
\]

- TETHER DEPLOYS BECAUSE OF CENTRIFUGAL FORCES

- TETHER OSCILLATES LONGITUDINALLY BECAUSE OF ELASTIC PROPERTIES
CONCLUSIONS

- DEPLOYMENT SCENARIO NOT ADEQUATE
  - EXCESSIVE TETHER OSCILLATIONS INDUCED
  - LATERAL AND LONGITUDINAL DYNAMICS SIGNIFICANT
  - POWER LEVELS FLUCTUATE WIDELY

- LARGE POWER LEVELS ACHIEVABLE
  - CLOSED LOOP CONTROL REQUIRED
  - DEPLOYMENT RATE CONTROLLER PROBABLY BEST

- MORE ANALYSIS REQUIRED
  - DEPLOYMENT SCENARIO
  - ENERGY "CHARGE" TECHNIQUES
  - DAMPING REQUIREMENTS
  - EFFECTS OF TAPERED TETHERS
APPENDIX B

TETHERED SATELLITE SYSTEM DEPLOYER TESTS
PERFORMED AT KSC
The final tests of the TSS 1 deployer hardware were performed at the Kennedy Space Center in August of 1991. Dr. J. Glaese of Logicon Control Dynamics, Inc. was an observer for these tests during the period 15-23 August. The tests were conducted in a large, environmentally controlled room in the O&C building.

Several anomalies were noted during the tests. In the first test, deployment stopped after 1.8 m of tether had been reeled off the spool (approximately 140 seconds). It was found to be caused by rubbing in the brake. Speculation was that the technicians who installed the hardware did not follow the alignment procedures which were supposed to prevent this. Ray Head of Martin Marietta adjusted the installation and confirmed it with a short deployment test. These extra activities delayed the test series by two days.

The deployment test was restarted on the 17th of August. Deployment then proceeded normally with no noticeable motor mode / generator mode switching as had occurred during the hardware/software integration tests (HSIT) before the control gains were changed to prevent this. The new gains seem to be working as expected.

There was a strange event that occurred at the twenty minute point of deployment in which noticeable slack appeared in the tether. Nothing appeared to be wrong with any of the flight hardware and after one minute the slack gradually went away. The best estimate of what had happened was that a glitch in the take-up reel (TUR) system occurred to cause this event. Except for this apparent TUR anomaly, deployment proceeded nominally. Test procedures called for keeping in-line thrusters on through 1800 m for test safety reasons. This is different from the planned flight profile which will probably call for cut-off at 1200 m (~4 N tension). Though concern was expressed about this variance from expected flight procedures, I felt this was not a problem.

Retrieval tests were delayed by difficulties loading the new retrieve profile into the DACA, but once this was accomplished, the retrieval proceeded as expected with tension deviations from expected flight conditions due to TUR friction limitations. Also, at end of retrieval, the TUR and flight encoder readings were 80 m different. This was ascribed to TUR calibration and was not considered a problem. As part of on station one reel-in / reel-out tests, an event occurred which was not noted at first by the dynamics observers. An instantaneous switchover was made from proportional control (length and rate only) to basic control which includes tension feedback. The current transient apparently tripped the reel motor power to the off position. This explained why these test results were so different from comparable HSIT results. The instantaneous switchover is not normal procedure and so we did not know what to expect dynamically. Thus, it was not clear at first that anything unusual had happened. We did notice that the tether was reeling out at a slow, steady pace and remarked at how smooth it was. No one was able to understand this behavior but since basic control behaves so erratically, we felt that there was just a large
error which was in the process of being taken out. It turned out that the motor controller assembly (MCA) was held at zero volts in the powered off position. This behaved like an electrical short across the reel so that the back EMF of the motor kept the reel rate to a low value. The lesson here is that we need to be sensitive in flight to this so that we never command an instantaneous switchover from proportional to basic control. Since basic control seems marginally effective at best due to internal friction in the deployer and the consequent effect on tension measurement, use of this mode should be minimized and carefully considered.

Low tension flyaway tests were performed following procedures developed for HSIT. Everything went well during these tests giving us confidence that the friction in the deployer was not excessive and it would behave adequately during the low tension phases of the mission.

