Correlation of Analytical and Experimental Hot Structure Vibration Results

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ABSTRACT

High surface temperatures and temperature gradients can affect the vibratory characteristics and stability of aircraft structures. Aircraft designers are relying more on finite-element model analysis methods to ensure sufficient vehicle structural dynamic stability throughout the desired flight envelope. Analysis codes that predict these thermal effects must be correlated and verified with experimental data. This paper presents experimental modal data for aluminum, titanium, and fiberglass plates heated at uniform, nonuniform, and transient heating conditions. These data are compared with vibration analysis results for the same heating conditions. The data show the effect of heat on each plate's modal characteristics, a comparison of predicted and measured plate vibration frequencies, the measured modal damping, and the effect of modeling material property changes and thermal stresses on the accuracy of the analytical results at nonuniform and transient heating conditions.

NOMENCLATURE

DFRF  Dryden Flight Research Facility, Edwards, CA
FEM  finite-element model
NASP  National Aero-Space Plane
STARS  STructural Analysis RoutineS

INTRODUCTION

Aircraft that fly at hypersonic speeds will be subjected to high surface temperatures and large temperature gradients. These conditions will affect the modal characteristics of the structure and can seriously affect the aeroelastic and aeroservoelastic stability of the vehicle. If finite-element model (FEM) analysis methods are to be relied upon to predict these instabilities, accurate determination of the effect of heat on the modal characteristics of these structures is vital.

In the late 1950's and early 1960's, the effect of heat on the modal characteristics of structures was investigated for simple panels [1] and on a prototype wing for the X-15 vehicle [2]. Results indicated that thermal stresses generated from heating the structure could have significant effects on structural stiffness in addition to the material property changes.

There is renewed interest in vibration testing of heated structures with the National Aero-Space Plane (NASP) program [3] and other hypersonic research programs. Design and flight test considerations dictate that analysis methods be accurate enough to predict the structural stability of the vehicle when it is subjected to high temperatures. Confidence in analytical results can only come through correlation and verification with experimental data.

A series of heated structure vibration tests have been conducted on aluminum, titanium, and fiberglass plates at the Dryden Flight Research Facility (DFRF). Modal data were acquired from these plates heated at uniform, nonuniform, and transient temperature profiles. A corresponding vibration analysis of each plate using finite element models was accomplished to correlate with this set of experimental data. This paper presents experimental modal data which show the effect of heat on the modal characteristics of each plate, and analytical vibration results which show the effect of modeling the structural material property
changes, and the thermal stresses on the accuracy of the results for nonuniform and transient heating profiles.

TEST ARTICLES

Four plates were selected as test articles. Each article was 30.5 cm (12 in.) high and 127 cm (50 in.) long. Plates of three different materials were tested. The materials were 2024 aluminum (0.48 cm (0.19-in. thickness)), A110 titanium (0.51 cm (0.20-in. thickness)), and fiberglass.

Two fiberglass plates were manufactured using a wet lay-up method. The first plate (#1) was 0.38 cm (0.15 in.) thick, and had a total of 24 plies (0,±45,90)3s. The second plate (#2) was 0.48 cm (0.19 in.) thick, and also had 24 plies (0,±30,±60,90)2s. Plates with different plies were selected to determine the effect of ply orientation on the change of modal characteristics at elevated temperatures. Each plate was manufactured using Shell EPON DPL 863 resin, Shell EPON curing agent W, and HEXEL style 7715 unidirectional fabric. This fabric is 90-percent unidirectional E-glass. Each plate was post cured at 191 °C (375 °F) for 2.5 hr. The laminate was quasi-isotropic to minimize thermal stresses.

The length of each plate was divided into three equal length zones for instrumentation and heating purposes as shown in Figure 1. Each plate was instrumented with 18 accelerometers and 30 thermocouples. Each plate zone contained six accelerometers and nine thermocouples on the front of the plate. The rear side of the plate had one thermocouple in the center of each zone to measure the temperature gradient across the thickness of the plate. The maximum operating temperature of the thermocouples was 1149 °C (2100 °F).

