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A Combined Algorithm for Minimum Time Slewing of Flexible Spacecraft*

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ABSTRACT

The use of Pontryagin's Maximum Principle for the large-angle slewing of large flexible structures usually results in the so-called two-point boundary-value problem (TPBVP), in which many requirements (e.g., minimum time, small flexible amplitude, and limited control powers, etc.) must be satisfied simultaneously. The successful solution of this problem depends largely on the use of an efficient numerical computational algorithm. There are many candidate algorithms available for this problem (e.g., quasilinearization, gradient, and shooting, etc.) In this paper, a proposed algorithm, which combines the quasilinearization method with a time shortening technique and a shooting method, is applied to the minimum-time, three-dimensional, and large-angle maneuver of flexible spacecraft, particularly the orbiting Spacecraft Control Laboratory Experiment (SCOLE) configuration.

Theoretically, the nonlinear TPBVP can be solved only through the shooting method to find the "exact" switching times for the bang-bang controls. However, computationally, a suitable guess for the missing initial costates is crucial because the convergence range of the unknown initial costates is usually narrow, especially for systems with high dimensions and when a multi-bang-bang control strategy is needed. On the other hand, the problems of near minimum time attitude maneuver of general rigid spacecraft and fast slewing of flexible spacecraft have been examined by the authors through a numerical approach based on the quasilinearization algorithm with a time shortening technique. Computational results have demonstrated its broad convergence range and insensitivity to initial costate choices.

Consequently, a combined approach is naturally suggested here to solve the minimum time slewing problem. That is, in the computational process, the quasilinearization method is used first to obtain a near minimum time solution. Then, the acquired converged initial costates from the quasilinearization approach are transformed (tailored) to and used as the initial costate guess for starting the shooting method. Finally, the shooting method takes over the remaining calculations until the minimum-time solution converges. The nonlinear equations of motion of the SCOLE are formulated by using Lagrange's equations, with the mast modeled as a continuous beam subject to three-dimensional deformations. The numerical results will be presented and some related computational issues will also be discussed.

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INTRODUCTION

* Future space missions (target acquisition, target tracking, and surveying multiple targets, etc.) require:

  - large-angle rotational (attitude) maneuver (slew);
  - 3-dimensional (3-D, 3-axis) maneuver;
  - large flexible spacecraft maneuver;
  - *minimum time maneuver.*

* Application of Pontryagin's Maximum Principle to the nonlinear slewing problem:

I. Non-Minimum-Time Slews:

1. 3-D Rigid Spacecraft
   Junkins, Turner, Vadali, Wie, Bainum and Li, etc.

2. 2-D (Single-Axis Rotation) Flexible Spacecraft
   Turner, Junkins, Vadali, Chun, Thompson, Bainum and Li, etc.

3. 3-D Flexible Spacecraft (SCOLE)
   Bainum, Li and Tan.

II. Minimum-Time (Near-Minimum-Time) Slews:

1. 3-D Rigid Spacecraft
   Bainum and Li, Vadali, Wie, etc.

2. 2-D Flexible Spacecraft
   Singh, Junkins, Vadali, Byers, Bainum and Li.

3. 3-D Minimum-Time Flexible Spacecraft; Using *Quasilinearization Method and Shooting Method*: present paper.

OUTLINE

1. 3-D Dynamics of Flexible Spacecraft
   - State Equations
2. Time Optimal Control Problem Formulation
   - Two-Point Boundary-Value Problem (TPBVP)
3. Quasilinearization Method for Near Minimum Time Slew
4. Shooting Method
5. Initial Costate Transformation
   - Scale Factors
   - Combined Algorithm
6. Numerical Examples
7. Conclusions

