Mechanical Failure Probability of Glasses in Earth Orbit

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I. Abstract

Results of five years of Earth-orbital exposure on mechanical properties of glasses indicate that radiation effects on mechanical properties of glasses, for the glasses examined, are less than the probable error of measurement. During the 5 year exposure, 7 micrometeorite or space debris impacts occurred on the samples examined. These impacts were located in locations which were not subjected to effective mechanical testing hence limited information on their influence upon mechanical strength was obtained. Combination of these results with micrometeorite and space debris impact frequency obtained by other experiments permits estimates of the failure probability of glasses exposed to mechanical loading under earth-orbit conditions. This probabilistic failure prediction is described and illustrated with examples.

II. Introduction

Design of glass components employed in earth orbit has a long and rich history and glass elements of orbital systems have not contributed to any known system failures. Growth of orbital debris and reports of micrometeorite impacts on space systems have heightened concern for safe design of glass components of orbital systems. To the authors knowledge no analytical techniques are currently available to predict failure probabilities of glass components in orbital systems.

Extensive data is available from the NASA LDEF (Long Duration Exposure Facility) and other flights to permit characterization of the size and frequency of micrometeorite and space debris impacts on orbital systems as is shown in Figure 1. The authors recently completed analyses of mechanical strengths of a group of glass samples exposed on LDEF. The total number of impact events suffered by these glasses was small and the data obtained are insufficient to permit statistically valid predictions of failure probabilities of glass in space. This paper combines the results of careful analysis of a small group of impact events on glasses with the larger collection of micrometeorite and debris impact data to produce a prediction of the probability of failure of a glass exposed in earth orbit as a function of stress on the glass.

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Ideally the determination of failure probability of a stressed glass sample in earth orbit would be determined by exposure of a large group of stressed samples for a period of time sufficient to obtain statistically valid exposures to the impact of a large number of micrometeorites or space debris. Unfortunately such experiments would require a large number of mechanical loading fixtures and associated instrumentation to ascertain the failure times for a variety of stress levels. Such an experiment is beyond the capability of existing hardware.

The experiment which has been completed is the exposure of 120 glass and glass ceramic samples in a passive unloaded state for a period of 5.8 years in earth orbit as previously described. These experiments resulted in the conclusion that the mechanical strength and optical transmission of those samples was, within the limitations of the measurement and experiment, unchanged by earth-orbit exposure. The samples did however suffer seven micrometeorite or debris impacts which clearly degraded the mechanical strength at the impact location. These impacts, because of their statistically determined locations, did not occur at locations permitting proper determination of their effect on the glass mechanical properties.

It was possible to determine the maximum stress concentration for several impacts, and this data provides the basis of the calculations reported here. The conclusion of this analysis was that the damage field associated with the impacts had a depth to diameter of crater of less than 1/4 which resulted in a stress concentration of not more than 2. Data gathered by other LDEF investigators on the flux of particles sharing the orbit with spacecraft was incorporated into the mechanical damage calculations. The results enable prediction of the life expectancy of glass components in space. The design of glass components for earth orbiters may be refined by application of this result.

III. Experimental Procedure

Samples, from which the strength measurement results (Table I) of glasses exposed on the LDEF (Table II) were obtained, were examined for impact sites from micrometeorite or debris. The stress field during biaxial flexure testing was related to the position of the micrometeorite or debris impacts. An estimate of the maximum stress at the site of the impact was calculated from the failure strength of the sample containing the impact. Fracture mechanics were applied using the estimated stresses to determine the maximum flaw size capable of withstanding the stresses before failure. The crack depths determined from the relation between critical flaw size and applied stress, were compared to radial crack lengths and crater diameters measured from scanning electron micrographs of the impacts. Particle size was calculated from crater size using cratering mechanics. Statistical information on particle size and flux was incorporated into the probability of failure calculation. A model of a circular window exposed to conditions in a low earth-orbit environment was developed. The stresses on the surface of an 80 cm diameter window under a pressure of 1 atmosphere were deter-
mined. Equations were developed relating the stress surrounding an impact to the flux of particles in a low earth-orbit. These equations were integrated with respect to the dimensions of the window to determine the expected time to failure. Probability of failure was calculated using the Poisson distribution.

