NASA-CR-196143

FINAL
IN-44-CR
OCIT.
16047
P. 219
426934

MARS VERTICAL AXIS WIND MACHINES

The Design of a Darreus and a Gyromill for Use on Mars

Work done under NASA Ames Grant No. NCC 2-710
Thomson Research Projects, Inc.
Lake Mills, Wisconsin
(A Non-Profit Organization)

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FINAL REPORT

(NASA-CR-196143) MARS VERTICAL AXIS WIND MACHINES. THE DESIGN OF A DARREUS AND A GIROMILL FOR USE ON MARS Final Report (Thomson Research Projects) 219 p

N94-36807

Unclas

G3/44 0016047

Preface

This report contains the design of both a Darrieus and a Giromill for use on Mars. The report has been organized so that the interested reader may read only about one machine without having to read the entire report. Where components for the two machines differ greatly, separate sections have been allotted for each machine. Each section is complete; therefore, no relevant information is missed by reading only the section for the machine of interest. Also, when components for both machines are similar, both machines have been combined into one section. This is done so that the reader interested in both machines need not read the same information twice.

Excellent

Theoretical analysis of Machines excellent. Detail design of machines with assembly dwgs etc. is piecemeal - Need summary of Component wts, dimensions, materials in report - Need overall assembly dwg with dimensions. /BM/

"Design not Finished?"

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Excellent section

Also!

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nice touch

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Statement of Purpose

The purpose of the Mars Vertical Axis Wind Turbine group was to design a vertical axis wind turbine which would provide power to an unmanned Mars research station. The research package for such a station might include meteorology equipment, a seismometer, surface chemistry equipment, and imaging equipment. The power requirement of this package is one watt while in normal operation, non-transmitting mode (p. A-1 to A-2). It is assumed that a lander is provided as a platform, with landing characteristics similar to those of the Martian Egg Lander EM 569 project of 1990 [19], and that the lander is equipped with a battery-type energy storage device. The turbine should be able to operate in the Martian environment for three years, which is the amount of time between launch windows to Mars; continuous data is desirable, and it would be three years before another research package could be landed on the surface. A complete list of the needs and functions of the design, as well as a FAST diagram appear on pg. A-3 to A-5.

Environmental Concerns

The Martian atmosphere differs greatly from the Earth's atmosphere. These differences strongly affect many aspects of our design. The most important differences are the temperature, pressure, and density of the air and the surface gravity. These values for Mars are:

Temperature:	140 - 240°K
Pressure:	6.5 - 10.5 millibars
Density of air:	1.667×10^{-5} g/cm^3
Surface gravity:	3.70 m/s^2

The values for pressure and density on Mars are between 1/75 and 1/100 of their respective values on Earth. Even at these low densities, it has been shown that the Martian atmosphere may be modelled as a continuum rather than as a collection of discrete particles [5]. This fact enables us to use the same laws of aerodynamics as we would use for a wind turbine on Earth. ✓

Another important environmental concern is that of dust storms on Mars. The low density Martian air is able to transport only small dust particles (approximately 0.1 mm). The dust storms affect our design in two major ways. First, the airfoils will have to be made of relatively hard material to resist abrasion. Second, a shield will be used to protect the generator and the gears.

I expect 0.1 mm dust to be insidious

The final environmental consideration is that of wind speeds. The next section covers this topic in detail.

If you say or but it doesn't feel that way to me!

Wind Speed Design Decisions

Two important decisions needed to be made regarding the Martian wind speeds. First, a range of wind speeds likely to occur most of the time needed to be determined. A good estimate for the maximum wind speed likely to occur was also necessary.

The range of wind speeds needed to be determined in order to optimize the aerodynamic performance of our wind turbine. Because there is little data on Martian wind speeds, this decision needed to be based on a combination of analysis of the data and engineering judgement. The existing data consists of measurements taken at the Viking lander site and several meteorological studies.

It is quite difficult to make any decisions based on the Viking data. The experiments measured north-south winds separately from east-west winds with no correlation between the two. The best estimate that we could obtain from this data is that the wind speed was greater than 4 m/s about 80% of the time [see pg. A-6].

Studies done by meteorologists [5] show that the Viking lander site was a non-optimal site as far as wind speeds go. They are able to locate several areas where they believe the average wind speed to be above 8 m/s, and a couple where the average could be as high as 14 m/s [5]. They also state that the global average wind speed is 6.5 m/s.

Since the mission will have other purposes other than just generating power from the wind, it is doubtful that the landing site would be at the optimal wind speed location. However, we believe if wind energy is to be used, the landing site should be a place of at least moderate wind speeds. Therefore we decided to design for an intermediate value between the Viking data and the meteorological studies.

The actual numbers that we have selected are listed below

1) Generate 1 Watt of power from all wind speeds greater than 6 m/s OK!

2) Operate at peak efficiency for wind speeds between 5 m/s and 8 m/s

3) Generate some power for all wind speeds greater than 4 m/s

We believe that these numbers will provide the best overall performance of our wind turbine. We see that even if we landed at a site similar to the Viking site, we would still be generating some power 80% of the time.

The maximum wind speed likely to occur is very important for determining the extent of structural support required by our turbine. Wind speeds of up to 100 km/hr have been recorded on Mars for only a few hours [3]. Meteorologic studies estimate the maximum wind speed likely to occur on Mars at 250 km/hr [4].
Once again, engineering judgement needed to be combined with the data.

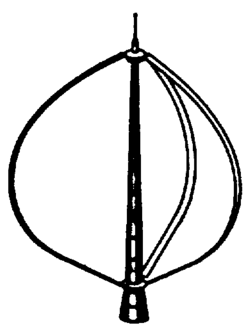
The meteorological study here is somewhat questionable due to some of the assumptions it has made [4]. Also, our turbine is expected to operate for only three years, so the probability ~~possibility~~ of it seeing wind speeds of 250 km/hr is quite small. We decided to design our wind turbine to withstand all wind speeds up to 150 km/hr.

Selection of Wind Turbines

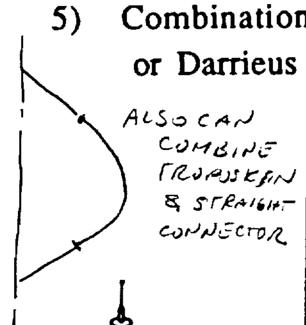
Vertical-axis wind turbines appear to be the most feasible means for generating power from Martian winds. Since vertical-axis wind turbines are always oriented into the wind, there is no need for vanes to rotate the blades into the wind. This eliminates the presence of large gyroscopic forces found in horizontal-axis wind turbines.

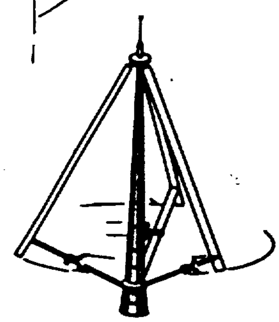
Several vertical-axis wind turbines were considered. The five possibilities that were given the most thought were:

1) Φ-Darrieus
2) Δ-Darrieus
3) Giromill
4) Split-Savonius
5) Combination of Split-Savonius with either Giromill or Darrieus

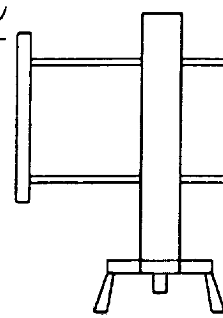


ϕ-Darrieus



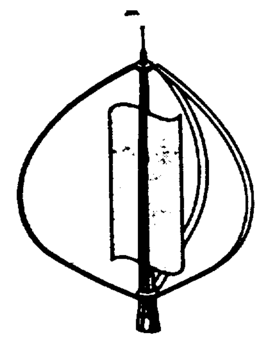


Δ-Darrieus



Giromill

Split Savonius



Savonius/ϕ-Darrieus

A turbine that is powered by aerodynamic drag forces, such as a Split-Savonius, would need to be very large to extract enough power from the thin Martian air. Therefore, this choice was determined to be less desirable than the other four.

Of the three lift-type turbines, the Φ-Darrieus and the Giromill were determined to be the best choices. Also, in order to minimize mass, each turbine will have two blades. Both the Φ-Darrieus and the Giromill possess strong advantages that were not found in the Δ-Darrieus. The main advantages of the two turbines are listed below.

<u>Φ-Darrieus</u>

1. The blades are designed to eliminate bending stress
2. Light weight blades can be built since no bending stresses are present

<u>Giromill</u>

1. Blade material positioned for maximum aerodynamic torque
2. Has deployment benefits since relatively easy to "compress" into small area

A combination of the Split-Savonius with one of these two machines was also considered. This type of design would enable the turbine to be self-starting. However the large size of Split-Savonius that would be required prevented us from selecting this option. The use of the generator to start our turbine will enable a much lighter design than if a Split-Savonius were used for this purpose. ✓ OK

For convenience, the rest of the report will drop the ϕ prefix from the ϕ-Darrieus.

Darrieus Blade Design

- Size Determination
- Development of troposkien shape for varying cross-sectional area of blade
- Optimization of Shape
- Airfoil Selection
- Material Selection
- Structural Analysis

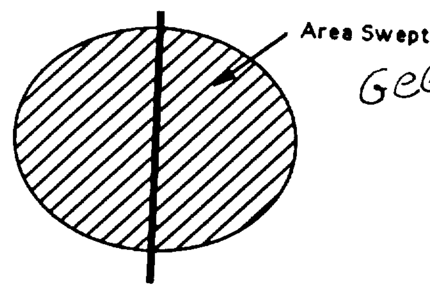
Introduction

When G. J. M. Darrieus received his patent for a vertical-axis wind turbine [2], he stated that the blades should take "the form of a skipping rope." A blade placed in this shape will be in a state of nearly pure tension, i.e. negligible bending stress. By eliminating the bending stress, the amount of material required for support is drastically reduced, and very light-weight blades may be designed. The word troposkien (from the Greek: τροπος, turning and σχοινιον, rope) is now used to describe the shape assumed by perfectly flexible cable that is spun about a vertical axis at constant angular velocity.

All previous Darrieus turbines have been designed with a constant cross-sectional area along the length of the blade. Although these solutions ease manufacturing of the blades, much material is wasted. From previous designs, it has been found that the tension, and therefore the stress, varies with horizontal (radial)? position squared [1]; the stress is highest at the top and lowest in the center. It was noticed that material could be saved by varying the cross-section along the length of the blade. Namely, it would be desirable to place more material where forces are higher, and remove some material where forces are lower. Our goal was to have a constant stress everywhere along the length of the blade, thus eliminating wasted material. Since the shape and the stresses depend upon how we vary our cross-sectional area, we realized that we could probably could not achieve exactly constant stress, but would try to come as close as possible.

Size and Tip-Speed Calculations

Using wind energy theory, together with experimental results from existing Darrieus turbines (see pg. T-1 to T-8), we were able to determine the necessary size for our Darrieus turbine. In order to generate 1 Watt from a 6 m/s wind, the blades must have a swept area of 2 m^2. Due to the low density of the Martian air, this is about 100 times larger than would be required on earth. The following figure shows what is meant by area swept.



Gee!

The aerodynamic performance of a wind turbine is strongly effected by its tip-speed. The tip-speed of a windmill is the speed at which the blades are moving ~~at~~. Since, for a Darrieus blade, every position has a different speed, the tip-speed is defined as the maximum speed of the blade. This is by definition, the product of the distance b times the angular velocity ω (the distance b can be seen on Figure 0, pg. T-10). Closely related is the tip-speed ratio, which is the tip-speed divided by the ambient wind velocity.

Using a suggested wind-speed probability relationship, the tip-speed was determined so that our turbine would generate the maximum average power (see pg. T-1 to T-8). The average power was maximized so that we are able to recharge our batteries with the most energy over time. This was done while also assuring the generation of 1 Watt from a 6 m/s wind speed. The required tip-speed was found to be $b\omega=31.3$ m/s.

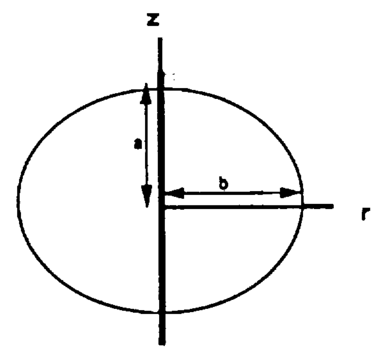
Troposkien Shape and Optimization

The equations governing the shape of a varying cross-sectional troposkien are developed in complete detail in the appendix (see pg. T-9 to T-17). After considerable manipulation, the equations may be reduced to one first-order differential equation, one unknown, and two boundary conditions. The equations are in the form shown below.

$$\frac{dr}{dz} = f(r,b,\zeta,K)$$

$$r(z=a) = 0$$

$$\frac{dr}{dz}(z=0) = 0$$



where

- r = horizontal coordinate
- z = vertical coordinate
- a = height, see above figure
- b = width, see above figure
- ζ = a measure of how much we vary our cross-section along the length of the blade
  - a low ζ ($\zeta \approx 1$) means that there is a strong difference in cross-sectional area between the top and the middle of the blades
  - a high ζ ($\zeta \approx 20$) means that the cross-sectional area is nearly constant along the length of the blade (this value for ζ will be used to approximate the constant cross-sectional area solution)
- K = a parameter which depends on many terms, the most physically significant are:
  - K is proportional to angular velocity squared.

• K is proportional to mass per unit length of blade
f = a function, the actual function may be found on pg. T-17

The values for a and b are not independent, they must be chosen so that the area swept is 2 m^2. In order to solve for the shape r(z), we are free to choose any combination of a and b that sweeps out 2 m^2, and any value of ζ. Since we have this freedom, we must select the values that give the best results.

What our solution needs to minimize is the required arclength of the blades so that the area swept is 2 m^2. The arclength is a measure of how much material is needed and therefore minimizing the arclength will minimize the blade mass. / for ρ=const

The function f in the differential equation is quite complex (see pg. T-17) and no analytical solution exists. Therefore, we have developed a numerical solution with a computer program. Our computer solution will be developed for several values of a and b. The selected values of a and b will not sweep out an area of exactly 2 m^2. Therefore our attempt will be to maximize the value of area swept divided by arclength; this is essentially a power-to-mass ratio. This concept can clearly be seen by observing the tabular data (pg. T-28 to T-31). Once this value is optimized, we may adjust a and b slightly in order to obtain the swept area equal to 2 m^2.

The computer solution (pg. T-18 to T-27) is carried out by first selecting values for a,b, and ζ, and then solving for the shape r(z) and the parameter K. The shape is determined pointwise and may be developed for as many points as desired. Since the equation is a first order differential equation with two boundary conditions, we need to leave K as an unknown in order to guarantee that the solution will "fit" both boundary conditions.

The program uses a combination of the fourth order Runge-Kutta method for numerical integration and the bisection method for root finding (see pg. T-24 to T-27). The program is able to determine the shape, r(z), and the value of K simultaneously. For each selected value of ζ, the solution is repeated for 30 different combinations of a and b that sweep out nearly 2 m^2. The whole procedure is then repeated for different values of ζ, with the goal being a maximum ratio of area swept to arclength.

After trying several values of ζ, we notice that the lower the ζ, the higher the value of area swept to arclength for all values of a and b. The following graph shows this quite clearly. The ratio of area swept to arclength is shown on the vertical axis. The height-to-width ratio (a/b) is plotted on the horizontal axis. The curve for ζ=20 is essentially the case of a constant cross-sectional area along the length of the blade.

POWER-TO-MASS

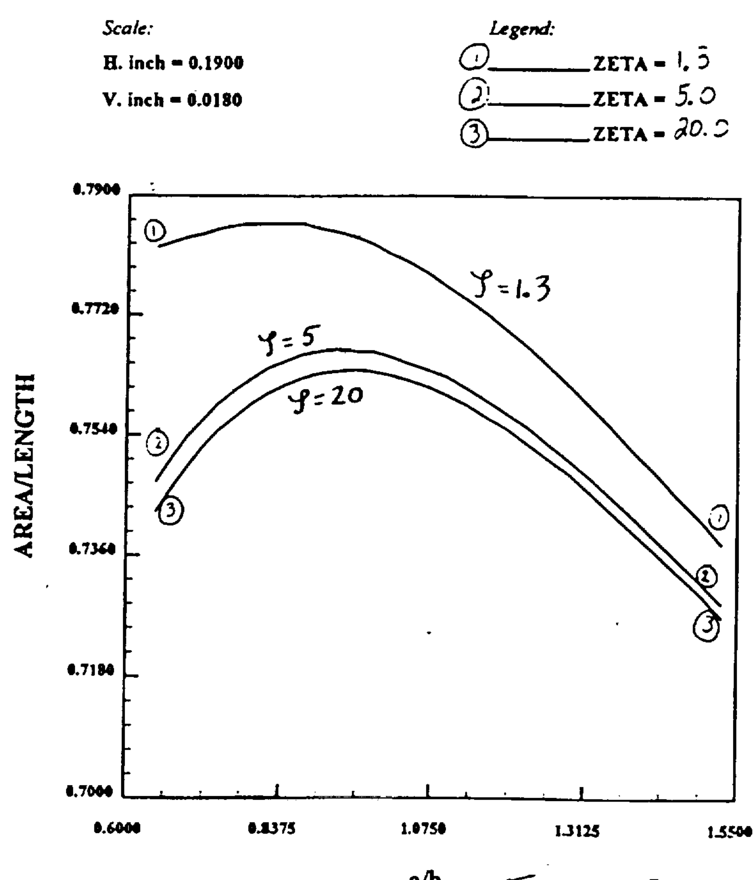


Figure 2

From these curves we see that by varying the cross-section, we get the added benefit of a better shape. This makes our results better than we had even hoped for. The concept of obtaining a better shape also makes sense physically. By placing more material near the top, we increase the centrifugal force there, and obtain a shape that is more square-like. This shows up well on the following graph. The graph shows the shape obtained by varying the cross-section (ζ=1.3), the shape obtained with a constant cross-section (ζ=20), and a circle for comparison. As expected, the varying cross-section case is "pushed out" more near the top of the blade, and the constant cross-section case is "pushed out" more towards the center of the blade (which is the bottom on this graph). All of these curves have the same arclength.