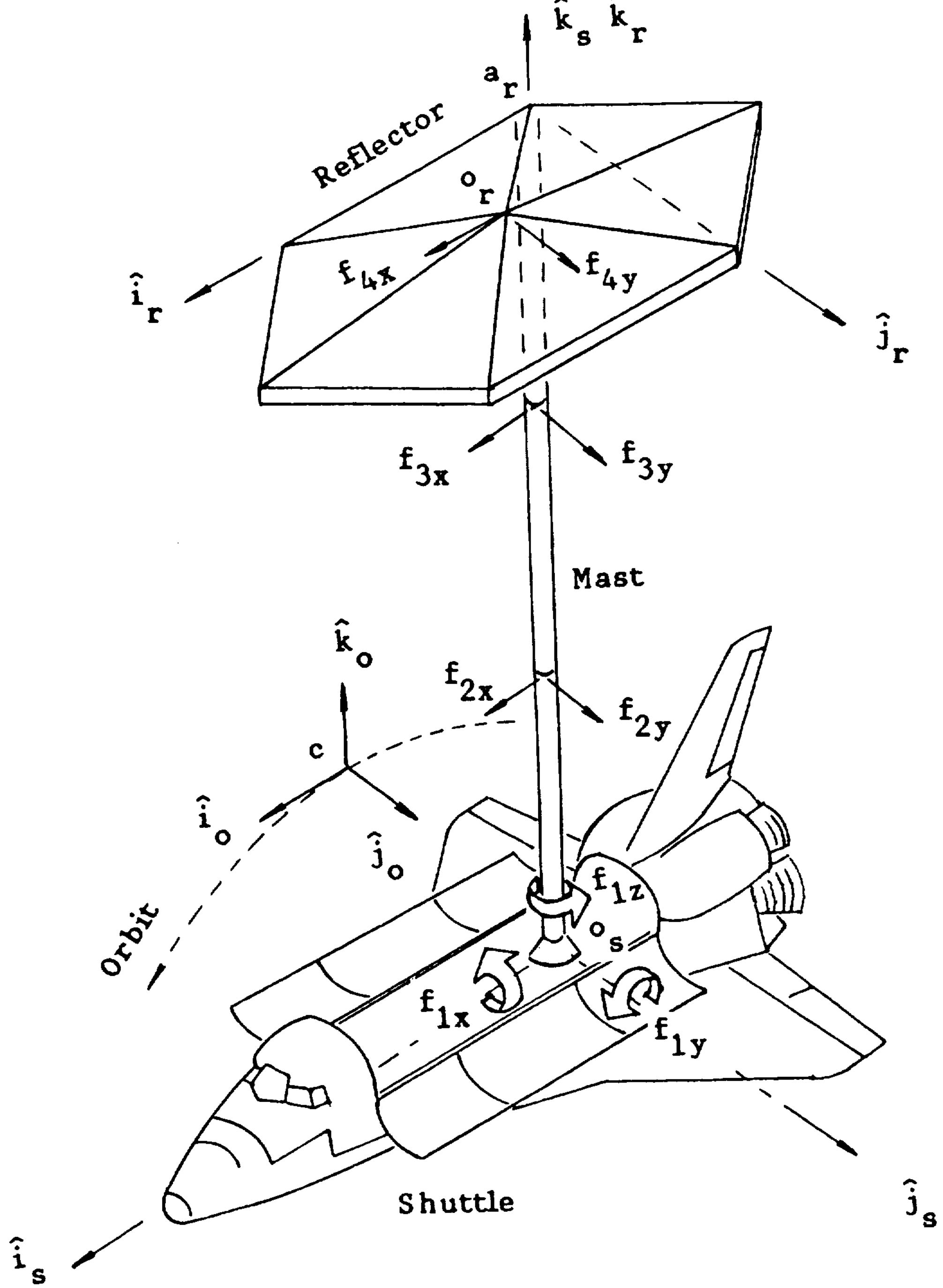


Figure 1. Drawing of the orbiting SCOLE configuration.

3-D DYNAMICS OF FLEXIBLE SPACECRAFT

I. 3-D Deformations of the SCOLE Mast (modal superposition):

$$U=\sum_i \xi_i(z)\,\alpha_i(t), \quad V=\sum_i \eta_i(z)\,\alpha_i(t), \quad \phi=\sum_i \zeta_i(z)\,\alpha_i(t) \tag{1}$$

where

U, V - bending in x and y directions;

ϕ - torsion in z direction;

ξ_i, η_i, and ζ_i - modal shape function vector components;

α_i - a scaled modal amplitude associated with the ith mode;

z - coordinate.

II. State Equations:

$$\dot{q} = \frac{1}{2}\underline{\tilde{\omega}}q, \quad \text{where} \quad \underline{\tilde{\omega}} = \begin{bmatrix} 0 & -\omega_1 & -\omega_2 & -\omega_3 \\ \omega_1 & 0 & \omega_3 & -\omega_2 \\ \omega_2 & -\omega_3 & 0 & \omega_1 \\ \omega_3 & \omega_2 & -\omega_1 & 0 \end{bmatrix} \tag{2}$$

$$\dot{\alpha} = \beta \tag{3}$$

$$\dot{y} = \begin{bmatrix} \dot{\omega} \\ \dot{\beta} \end{bmatrix} = (A + B_\alpha)\bar{\bar{\omega}} + (C_\beta)\omega + D\alpha + (E + F_\alpha)u \tag{4}$$

where

q is the 4×1 quaternion vector,

$B_\alpha = [B_1\alpha \mid B_2\alpha \mid \ldots \mid B_6\alpha]$,

$C_\beta = [C_1\beta \mid C_2\beta \mid C_3\beta]$, $\quad F_\alpha = [F_1\alpha \mid F_2\alpha \mid \ldots \mid F_9\alpha]$,

A, B_i, C_i, D, E, F_i - constant matrices;

$u = [f_{1x} \;\; f_{1y} \;\; f_{1z} \mid f_{2x} \;\; f_{2y} \mid f_{3x} \;\; f_{3y} \mid f_{4x} \;\; f_{4y}]^T$.

TIME OPTIMAL CONTROL PROBLEM FORMULATION

Two-Point Boundary-Value Problem (TPBVP)

Initial States and Final Required States:

$$q(0),\quad \alpha(0),\quad \omega(0),\quad \beta(0) \tag{5}$$

$$q(t_f),\quad \alpha(t_f),\quad \omega(t_f),\quad \beta(t_f) \tag{6}$$

Cost Function:

$$t_f = \int_0^{t_f} (1)\, dt \tag{7}$$

Saturation-Bounded Controls:

$$|\, u_i \,| \le u_{ib},\quad i = 1,\ 2,\ \dots,\ 9. \tag{8}$$

Hamiltonian:

$$H = 1 + \gamma^{\mathsf{T}}\beta + \lambda^{\mathsf{T}}[(A + B_\alpha)\bar{\bar{\dot{\omega}}} + C_\beta\omega + D\alpha + (E + F_\alpha)u] \tag{9}$$

$p,\ \gamma,\ \lambda=[\lambda_1\ \lambda_2]^{\mathsf{T}}$ - costate vectors associated with $q,\ \alpha,\ \omega,\ \beta$.

Costate Equations (by Pontryagin's Maximum Principle):

$$\dot{p} = -\frac{\partial H}{\partial q} = \frac{1}{2}\underline{\tilde{\omega}}p \tag{10}$$

$$\dot{\gamma} = -\frac{\partial H}{\partial \alpha} = -D^{\mathsf{T}}\lambda - (B_\alpha^{\mathsf{T}}\lambda)\bar{\bar{\dot{\omega}}} - (F_\alpha^{\mathsf{T}}\lambda)u \tag{11}$$

$$\dot{\lambda}_1 = -\frac{\partial H}{\partial \omega} = -\frac{1}{2}[q]p - [\lambda^{\mathsf{T}}(A + B_\alpha)]\omega - (C_\beta)^{\mathsf{T}}\lambda \tag{12}$$

$$\dot{\lambda}_2 = -\frac{\partial H}{\partial \beta} = -\gamma - (C_\beta^{\mathsf{T}}\lambda)\omega \tag{13}$$

Constraint Condition (a terminal condition to determine t_f):

$$H \equiv 0,\quad o \le t \le t_f \tag{14}$$

Optimal control:

$$u_i = -u_{ib}\, sign[(E+F_\alpha)^{\mathsf{T}}\lambda]_i,\quad i = 1,\ \dots,\ 9. \tag{15}$$

QUASILINEARIZATION AND TIME SHORTENING METHOD (QTS)

The near minimum time slewing problem can be solved by using an iteration approach based on the quasilinearization method.

Different Cost Function (Quadratic Cost Functional):

$$J = \frac{1}{2}\int_0^{t_f} (u^T R u)\, dt \tag{16}$$

where R are weighting matrices, t_f is the slewing time.

Same Costate Equations Eqs. (10)-(13).

Different Control Expression:

Unconstrained Optimal Control:

$$\frac{\partial H}{\partial u} = 0, \quad \rightarrow \quad u = -R^{-1}(E + F_\alpha)^T \lambda \tag{17}$$

Constrained Optimal Control:

$$u_i = \begin{cases} u_{ic}, & if\ |u_{ic}| < u_{ib} \\ u_{ib}\, \mathrm{sgn}(u_{ic}), & if\ |u_{ic}| \geq u_{ib} \end{cases} \tag{18}$$

$$u_{ic} = -[R^{-1}(E + F_\alpha)^T \lambda]_i, \quad i = 1, 2, \ldots, 9. \tag{19}$$

t_f can be obtained by sequentially shortening the slewing time.