Most lightweight accelerometers do not function above 260 °C (500 °F). Therefore, at test temperatures above 260 °C, the test article structural response was measured with a single point staring laser vibrometer.

The mass and stiffness effect of the thermocouple and accelerometer wires on each test article was considered to be negligible. The weight of the wires did not rest directly on the surface of the test article and the wires were fairly flexible. Test data verified this assumption to be correct.

TEST SETUP

The overall setup of the experimental equipment required to measure each plate's modal characteristics is shown in Figure 2. This schematic shows the interconnection of the oven, the test article heater control, the thermocouple data acquisition equipment and the accelerometer–laser vibrometer data acquisition equipment. This system provided closed-loop temperature control of the plate, and display and storage of the thermocouple and plate frequency response data. A detailed description of the experimental equipment used, including oven construction, remote satellite, and modal analysis computer functions is given in refs. [4–6].

Each plate was supported in the oven by a combination of bungee cord and steel cables (fig. 1) to provide a free-free boundary condition. The portion of the suspension cables that were inside the oven were made of steel to withstand the heat. The steel cable was kept as short as possible to avoid affecting the plate's modal characteristics. See Reference 5 for a more detailed description of the test article support system.
A free-free boundary condition was selected to minimize the complexity of the test setup. This type of boundary condition alleviated the uncertainties associated with effects of heat conduction and thermally-induced stresses at the mounting frame interface. Correlation of experimental data with analytical predictions is more accurately accomplished with these effects removed.

**TEST TECHNIQUE**

**Excitation**

Impact excitation was used to excite the plate by striking a rod, which was attached to the plate and extended through an opening in the oven, with a hammer calibrated to measure input force. Impact excitation provided a way to excite the plate in the shortest amount of time, which was essential during transient heating of the plate.

**Heating Profiles**

The plate was subjected to three heating profiles; uniform, nonuniform, and transient. Uniform heating of each plate was done by heating each zone of the plate to the same temperature. The thermocouple readings were monitored to ensure that the plate was uniformly heated.

The aluminum plate was uniformly heated at 93, 149, 177, 204, and 246 °C (200, 300, 350, 400, and 475 °F). The titanium plate was uniformly heated to 93, 149, 204, 260, 316, and 371 °C (200, 300, 400, 500, 600, and 700 °F). The fiberglass plates were uniformly heated to 66, 93, 121, 149, and 177 °C (150, 200, 250, 300, and 350 °F).

Nonuniform heating of each plate consisted of heating the plate's three zones to a different temperature. Each plate was uniformly heated until the lowest target temperature of a particular zone was reached. The oven then tried to maintain the temperature of that zone while heating the two remaining zones to the next highest target temperature. Once the target temperature of the second zone was reached, the oven heated the remaining zone, while attempting to maintain the temperatures of the two other zones. Data were acquired when the final zone was heated to its maximum target temperature. The average temperature profile along the length of each test article that was nonuniformly heated is shown in Figure 3.

Transient plate heating was conducted by heating an end zone of the plate. The zone was heated at a specified rate up to the desired maximum temperature. The maximum heating rate available was 3.9 °C/sec (7 °F/sec). The plate was continually excited while it was heated and each plate time history response was stored directly to the modal analysis system disk. The modal characteristics for each plate were determined from the data acquired at the maximum temperature condition of the heated zone. The average temperature profile along the length of each test article for transient heating is shown in Figure 3.

**EXPERIMENTAL DATA ANALYSIS METHOD**

Once data acquisition was completed for a given heating profile, frequency and damping values for the first four modes were estimated. These modes were first plate bending (mode 1), first plate torsion (mode 2), second plate bending (mode 3), and second plate torsion (mode 4). The technique used operated on a single frequency response function. The frequency response function at the plate corner was selected
because it contained the best response of the first four modes. Only the first four modes were selected because the number of accelerometers was insufficient to determine mode shapes above this number. The modal parameter estimates were obtained by fitting a second-order polynomial to each frequency peak in the selected frequency response function [7].

Mode shapes were generated using a single-degree-of-freedom technique. This technique extracted amplitude and phase information from each plate frequency response function at the specified modal frequency. The information was then used for viewing animated mode shapes and static deformation plots.