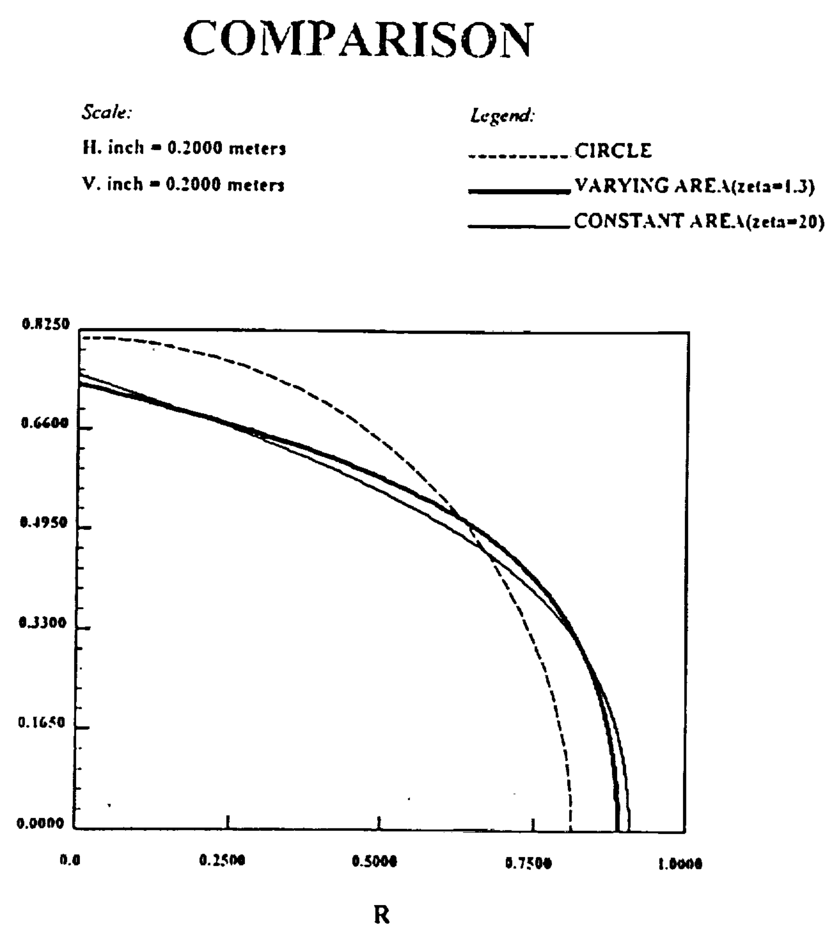


Figure 3

We see from figure 2 that for each value of ζ, there is an optimal combination of a and b. We are then left with determining the best value for ζ. By definition, ζ has a minimum possible value of approximately 1.0. With this fact and the above curves in mind, we see that we want ζ very close to 1.0.

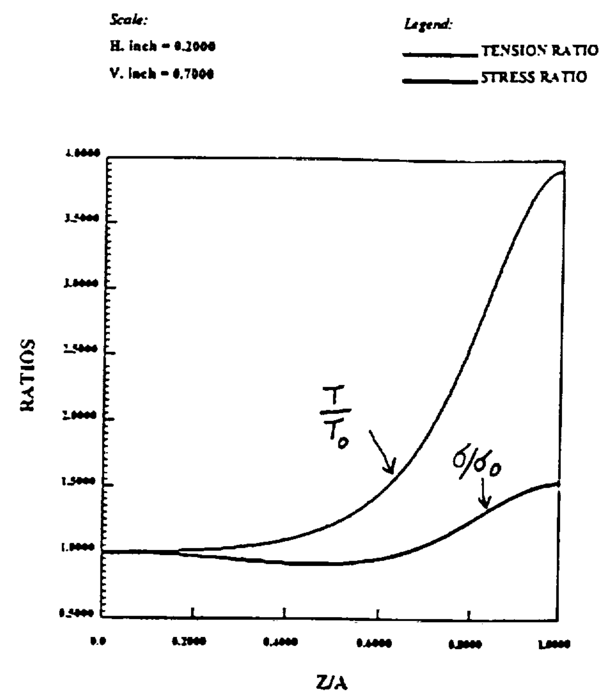
Besides having the best shape, we must remember our original intent to have nearly constant strength (i.e. constant stress) along the length of the blade. It is important to understand the differences between the two measures. A constant strength reduces required material along the length of the blade, whereas a better shape reduces the length of the blade.

After several iterations, we see that if ζ is too close to 1.0, we are not able to approach constant strength and that if ζ is much greater than 1.0 we quickly lose the better shape and the ability to obtain constant strength. The optimal value, determined by trial and error, was found to be $\zeta=1.3$.

With this value selected, and the optimal values for a and b determined from the tabular data (pg. T-28 to T-31), we have the complete solution for our shape and the value of K. With b known, we are able to determine the required angular velocity from our earlier specified tip-speed of $b\omega=31.3$ m/s. Then with K and ω known, relationships may be written in order to determine the tension and stress at any point along the blade. This is all done on pages T-32 to T-36.

From these relationships, we are able to plot the tension and stress along the length of the blade. The values are normalized for ease of understanding. The tensile value at any point is divided by the tension at the blade's midpoint (z=0), which is the minimum tension. The stress value is also divided by the stress at the blade's midpoint. These ratios are written as T/T_o and σ/σ_o, respectively, where the subscript o refers to the value at z=0. These ratios are measured along the vertical axis on the following plot. Along the horizontal axis is the non-dimensionalized vertical coordinate (z/a). We see that we have taken a maximum loading ratio (T/T_o) of nearly 4 and reduced it to a stress ratio (σ/σ_o) of approximately 1.5. We also see that we have come very close to achieving constant strength ($\sigma/\sigma_o=1$) throughout the length of the blade. For the traditional, constant cross-sectional area Darrieus designs, the stress ratio would follow the same curve as the tension ratio.

TENSION AND STRESS RATIOS



GOOD

Figure 4

Although all of these figures give great promise to the idea of varying the cross-sectional area along the length of the blade, we needed to estimate the total saving of mass to see if the idea was really worthwhile. Remember that benefits occur for three different reasons. First, we reduce mass by varying the cross-section along the length of the blade and removing material where it is not necessary. Second, we obtain a better shape (shorter length of blade) which requires less material. Finally, by reducing the total blade mass, we reduce stresses everywhere. When our solution was compared to the constant cross-sectional area solution we found that by varying the cross-sectional area we reduced blade mass by over 34% while also achieving a 54% reduction in maximum stress (see pg. T-36). These results verify that it is worthwhile to vary the cross-section along the length of the blade.

Great! That is a fine result!

Darrieus Airfoil Selection

Chord Length

The optimum performance of the wind turbine depends upon the size and shape of the machine itself and the size and type of airfoil used for the blades. A measure of the turbine's performance can be given by

$$C_p = 0.25n(c/r)k\lambda V^2 - 0.5n(c/r)C_d\lambda^3$$

where:

C_p = coefficient of performance, a measure of efficiency
n = number of blades for the turbine
c = chord length of the blades
r = distance from the chord line to the center line of rotation
λ = the tip-speed of the machine
V = the incident wind velocity acting on the machine
C_d = the average drag coefficient for the blades

The incident velocity, V (Fig. 6 and appendix p. T-41; [24]), is given by the equation

$$V = 1 - 0.0625n(c/r)\lambda(k+3C_d)$$

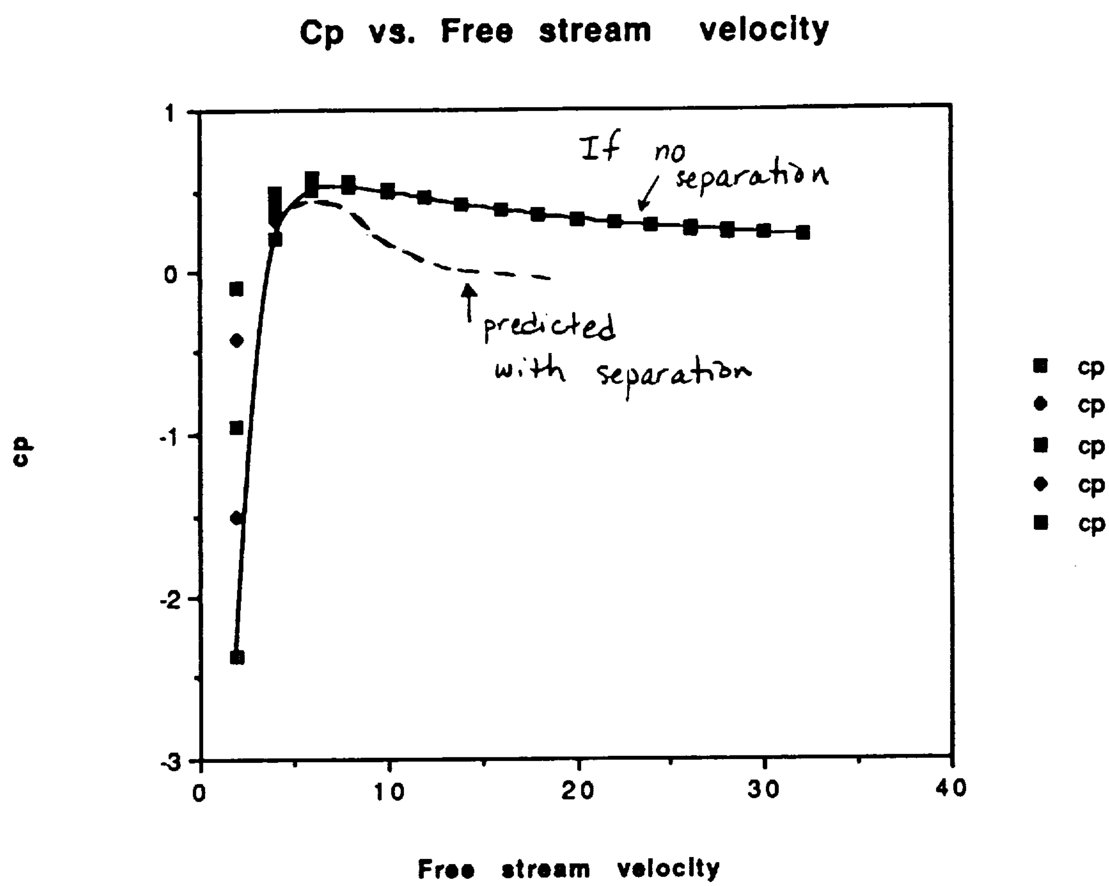
Substituting this equation into that of C_p, differentiating with respect to the tip speed, λ, and assuming the atmosphere acts as an ideal fluid (C_d=0) results in the expressions

$$\lambda = 16r/(3nck)$$

and

$$V = 2/3$$

The latter simply means that during optimal operation, the incident velocity acting on the wind turbine is equal to 2/3 the free stream velocity. For the darrieus machine, the first equation, used to find the chord length, is now a function of two variables, c and r; unlike the giromill. Considering that the torque generated by the the outermost part of the blade will be much greater than that of inner sections, the value of r used was 0.839 m. Substituting values for λ,r,n, and k results in an optimum chord length of 6.83 cm.



The predicted curve comes from experimental results done on several Darrieus turbines. An example of these experimental curves is on pg. T-4.

Figure 5

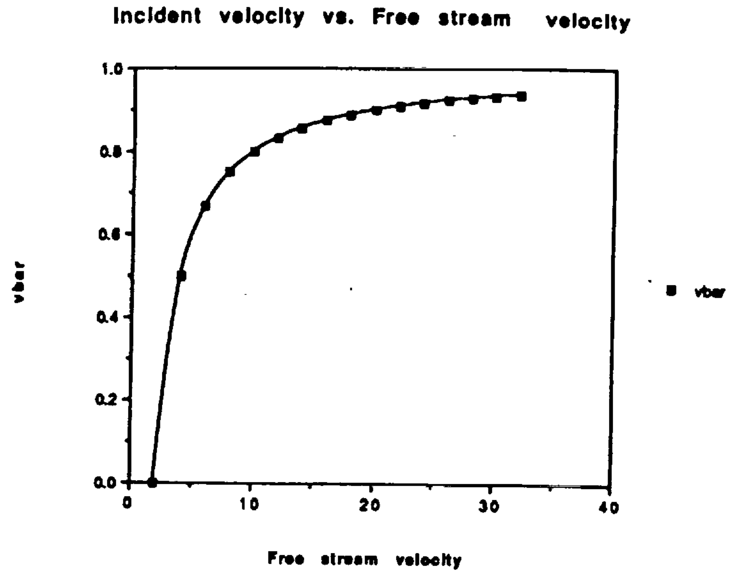


Figure 6

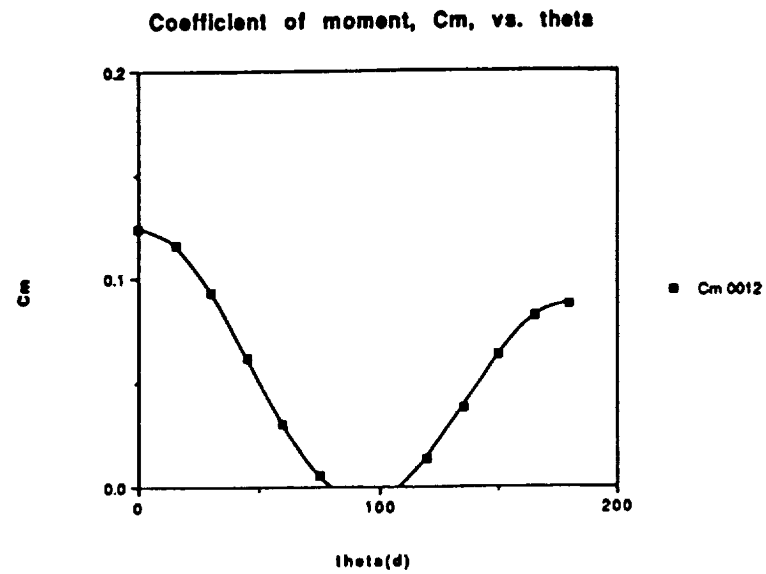


Figure 7

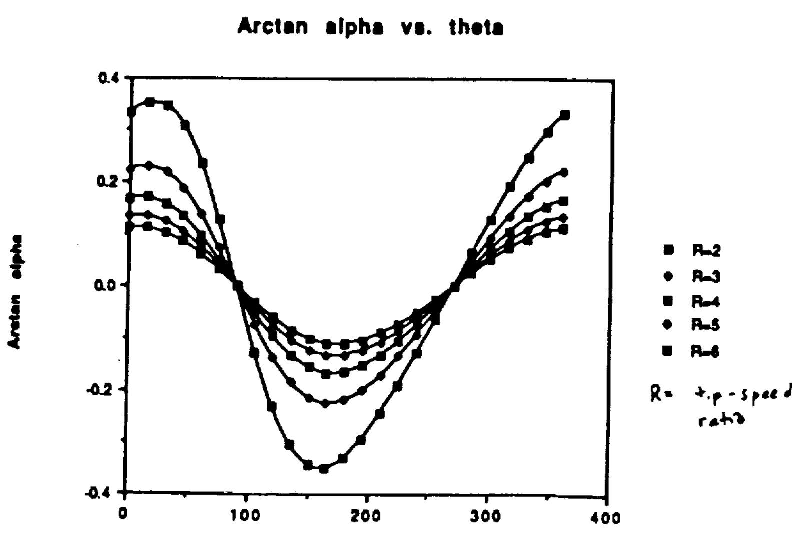


Figure 8

Thickness

The thickness of the airfoil can be determined by finding the maximum value of the average coefficient of moments (Fig. 7) which is given by

$$C_{ma} = (c/r)[(p/2)V^2 - C_d\lambda^2]$$

By inspection, it can be seen that the thickness of the airfoil should only be dependent on finding the lowest value of C_d. As before, the radius also affects these calculations. However, as r decreases, the contribution to the moment decreases as well and so we make the same assumption of r as when finding the chord length. Using experimental data [8] to compute a value for these drag coefficients gives the following as examples:

Airfoil type	C_d
NACA0006	0.0098
NACA0009	0.0079
NACA0012	0.0076
NACA1410	0.0081

As can be seen, there is very little difference among these drag values which results in moment coefficients which differ by only thousandths; the NACA 0012 airfoil being only slightly better than the others. Initially it was thought ignoring these differences and using the thinnest section possible (to cut mass requirements) would be best. The range of usable angle of attacks (Fig. 8 and appendix p. T-42; [24]) for each airfoil, however, does show that the NACA 0012 section is the best choice, especially at higher wind speeds.