Motivation:

Is this bang-bang control the same as that obtained by using the shooting method? (Do these controls have the same time histories?) If the answer is yes, the results from the QTS approach may be used as the starting solution for the shooting method. (Here, we use the numerical results to prove the equivalence.)

SHOOTING METHOD

Formulation of the TPBVP:

$$\dot{X}(t) = F[X(t),\ u(t)], \quad 0 \le t \le t_f \tag{20}$$

$$X(0) = K[D] \tag{21}$$

$$L[X(t_f),\ D,\ t_f] = 0 \tag{22}$$

$$u_i = -\text{sign}\ \{g_i[X(t)]\}, \quad i = 1,\ \dots,\ m. \tag{23}$$

D - the $n \times 1$ unknown initial costate vector;

$L[X(t_f),\ D,\ t_f]$ - $(n+1) \times 1$ terminal constraint vector;

g_i $(i=1,\ \dots,\ m)$ - the switching functions.

Initial Boundary Conditions Correction Process:

To satisfy: $L[X(t_f),\ D,\ t_f] = 0$, D and t_f need to be corrected at each iteration:

$$\begin{bmatrix} D^{(k+1)} \\ t_f^{(k+1)} \end{bmatrix} = \begin{bmatrix} D^{(k)} \\ t_f^{(k)} \end{bmatrix} + \begin{bmatrix} \Delta D^{(k)} \\ \Delta t_f^{(k)} \end{bmatrix} \tag{24}$$

where

$$\begin{bmatrix} \Delta D^{(k)} \\ \Delta t_f^{(k)} \end{bmatrix} = -\alpha_k \begin{bmatrix} \delta D^{(k)} \\ \delta t_f^{(k)} \end{bmatrix} \tag{25}$$

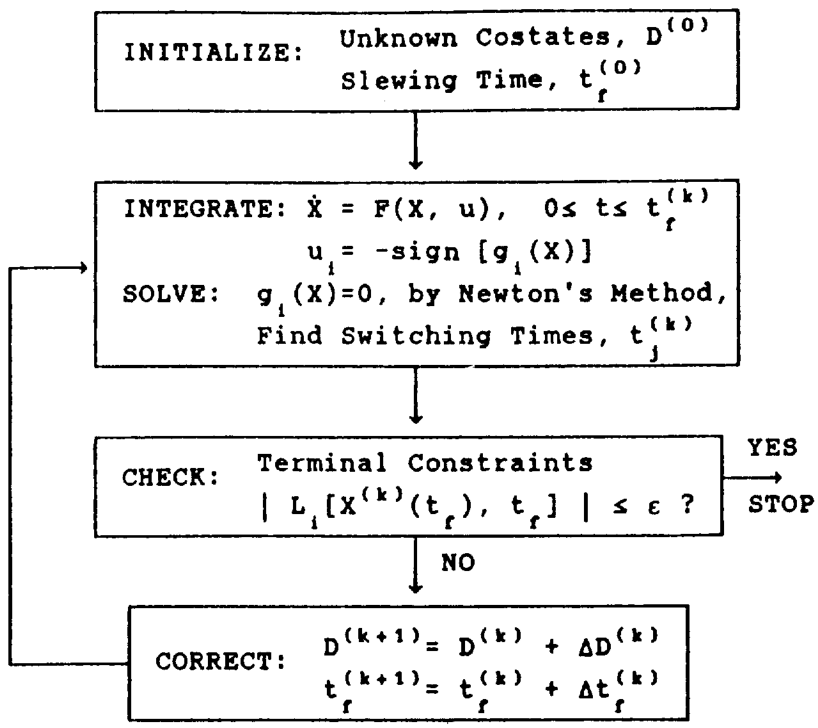
$$\begin{bmatrix} \delta D^{(k)} \\ \delta t_f^{(k)} \end{bmatrix} = \begin{bmatrix} \dfrac{\partial L}{\partial D} & \dfrac{\partial L}{\partial t_f} \end{bmatrix}^{-1} L[X^{(k)}(t_f),\ D^{(k)},\ t_f^{(k)}] \tag{26}$$

$D^{(k)}$ and $t_f^{(k)}$ - the values of D and t_f at iteration k;

scalar α_k $(0 \le \alpha_k \le 1)$ is chosen as:

$$\alpha_k = \min \left\{ 1,\ \frac{\rho \| [D^{(k)},\ t_f^{(k)}] \|}{\| [\delta D^{(k)},\ \delta t_f^{(k)}] \|} \right\}, \quad 0 < \rho < 1 \tag{27}$$

A general algorithm suggested by Lastman has been used in our calculations and can be recast into the following block diagram:



Summary:

I. Difficulty in initialization for the present nonlinear, multi-input system control problem. Improper choice of D and t_f will result in singular correction matrix, and program diverges.

II. Advantages and disadvantages of the two methods:

- Quasilinearization method and time shortening technique has good convergence properties;
- Shooting method generates more accurate final results, but is sensitive to starting solution;
- A combined technique is needed.

INITIAL COSTATE TRANSFORMATION

Different Costate Solutions:

Although the costate equations are the same, the initial costates from these two different formulations of the same problem are different:

1. Initial costates, $\bar{p}(0)$, $\bar{\gamma}(0)$, and $\bar{\lambda}(0)$ from the QTS method are large;
2. $p(0)$, $\gamma(0)$, and $\lambda(0)$ from the shooting method are finite.

Assumed Relation Between Them:

$$p(0) \approx k_1 \bar{p}(0), \quad \lambda_1(0) \approx k_1 \bar{\lambda}_1(0),$$
$$\gamma(0) \approx k_2 \bar{\gamma}(0), \quad \lambda_2(0) \approx k_2 \bar{\lambda}_2(0). \tag{28}$$

where $k_1 < 1$, and $k_2 < 1$ are scale factors to be determined.

Scale Factors

Assumed Eigen-Axis Rotation for Rigid Spacecraft:

$$\omega = e\dot{\theta}, \quad \dot{\omega} = e\ddot{\theta} \tag{29}$$

$e = [e_1\ e_2\ e_3]^T$ - a unit vector representing the eigen-axis,

θ - the rotation angle about this axis.