IV. Results

Stresses at the point of the micrometeorite or space debris impacts were found for each of the impacted samples described in Table I. Table III reports observed crater diameters, spall diameters and melt pit diameters. Particle size ($D_o$) was computed from crater diameter ($d_o$) with the aid of defined cratering relationships. Cratering mechanics provides established relationships between projectile size and crater size for projectiles with similar densities and velocities. A relation was found between radial cracking, melt pit diameter, and crater diameter for the impacted LDEF glasses. According to the data in Table III, radial crack lengths are on the order of five times the melt pit diameter, and between two and three times the crater diameter. Through this series of relationships the depth of the surface flaw created during impact in glass may be estimated for any given particle size assuming the particles have similar densities and velocities. Particle flux versus particle size data reported by Kinard' as illustrated in Figure 1 was integrated with the cratering data for the 7 impacts. The surface damage per unit time per unit area leading to failure was determined for a glass component in the earth-orbit environment. The expected time to failure depends on the surface stresses of the component and the depth of impact damage.

Failure of glass occurs when the stress surrounding a flaw exceeds the strength of the glass. Maximum tensile stresses due to bending occur at the surface where flaws intensify local stresses. The flaw depth and crack tip radius determine the stress concentration associated with the flaw. A flaw created by the impact of a micrometeorite or space debris particle with an orbiter window will cause failure if the stress concentration factor at the impact site and the stress applied at that location exceeds the strength of the glass. Particles producing flaws with depths less than the critical crack length will not cause failure, thus failure from impact depends on flaw depth and local stress. Local surface stress is determined by the applied pressure and window geometry. Particles producing flaws larger than the critical size will cause failure in glass, thus the threshold particle size depends on the specific stress of each surface element. The total number of impacts leading to failure for a window depends on the product of the area under constant stress, the time of exposure, and the flux of particles exceeding the threshold size to create critical flaws. The average cumulative flux ($\phi$) of micrometeorites and space debris arriving over a period of one year, per square meter of area was found from Kinard's data to fit a line approximated by:

$$\phi = bD^m$$

where $b = 4 \times 10^4$, $m = 2.1$, and $D$ is the particle size.
The critical particle size \( (D_c) \) is the particle size which will produce a flaw with a depth exceeding the critical crack length. \( D \) in equation (1) may be replaced by \( D_c \) to yield an equation which describes the flux of particles causing failure. The critical particle size was determined from cratering relations and the critical flaw size. The critical flaw size \( (a_c) \) was found by applying the stress concentration equation for a semielliptical surface flaw which is given as:

\[
\frac{\sigma_{th}}{\sigma_a} = 2\sqrt{\frac{a_c}{\rho}}
\]

where \( \sigma_a \) is the theoretical strength, \( \sigma_a \) is the applied stress, \( a \) is the crack depth, and \( \rho \) is the curvature of the crack tip. Failure occurs when the crack depth equals the critical flaw size needed to increase the stress at the crack tip to the theoretical strength of the glass. This occurs in glasses impacted by micrometeorites or space debris when the particle size and velocity is great enough to produce damage greater than or equal to the critical flaw size. The average velocity of particles for the trailing-side of LDEF was estimated\(^a\) to be on the order of 13 km/s. Flaw depth may be determined from projectile size, assuming the velocity of impacting particles for the samples in this investigation is constant. The critical particle diameter \( (D_c) \) may be substituted in the stress concentration equation along with the radius of curvature for the median cracks. The radius of curvature was calculated by the authors\(^3\) to be about \( 4 \times 10^3 \) \( \mu \)m. The equation as applied to the impacted glasses becomes:

\[
\sigma_a = \sigma_{th} \sqrt{\frac{\rho}{4D_c}}
\]

which yields the critical stress leading to failure for a given particle size. The particle size creating a critical flaw is therefore:

\[
D_c = \left( \frac{\sigma_{th}}{2\sigma_a} \right)\frac{\rho}{4}
\]