The overall implications from the tests were that the deployer would perform the functions required of it. It was also clear that precautions must be taken to assure that the brake is properly aligned with the reel so that there is no dragging. Assurances were given that the vibration environment of launch would not be sufficient to disturb the critical brake alignment with the reel resulting in brake drag which is sufficient to stop deployment. The hardware was to be disassembled for reassembly on the flight pallet in the orbiter cargo bay. Assurances were given that alignment would be reverified after completion of this reassembly. Based on these assurances and the observed behavior of the equipment during HSIT and the current tests, it was concluded that the hardware is capable of performing its mission.
APPENDIX C

DEVELOPMENT OF THE COMPLEX FREQUENCY DOMAIN
SKIPROPE OBSERVER
The tether motion called skiprope can be described as a lateral string vibration in the two transverse directions with respect to the tether. It is nominally circular in cross section but can be highly elliptical with the major axis of the ellipse precessing. Higher order string modes may participate in this motion thereby increasing the complexity. No direct methods are available to observe this motion except at very short tether lengths where orbiter TV cameras may make it observable. Since skiprope motion affects satellite rotational motion in a characteristic way, observer software can be developed which estimates skiprope dynamics.

Observers have been developed to operate in the time domain and in the frequency domain. John Tietz at Martin Marietta/Denver Aerospace developed a time domain skiprope observer (TDSO) which was used as mission support software for the TSS-1 flight. The original frequency domain skiprope observer (FDSO) was jointly developed by Stan Carroll and Keith Mowery at NASA/MSFC and Dr. George Ioup, et al. at University of New Orleans. This FDSO was also used to support the TSS-1 flight as a backup/alternative to the TDSO. The FDSO is based on taking separate discrete fourier transforms (DFT's) of the satellite x and y angular velocities, searching the result over the frequency band appropriate to skiprope motion and computing the skiprope
motion which would be required to produce the observed amplitude and phase. The working assumption for this software is that the skipooppe frequency is well below the pendulous frequency of the satellite so that the satellite motion tracks the tether. The line between the tether attachment point and the satellite center of mass will track the tangent to the tether at the satellite end.

A modification of the treatment of the angular rate data in the DFT leads to a significant improvement in the performance of the FDSO. This modification of the FDSO is devised to take advantage of the following additional concepts: 1. Complex rotating phasors; 2. DFT of complex body rate and use of negative rotational frequencies; 3. gyrocompassing; 4. Body rotational dynamics transfer functions; and 5. Extension to higher order string modes. First, the notion of the complex, rotating phasor arises in the mathematical study of physics and electrical engineering. Since the skipooppe can be resolved into a composite of circular motions of various frequencies, rotating in both directions, the rotating phasor is a natural way to represent the behavior. Second, The close coupling and phase relationships between satellite X and Y body rates is nicely treated in the complex arithmetic domain with positive and negative motions said to have a positive or negative rotational frequency. Third, gyrocompassing is the term used here to describe the way in which the satellite yaw angle can be determined by noting how the orbit rate portion of measured satellite rate is resolved into satellite X and Y body rates just as the direction of the earth's spin axis is determined by gyro measurements on a ship to determine geographic north. Fourth, effects of satellite rotational dynamics on measured body rates arising from skipooppe motions can be determined and compensated to improve the accuracy of the calculation. Fifth, as previously stated, each string mode
frequency is distinct from the others and can be studied separately. The mathematics of these assumptions and concepts will be treated in the following paragraphs.

**Figure 1. Skiprope Geometry.**

Figure 1 shows an illustration of the satellite, tether and shuttle geometry. The Y axis is aligned with the negative of the orbit normal. The Z axis lies along the local vertical pointing toward the earth. The X axis completes the right handed set pointing in the general direction of the orbital velocity vector. This orientation is parallel with the LVLH frame. Assuming that \( \frac{A}{L} << 1 \), the angle which the tether makes with the local vertical at the satellite end in the nth mode can be approximated by \( \theta = n\pi \frac{A}{L} \) (radians). If satellite motion tracks the tether, then the satellite will also be rotated through \( \theta \) with respect to LVLH. We can resolve the planar vector displacement \( \mathbf{A} \) into components \( A_x \) and \( A_y \). Assuming that satellite X and Y are nominally parallel to LVLH X and Y, we can write \( \theta_x = -n\pi \frac{A_y}{L} \) and \( \theta_y = n\pi \frac{A_x}{L} \). Since \( \frac{A}{L} << 1 \), \( \theta_x \) and \( \theta_y \) are small angles. Thus, we can approximate the satellite angular velocity components by \( \omega_x = \dot{\theta}_x \) and \( \omega_y = \dot{\theta}_y \). For now, we ignore orbital rate and satellite
spin. These will be included later. Let us first consider a circular skipsquare motion:

\[ A_x = A \cos(\omega sk t + \phi) \quad \text{and} \quad A_y = A \sin(\omega sk t + \phi) \, . \]