Resulting Four Related Equations (from rigid dynamic equations) :

$$e\ddot{\theta} = f\dot{\theta}^2 + I^{-1}Bu \tag{30}$$

f - 3×1 constant vector; and

$$\ddot{\theta} = e^T f\,\dot{\theta}^2 + e^T I^{-1} B u \tag{31}$$

- Let "p" = the "principal" axis among the axes 1, 2, and 3, about which the rotation requires the largest t_f;
- Let "4" = Eq. (31).

Further Simplification of Equations:

$$\ddot{\theta} = b_i v_i, \quad b_i > 0, \quad |v_i| \le 1, \quad i = 1, 2, 3, 4.$$

"Average" Values of the Initial Costates:

$$p_a = 2/(b_a \theta_f)^{1/2}, \quad \lambda_a = 1/b_a, \quad t_{fa} = 2(\theta_f / b_a)^{1/2}$$

$$b_a = k_p b_p + k_4 b_4, \quad k_p + k_4 = 1$$

θ_f - the required rotation angle about the eigen-axis.

k_1 and k_2:

$$k_1 = p_a / |\bar{p}_p(0)|, \quad k_2 = \lambda_a / |\bar{\lambda}_p(0)|$$

The Initial Costates for Starting the Shooting Method:

$$p(0) = k_1 \bar{p}(0), \quad \lambda_1(0) = k_1 \bar{\lambda}_1(0),$$

$$\gamma(0) = k_2 \bar{\gamma}(0), \quad \lambda_2(0) = k_2 \bar{\lambda}_2(0).$$

NUMERICAL EXAMPLES

Given Slewing Conditions (for all cases considered here):

1. Rest-to-rest slews, i.e.,

$$\omega(0)=0, \quad \omega(t_f)=0,$$

$$\alpha(0)=0, \quad \beta(0)=0, \quad \alpha(t_f)=0, \quad \beta(t_f)=0.$$

2. Three (3) control variables are used.

Example 1 (a scaled rigid spacecraft, $\lambda = \lambda_1$):

$q(t_f)=[.877582561,\ .434965534,\ .142572492,\ .142572492]^T$,

$q(0)=[1.0,\ 0.0,\ 0.0,\ 0.0]^T$, $\theta_f=1$ rad, $I = \text{Diag}(1.0,\ 0.9,\ 0.6)$,

$R = \text{Diag}(1.0,\ 0.7,\ 0.4)$, $p = 1$, $k_1 = k_p = k_4 = 0.5$.

QTS Method Results:

$t_f = 1.8$ sec. By transformation, the initial costates for starting the shooting method are obtained:

$$p(0) = \begin{bmatrix} -1.67968 \\ -.248420 \\ .415782 \end{bmatrix}, \quad \lambda(0) = \begin{bmatrix} -.705333 \\ -.0955727 \\ .0481507 \end{bmatrix}, \quad \begin{matrix} t_f=1.71209 \\ (s) \end{matrix}$$

Shooting Method Results:

$\rho = 0.1$, solutions are obtained in 6 iterations (to 5 digits):

$$p^*(0) = \begin{bmatrix} -1.74008 \\ -.267243 \\ .462349 \end{bmatrix}, \quad \lambda^*(0) = \begin{bmatrix} -.770403 \\ -.115614 \\ .0606796 \end{bmatrix}, \quad \begin{matrix} t_f^*=1.76403 \\ (s) \end{matrix}$$

The converged values of the switching times are:

u_i	u_3	u_2	u_1	u_3	u_2
t_i(s)	0.314356	0.701830	0.874531	1.18114	1.53158

Example 2:

Maneuver of the rigidized SCOLE model. The scaled inertial matrix (set $I_{33} = 1.0$):

$$I = \begin{bmatrix} .16902 & -.001061798 & .01619427 \\ -.001061798 & .9948471 & -.007354633 \\ .01619427 & -.007354633 & 1.0 \end{bmatrix}$$

- Simultaneous 75°, 30°, 45° slew about roll, pitch, yaw axes;
- u = three torquers on the Shuttle, u_{ib} = 10,000 ft-lb;
- R = Diag(1, 1, 1);
- $k_p = k_3 = 0.75$, $k_4 = 0.25$.

Average Values:

$$p_a = 1.01466, \qquad \lambda_a = .461915$$

Initial Costates for Starting Shooting Method:

$$p(0) = \begin{bmatrix} .00238330 \\ 1.24510 \\ -1.01466 \end{bmatrix}, \quad \lambda(0) = \begin{bmatrix} -.00709927 \\ .456237 \\ -.461519 \end{bmatrix}$$

The t_f = 1.6407 sec from the QTS method is used as $t_f^{(0)}$ in the shooting method, $\rho = 0.0035$. The final converged initial costates:

$$p^*(0) = \begin{bmatrix} .00237568 \\ 1.32922 \\ -1.08389 \end{bmatrix}, \quad \lambda^*(0) = \begin{bmatrix} -.00773848 \\ .496600 \\ -.502920 \end{bmatrix}, \quad \begin{matrix} t_f^* = 1.64066 \\ (s) \end{matrix}$$