**VIBRATION ANALYSIS**

The analytical predictions were done using the STructural Analysis RoutineS (STARS) program [8]. This program used finite-element modeling to determine the change in vibration frequency caused by various temperature profiles. The temperature profiles were obtained from the thermocouple data acquired during heating tests.

A finite-element representation of each plate was created using 300 triangular elements and 182 node points. Each node point had six degrees of freedom (free-free boundary conditions) which resulted in a model with a total of 1092 degrees of freedom. Each accelerometer on the plate was modeled as a concentrated mass. Each plate, including the fiberglass plates, was assumed to be an isotropic material.

The material property of each plate was modeled for each temperature profile. For uniform heat, the modulus of elasticity value at each temperature was determined either from a materials handbook [9] or experimentally [10]. For nonuniform and transient temperature profiles, the plate temperature distribution was determined from the thermocouple data. The length of the plate was divided into 25 strips to represent the temperature distribution in the FEM. The average temperature of each strip was used to determine the value of the elastic modulus to be used for each strip. A typical temperature profile and the division of the plate into strips is shown in Figure 4.

The thermal stresses caused by temperature gradients along the length of the plate were modeled in a similar manner as the material property. The average temperature of each strip was used to specify a delta temperature above room temperature. The internal forces were then calculated along each element edge by taking into account the effect of temperature and thermal expansion [11].

**RESULTS AND DISCUSSION**

Multiple heating tests were conducted on each plate except the titanium plate. No significant differences in the modal data were found between repeated test results. Repeatability of experimental test results was typically within 2 percent or less. The data presented in the following section were acquired on one test run and are not averaged values.

Mode shapes are not presented in this paper. The mode shapes were determined for the first four modes for uniform, nonuniform, and transient heating profiles. It was found from the aluminum and fiberglass plate data that the mode shapes did not change due to the effects of heating. These results are reported in Reference 5.
The titanium plate data were acquired with a laser vibrometer since test temperatures were above 260 °C (500 °F). A corner of the plate was selected as the measurement point because it contained the best response for the first four modes. Mode shapes were therefore not determined for the titanium plate.

**Aluminum Plate**

**Uniform Heating.** Figure 5 shows a comparison of analytical and experimental frequencies for the aluminum plate heated at uniform temperatures to 246 °C (475 °F). The trend shown by the data was that frequency decreased as the temperature increased. The average decrease in experimental frequency from room temperature to 246 °C was 7 percent for each mode. The frequency change was solely a function of changes in the material properties. The modulus of elasticity decreased as temperature increased, causing a reduction in plate stiffness.

In general, there was good agreement between the predicted and measured values of frequency for each mode. However, the data showed that for temperatures greater than 171 °C (340 °F), the analysis predicted greater rates of frequency change as a function of temperature than were actually measured. This discrepancy occurred in a temperature range where the handbook modulus does not vary linearly with temperature. It is possible that material property variations may be affecting the accuracy of the comparisons.

Figure 6 shows the experimental modal damping estimates as a function of temperature. The trends were fairly flat for temperatures to 149 °C (300 °F). Above 149 °C, the damping trend for each mode increased with an increase in temperature. This was a result of the plate becoming more viscous as it was heated above 149 °C. The increase in viscosity allowed the plate to dissipate more energy which caused an increase in modal damping as the plate temperature increased.

**Nonuniform Heating.** Figure 7 compares analytical and experimental percent frequency change from room temperature for a nonuniform heating profile. The experimental data were acquired from a test which consisted of heating the plate to 38, 93, and 204 °C (100, 200, and 400 °F) in zones 1, 2, and 3, respectively.

The experimental data revealed that the frequency decreased for all the plate modes by an average of 3.3 percent. The frequency reduction was the result of a change in stiffness associated with a change in material properties brought about by the increased temperature. To some degree, the thermal stress associated with the temperature gradient across the length of the plate also affected the material properties.