Lift and Drag

Due to the extremely low density of Mars' atmosphere and the small planform area of the blades (0.18 m^2) the lift and drag forces acting on the blades are small. At a wind speed of 6 m/s the lift force on the blades is only 0.72 N (appendix p. T-42). The drag forces have been neglected due to the fact that they are typically on the order of one percent of the total lift force.

Do all the materials on M1&M2 work at these temps?

Darrieus Material Selection

The material selection for the blades was limited by two main factors, one being the low temperature range (140K-240K), and the second being the need for a high strength-to-weight ratio. The materials collected within those limits are found in Table 1 and Table 2 on pg. M-1 and M-2 in the appendix.

The Darrieus blades are in pure tension while the machine is operating, thus a light weight material with strong tensile properties is needed. These two criteria are best exhibited by unidirectional composites (Table 2 (pg. M-2) in the appendix shows properties of such composites). WHICH KEVLAR? (#)

THERE ARE OTHERS - e.g. POLYETHYLENE "SPECTRA 1000"

Kevlar had the highest strength-to-weight ratio of all the materials. It also exhibits superior properties of fatigue. Kevlar best suited the need for the Darrieus blades and thus was chosen for the blade material. Verification of the material selection can be found on page M-4 in the appendix.

IS THIS Ti6Al4V?

All connector pins are made out of titanium alloy 6Al4V. This material was chosen because of its extremely high shear strength. Its low coefficient of expansion is also very important so that it won't interfere with the materials it is pinning. Properties of this titanium alloy can be found in Table 1 (pg. M-1) in the appendix.

Troposkien Blade Analysis

The solution of the troposkien equation for a given set of parameters was completed. These parameters constitute a set of "governing equations" that determine the dimensions of the blade. The "governing equations" have been included in the appendix(T-33, T-34). The subsequent analysis of the blades included the following:

(1) Centrifugal loading analysis,
(2) Aerodynamic loading analysis,
(3) Thermodynamic loading analysis.

These will be discussed further in the following, additionally, sample calculations have been included(T-43 - T-62). The troposkien blade specifications will be discussed first.

Troposkien blade specifications

The final configuration of the troposkien blade was a hollow shell with approximately constant wall thickness. Many alternative configurations were considered(T-60,T-61), but in the end the thin, hollow shell proved the best choice(T-63).

Outer shell shape - NACA0009 pg 18 says NACA 0012?

At the equator	At r = 6 cm
cord = 7.4 cm	cord = 9.0 cm
wall thickness = 0.5 mm	wall thickness = 1.0 mm

total mass of each blade = 0.486 kg.

Addition information on the geometry of the blade can be found in the appendix(T-43 - T-45).

Centrifugal Loads

The dominant loads acting on the troposkien airfoil are the centrifugal forces. The troposkien blades are in a state of pure tension for centrifugal loading. The solution of the troposkien solution yields specific parameters which must be satisfied; these parameters may be found on pg. T-33 and T-34. Manipulation of these equations yields specific information about the stress in the blade at any location (T-39). Remarkably, we find that the tensile stress is completely independent of the cross-sectional area, as long as the area is varied according to the mass per unit length equations on pg. T-14. This is because as the cross-sectional area is increased, the centrifugal force is increased, and the tensile force increases. For example, the uniaxial stress acting at the blade's equator is

$$\sigma_o = 61.1 * \delta = T_o / (A_{cs})_o$$

where:

T_o = tension at equator
δ = mass density (mass/length^3) of material
$(A_{cs})_o$ = cross-sectional area at the equator

The density of the blade material is 1380 kg/m^3. This yields a uniaxial stress of 84.3 kPa at the equator. The maximum stress is 129.8 kPa, occurring at the roots of the blades. Because of the high strength of Kevlar, the factor of safety under normal operating conditions is 10,870. The only way to reduce the factor of safety is to use a weaker and/or heavier material!

The source of greatest concern with respect ~~the~~ to the centrifugal loads is the location of the pin that will secure the blade to the hub of the shaft (T-46 to T-48). A stress concentration will occur around the hole. The stress concentration factor [20] is defined as,

$$k = 2 - (1-w/d) + [1.5(1-w/d) / (1+w/d)]$$

I would like to see this expanded

The stress intensity factor evaluated at the pin location is k=1.725. ?

The maximum stress is computed by $\sigma_{max} = k\ P/t(w-d)$. The load P is determined to be T/2 = 9.0 N, so we obtain

$$\sigma_{max} = 181.1 \text{ MPa}$$

The ultimate stress for the material is given to be 1400 MPa, giving a factor of safety of 7.7.

Aerodynamic loading

The aerodynamic loads acting on the airfoil were found to be negligible compared to the centrifugal loads. Considered in the analysis, were the pressure acting over the surface of the airfoil, which could potentially deform the airfoil shape, and the bending moment acting on the blade due to the total aerodynamic load.

The pressure acting on the surface of the blade was analyzed by modeling the blade surfaces as infinite plates fixed on the leading and trailing edges.

The equation for the maximum deflection of the center of the plate[22]is

$$y_{max} = \alpha \, q \, b^4 / E \, t^3$$

where

$\alpha = 0.0285$
q = pressure acting on the surface
b = cord of airfoil   modulus
E = flexural modules of material
t = wall thickness of the blade

Assuming the maximum deflection of the plate to be equivalent to half the maximum thickness of the airfoil, a pressure can be calculated. This pressure acts as a residual force against any restoring pressure. Thus, for $y_{max} = 0.00333$ m,

$$q = 38.96 \text{ MPa}$$

The pressure acting on the blade due to the aerodynamics of the airfoil is estimated(T-51 - t-55)[8] and subtracted from q.

$$Q = q - p = 38\,94 \text{ MPa}$$

This yields a maximum deflection of

$$y_{max2} = 0.003328 \text{ m}$$

The difference between the maximum deflections is 0.05%.

The aerodynamic loading was estimated to be 0.06 N-m about the vertical axis. The moment acting on the blade at r = 0.06m would be approximately equal to that acting about the center(T-56 - T-57).

The relevant moment of inertia and centroid at r = 0.06m is

$$I_{yy} = 514.08(10)^{-9} \text{ m}^4$$
$$x = 0.042c$$

Substituting into $\sigma = Mx / I_{yy}$, yields

$$\sigma = 6.1 \text{ kPa.}$$

This stress is less than 5% of the uniform stress due to the centrifugal loads.

Thermodynamic loading

The martian atmosphere experiences a diurnal temperature change of ~100 K. The strains that result from these temperature fluctuations is negligible(T-58, T-59).

The material has an axial and a transverse coefficient of expansion. They are:

$$\varepsilon_{axial} = 100\ K\ (-2.0(10)^{-6} / K)$$
$$\varepsilon_{trans} = 100\ K\ (60(10)^{-6} / K)$$

In the axial direction, the total length is 2.54 m. In the transverse direction the length is approximately the cord length. The total change in overall length is

$$\delta = 53.4\ \mu m, \text{ axially; and}$$
$$\delta = 444\ \mu m, \text{ transversely.}$$

BOTH ARE INCONSEQUENTIAL

The expansion in the axial direction is negligible, and the expansion in the transverse direction is less than 1%.

Giromill Blade Design

- Size Determination
- Strut Positioning
- Blade Structure
- Strut Connections
- Stresses
- Airfoil Selection
- Material Selection

GIROMILL

Dimensions and Tip Speed Ratio

In determining Giromill dimensions the initial step comprised of an approximate Swept Area(A_{sw}) (see Pg.G1) needed to produce one Watt of mechanical power. From A_{sw} dimensions of height and radius of Giromill were obtained.

From wind energy theory, it is possible to deduce a rough idea of A_{sw}. By considering kinetic energy of moving air, the following equation for maximum mechanical power(P_{mech}) that can be extracted was derived:(for complete derivation see pg.T-8)

Eq.(1) $P_{mech}=C_p(.5*\rho*A_{sw}*V_0\text{^}3)$

C_p=>coefficient of power
ρ=>density of Martian air
V_0=>free stream velocity on Martian surface

Wanting to maximize P_{mech}, a peak C_p and related Tip Speed Ratio(T) where needed. Numerous numerical data showed an approximate $C_{p/max}=0.5$ in accordance with a T=3.0. [9](see pg. G2)

Solving Eq.(1) for A_{sw}, setting $P_{mech}=1.0$ Watt, and plugging in appropriate quantities gives:

Eq.(2) $A_{sw}=1.389$ [m^2] (see pg.G3 for calculation)

It is also know that the A_{sw} can also be represented as a function of height(h) and radius(R) of the Giromill Eq.(3) below. Another equation which is a function of h and R is the perimeter of the Giromill(P) Eq.(4) below.

Eq.(3) $A_{sw}(h,R)=2hR=1.389$[m^2]
and
Eq.(4) $P(h,R)=4R+2h$

If Eq.(4) is minimized to ensure smallest amount of material used, Eqs. (3) and (4) can be solved for Giromill dimensions **h** and **R**.

R=.5546[m]
h=1.1786[m]

(see pgs.G3-G4 for calculation)

The **T** related to **Cp/max** can be used to obtain the operating angular velocity of the Giromill, with the correct **R** determined above. The equation for **T** is as follows:

Eq.(5) $\mathbf{T} = \omega \mathbf{R}/V_0$

ω=> operating angular velocity of the Giromill

Setting Eq..(5) equal to 3.0 and solving for ω:

ω=32.45[r/s]
(see pg.G5 for calculation)

These first calculations for dimensions and angular velocity became very important for later calculations in Giromill design.

Strut Positioning

The decision for a two strut per blade design was determined from two factors. Factor one was two struts ~~are~~ provide a simpler design than then three or more. Factor two was two struts reduced the maximum bending moment in the one strut case by 17.1%.(see appendix pg.G15)

For the two strut design the positions of the struts along the blades was crucial. The position was in direct relation to the bending moments and stress on the blade. influences max

In the blade and strut arrangement, the blade was modelled as a beam with a uniform distributed load(w) acting in the plane of the struts.(see appendix pg.G9) Looking at the bending moment diagram we see three large moments:

$M_A = M_B = wa^2/2$
$M_C = (wh/8)(-h+4a)$

a=>position of struts from end of blade
h=>length of blade
(see appendix pg.G12-G13)

With the maximum bending moments determined, it was desirable to minimize them as much as possible. This in turn minimizes the stress on the blade. If the moments M_A and M_C can be balanced the desired minimal stress will result. By determining the ratio of **a/h** to make the following equality true:

Eq.(1) $|M_A|=|M_C|$

the smallest stresses possible are achieved.

Substituting ?

Plugging the appropriate equation for the moments in Eq.(1) from above ~~the~~ following equation for **a** and **h** result:

Eq.(2) a^2+ha-.25h^2=0

Solving Eq.(2) give the ratio **a/h**=.207 or **a** is 20.7% the length of the blade under a uniform load.(see pg.G13-G14)

Blade Structure

A NACA-0012 airfoil shape was chosen for reasons to be mentioned later under the title Giromill Airfoil Selection. The material aluminum boron was chosen for the Giromill, which includes the blades, for reasons which will be mentioned under the title of Giromill Material Selection.

When considering blade structure, centrifugal loads acting under operating conditions become the key design consideration. The aerodynamic loads are negligible compared to the centrifugal loads.(see pg.G18)

Centrifugal distributed load:

Eq.(1) w=(ω^2**R/h**)Mass|one blade
(see appendix pg.G19)

This relates to the maximum bending moment on the blade of:

Eq.(2) M_A=wa^2/2=((ω^2**R**a^2)/**h**)Mass|one blade
(see pg.G12)

Notice Eq.(2) M_A is a function of mass, if this mass can be reduced, the stress on blade will also reduce.

To reduce the mass the blade was hollowed out to a thickness of .5[mm]. This dimension was determined on a manufacturability of aluminum boron.

The consideration of a ribbed blade was omitted due to the fact that, with the rib, stress and deformation of blade increased.

<u>Stresses</u>(using σ_x on pg.G21)

w/o rib $\sigma_x|_{w/o}$=1.6297*10^7[Pa]

w/rib $\sigma_x|_{w/}$=1.6899*10^7[Pa]

<u>Deflection</u>(see calculation on pgs.G28-G29)

$y_E|_{w/o}$=(.9644)$y_E|_{w/}$

The final overall dimension of the blade are as follows:(see pg.G4, pg.G30 and section Giromill Airfoil Selection)

C(cord length)=8.13[cm]
T(Max. thickness)=9.756[mm]
D(blade thickness)=.5[mm]
h(blade length)=1.17857[m]

<u>Connections of Struts</u>

To connect the blades to the struts three points must be consider. Point one the struts must be pinned for deployment and to reduce bending moments on blade. Point two struts must be positioned at 42.04% of the cord length from the leading edge. This will insure no resulting moment due to centrifugal loading. Point three struts must be positioned at 20.7% of blade length from end of blade.(see pg.G14)

To comply with point one small cylinders of radius=1.5[mm] and length=7.0[mm] are attached to the end of each strut. This small cylinder is then fitted into a U-shaped fitting, which is welded to the blade. See appendix pgs.G35-G37 for complete dimensions and shape of pin connections.

Position for points two and three are found in appendix pg.G34.

Stresses and Factor of Safety

In the final stress analysis there are two points to consider. Point one is the the centrifugal loads dominate the aerodynamic loads.(see pg.G18). Point two the centrifugal loads act perpendicular to the blade and at the center of mass.

Keeping these two points in mind, the maximum stress(σx) found in the blade will occur at the point of maximum bending moment Calculated σx during operating condition:

σx=1.62960792*10^7[Pa]=16.296[MPa]

with a factor of safety of:

F.S.=81.0

(see pg.G41 for sample calculation)

wording'

Checking shear stress in the blade is approximately during operating conditions:

τ=4.1[MPa]

with a factor of safety of:

F.S.=19.5

(see pgs.G42-G44 for sample calculation)

Notice the large factor of safety. Due to the nature of the centrifugal loads and moment of inertia dependance on blade thickness(D) this can not be helped. The stresses depend on (D) in such a way that they decrease with decreasing (D). (see pg.G39 for justification)

The final consideration looked at concerning the blade was the pressure load due to aerodynamic loads. It was thought that with such a small thickness of the blade (D=.5[mm]), this pressure load may deform the blade. This was modelled and determined to be negligible.(see pg.T51-T55 for calculations)

The stresses concerning the struts and connections are of very high factors of safety. This is due to the small forces applied to the strong aluminum boron material. See appendix pg.G31-G34 for strut considerations and dimensions.

Total Mass of Blades and Struts

The Giromill contains two blades, four struts, and four pin connections. The resulting mass of Giromill(M_T):

$$M_T=2M_B+4M_S+8M_U+4M_{CL}=0.42887[Kg]$$
(see pgs.G45-G46)

Giromill Airfoil Selection

Chord Length

The optimum performance of the wind turbine depends upon the size and shape of the machine itself and the size and type of airfoil used for the blades. A measure of the turbine's performance can be given by

$$C_p = 0.25n(c/r)k\lambda V^2 - 0.5n(c/r)C_d\lambda^3$$

where:

C_p = coefficient of performance, a measure of efficiency
n = number of blades for the turbine
c = chord length of the blades
r = distance from the chord line to the center line of rotation
λ = the tip-speed of the machine
V = the incident wind velocity acting on the machine
C_d = the average drag coefficient for the blades

. The incident velocity, V (Fig. 6 and appendix p. G-54; [24]), is given by the equation

$$V = 1 - 0.0625n(c/r)\lambda(k+3C_d)$$

Substituting this equation into that of C_p, differentiating with respect to the tip speed, λ, and assuming the atmosphere acts as an ideal fluid ($C_d=0$) results in the expressions

$$\lambda = 16r/(3nck)$$

and

$$V = 2/3$$

The latter simply means that during optimal operation, the incident velocity acting on the wind turbine is equal to 2/3 the free stream velocity. For optimum performance, the first of these results has only one variable, c. For the case of the giromill, Substituting values for λ,r,n, and k results in an optimum chord length of 8.13 cm.