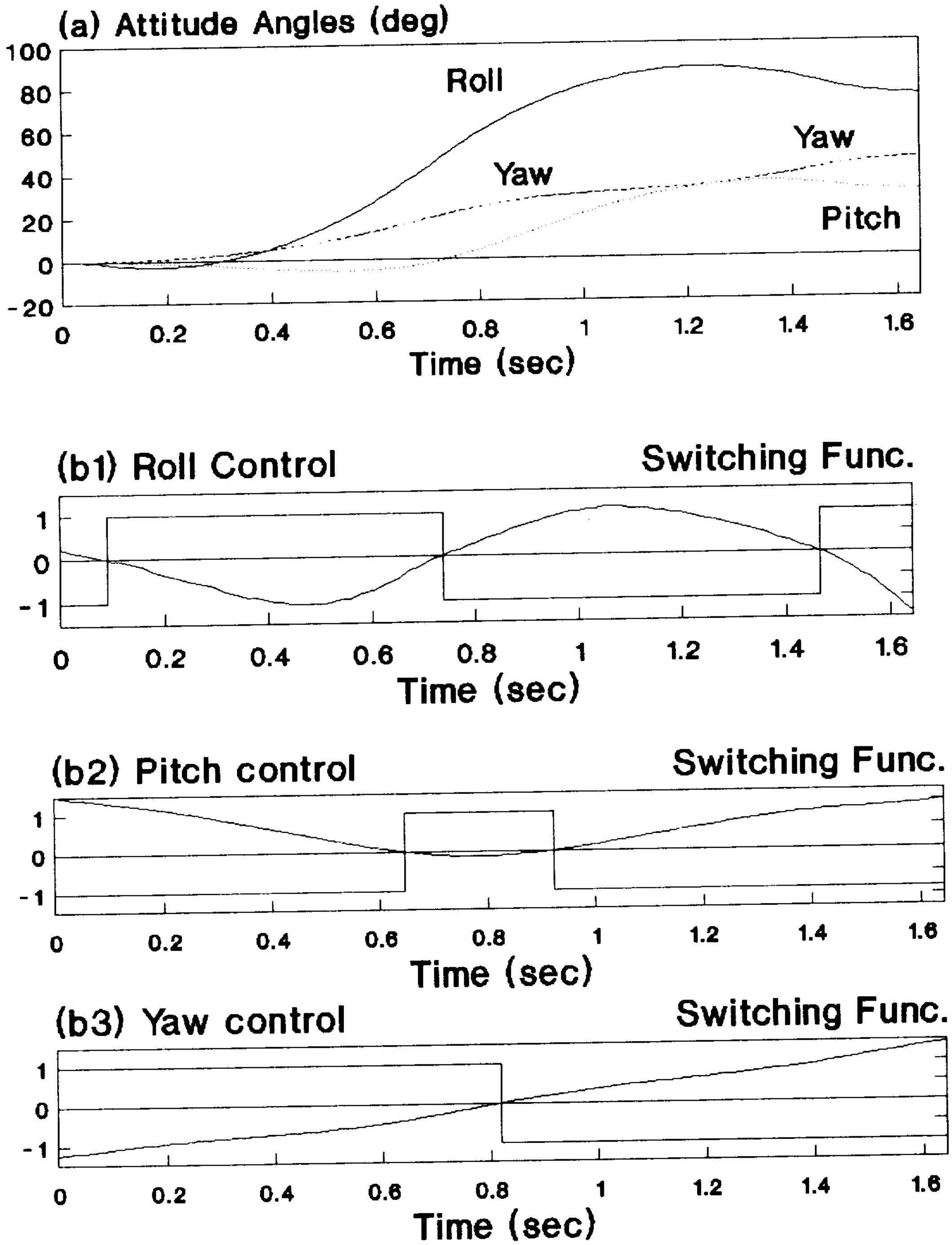


Fig. 2. Rigid SCOLE Maneuver (Scaled Model)
75-30-45 (deg) (Shooting Method)

Example 3:

The maneuver of the both rigidized and flexible orbiting SCOLE model is considered (using the original SCOLE challenge parameters).

- 90° slew about roll axis. Although the expected motion is "single-axis" rotation, the minimum-time dynamic maneuver process is not necessarily a single-axis rotation because of the offset inertia distribution of the SCOLE model. (The three axes for the three control torquers are not principal axes). Therefore, the present slew is a 3-D slew.
- u = three torquers on the Shuttle, u_{ib} = 10,000 ft-lb;
- R = Diag(1.E-4, 1.E-4, 1.E-4);
- $k_p = k_1 = 1$.
- Two flexible modes (the first and the second) are included.

Average Values of Initial Costates:

$$p_a = 24.7475, \qquad \lambda_a = 120.242$$

Initial Costates for Starting Shooting Method:

p(0)		$\gamma(0)$	$\lambda_1(0)$		$\lambda_2(0)$
(Rigid)	(Flexible)		(Rigid)	(Flexible)	
.00000E0	.00000E0	-.90512E-3	.12024E3	.12024E3	.10717E-1
.24747E2	.24747E2	.46390E-2	.67830E1	.51236E1	-.35993E-1
-.19219E0	-.11478E0		.11671E2	.10969E2	
.25248E1	.23758E1				

The t_f = 27.3992 seconds from the QTS method is used as $t_f^{(0)}$ in the shooting method. The final converged initial costates:

p(0)		$\gamma(0)$	$\lambda_1(0)$		$\lambda_2(0)$
(Rigid)	(Flexible)		(Rigid)	(Flexible)	
.00000E0	.00000E0	-.87221E-3	.11894E3	.11587E3	.10331E-1
.24422E2	.23767E2	.44563E-2	.67130E1	.49375E1	-.34685E-1
-.18880E0	-.11001E0		.11544E2	.10570E2	
.24825E1	.22818E1				

The Hamiltonian, H, is observed as a constant during each iteration and is iteratively reduced to the final value:

$$H = 1.2000E-9$$

The time histories of the slews are plotted in Fig. 3 to Fig. 6. In these figures, the results for the attitude angles, the mode amplitudes, the control torques, and the switching functions are presented, whenever applicable.

Fig. 3 shows the rigid SCOLE maneuver by using the QTS method and Fig. 4 shows the rigid SCOLE maneuver by using the shooting method. The results show that the solutions by using the two methods are very close. It is also noted that, during the slew, the yawing control, u_3, switches twice consecutively before other controls (rolling control u_1 or pitching control u_2) switch.

Figs. 5 and 6 show the flexible SCOLE maneuver by using the QTS and the shooting methods, respectively. Again, the results from both methods are close. Due to the inclusion of the flexible modes, the switching number for every control is tripled or even more (23 for u_3) compared with the results for the rigid SCOLE maneuver. The modal amplitudes are very small and the associated vibration of the reflector of the SCOLE and the "Line of Sight" are also very small.