The plate was analytically modeled using two techniques. First, only the change in modulus of elasticity caused by the temperature change was modeled. Then, the change in modulus of elasticity and the thermal stresses caused by the temperature gradient were modeled. The results show that when only the modulus was modeled, the frequency change was consistently predicted to be less than that measured. When the thermal stresses were added to the analytical representation, the analysis predicted greater changes in frequency than were measured. These results were in much better agreement with the experimental values than those with only the modulus modeled. However, the percent difference between the results of the two modeling techniques was small.

**Transient Heating.** Figure 8 compares the analytical and experimental percent frequency change from room temperature for a transient heating profile. The experimental data were acquired from a test which consisted of heating the plate to 246 °C (475 °F) in zone 1 at a rate of 3.9 °C/sec (7 °F/sec).
The experimental data indicated that the frequency decreased for all the plate modes. The predictive vibration analysis indicated that for this case the dominate cause of the frequency reduction was the induced thermal stresses.

The plate was again modeled for changes only in modulus of elasticity and effective stiffness changes caused by the thermal stresses. The predicted results show a large difference between the two modeling techniques. The experimental results were in closer agreement with the analysis results that incorporated the modeling of the modulus and the thermal stresses. The temperature gradient was much larger than that for the nonuniform heating profile and produced much greater thermal stresses. The effect of not modeling these stresses accounts for the differences shown in Figure 8. Overall, the correlation between experimental and analytical results was good but not as good as the nonuniform heating case.

Titanium Plate

**Uniform Heating.** Figure 9 compares analytical and experimental frequencies for the titanium plate heated at uniform temperatures to 371 °C (700 °F). The trend shown by the data was that frequency decreased as the temperature increased. The average decrease in frequency from room temperature to 371 °C for the first and fourth modes was 9 percent while the average decrease for the second and third modes was 3 percent. As in the case of the aluminum plate, the change in frequency was solely a function of changes in the material properties. The experimental data also showed a slight increase in frequency for the second and third modes from 316 to 371 °C (600 to 700 °F) which was not predicted. The cause of this effect is not understood.

In general, there was excellent agreement between predicted and measured values of frequency for all the modes. The disagreement seen at 371 °C is a result of a slight increase in frequency of the experimental data.

The experimental modal damping estimates as a function of temperature are shown in Figure 10. It is interesting to note that the damping trend for the first and fourth modes was flat as temperature increased and the second and third mode damping trends showed a slight decrease.

**Transient Heating.** Figure 11 compares analytical and experimental percent frequency change from room temperature for a transient heating profile. The experimental data were acquired from a test in which the plate was heated in zone 1 to 371 °C (700 °F) at a rate of 3.9 °C/sec (7 °F/sec).

The experimental data indicated a decrease in frequency greater than 10 percent for each mode. Overall, there was not good agreement between the experimental and analytical data with the exception of the first mode. The analytical results consistently under-predicted the frequency change. The analytical results that incorporated the material property changes and the thermal stress effects agreed the best with the experimental data. However, the difference between these results and those from the analysis using only the material property changes was small.

The temperature distribution across the length of the titanium plate was similar to the nonuniform heating temperature profile for aluminum. The temperature distributions for the aluminum plate nonuniform and transient heating profiles compared with the titanium plate transient heating profile are shown in Figure 3. The largest local temperature gradient was for the aluminum plate transient heating profile.
11.4 °C/cm (52 °F/in.). This set of data indicated that the temperature gradient must be large before the modeling of the thermal stresses becomes important. The temperature gradient for the titanium plate transient heating profile was in between the transient and nonuniform heating profiles for aluminum. The temperature gradient is not large enough to have a significant effect on the results when the thermal stresses are modeled in the analysis. The prediction of the effect of the thermal stresses appears to be a weak part of the analysis.

Fiberglass Plates

Uniform Heating. A comparison of the experimental and predicted frequency values for plate #1 and plate #2 is shown in Figures 12 and 13, respectively. Each plate was uniformly heated to 177 °C (350 °F), which was 15.6 °C (28 °F) above the manufacturer's specified glass transition temperature.