Thickness

The thickness of the airfoil can be determined by finding the maximum value of the average coefficient of moments (Fig. 7 and appendix p. G-54 to G-55; [ibid.]) which is given by

$$C_{ma} = (c/r)[(p/2)V^2 - C_d\lambda^2]$$

By inspection, it can be seen that the thickness of the airfoil should only be dependent on finding the lowest value of C_d. Using experimental data [8] to compute a value for these drag coefficients gives the following as examples:

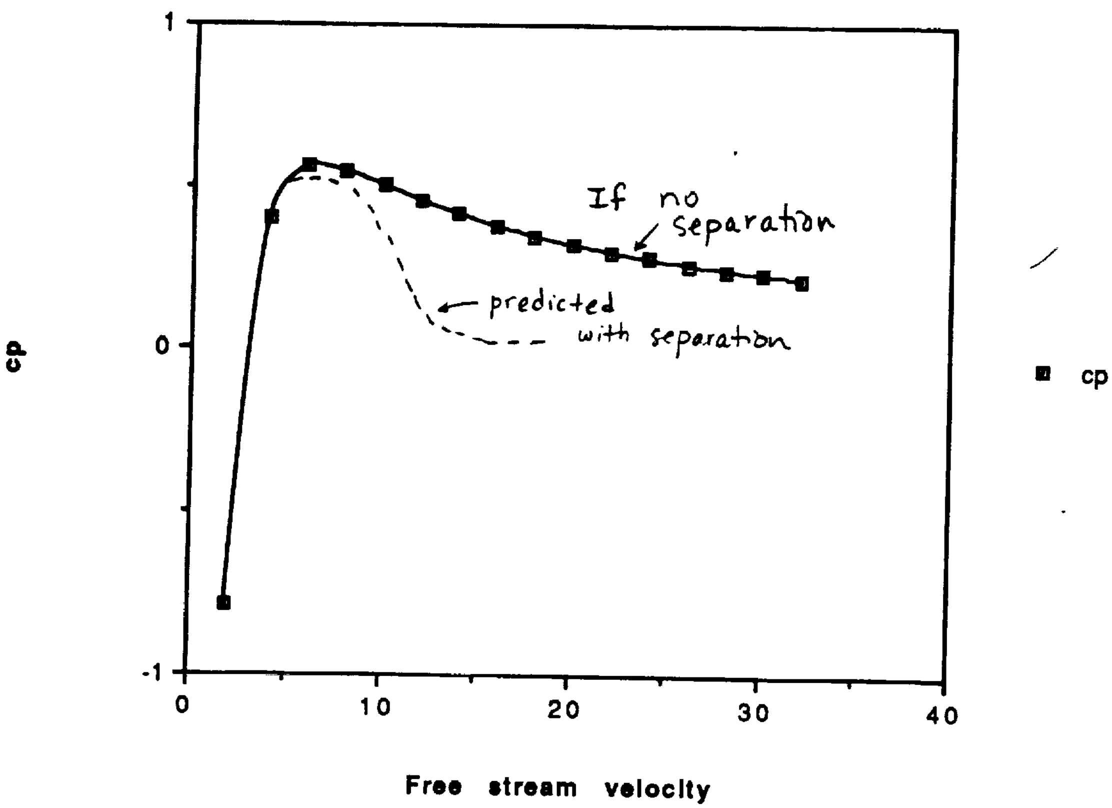
Airfoil type	C_d
NACA0006	0.0098
NACA0009	0.0079
NACA0012	0.0076
NACA1410	0.0081

as before

As can be seen, there is very little difference among these drag values which results in moment coefficients which differ by only thousandths; the NACA 0012 airfoil being only slightly better than the others. Initially it was thought ignoring these differences and using the thinnest section possible (to cut mass requirements) would be best. The range of usable angle of attacks (Fig. 8 and appendix p. G-54 to G-55; [24]) for each airfoil, however, does show that the NACA 0012 section is the best choice, as below especially at higher wind speeds. Peculiar to the darrieus type wind machines is the fact that regardless of the wind speed, chord length, and thickness, there will always be a section of the blade that will be stalled during some portion of the rotation. As r decreases or as the free stream wind velocity increases, the tip speed λ decreases. This results in an increasing angle of attack until the stalling angle of the airfoil is reached. At $\lambda = 1$, the blade speed is equivalent to the wind speed and the lift and drag forces are equal to zero at that instant. At optimum levels, this point occurs at roughly 16 cm from the axis of rotation.

Lift and Drag

Due to the extremely low density of Mars' atmosphere and the small planform area of the blades ($0.09 m^2$) the lift and drag forces acting on the blades are small. At a wind speed of 6 m/s the lift force on the blades is only 0.23 N (appendix p. G-54 to G-55). The drag forces have been neglected due to the fact that they are typically on the order of one percent of the total lift force.



The predicted curve comes from experiments done on many Giromill machines. A similar curve can be found in any book on vertical-axis wind turbines.

Figure 5

Incident velocity vs. Free stream velocity

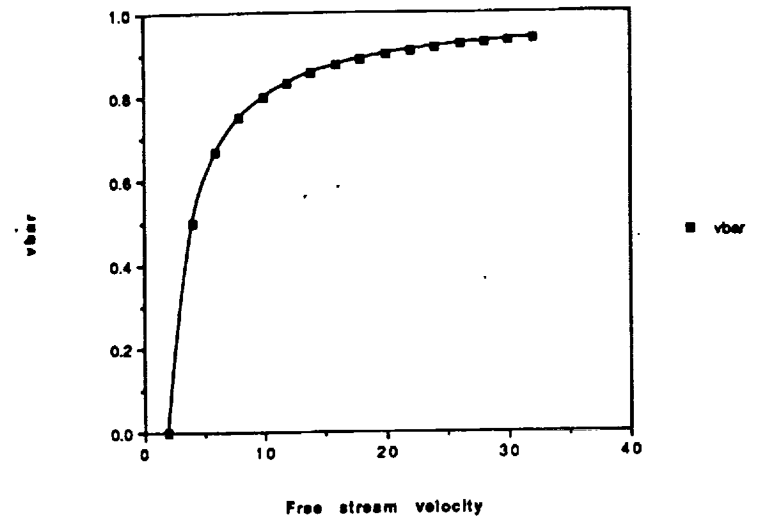


Figure 6

Coefficient of moment, Cm, vs. theta

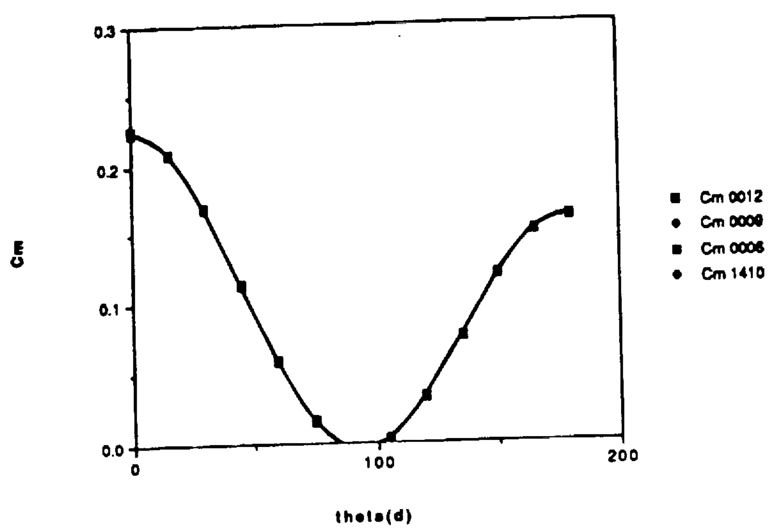


Figure 7

Arctan alpha vs. theta

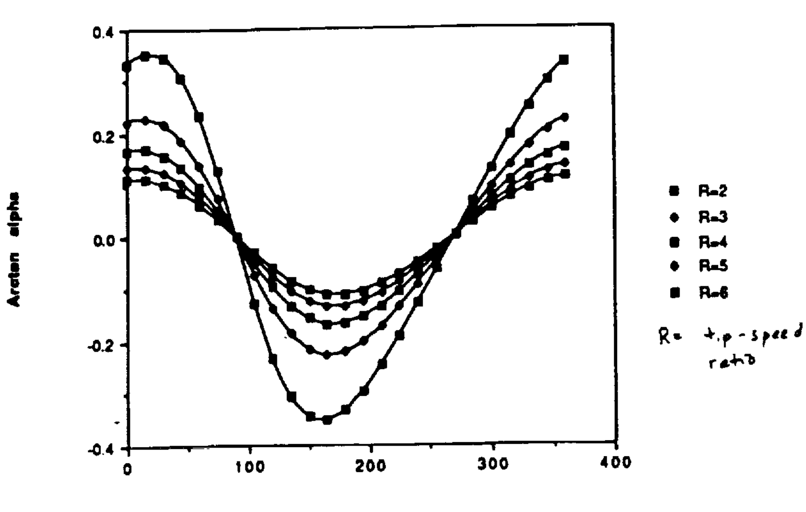


Figure 8

Giromill Material Selection

The material selection for the Giromill was limited by two main factors, one being the low temperature range and the other being the need for a high strength-to-weight ratio. The materials collected within those limits can be found in Tables 1 and 2 on pg. M-1 and M-2 in the appendix.

The material selection for the Giromill blades and struts must also take in consideration bending, and shear. A high modulus of elasticity is also needed to withstand any deflection due to the location of the struts. The material which exhibited the best properties in all categories was boron reinforced aluminum. This is a metal matrix composite characterized by high tensile strength and shear modulus, dimensional stability, joinability,* high ductility, and toughness. For further verification of the Giromill material selection, see page M-6 in the appendix.

All connector pins are made out of titanium alloy 6Al4V. This material was chosen because of its extremely high shear strength. Its low coefficient of expansion is also very important so that it won't interfere with the materials it is pinning. Properties of this titanium alloy can be found in Table 1 on page M-1 in the appendix.

* [weldability] OK

Shaft Design

The loading on the shaft is as follows (pp. S-1 to S-8):

- •Bending due to wind drag on the turbine blades
- •Torsion due to power transmission from blades to generator
- •Axial/buckling due to weight of blades

SHOULD ALSO CONSIDER TORSIONAL BUCKLING OF THIN TUBE

The shaft configuration of Figure 9a was chosen because fatigue is eliminated from the shaft. The non-rotating inner shaft takes the bending moment, while the outer shaft takes the torsional load. Because the rotating shaft does not see any bending, a fatigue situation is avoided.

The bending moment of approximately 18 N-m led us to an aluminum-boron composite (AlB) inner shaft with a diameter of 10mm. The outer rotating shaft, also of AlB, has an inner radius of 19mm and an outer radius of 19.5mm. The bearing races are integral to the shafts to avoid problems with adhesion over the low, wide temperature range. The dimensions of the bearings were based on an SKF ball bearing which was selected for its ability to handle small thrust loads [p. S-13]. The races are coated with a film of molybdenum disulfide for lubrication.

A first approximation of the torsional natural frequency of the Darrieus machine was found to be 2.5 Hz, while that of the giromill was 4.3 Hz (pp. S-9 to S-12). The main torsional forcing function for these machines is caused by "tower shadow," which is illustrated in Figure 10. Each blade passes through a region of reduced wind velocity, caused by the interference of the tower, once per revolution. When the blade passes through this tower shadow, it experiences a dip in lift and therefore in the torque transmitted to the shaft. Because there are two blades, the frequency of this dip is twice the rotational speed of the machine, or 5.6 Hz for the Darrieus

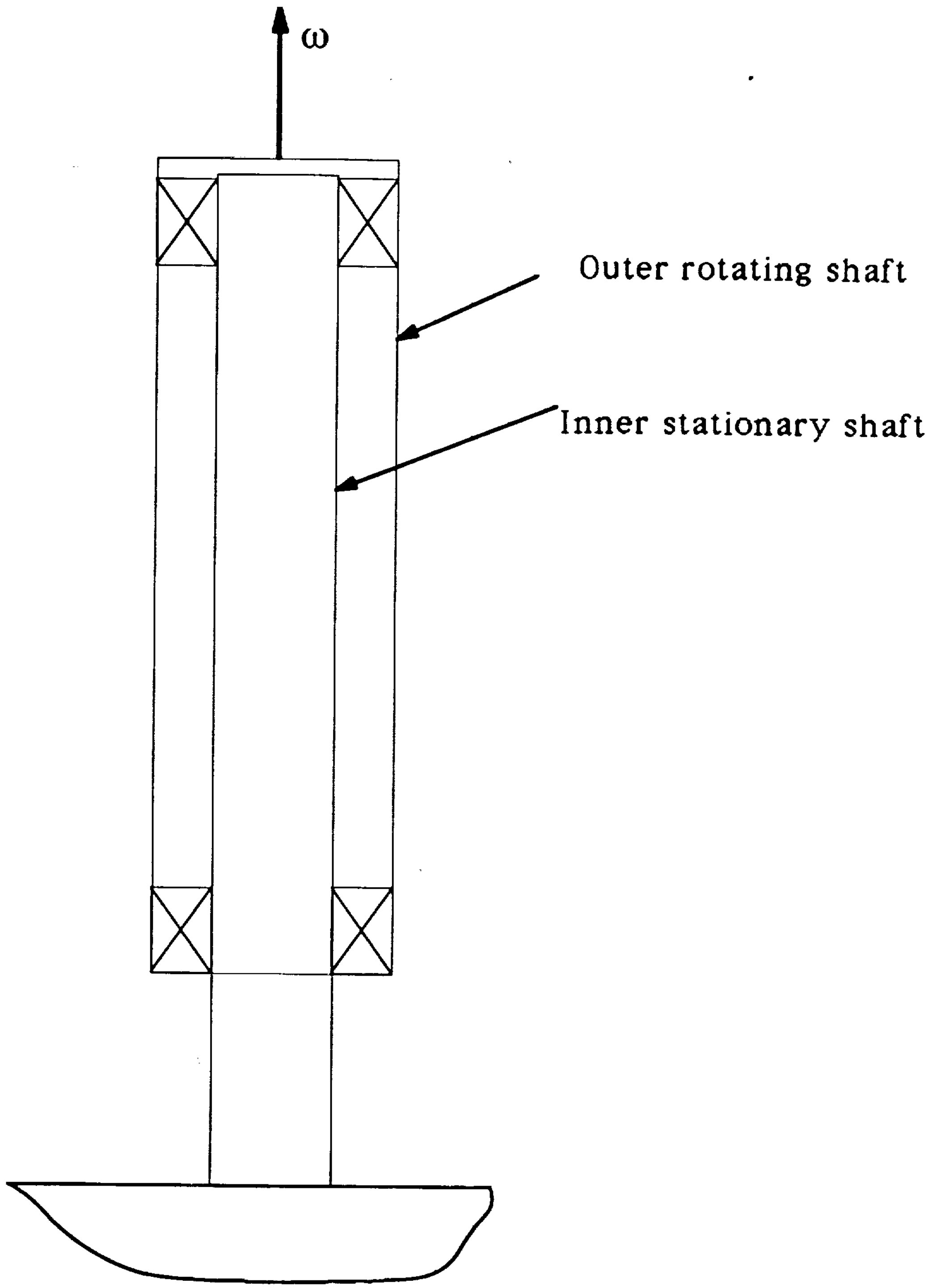


Figure 9 a: Stationary Inner Shaft Configuration

Note that the inner shaft bends while the outer shaft rotates and transmits power to the generator--no fatigue.