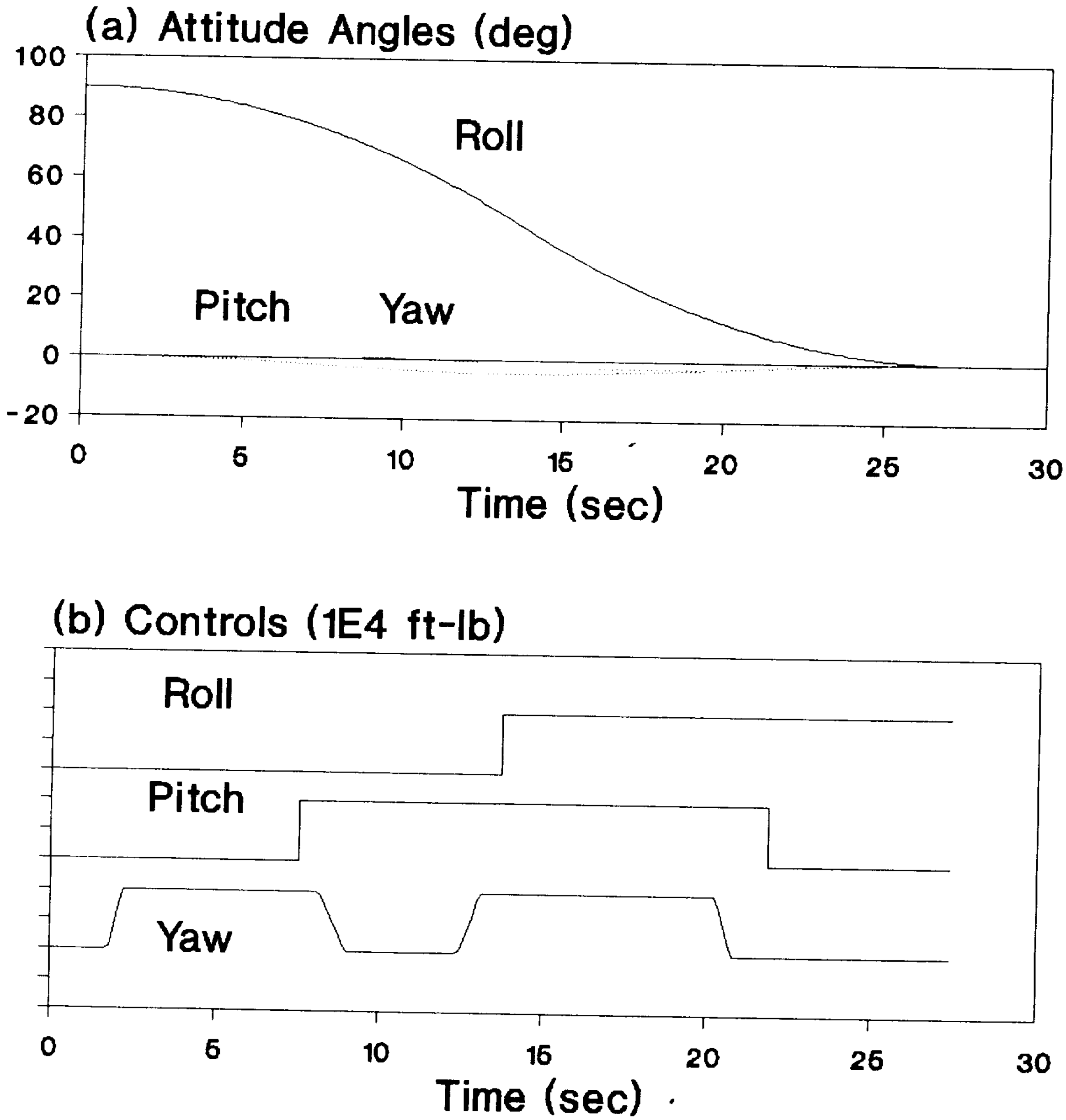


Fig. 3 Rigid SCOLE Maneuver,
Roll Angle = 90° (QTS Method)

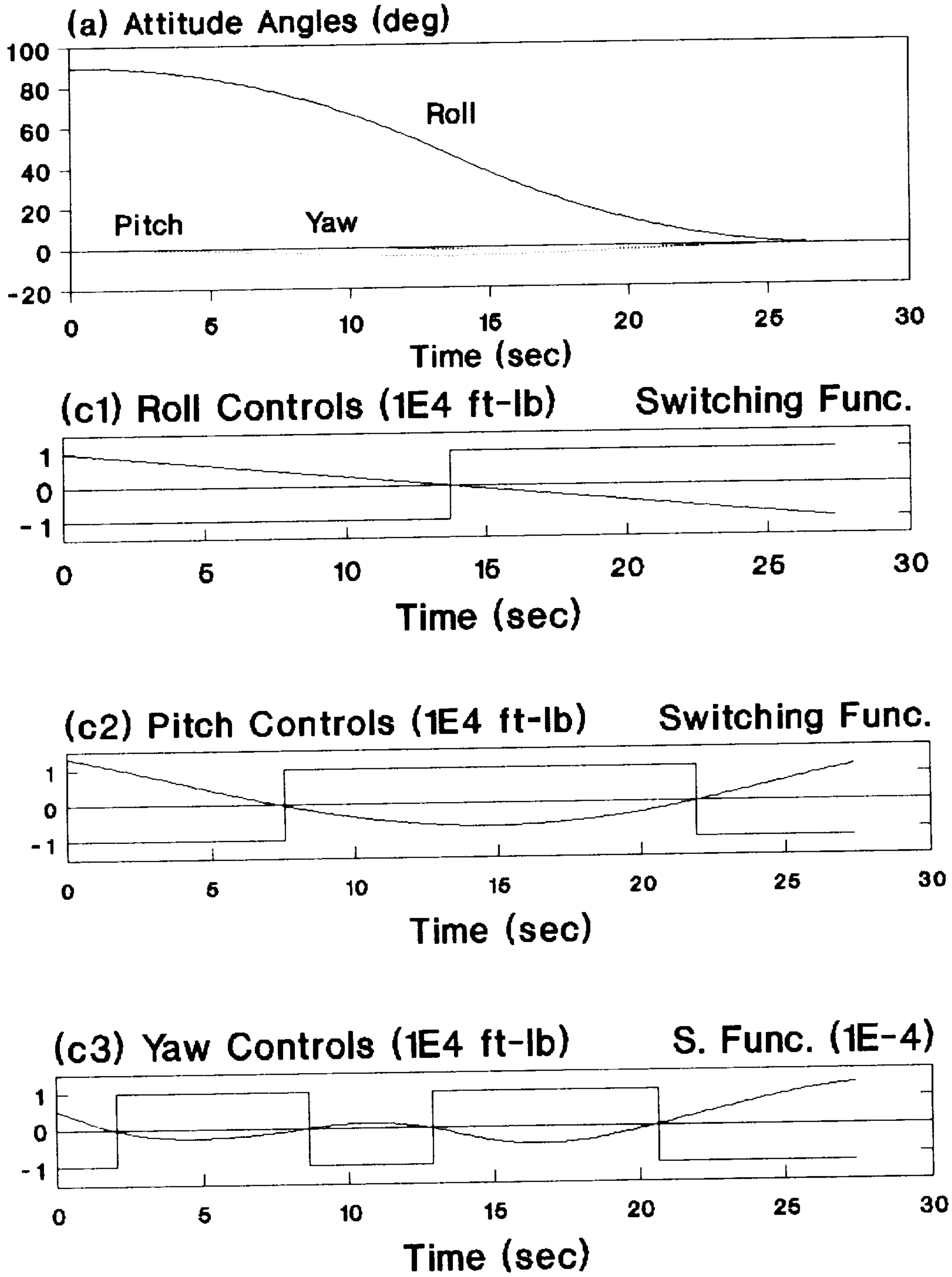


Fig. 4 Rigid SCOLE Maneuver,
Roll Angle = 90° (Shooting Method)