Equation (1), written in terms of the critical particle size, defines the cumulative particle flux causing failure in earth-orbit glasses. Substituting equation (4) for \( D_c \) in equation (1) yields a flux equation, which is dependent on the applied stress, given by:

\[
\phi_{(\sigma)} = b \left[ \left( \frac{\sigma_{th}}{2\sigma_a} \right)^2 \frac{\rho}{4} \right]^{2.1}
\]

The expected number of impacts causing failure per unit time per unit area was found by applying the stress-dependent flux equation over the surface of the glass. The calculation is the product of \( \phi_{(\sigma)} \) and the area under constant stress. Generally, glasses used in space applications would not be expected to have constant stress over the entire surface
exposed to the space environment; hence, the total number of expected fatal impacts per unit time is found by integrating the flux equation for particles greater than or equal to the critical size over each area increment under critical stress. The expected number of impacts leading to failure was determined by integrating over the surface:

\[ \iint \phi_{(a)} \, d\sigma \, dA \]  

(6)

Application to an Orbiter Window

The stress distribution on the surface of a glass window with circular geometry is determined by:

\[ \sigma_{(x)} = \frac{6 \, P \,(3 + \nu)}{16 \, t^2} \,(R^2 - r^2) \]  

(7)

where \( P \) is the applied pressure, \( R \) is the outer radius, \( r \) is the radius, and \( \nu \) is Poisson's ratio, which for silicate glasses is about 0.3. The maximum stress occurs in the geometric center of a simply supported circular window. The stress decreases as \( r \) approaches the outer radius and is constant in any annulus. The area of each annulus with constant width increases as \( r \) increases; thus area limits the number of particles causing failure in the near geometric center of the window, and stress, which tends to zero, limits the number of particles causing failure in the outer annuli. This premise is valid for particle sizes less than those required for penetration, which limits the flux of critical particles to be greater than the penetration flux. The flux of critical impacts is illustrated by a graph of the number of impacts causing failure per annulus per year as a function of radius. This function is represented by Figure 2 for a window under 1 atmosphere of pressure with a diameter of 0.8 meters (area = 0.5 m²) and a thickness of 3 cm. The area under the curve is the total number of impacts which the window is expected to receive per year. The surface integral giving the total number of expected fatal impacts for the circular window is:

\[ \int_0^{2\pi} \int_0^r \phi_{(x)} \, r \, dx \, d\theta = 2\pi \int_0^r \phi_{(x)} \, r \, dx \]  

(8)

where \( \phi_{(x)} \) is a function of the cumulative particle flux, surface tensile stress, and annulus area, the latter two being dependent on the radius \( r \). The value of this integral is well approximated by taking the sum of the impacts causing failure per year per annulus over all annuli by:
The number of years expected before failure is the inverse of this sum.

The probability of failure may be determined from the cumulative number of impacts causing failure. The impact events may be considered as Poisson arrivals over a given period of time. The Poisson probability function is given by:

\[ P(x, \lambda) = \frac{\lambda^x e^{-\lambda}}{x!} \]  

where \( \lambda \) is the sum found by equation (9) and \( x \) is the Poisson variable equal to 0 if no failure occurs. The probability that failure will occur in time \( t \) is given by:

\[ P(t)_{\text{failure}} = 1 - e^{-\lambda t} \]  

Figure 3 shows the expected years to failure as a function of applied pressure for several different thicknesses. Windows with no applied stress are limited by the flux of particles large enough to cause penetration. The expected time for one impact causing failure in glass with a thickness of 3 cm, a diameter of 0.8 meters, under a pressure of 1 atmosphere is 35 years, which is a reasonable requirement for earth-orbit space station components. Equation (11) estimates a 0.84 probability that the glass would fail in 35 years and a 5% chance of failure in one year. A window with similar geometry requires a thickness over 6.5 cm to achieve a 0.008 failure probability in 35 years. A 4.7 cm thick window with no applied stress yields an equivalent failure probability.