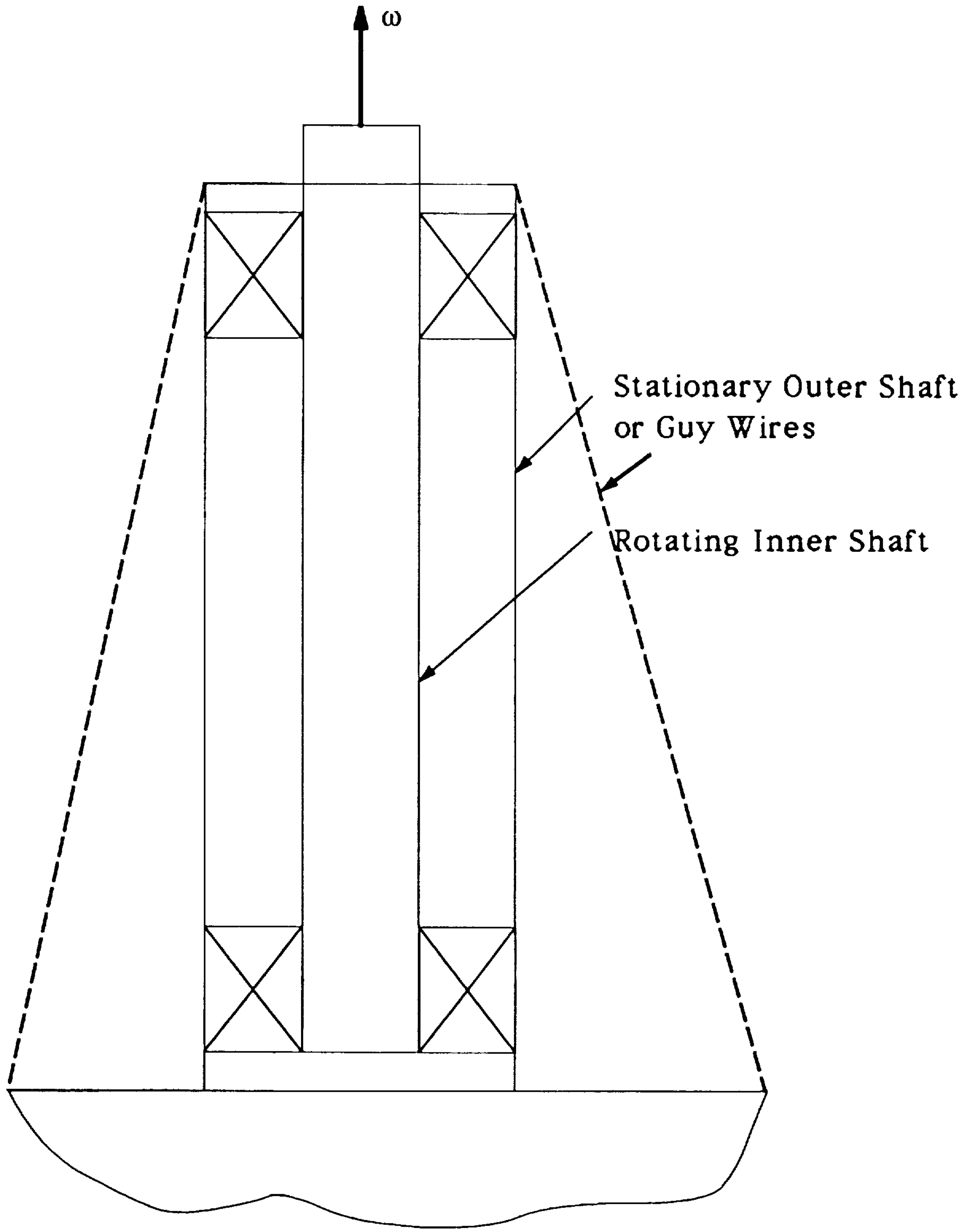


Figure 9 b: Stationary Outer Shaft or Guy Wires

Note difficulty of power transmission from inner shaft to generator outside of outer shaft.

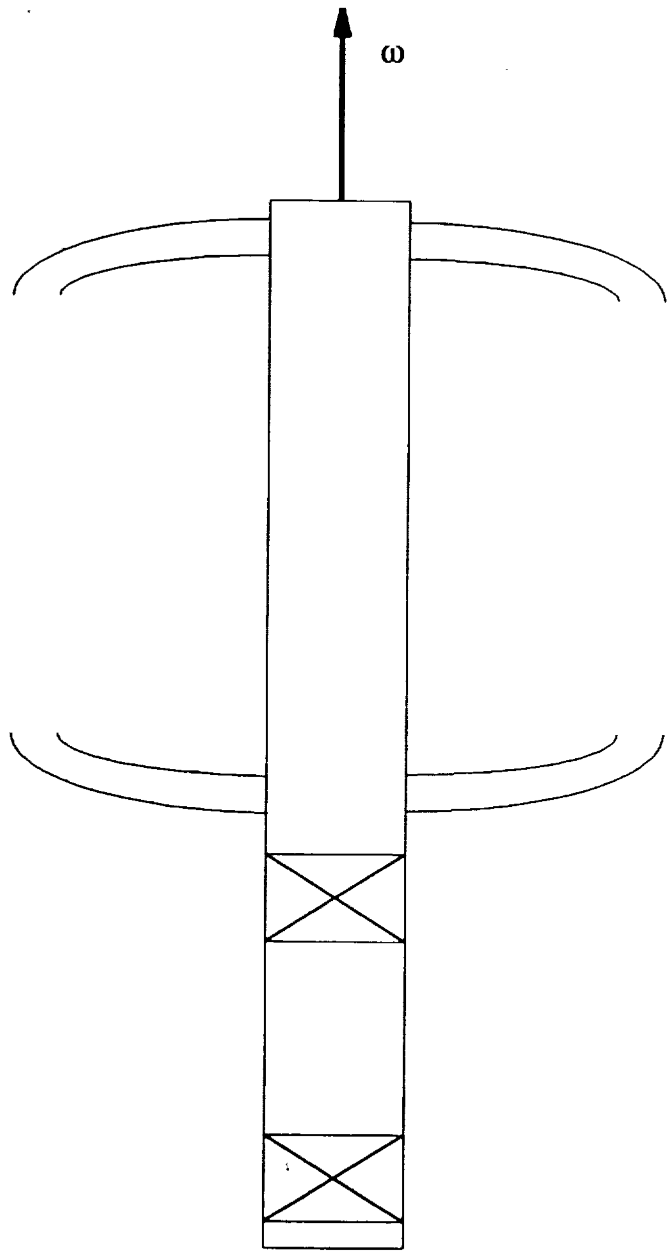


Figure 9 c: Single, Rotating, Overhung Shaft

Note that the rotating shaft is subject to bending and there is therefore fatigue of the shaft, so a larger diameter is required.

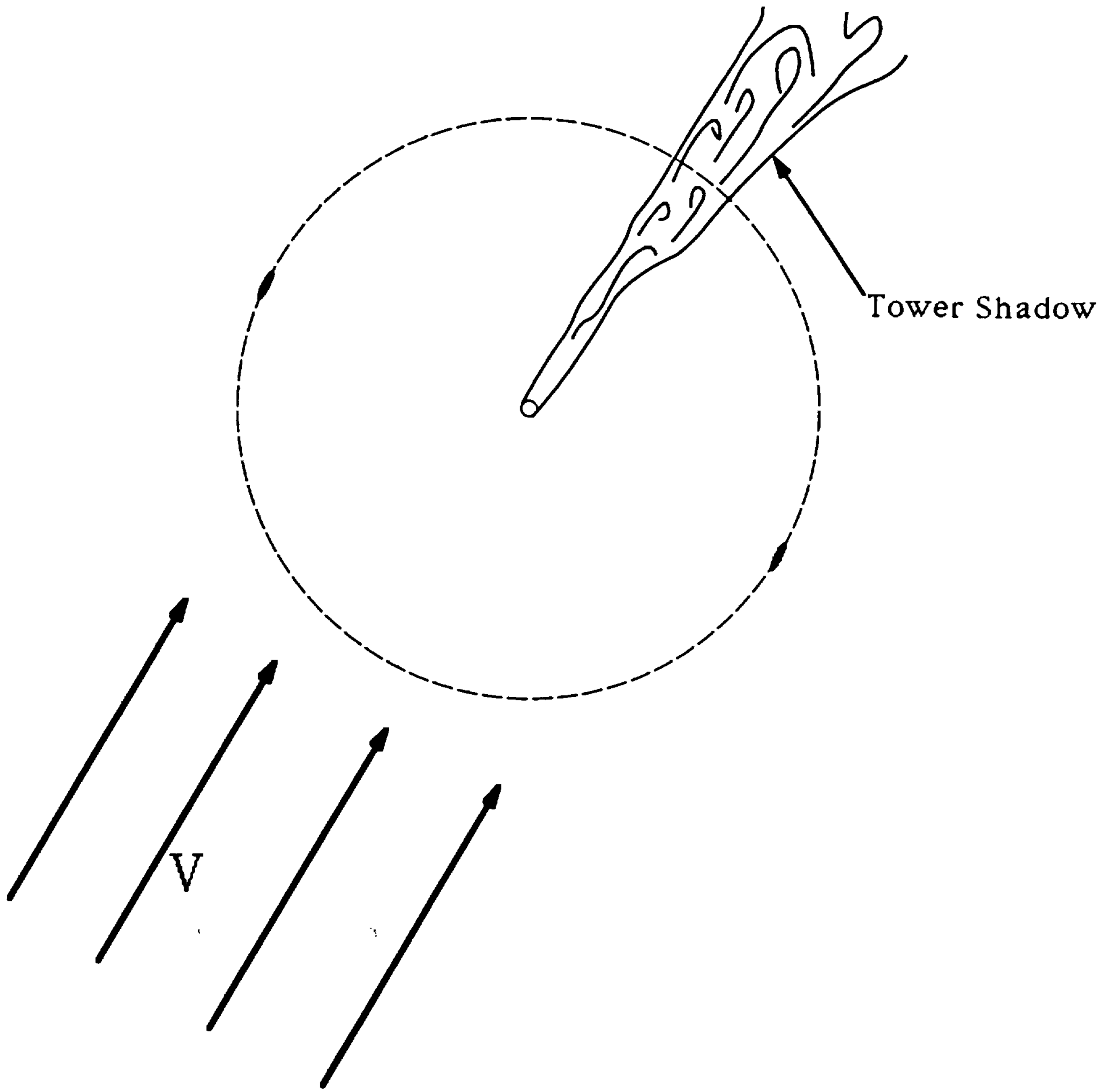


Figure 10: Illustration of Tower Shadow. Our shaft is so small in diameter that this effect is minute at best.

and 5.2 Hz for the giromill. If the torsional natural frequency of the shaft is near the frequency of this forcing function, a damaging condition of resonance could occur; however, the amplitude of the forcing function depends directly on the width of the tower. Our tower is so slender (less than two percent of the total width of the machine) that it is questionable whether a turbulent shadow would even exist behind our shaft. Regardless, the resonant frequency of the Darrieus machine is considerably lower than the frequency of the forcing function; that of the giromill is closer but still less than half of the forcing function frequency. We have therefore concluded that the major vibrational forcing function of tower shadow should not present a problem. OK!

The total mass of the shaft for the Darrieus is 0.396 kg, and for the giromill is 0.300 kg. Development of shaft loading and dimensions, and natural frequency calculations appear in greater detail in Appendix S. Figures 11 and 12 show the configurations of both machines. ✓

Should estimate bending frequencies of gyromill blades

Fig 17 use 5:1 ?

Gears ,Bearings and Lubrication

The bearings are integral to the rotating and stationary shafts. The power is transmitted from the rotating shaft to the generator through a gear system. The large gear is integral to the rotating shaft and the small gear is connected to the generator. A gear ratio of 1:6 is needed to accommodate the generator. A detailed description of the gears and bearings is found on pages M7 and M8 in the appendix.

Lubrication becomes a complex problem when dealing with the low temperature range of Mars. Oils and greases do not operate at these low temperatures, therefore solid lubricants must be used. Of the solid lubricants available, Molybdenum disulfide (MoS_2) is the best suited for our purposes. MoS_2 is effective in a vacuum, dependable at low temperatures, dimensionally stable, and is not damaged by radiation. Other solid lubricants , such as graphite, are ineffective in at least one of these area[11,12]. MoS_2 has a comparably low coefficient of friction and one of the highest wear lives (see Table 3, pg. M-3 in appendix). MoS_2 performs best when used as a film. To increase wear life, MoS_2 should be bonded to the surface with a phenolic resin.

resin bonded MoS_2 film	wear life= 9,860,000 (35ksi cycles)
non-bonded MoS_2 film	wear life= 103,680

This MoS_2-resin combination works well with aluminum, which is the material that we are using for our gears and bearings. The thickness of a typical film for our case would range from .07mm - .001mm.

Generator and Electrical System

The electrical system for this project has three primary functions These functions include:

1.) Rotational start-up

2.) Speed control

3.) Electrical power generation

To achieve each of these objectives simultaneously, a rather complex system will be employed. A schematic of this system is shown in fig. 17. (pg 37) The system includes a miniature DC motor with a step-up gearhead, a speed control card with potentiometer, a battery bank with charging unit, an external anemometer, and two control devices. Each function, along with its individual components is examined in detail in the pages that follow.

Rotational Start-Up

Neither the Giromill nor the Darrieus Vertical Axis Wind Machine (VAWM) are expected to be able to self-start with any degree of certainty. To overcome this problem, the motor will be used to start the rotation of both machines.

The estimated wind speed at which both machines would be able to produce energy is 4 m/s (see p. T-4). The anemometer would be used as a means to detect when the free stream wind speed reaches this cut-in value. Upon reaching the cut-in wind speed, the cut-in control would be used to forward-drive the motor and begin rotating the blades of the VAWM.

To forward-drive the motor, the battery bank would have to be used as a power supply for the motor. This type of operation would present the potential problem of completely exhausting the batteries. To prevent this, another control device would be used that would limit the level to which the batteries could be depleted.

Does this mean that we protect the batteries but can never start the wind machine?

Speed Control

Speed control is a very important part of our VAWM design. For the case of the Darrieus, the moment-free shape of the troposkien is only valid for one rotational speed. The maximum coefficient of power is also attained at a constant tip-speed-ratio, which, for a constant wind speed, requires a constant rotational speed. The final reason for constant rotational speed is structural integrity. If the machines were allowed to spin uncontrolled, the stresses developed would become self-destructing and eventually cause failure.

To maintain a constant rotational speed, a commercially available speed controller card, the Instech 1100, will be used (see reference [25]). The rotational speed of the motor (back-driven to act as a generator) is linearly related to its output voltage by the generator's velocity constant. By measuring the output voltage and knowing the velocity constant of the generator, the card is able to use an external potentiometer to regulate the rotational speed of the generator to a specified value.

Examining the figure on p. T-4, we see that output power increases with increasing wind speed up to a point after which the output power decreases. In general, we find that, "If the fixed speed load is able to accept the maximum possible mechanical power, no additional braking or loading is necessary as the wind speed increases above its rated value."[7]

seems hard to believe but that's what T4 says!

By scaling the figure on p. T-4, we estimate a maximum generator output capacity of 3.1 Watts will be required for adequate braking. For confirmation of these findings, however, wind tunnel tests should be performed. yes - of course!

So we really have a 3.1 watt generator.

Electrical Power Generation

For braking, we found that a 3.1 Watt generator is required. NASA also requires that the system operates at 12 Volts. This information allows us to examine available miniature motors. A commercial miniature motor catalogue indicates that a 12 Volt, 3.7 Watt motor would typically have an efficiency of 85%, a velocity constant of 714 r.p.m./volt, a no-load speed of

9,000 r.p.m., and a mass of 0.058 kg [26].

For the operating speed of 300 r.p.m. to generate 12 Volts, a gear ratio of 30:1 will have to be used. This will be achieved by a 6:1 increase from the blade rotational shaft to the shaft of a 5:1 gearhead. Again referring to a commercial motor catalogue, we find an efficiency of 80% and a mass of 0.065 kg for such a gearhead [26]. ok!

Calculating the driving torque imposed on the central shaft by the blades, we find a value of 0.068 Nm (pp. P-1, P-2). At the normal operating speed of 300 r.p.m., we find an output power of 1.48 Watts(p. P-2). This value does not take the efficiencies of the bearings into account, and will be slightly higher than the actual value.

Other Considerations

1.) Conventional lubrication will probably not be effective at such extreme temperatures. Investigation of alternatives should be made.

2.) Calculations show that at 15% inefficiency, a 3.7 Watt motor would reach steady state 17.21 K warmer than the surroundings. (p. P-3) This does not present a problem for overheating.

3.) The total mass of the electrical system (excluding batteries and charging unit) is 0.160 kg.

What assumptions did you make to justify this?

Deployment

The Darrieus and giromill wind machines will be stored in the lander in a very space efficient manner.

DARRIEUS

The Darrieus machine has blades that fold into the center shaft when stored. See figure 13a. for stored position. Upon landing on the surface of Mars, the lander lid will open and the machine will be elevated out of the lander compartment. The stationary shaft of the machine will be secured to the elevated platform which rises and acts as the base of the wind machine. This platform will have a blind hole that the windmill's stationary shaft will be inserted into. Once the elevator has deployed the machine from the lander compartment, the blades will fall into their appropriate operational positions due to gravitational effects. See figure 11 for the elevated working position. The blades simply roll around a countersunk pin hole into their locked and working positions. This countersunk hole is higher at one side with an increasing slope leading to a slot that the connecting pin will lay in for operation. See figure 13b. for detailed drawing of blade connectors. The blades of the Darrieus are connected to the collar, which is integral with the rotating sleeve, by an airfoil shaped piece that slides into the hollow blades and is secured with a rivet. See figure 13c. for detailed and dimensioned drawings. See appendix page D-1 for the calculations determining appropriate pin sizes.

GIROMILL

The giromill has hinged blades that fold up into the center shaft when stored. See figure 13a. for stored position. Like the Darrieus, upon landing, the lid will open and the giromill will be elevated out of its lander compartment. The giromill struts are hinged to the rotating sleeve which allows them to fall into their operational positions once out of the lander compartment. See figure 12 for elevated working position. A diagonal wire, made of titanium, connecting the top strut to the bottom strut keeps the struts in their desired positions and prevents them from falling back down into the shaft. This wire has a ball and socket connection at the shaft. See figure 13d. for detailed drawings and dimensioning. See appendix page D-2 for the calculation determining appropriate pin sizes.