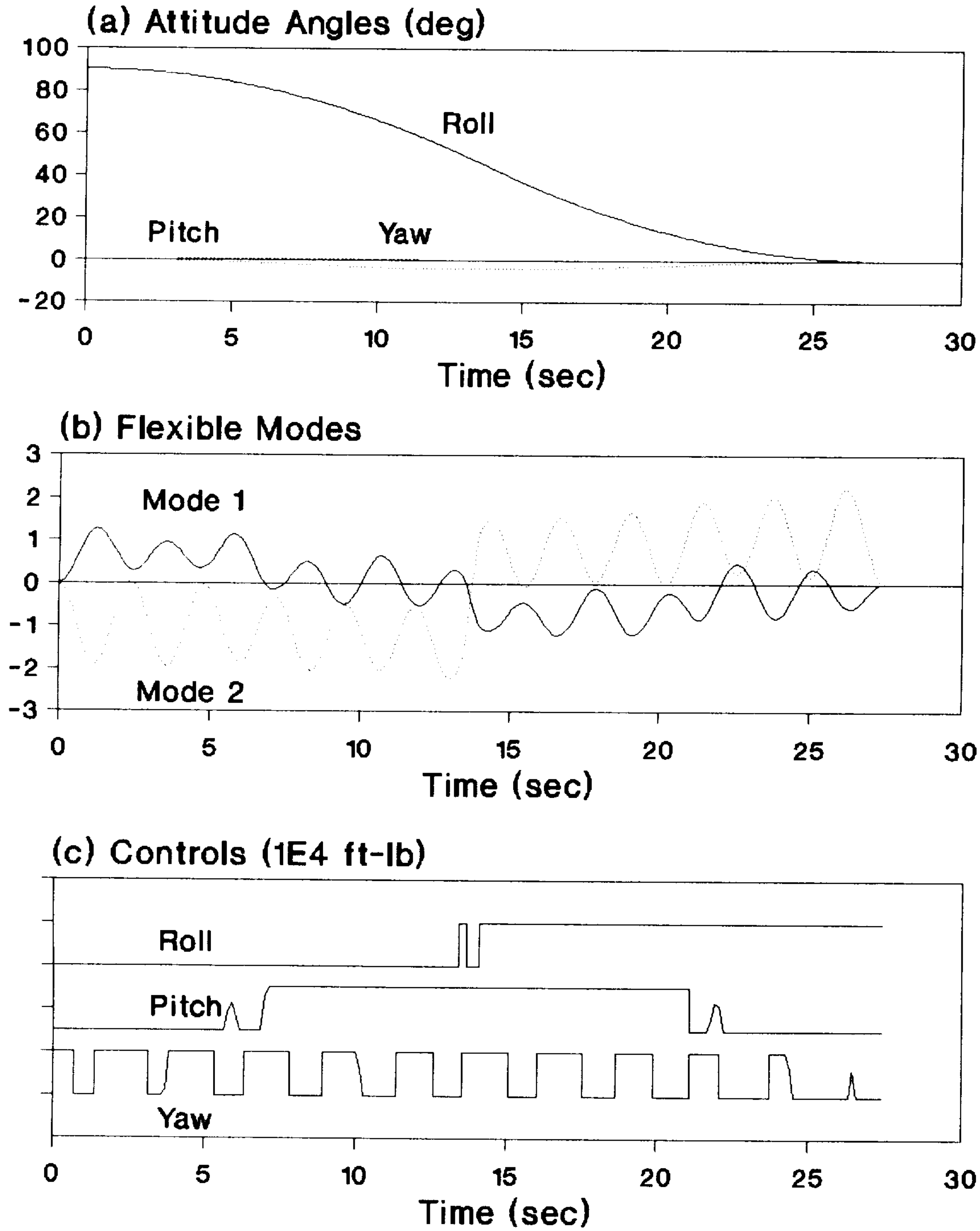


Fig. 5 Flexible SCOLE Maneuver, Roll Angle = 90° (QTS Method)