Thus, \( \theta_x = -n \pi \frac{A}{L} \sin(\omega sk t + \phi) \)

and \( \theta_y = n \pi \frac{A}{L} \cos(\omega sk t + \phi) \, . \) With this we can express the skipsquare induced satellite X and Y angular velocity components, \( \omega_x = -n \pi \frac{A}{L} \omega sk \cos(\omega sk t + \phi) \) and \( \omega_y = -n \pi \frac{A}{L} \omega sk \sin(\omega sk t + \phi) \) expressed in LVLH.

Orbit rate must be added along the negative y axis of LVLH to account for motion of this frame which the satellite is compelled to track by tension forces. Adding orbital rate along the Y axis, we obtain \( \omega_y = -n \pi \frac{A}{L} \cos(\omega sk t + \phi) - \omega_o \). If the satellite is spinning about its Z axis at a rate \( \omega_s \), the rate components in the satellite frame S will be:

\[
\begin{align*}
\omega^S_x &= -n \pi \frac{A}{L} \omega sk \cos(\omega sk t - \omega_s t + \phi - \phi_s) - \omega_o \sin(\omega_s t + \phi_s) \\
\omega^S_y &= -n \pi \frac{A}{L} \omega sk \sin(\omega sk t - \omega_s t + \phi - \phi_s) - \omega_o \cos(\omega_s t + \phi_s)
\end{align*}
\]

At this point, inspection of these expressions suggests that they could be represented more concisely in complex form by letting \( \omega_c = \omega_x + i \omega_y; \) \( (i = \sqrt{-1}) \). With this substitution, we have \( \omega_c = -n \pi \frac{A}{L} \omega sk e^{i(\omega - \omega_s) t + \phi - \phi_s} - \omega_o e^{-i(\omega_s t + \phi_s)} \). This complex angular velocity can now be interpreted as the sum of two circular rotating complex phasors: The first is rotating with angular frequency \( \omega = \omega sk - \omega_s \) and complex amplitude \( -n \pi \frac{A}{L} \omega sk e^{i(\phi - \phi_s)} \); The second is rotating with frequency \( -\omega_s \) and complex
amplitude $-i\omega_0 e^{i\omega t}$. This result is easily generalized to precessing, elliptical motions by noting that an ellipse can be constructed from two circular motions of different amplitudes turning in opposite directions at equal frequencies, i.e. frequencies that are negatives of each other. If the magnitudes of these frequencies are different, the ellipse precesses with the rate determined by half the sum of the frequencies including their signs. Thus, an elliptical skipoare motion with precession is represented by

$$\omega_c = -e^{-i\delta} \left( i\omega_0 e^{-i\omega_0 t} + n\frac{A_r}{L} \omega_r e^{-i(\omega_r - \omega_0) t + \phi_r} + n\frac{A_b}{L} \omega_b e^{-i(\omega_b - \omega_0) t + \phi_b} \right)$$

Let $a_s = -i\omega_0 e^{-i\delta}; a_r = -A_r e^{i\delta}; a_b = -A_b e^{i\delta}$. We can now use DFT's to solve for $a_s, a_r, a_b$. The amplitude $a_s$ expresses orientation of orbit normal at $t=0$. The amplitudes $a_r$ and $a_b$ are "forward" and "backward" skipoare amplitudes respectively. Forward means positive rotational sense and backward means negative rotational sense.