The frequency of each mode did not vary significantly below 121 °C (250 °F). Above this temperature, there was a noticeable decrease in frequency as temperature increased. This data indicated that the glass transition temperature was between 121 and 149 °C (250 and 300 °F), since it was in this range that the frequency decreased significantly.

The analytical results also reflect the change in frequency above 121 °C (250 °F). This change is frequency was due to the variation in the modulus with temperature. In the case of the fiberglass plates, the modulus was experimentally determined and indicated a large decrease in modulus as temperature was increased above 121 °C.

There was a fair comparison between the experimental and analytical data results. The analytical data came from a plate model using isotropic elements rather than more complex composite elements. This may have degraded the analytical results. In general, the frequencies were predicted to decrease slightly as the temperature was increased to 93 °C (200 °F). Above 93 °C, the predicted rate at which frequency changed increased. The frequency was predicted to remain constant from 149 to 177 °C (300 to 350 °F). This trend was not unexpected since the experimentally determined modulus of elasticity for this material [10] exhibited a similar trend with temperature. The predicted frequency values were solely a function of the modulus of elasticity for the uniform heating profile.

The difference in experimental frequency values between plate #1 and plate #2 was primarily due to the difference in plate thickness, .38 cm vs .48 cm (0.15 in. vs 0.19 in.). The frequency trends as a function of temperature were similar between the two sets of data. The orientation of the E-glass plies appeared not to have any effect on the frequency data at elevated temperature conditions.

The experimental damping trends for each plate mode are shown in Figures 14 and 15, respectively. Above 121 °C (250 °F), the damping for each mode first increased and then decreased as the temperature was increased. These trends were verified through repeated tests in this temperature range. The damping increase between 121 and 149 °C (250 and 300 °F) indicated an increase in plate viscosity, which was another indication that the glass transition temperature was between 121 and 149 °C.

The reason for the damping decrease above 149 °C (300 °F) is not known at this time. The material was fully cured at 191 °C (375 °F); therefore, no post curing should have occurred that would have changed the material properties. It appears that the material matrix changes above 149 °C, causing the plate to become less viscous.
Nonuniform Heating. Figure 16 compares the experimental and analytical percent frequency change from room temperature for plate #1 for a nonuniform heating profile. The plate was heated to 149, 93, and 38 °C (300, 200, and 100 °F) in zones 3, 2, and 1, respectively.

The data indicated that the predicted change in frequency was more than 10 percent greater than the experimentally measured values. The inaccuracies of the analytical results are most likely caused by modeling the plate with isotropic FEM elements and not with composite elements.

The data also indicated that there was essentially no difference in the predicted results with only the material properties modeled or with the material properties and thermal stresses modeled. This was expected because of the small temperature gradient across the length of the plate.

Transient Heating. A comparison of experimental and analytical modal frequency change from room temperature for a transient heating profile is shown in Figure 17. The experimental data were acquired from a test which consisted of heating the plate to 177 °C (350 °F) in zone 1 at a rate of 1.7 °C/sec (3 °F/sec). Note that the temperature of the plate overshot to 204 °C (400 °F) which was above the curing temperature. The time that the plate was 204 °C was very short, however.

The data comparison was similar to the nonuniform heating profile data. Each mode's frequency change was predicted to be greater than that which was measured. The effect of modeling the thermal stresses in addition to the material properties can also be seen in Figure 17. The temperature gradient generated with the transient heating profile created thermal stresses which when modeled, affect the predicted frequency value. Overall, the experimental and analytical comparison was not good for the same reason stated for the nonuniform heating profile data comparison.

Plate Data Comparison Summary

Frequency. The STARS analysis code accurately predicted the frequency for the metal plates that were uniformly heated. The analytical results were less accurate for the fiberglass plates, most likely a result of modeling the plate with isotropic FEM elements and not composite elements.

The accuracy of the analysis to predict frequency changes caused by nonuniform and transient heating was less accurate than for the uniform heating for the metal plates. This may have been a result of heating the plates to temperatures where the material behaved nonlinearly.

The frequency changes were not accurately predicted for the fiberglass plate for either nonuniform or transient heating. Errors in excess of ten percent were common. This was most likely caused by modeling the fiberglass plate with isotropic FEM elements instead of composite elements.