The generator for both of these machines will be connected to the stationary shaft by a bracket.

Conclusions

Our main goal was to design a functioning vertical-axis wind turbine that would be lighter than the existing design of a tornado vortex wind turbine. At an estimated shipping cost of $20,000/kg, it is obvious why low mass is crucial to the design. The total masses of the three machines are listed below.

Tornado Vortex	≈30.0	kg
Darrieus	1.624	kg
Giromill	0.900	kg

Although the vertical-axis wind turbines we have designed would be quite expensive to produce (especially the Darrieus), the reduction in shipping cost is so large that the production cost is almost negligible.

We also see that the Giromill appears to be a better choice than a Darrieus. The small size of our machines leads to small stresses developed in the blades. Because of these small stresses, the structural advantage of the Darrieus is not a major benefit. If the machine were to be scaled to a larger size, the structural advantages of the Darrieus would have a much larger effect. As far as a 1 Watt turbine goes, the Giromill is a better choice than the Darrieus.

Although the varying cross-sectional Darrieus is not the best choice for a 1 Watt Mars turbine, it deserves serious consideration for future wind turbines on Earth. By varying the cross-section, both the required mass and the maximum stress are significantly reduced. Although initial production costs would be high, a mold could be built so that several of the turbines could be built at moderate cost.

Future Work

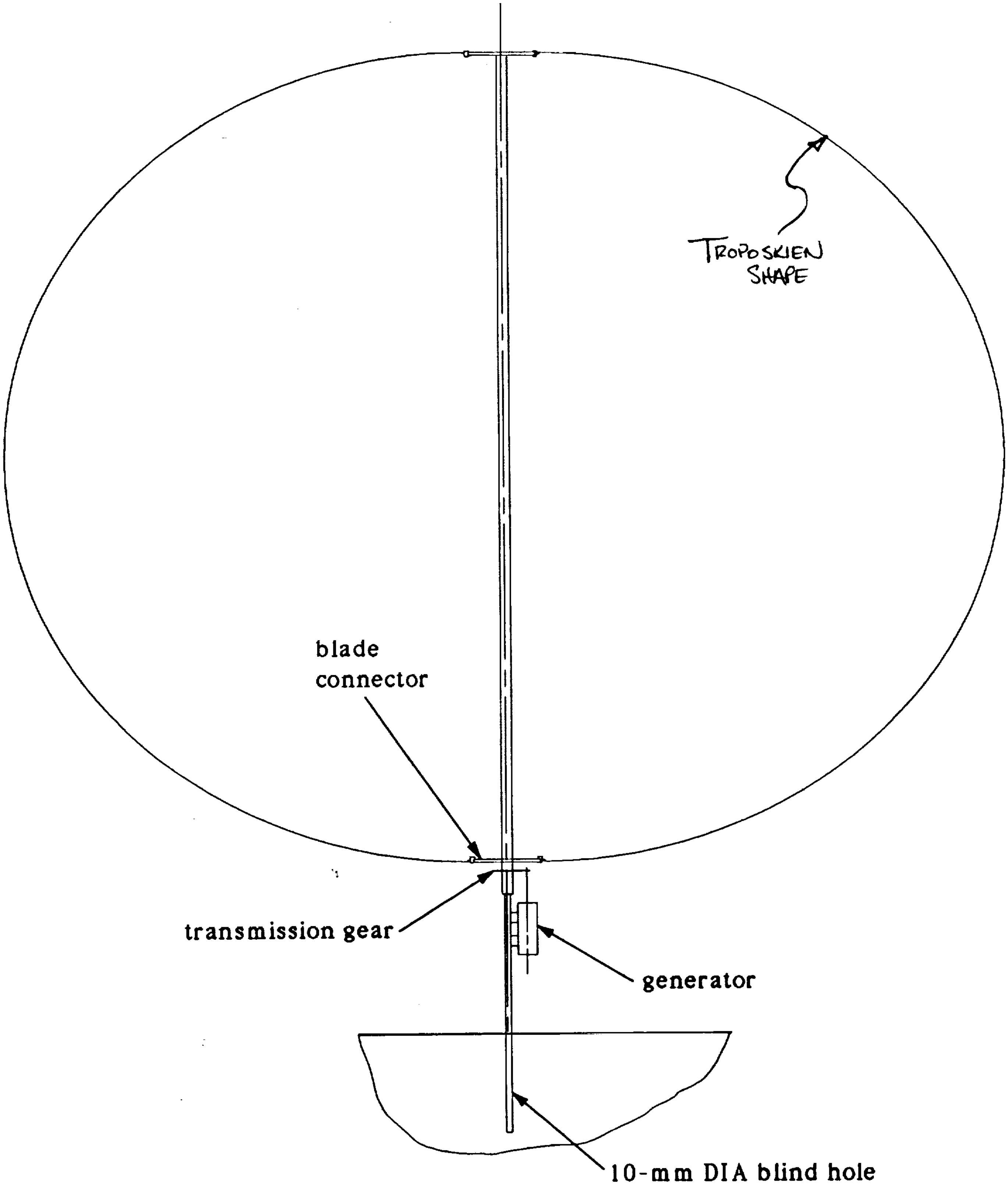
In the development of this design, several ideas came up which we were unable to incorporate because of temporal constraints. These included:

- Use of magnetic bearings instead of ball bearings. This would cut the mechanical losses considerably, but magnetic bearings do take a considerable amount of power. Perhaps if a material were created with a critical temperature of superconductivity above the range in which we're working...
- Use of pitch control, or "smart blades," on the giromill. By varying the pitch of the giromill blades as they progress around the shaft, either by an active control system (computer feedback loop and servo motors) or by ingeniously locating the pin between the blade and strut at a point other than the quarter-chord point, where a varying coefficient of moment could be used against a spring to change the pitch of the blade.

SWASH PLATE CONCEPT

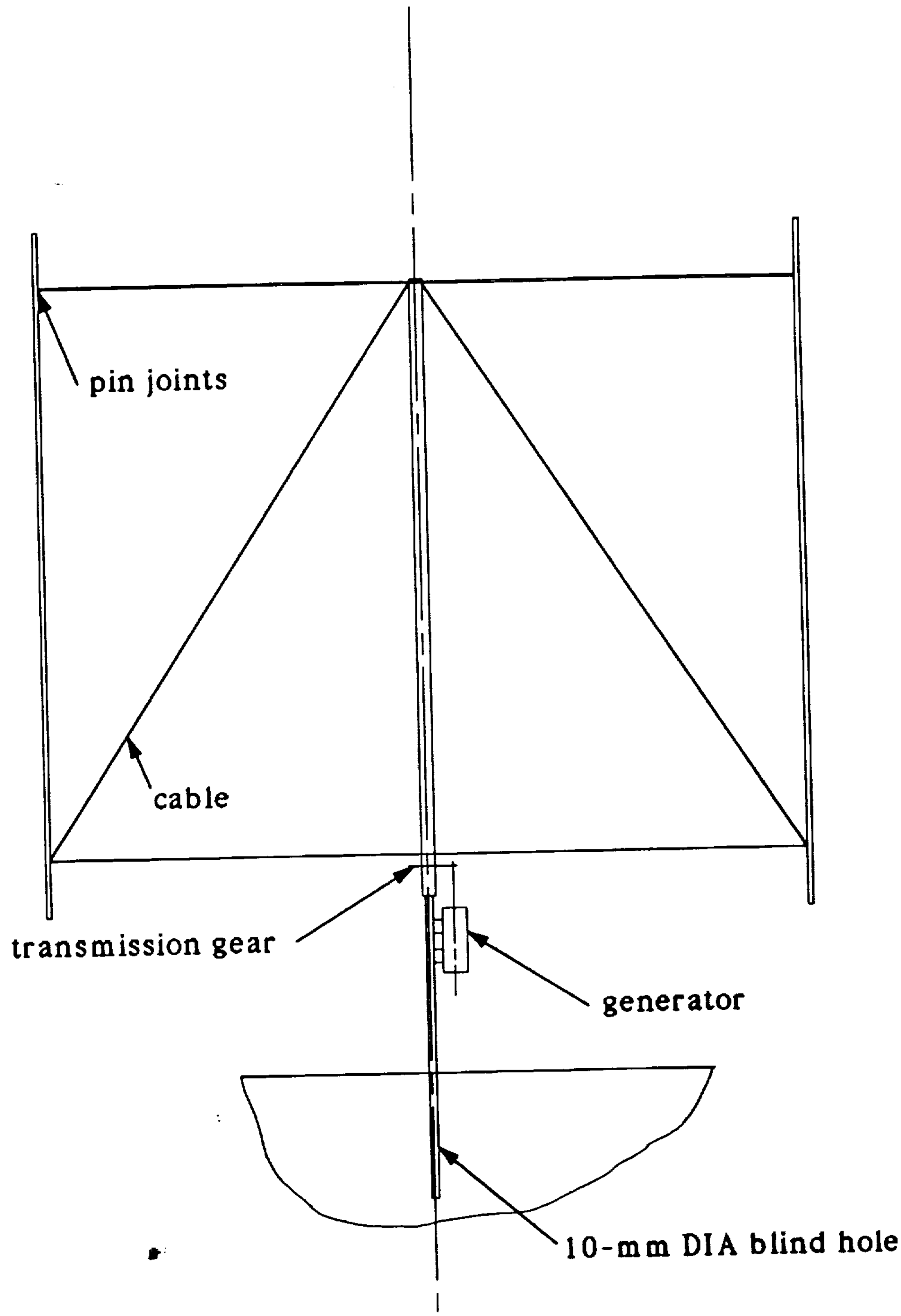
- Develop a shock absorber system to mount the wind turbine on while stored in the lander. The purpose of this would be to cut the acceleration of landing and further reduce the size of structural components.
- Scale up the wind turbine so that it could provide enough power to operate the research package in the transmitting mode (about eight watts; p. A-2).
- Wind tunnel test models of the machines to insure that overspeed will not occur. While the best information we could find has indicated that this will not occur, airfoil data does not cover angles of attack over approximately 20 degrees; in high winds, the angle of attack could range from 0 to 180 degrees. If overspeed did occur, we recommend the addition of a braking system, either aerodynamic or mechanical. Either could be deployed by centrifugal force acting on a governor.

Consider some passive damping.



Darrieus Machine Configuration

Figure 11



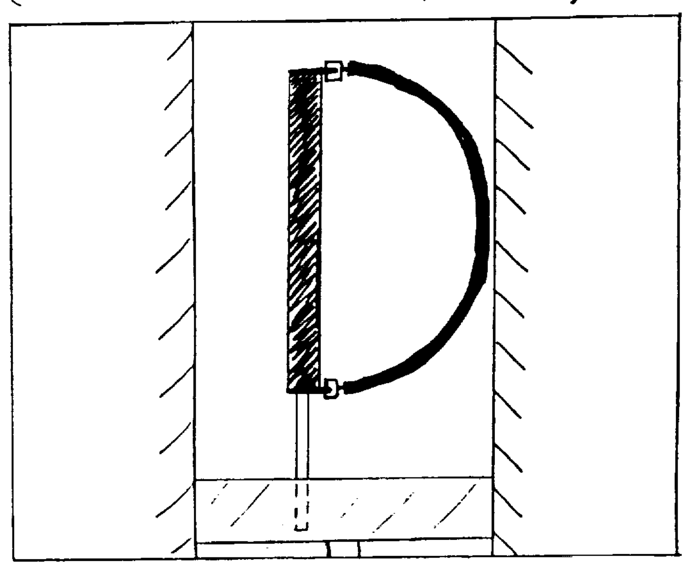
Giromill Machine Configuration

Figure 12

STORED POSITIONS

DARRIEUS

(cross-sectional view of lander)



GIROMILL

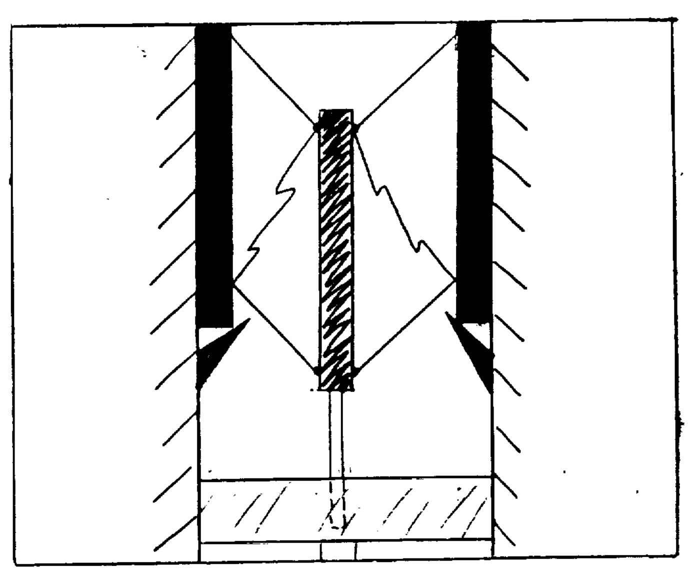


FIGURE 13a.

DARRIEUS BLADE CONNECTORS

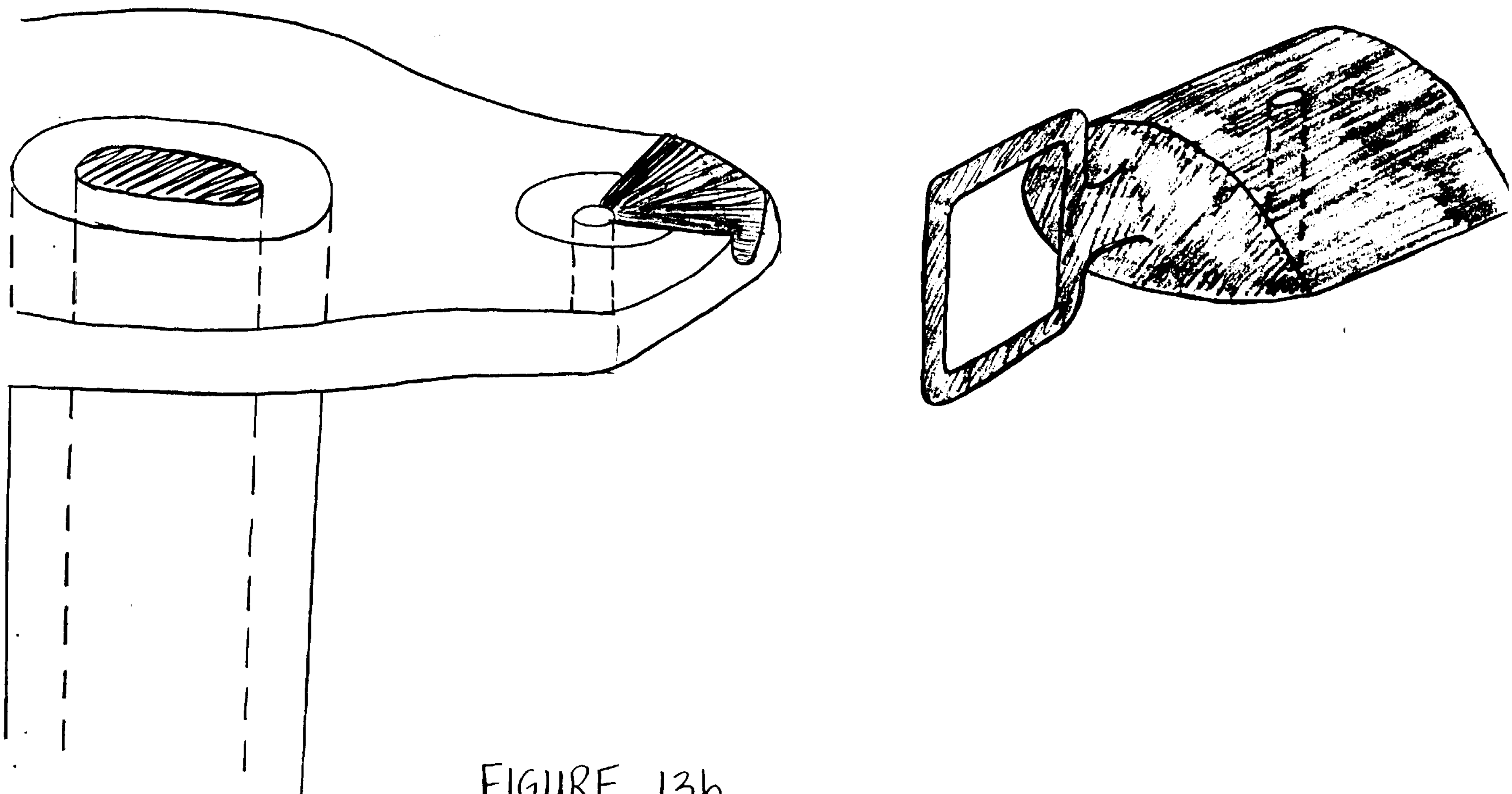


FIGURE 13b.