All satellite/tether oscillations of sufficiently small magnitude to satisfy linearity, fit this scheme. The mathematics discussed above forms the basis of a new, skipoare observer based on complex analysis. For this reason, it is being called the complex frequency domain skipoare observer (CFDSO). Many test cases developed from TSS-1 simulations have shown it to give excellent results for cases involving a spinning as well as a non spinning satellite. It also gives good phase and amplitude data for precessing ellipses and more complicated motions involving higher order tether modes. It was originally investigated as an enhancement for the FDSO for use in post flight evaluation but its performance was sufficiently superior to the FDSO while requiring no more elaborate calculations that an attempt was made to make it available to support the flight.
From the events of the flight, it became clear that the CFDSO software implemented was not going to be sufficient. As stated previously, the formulation was based on the assumption that pendulous frequencies would be high enough to be ignored. At the lengths TSS-1 actually achieved, this assumption is not valid. To treat skipoop at short tether lengths where skipoop frequencies are of the same order as the satellite pendulous frequencies, the satellite dynamics transfer function must be taken into account. Typically, at this length, the skipoop frequency is above the pendulous frequencies. The satellite moments of inertia and the radius from the tether attachment point to the satellite center of mass must be known to determine the transfer function for satellite transverse (X and Y) axes. The linearized equations of motion for the satellite transverse axes are

\[
l_x \dot{\omega}_x = (l_z - l_y) \omega_y \omega_y - RT(\theta_x + n\pi \frac{a_x}{L})
\]

\[
l_y \dot{\omega}_y = (l_x - l_z) \omega_z \omega_x - R T(\theta_y - n\pi \frac{a_x}{L})
\]

The corresponding kinematic relationships are

\[
\omega_x = \dot{\theta}_x - \omega_s \theta_y - \omega_o \sin(\omega_s t + \phi_s)
\]

\[
\omega_y = \dot{\theta}_y + \omega_s \theta_x - \omega_o \cos(\omega_s t + \phi_s)
\]

This set of dynamic and kinematic equations provides the basis for determining the
satellite dynamical transfer functions for skiprope.

\[-n\pi \frac{\omega + \omega_s}{L} A(\omega) = [1 - \frac{\omega (\omega + \omega_s)}{\Omega_x^2}] \omega_x + i[1 - \frac{\omega (\omega + \omega_s)}{\Omega_y^2}] \omega_y \]

\[-\omega_s (\omega + \omega_s) \left( \frac{l_z}{l_x^2} \omega_x + i \frac{l_z}{l_x^2} \omega_y \right) \]

\[+ i\omega_o \delta_{\omega_o - \omega_o} \]

The quantity on the left of the equality is the generalization of \( \omega_o \). The quantities \( \Omega_x \) and \( \Omega_y \) are the satellite pendulous frequencies for the X and Y axes respectively. This generalization accounts for the effect of satellite dynamics and spin. It also has generalized the results to include short tethers and low pendulous frequencies. As noted previously, the rotation of the LVLH frame and the action of the gravity gradient forces which compel the satellite to track it, provides a convenient method for determining the yaw angle since the LVLH frame rotates about the orbit normal which is approximately fixed in space. Corresponding to the previous equation for the skiprope amplitude is the equation for orbital rate amplitude. As before, the orbital rate component is the key to determining the phase of skiprope rotation.

\[-\lim_{\omega \to \omega_o} \left( \frac{n\pi}{L} (\omega + \omega_s) A \right) = i\omega_o e^{i\gamma_o} \]

A revision to the CFDSO has been implemented for post flight evaluation based on the above considerations. This software is now being used to analyze TSS-1 gyro data for presence and characteristics of skiprope. Our preliminary investigations show that skiprope oscillations of modest amplitudes are indeed present and show very small
tendencies to damp. These results will be included in our post flight investigation report.

An additional outcome of the CFDSO development has been greater insight into proper phasing of the orbiter yaw maneuvers which have been shown through simulation to be capable of reducing the amplitude of skiprope motions. For a truly linear system exhibiting skiprope motions at several amplitudes and frequencies, the resulting motion is the superposition of the several motions which can be determined separately. Thus, yaw maneuvers can be used to address these motions separately. From this logic, the proper phase, rate, direction and duration of the maneuver can be determined for each separate skiprope motion. These could in principle be used to null each mode in succession, though this is probably impractical and unnecessary. Only the largest skiprope amplitude needs to be addressed and its amplitude need only be adjusted to the minimum extent necessary. Let us define $t_0$, $t_{\text{now}}$, $t_{\text{start}}$, $t_{\text{dur}}$, $\omega_{\text{yaw}}$ as time of beginning of data set taken for CFDSO, present time, time to start the yaw maneuver for optimum phase, duration of yaw maneuver and angular rate to be held for maneuver respectively.