Modeling of the thermal stresses caused by temperature gradients was found to be important for gradients larger than approximately 11.4 °C/cm. (52 °F/in.). Modeling of the thermal stresses for local temperature gradients less than this did not have as significant an effect on the predicted frequency values.

Damping. The experimental damping trends for the metal plates remained fairly flat for the temperature range in which the modulus of elasticity varied linearly with temperature. The damping increased as temperature increased above this range indicating an increase in material viscosity. The damping trend for the fiberglass plates remained flat for temperatures below the glass transition temperature. Above this
temperature, the damping first increased and then decreased. This result was unexpected and the cause is still under investigation.

CONCLUDING REMARKS

Analysis codes that predict the aeroelastic and aeroservoelastic stability of proposed hypersonic vehicles at high temperatures must be correlated and validated with experimental data. Hot structure vibration tests for uniform, nonuniform, and transient heating were conducted on aluminum, titanium, and fiberglass plates to establish an experimental database for comparison with analytical predictions.

The results have shown that a reduction in stiffness was reflected in each test article by a reduction in modal frequency. The reduction in stiffness was primarily a function of material property (elastic modulus) changes for uniform heating profiles. A combination of property changes and thermal stresses created from the temperature gradient across the length of the plate caused the stiffness changes for nonuniform and transient heating.

Vibration analysis methods accurately predicted frequency changes in the temperature range where the modulus of elasticity varied linearly with temperature. Analyses results were less accurate beyond this range.

Thermal stresses created from nonuniform and transient heating profiles must be represented in the finite element models when large local temperature gradients are present across the structure. Large errors in the predicted frequency results occur when these stresses are neglected.

Each test article, particularly fiberglass, must have well known material properties at all test temperatures. This information would improve the modeling of each test article.

REFERENCES


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**Figure 1.** Test plate heating zones and instrumentation locations.
Figure 2. Oven control and data acquisition.

Figure 3. Plate temperature distributions for nonuniform and transient heating profiles.
Figure 4. Typical temperature profile and division of the test plate into strips.

Figure 5. Comparison of analytical and experimental frequencies for a uniformly heated aluminum plate.
Figure 6. Experimental modal damping estimates of a uniformly heated aluminum plate.

Figure 7. Comparison of analytical and experimental frequency changes for a nonuniformly heated aluminum plate.
Figure 8. Comparison of analytical and experimental frequency changes for a transientsly heated aluminum plate.

Figure 9. Comparison of analytical and experimental frequencies for a uniformly heated titanium plate.
Figure 10. Experimental damping estimates of a uniformly heated titanium plate.

Figure 11. Comparison of analytical and experimental frequency changes for a transiently heated titanium plate.
Figure 12. Comparison of analytical and experimental frequencies for fiberglass plate #1 at a uniform temperature profile.

Figure 13. Comparison of analytical and experimental frequencies for fiberglass plate #2 at a uniform temperature profile.
Figure 14. Experimental modal damping estimates for fiberglass plate #1.

Figure 15. Experimental modal damping estimates for fiberglass plate #2.
Figure 16. Comparison of analytical and experimental frequency changes for a nonuniformly heated fiberglass plate (#1).

Figure 17. Comparison of analytical and experimental frequency changes for a transiently heated fiberglass plate (#1).
High surface temperatures and temperature gradients can affect the vibratory characteristics and stability of aircraft structures. Aircraft designers are relying more on finite-element model analysis methods to ensure sufficient vehicle structural dynamic stability throughout the desired flight envelope. Analysis codes that predict these thermal effects must be correlated and verified with experimental data. This paper presents experimental modal data for aluminum, titanium, and fiberglass plates heated at uniform, nonuniform, and transient heating conditions. These data are compared with vibration analysis results for the same heating conditions. The data show the effect of heat on each plate’s modal characteristics, a comparison of predicted and measured plate vibration frequencies, the measured modal damping, and the effect of modeling material property changes and thermal stresses on the accuracy of the analytical results at nonuniform and transient heating conditions.