V. Discussion

Reliance upon a single LAYER window does not permit satisfaction of design lifetime criteria for earth orbiters. The probability of success for components affecting crew safety in the current shuttle program is currently 0.992. The failure probability of a window may be significantly decreased by using multi-layer windows. In fact, current window design for spacecraft in the shuttle program employs multi-layer fused-silica and alumino-silicate windows. Failure of the window is dependent on the joint probability of each component. An independent joint probability assumes that once the integrity of an outer layer is violated then the outer window offers no protection to inner windows. This assumption produces a conservative estimate of the failure probability for multi-layer windows. The outer glass in a multi-layer window requires a larger particle size to initiate failure as compared to the inner pane because the outer pane is not pressurized. This circumstance eliminates the dependence of failure on stress. Failure of the unstressed window due to micrometeorite or space debris interaction becomes dependent
on penetration of an impacting particle. Assuming the ratio of crack depth to particle size remains constant, the integrity of the window would be compromised for particles of 1/4 the glass thickness according to cratering analysis. This assumption is valid only if the velocity of particles is no greater than the velocity of those particles impacting the trailing edge of LDEF. Cratering calculations’ suggest penetration is dependent on impact energy; thus, smaller particles penetrate at higher velocities. Multi-wall protection of critical parts in space applications is a well known means of protection and has been well investigated. The outer plate is an important component since its purpose is to absorb the initial shock and fragment or contain the impacting particle. A failed outer layer not leading to immediate inner layer failure may be replaced, which reduces the chance that a particle will impact the inner window.

Probability of failure for a two layer window may be determined by applying the probability density function of the Poisson distribution to the joint probability of failure of two layers. The probability is calculated by integrating the probability density functions over time of service. The probability density function for the windows is determined from the integral of the joint probability function given as:

$$\int_0^\tau \lambda_1 e^{-\lambda_1 \tau} \lambda_2 e^{-\lambda_2 (\tau - t)} d\tau$$

where $\lambda_1$ and $\lambda_2$ are the flux of critical impacts for the inner and outer window respectively, $\tau$ is the time to outer layer failure and $t$ is the time to inner layer failure. To achieve a 0.992 probability of success in a two-layer window with an inner 3 cm thick window, the outer window would need to be 3.5 cm thick.

The Poisson probability function is convenient for approximating binomial functions which depend on several variables. The probability of failure found from the Poisson distribution approximates the value determined directly from the flux equation of the expected number of impacts leading to failure for less than 0.1 expected fatal impacts per year. For rates of more than 0.1 expected fatal impacts per year, the expected number of impacts is not valid for approximating the probability of a component failure. The Poisson limit accounts for survival chances at high probabilities of failure and is an accurate model when the total number of events is not defined; hence, this distribution is widely used to model "real world" phenomena.

Damage due to micrometeorite or space debris in the glasses incorporated in this study occurred at velocities on the order of 13 km/s. Mandeville has found that craters formed in glass in the hyper-velocity range to be primarily dependent on velocity. Dependence on projectile and target density was secondary. His findings determined the central pit diameter was on the order of 2.5 times the particle size. Estimations of crater diameters determined by explosive cratering studies have been determined by applying Pi-group scaling laws. The calculations were developed for cratering in rock
and soil, but were found by this author to approximate microcrater sizes on the order of the spall diameters listed in Table I. Assuming the scaling laws are valid for micrometeorite and space debris impacts over the range of particle velocities in low earth-orbit, flaw depths in glass may be estimated for impacts above 13 km/s using cratering dimensions and particle flux. Scaling laws predict that crater size is dependent on \( E^{1/3} \), where \( E \) is impact energy; thus, the damage from micrometeorite and space debris would be expected to increase as \( v^{2/3} \); however, this dependence has not been confirmed in earth-orbit glasses. Melting occurs in glass at impact velocities greater than about 10 km/s. Depth of damage may be reduced by melting and viscous flow since the dissipated energy may relieve stresses related to crack growth.