The calculation requires $a_r, a_b, \omega_r, \omega_b$ which is provided by the CFDSO. From these parameters, several calculations are made. The following algorithm calculates the parameters for the yaw maneuver:

1. $a_d = |a_r| - |a_b|$
2. if $a_d > 0$, then $\omega = \omega_r$, and $a = a_r$ else $\omega = \omega_b$, $a = a_b$ and $a_d = -a_d$
3. $t_{\text{dur}} = k a_d \frac{2\pi}{|\omega|}$
4. $\phi = \arctan(\frac{\text{imag}(a)}{\text{real}(a)}); \text{four quadrant}$
5. $t_n = t_0 - \frac{\phi - \phi_{\text{orb}}}{\omega} + (n + \frac{1}{4}) \frac{2\pi}{|\omega|}$

The nth opportunity to do a yaw maneuver is given by $t_n$. The next opportunity is the first
value of $t_0$ which is future with respect to $t_{\text{now}}$. This is the value to assign to $t_{\text{start}}$. The yaw rate for the maneuver is given by the value of $\omega$. The constant $k$ is the number of turns required to damp 1m of skiprope. Its value is approximately equal to 0.1 turns/m.

The algorithm above sizes the yaw maneuver to reduce the amplitude of the larger skiprope component to equal the smaller so that the resulting motion is approximately linear. Linear oscillations are expected to be damped effectively by action of the reel motor in length control mode. Other strategies may be employed by changing the amplitude to be damped, $a_d$, to the desired value. A few cases using this algorithm have been run with the Logicon Control Dynamics tether simulation and worked successfully but more extensive looking should be made to verify the conjectures made here. However, it is clear that the insights provided by this analysis have removed much of the mystery in elliptical, precessing skiprope including the effects of higher order modes.

Comments with respect to the information in this technical memorandum would be appreciated and may be addressed to John Glaese at Logicon Control Dynamics, Inc., 205-882-2650.
APPENDIX D

MISSION SUPPORT FOR THE TSS-1 FLIGHT
The flight of TSS-1 was marred by repeated problems with the deployer hardware. The main problems seem to have been the sticking of the U2 umbilical and the repeated snagging of the tether either in the reel or somewhere in the mechanism between LTCM and the UTCM. A contributing problem was the undersized and trouble-prone vernier motor. It was neither powerful enough to overcome these problems nor robust enough to take the abusive treatment such as "clutch popping" or backdriving required to free the various jams. The consequence was a failure to fully deploy the satellite and severe curtailing of the mission. This was a gross example of a false economy. It was known pre-flight that the vernier motor was marginally sized and somewhat fragile but tests did not reveal a real problem so no strong arguments other than prudence could be made to replace it.

For flight support, software was developed to observe and manage growth of skiprope oscillations. The time domain skiprope observer (TDSO) was a Kalman filter developed to use a variety of satellite sensor information to indirectly sense and reconstruct tether skiprope oscillations. The frequency domain skiprope observer (FDSO) used frequency domain signal processing techniques to perform a similar function. The FDSO was considered a backup to the TDSO because of its more limited capabilities. We developed an enhanced version of the FDSO which removed the limitations of the original FDSO for post flight evaluation during the final weeks prior to the TSS-1 flight. The performance of this observer was so promising that it would have made the TDSO unnecessary. Unfortunately, it was developed too late to be incorporated in the mission support software but was available in a contingency mode. The original observers were developed to treat circular skiprope oscillations and were not adequate to treat highly elliptical, precessing skiprope motions. The modified FDSO eliminated these restrictions. Circumstances obviated the need for the skiprope observer software. The updated FDSO will be described elsewhere. This software will be used to evaluate TSS-1 dynamics data and is available to support future TSS flights. In addition to the FDSO, we developed techniques for defining orbiter yaw maneuvers to damp highly elliptical, precessing skiprope motions.
This report summarizes the work done in the planning and development of the Marshall Space Flight Center, Large Space Structures Ground Test Facilities. Major topics include a summary of the reports written documenting the facility development, tether dynamics and control studies and simulations and test activities in support of the development of the Tethered Satellite System (TSS).