DARRIEUS BLADE CONNECTORS

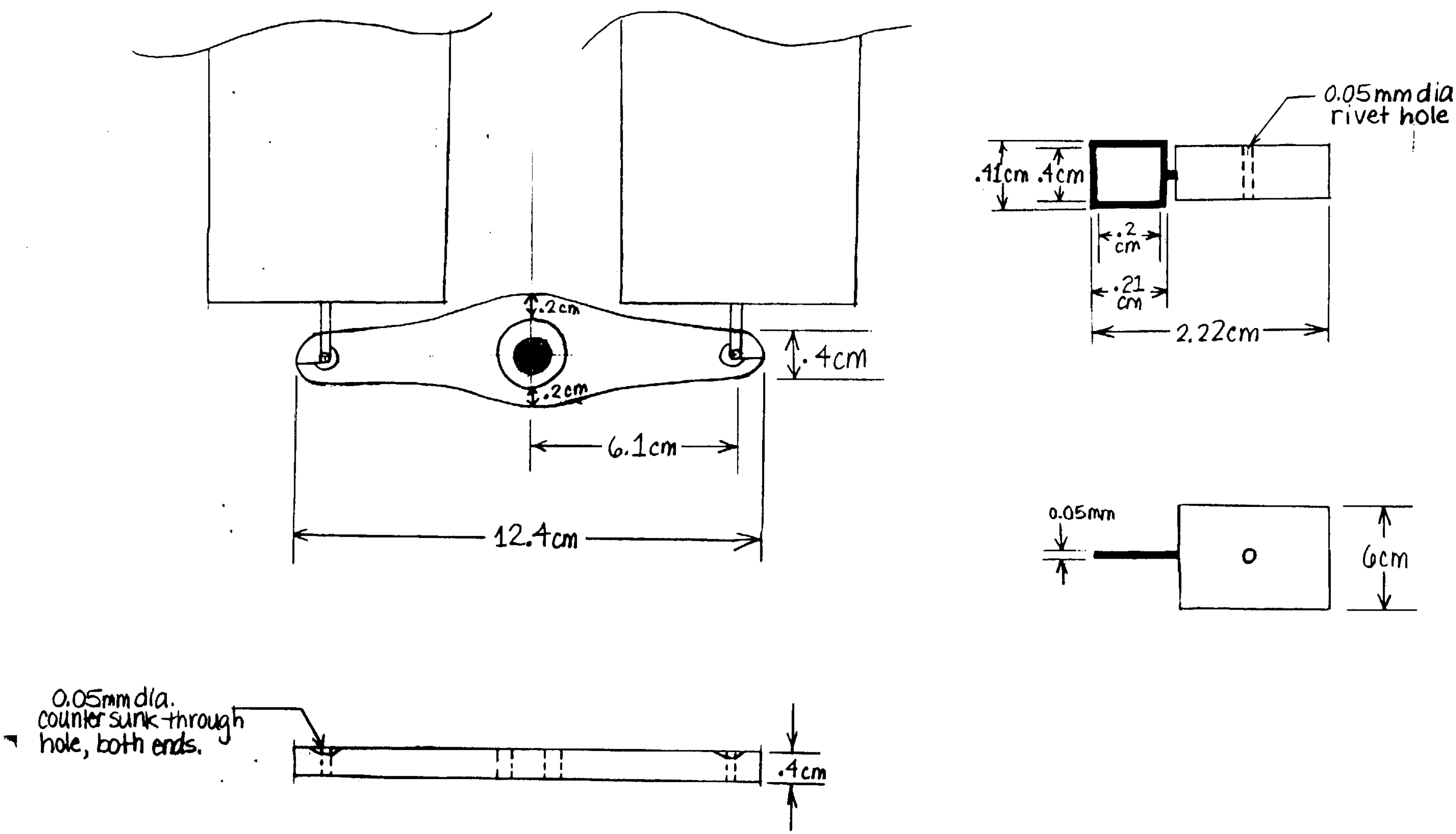


FIGURE 13c.

(Drawings not to scale.)

GIROMILL BLADE CONNECTORS

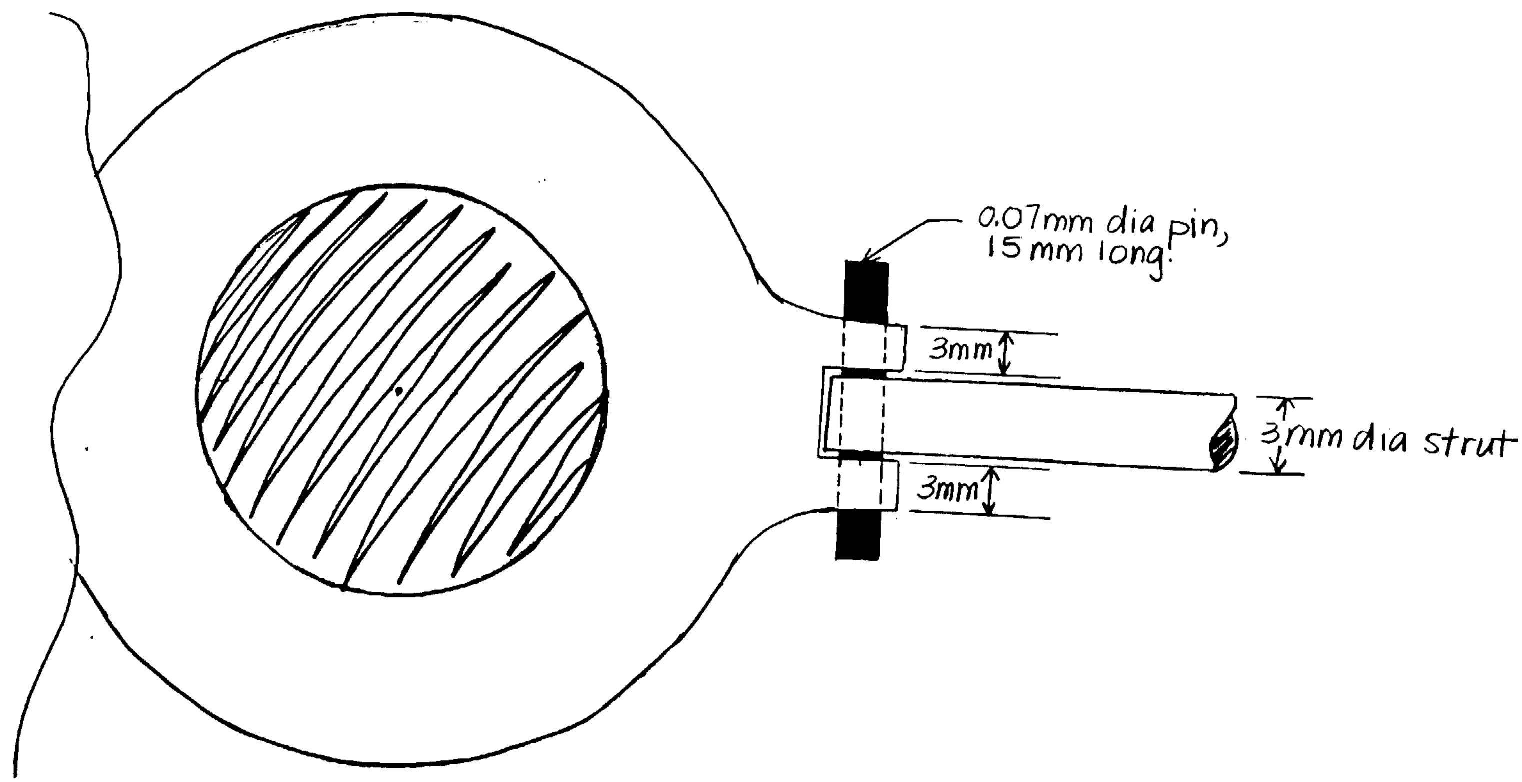


FIGURE 13d.

(Drawing not to scale)

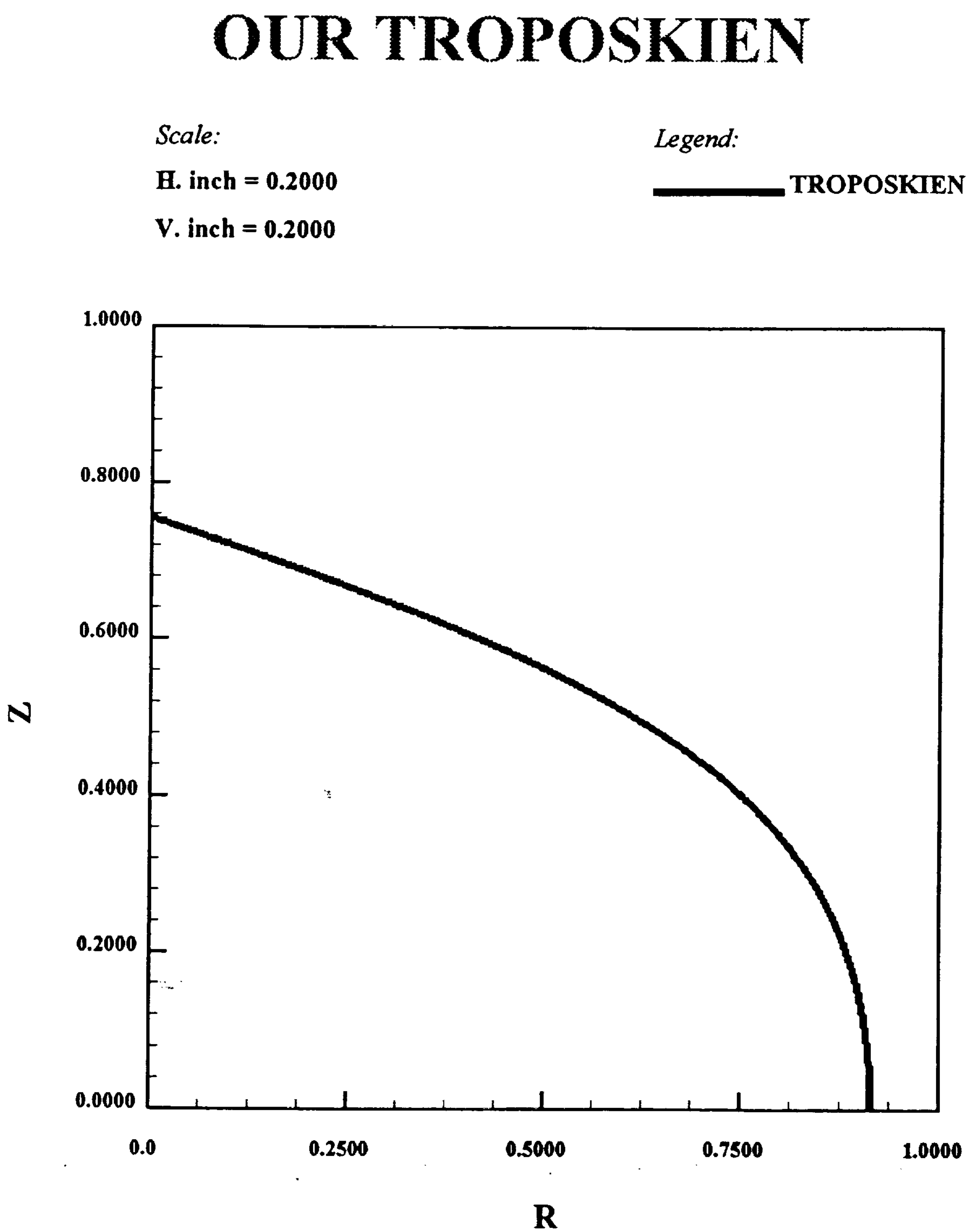

OUR TROPOSKIEN
Scale:
H. inch = 0.2000
V. inch = 0.2000
Legend:
TROPOSKIEN
1.0000
0.8000
0.6000
0.4000
0.2000
0.0000
Z
0.0
0.2500
0.5000
0.7500
1.0000
R

Figure 14

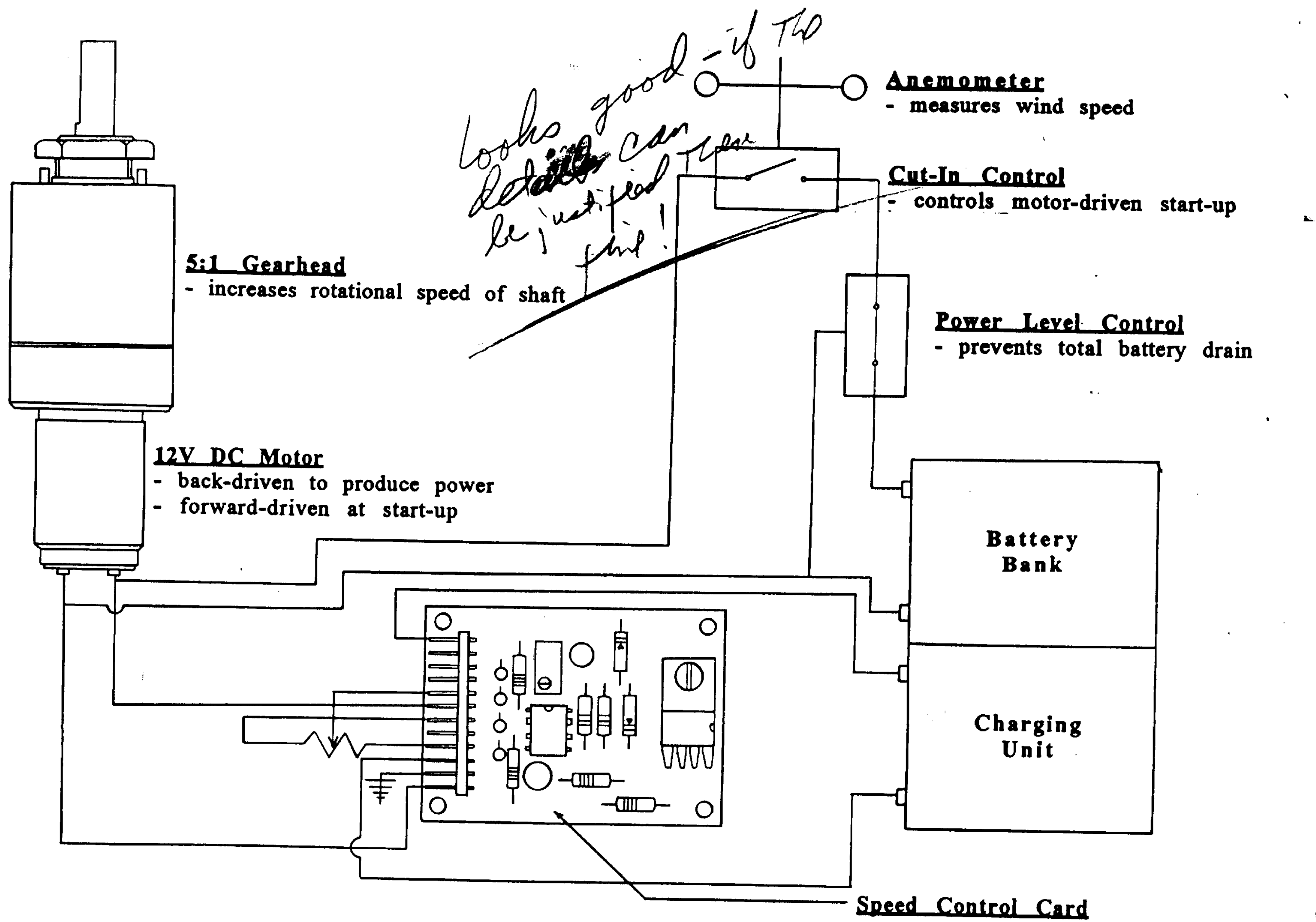

Anemometer
- measures wind speed
Cut-In Control
- controls motor-driven start-up
5:1 Gearhead
- increases rotational speed of shaft
Power Level Control
- prevents total battery drain
12V DC Motor
- back-driven to produce power
- forward-driven at start-up
Battery Bank
Charging Unit
Speed Control Card
- measures motor output signal
- regulates speed with potentiometer
Looks good - if the ~~details~~ can be justified then fine!

Fig. 17

References

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Good Proofreading!

Excellent!