The velocity of particles impacting glass on an earth orbiter depends on the orientation of the glass with respect to the ram direction of the orbiter. The velocity of particles impacting the ram side of earth orbiters has been estimated to be about 20 km/s; therefore, the flaw depths produced by micrometeorite and space debris should be a maximum at this orientation. The cratering process is dependent on the particle velocity; thus, the magnitude of melting and cratering produced by particles at velocities greater than 13 km/s would be expected to scale accordingly. Assuming the relation between crater size and depth of damage in the glasses is comparable for higher velocity impacts and that the flux of particles is constant, the probability of failure for glasses exposed to higher velocity particles may be calculated.

Windows of an earth orbiting space-station would be susceptible to hazards of the earth orbit environment. Glasses integrated into such systems must be designed to maximize the safety of the crew members for the duration of the orbiter’s flight. Proper window design must include the probability of a penetrative meteorite or space debris impact with the window systems. This study has produced a means of predicting the lifetime expectancy of such windows by applying cratering relationships and fracture mechanics to glasses exposed in the earth orbit environment. Further studies should examine impact parameters determined from higher velocity impacts in order to investigate crack morphologies of the higher energy impacts. Information on flaws in glass produced at higher velocity would be useful in representing particle impacts in glass components on the ram side of earth orbiting modules.

VI. Conclusions

Measured micrometeorite and debris flux, and particle size may be used to estimate failure probabilities of glasses in earth-orbit.

Failure probability of windows depends on design, pressure and geometry. The use of a multi-pane design significantly reduces the probability of window failure.

Earth orbit life expectancy of glasses may be computed from particle flux and cratering relationships.
Acknowledgements

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### Table I. Location of Impacts with respect to Stress Contours

<table>
<thead>
<tr>
<th>Sample Stress Type</th>
<th>Stress Contour</th>
<th>Failure Stress (MPa)</th>
<th>Minimum (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fused Silica</td>
<td>0.4</td>
<td>103</td>
<td>39</td>
</tr>
<tr>
<td>Zerodur</td>
<td>0.2</td>
<td>125</td>
<td>31</td>
</tr>
<tr>
<td>Zerodur</td>
<td>0.5</td>
<td>125</td>
<td>62</td>
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<tr>
<td>Zerodur</td>
<td>0.3</td>
<td>135</td>
<td>41</td>
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<tr>
<td>Pyrex</td>
<td>0.1</td>
<td>106</td>
<td>11</td>
</tr>
<tr>
<td>Soda-Lime</td>
<td>0.4</td>
<td>105</td>
<td>37</td>
</tr>
<tr>
<td>BK-7</td>
<td>&lt;0.1</td>
<td>130</td>
<td>13</td>
</tr>
<tr>
<td>Type</td>
<td>Comment</td>
<td>SiO₂</td>
<td>Na₂O</td>
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<td>------------</td>
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<td>------</td>
</tr>
<tr>
<td>BK-7</td>
<td>Optical Crown</td>
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<td>5.5</td>
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<tr>
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<td>Schott low expansion glass ceramic</td>
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<td>0.7</td>
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<td>Material</td>
<td>Melt Diameter (µm)</td>
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<td>Spall Diameter (µm)</td>
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<tr>
<td>BK-7</td>
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<td>200</td>
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<tr>
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<tr>
<td>Zerodur**</td>
<td>No Melt</td>
<td>100</td>
<td>275</td>
</tr>
</tbody>
</table>

* This sample received two impacts.
** This sample exhibited no melting.
Figure 1  Meteorite and Debris Environment Definition. (after Kinard, Ref. 4)
Figure 2  Number of Impacts/Yr Causing Failure as a Function of Window Radius
Figure 3 Expected Years to Failure for Varying Pressure and Window Thickness.
References