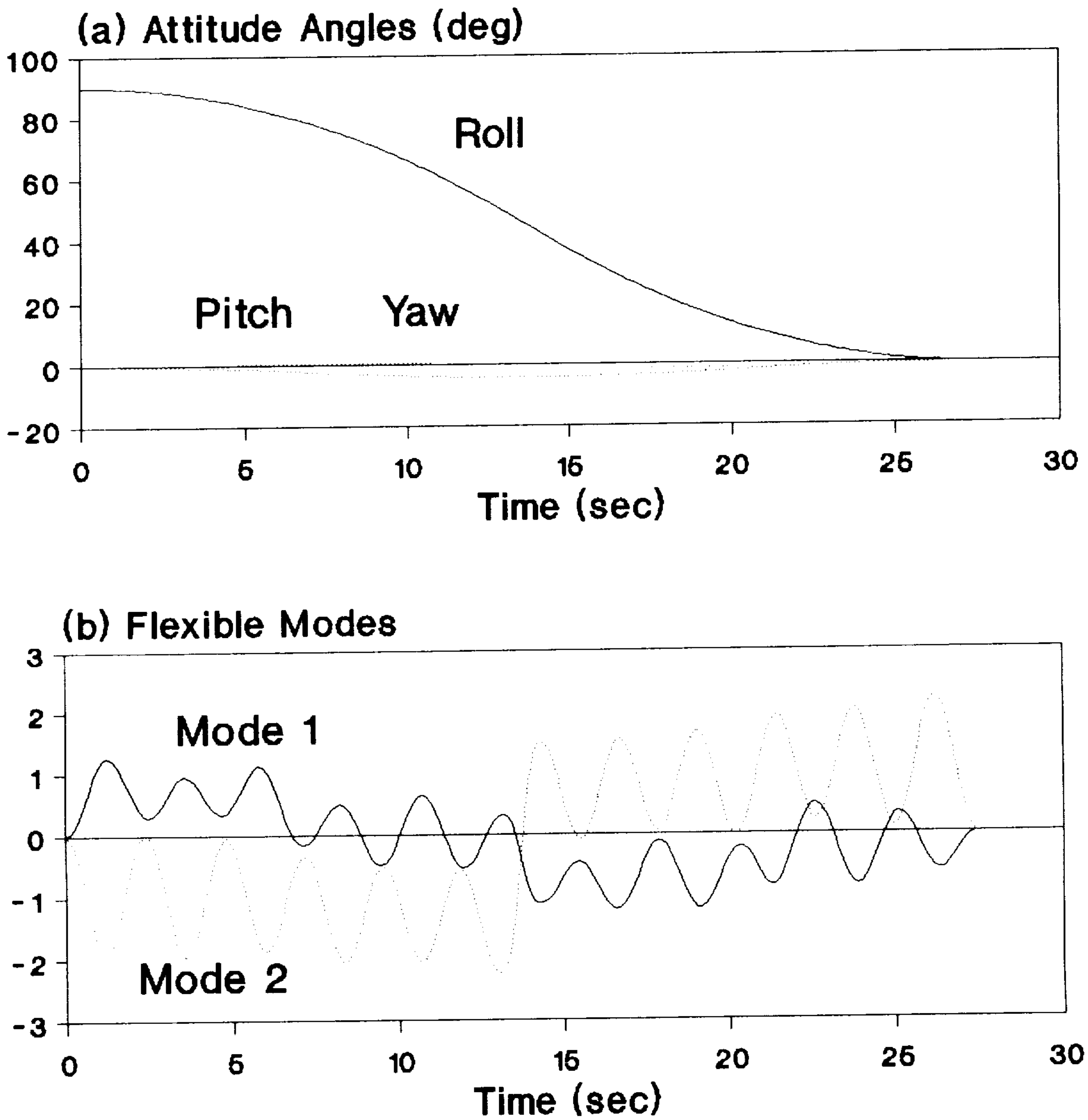


Fig. 6 Flexible SCOLE Maneuver,
Roll Angle = 90° (Shooting Method)

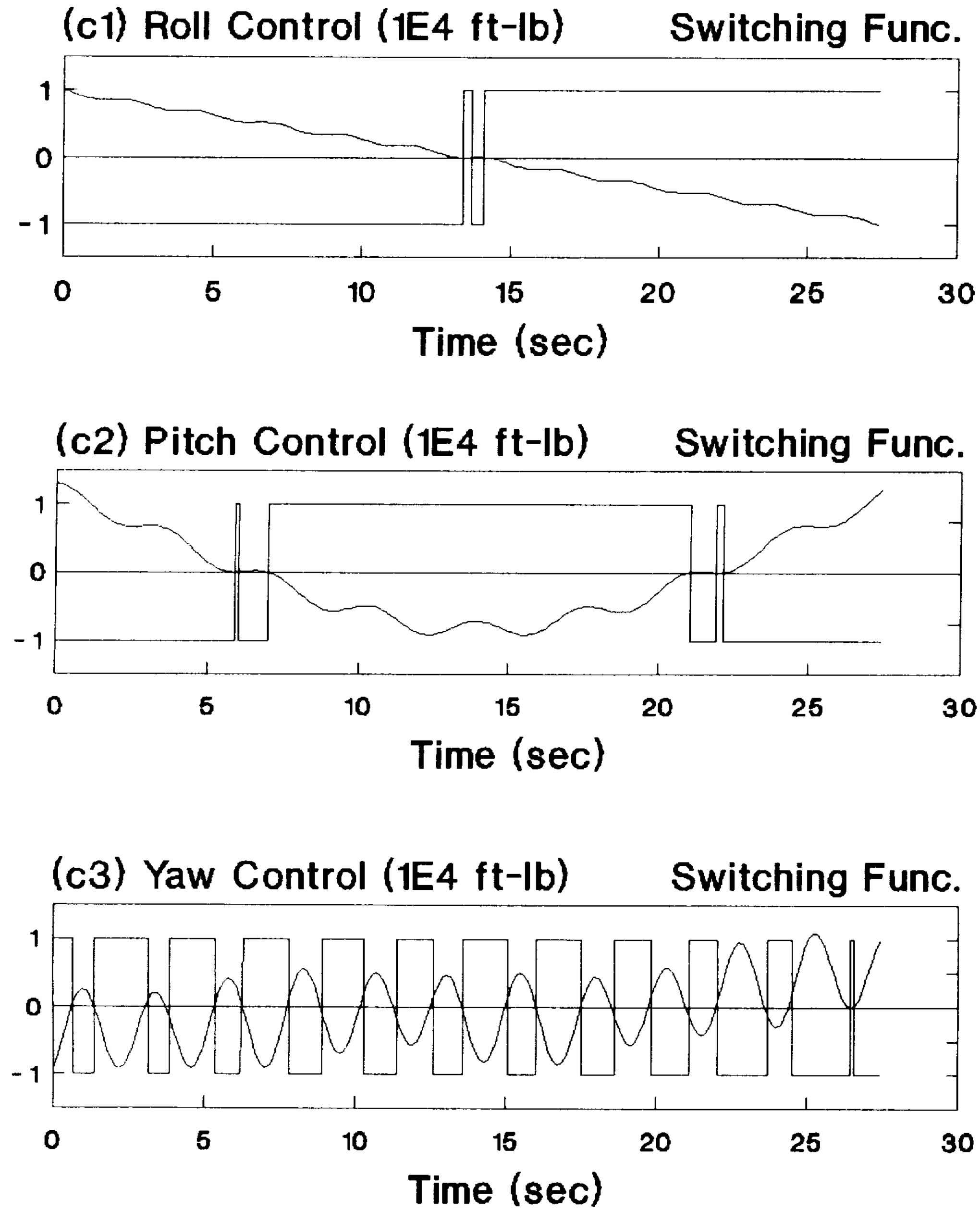


Fig. 6 (Cont'd) Flexible SCOLE Maneuver,
Roll Angle = 90° (Shooting Method)

CONCLUSIONS

1. The QTS method is stable for relatively coarse choices of the unknown initial costates, and the shooting method is not.

2. The QTS method usually results in very large values of the costates which may lead to the numerical overflow in the calculation process, if the "exact" (numerically) switching times are to be found, while the shooting method does not have this problem and exact switching times can be obtained iteratively.

3. A technique to combine these two methods is proposed.

4. The estimated initial values of the costates, $p(0)$, $\gamma(0)$, and $\lambda(0)$, based on the solution from the QTS method, are very close to the converged values of these parameters in the shooting method and hence the convergence of the shooting method has been improved.

5. The costates from both methods are proportional.

6. The control histories from both methods are the same and may imply the uniqueness of the control for the slewing problem.

7. The application of this method to the minimum time maneuver of other flexible spacecraft is suggested.

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