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STRAWMAN SCIENCE PAYLOAD

INSTRUMENT	MASS (kg)	POWER (W)	DATA	DATA CAPABILITY: WITH ORBITER	DATA CAPABILITY: DIRECT LINK TO EARTH: FREQUENCY	DATA RATE, BPS (CONTINUOUS)	TOTAL NO. OF BITS	HERITAGE
SURFACE METEOR.	0.60	0.15	88 B/HR.	CONTINUOUS	CONTINUOUS	0.0244		NEW/DEVEL-1,2,3
SEISMOMETER	1.20	0.20	1.25 MB/EVENT	3+ EVENTS/DAY	1 EVENT/MONTH	0.48		NEW/DEVEL-1,2,3
ATMOSPH. STRUCT.	0.80	0.20	20 KBITS	ONCE	ONCE		20000	PVO/GLL-4
SURFACE CHEM.	0.80	0.20	12 KBITS	FOUR TIMES	FOUR TIMES		48000	VIKING-1,3
ACCELEROMETER	0.10	0.06	15 KB/SEC.	2 SEC. (ONCE)	2 SEC. (ONCE)		30000	PVO/GLL-1,3
IMAGER:	0.50	1.80						
DESCENT			2 MB/EVENT	10 EVENTS (10:1 DATA COMP.)	10 EVENTS (10:1 D.C.)		2000000	NEW/DEVEL-1,2,3
SURFACE			2 MB/EVENT	12 EVENTS/YEAR (10:1 DATA COMP.)	5 EVENTS/YEAR (10:1 D.C.)	0.0322		
(NEUTRON SPECTR)	(0.89)'	(0.05)'	90 KB/EVENT	24 EVENTS	4 EVENTS		360000	NEW/DEVEL-5
INTEGRATION HWR	2.00							OFF/SHELF
THERMAL ANALYSIS INSTRUMENT	TBS	TBS	TBS	TBS	TBS	TBS	TBS	
TOTAL (Not Incl Therm. Anal. Exp.)	6.00 (6.89)'	2.61 (2.66)'		> 4Mbits per Lander per Day		0.5366	2458000	

1-Mars Surface Penetrator-System Description, NASA TM-73243, June 1977. Mass reflects 100% margin from report.
2-METEGG Surface Stations for Soviet Mars-94 Mission Pase A2 Study, Finnish Meteorological Institute, August 1989.
3-Mission Concept and Development for Mars Global Network Mission, JPL, Dec. 15, 1989.
4-Data Provided by Al Seiff
5-Data Provided by Steve Squyres

Mars Probe Power Statement

		Power During Cruise (W)	Average Power for a Martian Day (W/day): Separation & Descent	Normal Transmitting Mode 2 W Trans.	Low Power Non-Transmitting Mode
Payload	Imager	1.80	1.80	1.80	
	ASI	0.20	0.20		
	Surface Meteorology	0.15	0.15	0.15	0.15
	Surface Chemistry	0.20	0.20		
	Seismic Detector	0.20	0.20	0.20	0.20
	Accelerometer	0.06	0.06		
C & DH	Transponder				
	Transmitter	8.00	0.00	2.60	0.00
	Receiver	1.00	0.00	0.50	0.00
	Data Handling	3.00	3.00	3.00	0.75
	Controls	3.00			
	Total Avg Power (W)	17.61	5.61	8.25	1.10
	Energy (Whr/day)			202.95	27.06
	*Usable Power (W)	50.00		14.50	3.80
	*Usable Energy (Whr/day)			356.90	94.20

*Power is generated by spacecraft panel during cruise and by the lander on the surface.
Lander power is based on Viking Lander #1 data during the summer for the transmitting mode and during the winter for non-transmitting mode.

A-2

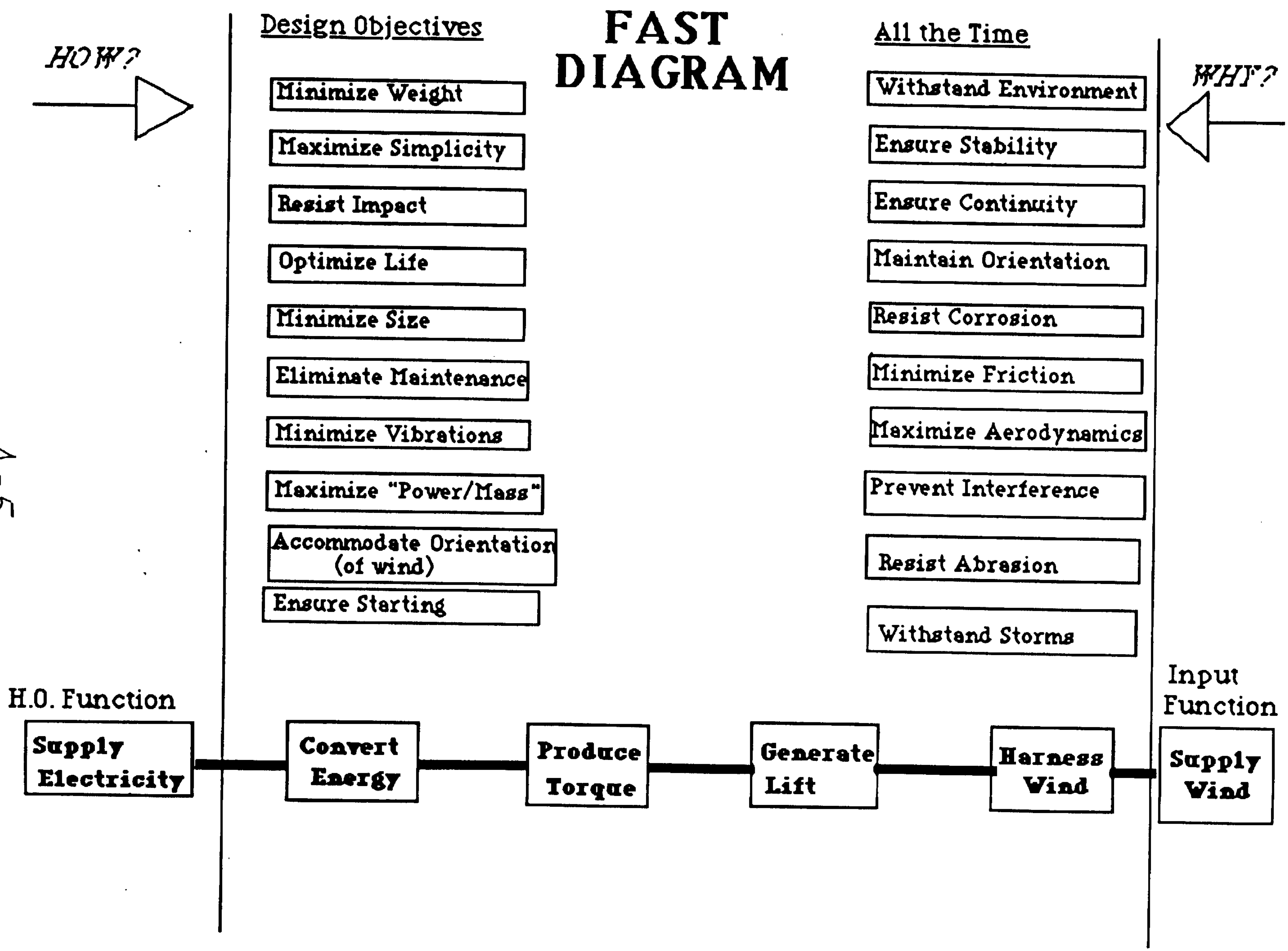
RPH - 5/1

NEEDS AND FUNCTIONS

Need	Function
Must operate in storms	Withstand Storms
Must operate in Martian environment	Resist Environment
Want a lightweight design	Minimize Weight
Want a simple design	Maximize Simplicity
Must withstand landing	Withstand Impact
Want sufficient life for mission	Optimize Life
Must produce 1 Watt of power	Provide Power
Want small design	Minimize Size
Must not interfere with experiments	Prevent Interference
Want to be adaptable to changing wind velocities	Promote Adaptability
Must convert energy from wind	Convert Energy
Want to provide power continuously	Ensure Continuity
Must provide means for starting	Ensure Starting
Want dynamic stability	Ensure Stability
Want machine to stay upright	Maintain Orientation
Want to achieve maximum power-to-mass ratio	Maximize "Power/Mass"
Must eliminate need for maintenance	Eliminate Maintenance
Want to resist corrosion	Resist Corrosion
Want to use wind from any direction	Accommodate Orientation
Want to minimize friction losses	Minimize Friction
Want maximum aerodynamic performance	Maximize Aerodynamics

Want to minimize mechanical vibrations	Minimize Vibrations
Want to minimize surface roughness (to minimize drag)	Resist Abrasion

FAST DIAGRAM



L_s (deg)	0.1%	1.%	10.%	50.%	90.%	99.%	99.9%	# Obs.
270-299	-13.3	-9.7	-4.5	0.7	7.4	13.3	15.6	2217
299-329	-12.0	-8.7	-4.0	0.8	9.3	16.3	19.5	2400
329-360	-17.4	-10.7	-6.2	-0.3	10.0	16.9	20.0	1892
0-29	-15.4	-8.5	-4.8	-1.1	3.7	10.0	14.8	1461
29-59	-6.7	-5.0	-2.6	-0.7	2.0	4.0	4.9	1615
59-89	-6.1	-4.9	-2.4	-0.7	1.5	2.5	3.1	289
89-119	-6.2	-4.9	-3.0	-1.0	1.0	2.5	3.3	1240
119-149	-6.3	-5.2	-3.1	-0.9	2.1	4.1	5.1	2843
149-179	-7.2	-5.6	-3.0	-1.0	3.0	5.2	6.7	2484
179-209	-13.9	-9.7	-5.5	-1.2	3.5	8.4	11.0	2187
209-239	-12.7	-9.0	-4.8	-0.1	7.7	14.6	18.4	2161
239-270	-10.6	-6.0	-1.8	1.6	7.8	14.7	16.5	2360

TABLE 2-3. - Zonal wind cumulative probabilities versus season (m/sec, + from west)

L_s (deg)	0.1%	1.%	10.%	50.%	90.%	99.%	99.9%	# Obs.
270-299	-15.1	-13.6	-7.7	-0.9	4.0	9.8	11.7	2217
299-329	-13.9	-11.9	-6.7	-0.9	5.0	10.4	13.7	2400
329-360	-17.6	-14.7	-9.9	-1.4	6.6	12.0	15.0	1892
0-29	-12.7	-11.2	-5.9	-0.3	3.3	6.9	9.0	1461
29-59	-4.6	-3.4	-1.9	-0.3	3.1	5.0	5.9	1615
59-89	-2.7	-2.5	-1.6	-0.3	2.9	4.0	4.6	289
89-119	-3.6	-3.0	-2.0	-0.4	3.1	4.5	5.0	1240
119-149	-5.8	-4.2	-2.4	-0.7	2.9	5.1	6.3	2843
149-179	-7.9	-5.5	-3.0	-0.5	2.3	5.4	6.7	2484
179-209	-16.2	-12.2	-7.6	-0.9	3.5	7.3	9.4	2187
209-239	-16.2	-12.5	-7.5	-0.5	5.5	9.7	11.7	2161
239-270	-10.8	-7.9	-4.2	0.5	5.1	9.8	14.1	2360

TABLE 2-4.- Meridional wind cumulative probabilities versus season (m/sec, + from South)

L is a measure of seasonality
- it is the angular position of Mars in its orbit around sun

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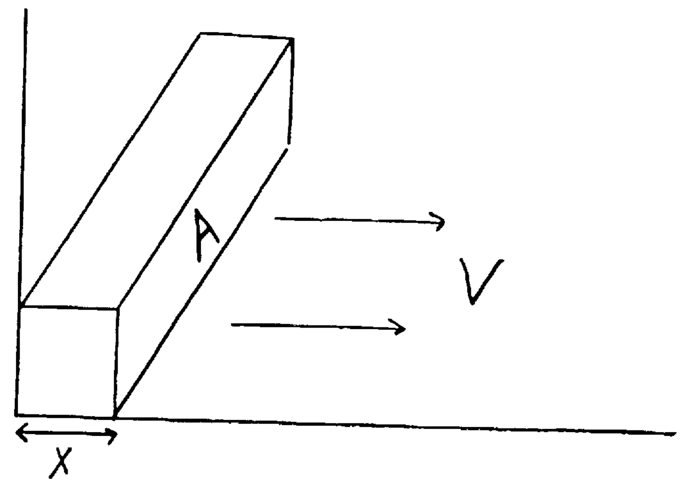
Wind Energy Theory and Probability: Size Calculations

A moving mass of air, as shown in the figure below, has kinetic energy of motion. For the air moving with the wind velocity, V, the kinetic energy is

$$T=0.5mV^2$$

where T=kinetic energy
m=mass of air
V=velocity of air



For our parcel of air, the mass is equal to the density times the volume, or:

$$m=\rho Ax$$

where ρ=mass density of the air
A=cross-sectional area of the parcel
x=length of the air parcel

Substituting this mass into our energy equation yields,

$$T=0.5(\rho Ax)V^2$$

The power in the air, P_{air}, is then given by the derivative of the kinetic energy with respect to time. Assuming incompressible flow, we have:

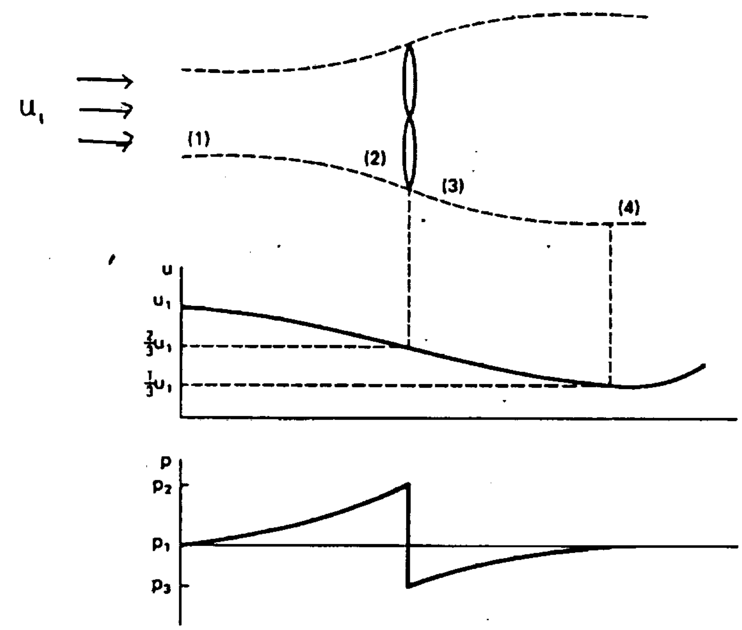
$$P_{air} = dT/dt = 0.5\rho AV^2(dx/dt) + 0.5\rho Ax(2V)(dV/dt)$$

For steady air flow, $(dV/dt)=0$, and $(dx/dt)=V$.

The power equation now reduces to

$$P_{air}=0.5\rho A V^3$$

When wind flows past a windmill or turbine, as in the diagram shown below, the maximum power that can possibly be extracted from the wind can be calculated.



Circular tube of air flowing through ideal wind turbine.

For this idealized case, the air is moving from point 1 to point 4 past the turbine. If the air flow is ideal, and the maximum possible power is extracted, it can be shown by momentum theory [6] that

$$V_2 = V_3 = (2/3)V_1 \qquad A_2 = A_3 = (3/2)A_1$$
$$V_4 = (1/3)V_1 \qquad A_4 = 3A_1$$

Now, taking an energy balance, for this idealized case,

$$P_{mech,ideal} = P_{air1} - P_{air4}$$

where $P_{mech,ideal}$= the maximum power that can be extracted
P_{air1} = the power in the air at position 1
P_{air4} = the power in the air at position 4

Substituting into our power equation,

$$P_{mech,ideal} = 0.5\rho(A_1 V_1^3 - A_4 V_4^3) = 0.5\rho(8/9)A_1 V_1^3$$

Expressing in terms of the physically meaningful terms, A_2 and V_1,

$$P_{mech,\ ideal} = 0.5\rho(8/9)(2/3)A_2 V_1^3 = 0.5(16/27)\rho A_2 V_1^3$$

where (16/27) is the Betz coefficient.

The actual mechanical power is typically defined as

$$P_{mech}=C_p(0.5\rho A_2 V_1^3)$$

where C_p is defined as the coefficient of power and must be less than the Betz coefficient. OK!

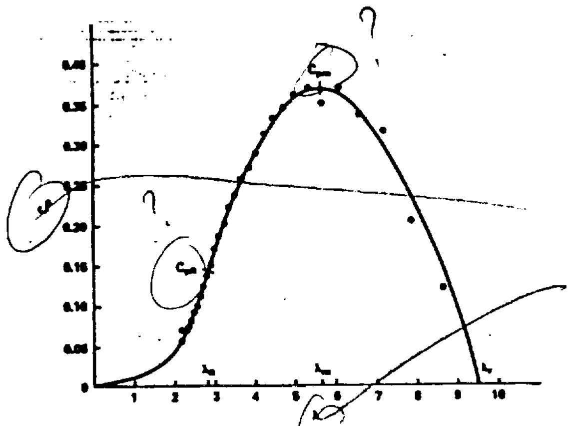
Wind speed probability relationships

The figures shown on the next page show the relationship between C_p, λ, and the shaft power, where λ is the tip speed ratio defined as:

$$\lambda = r_{max}\omega / V_\infty$$

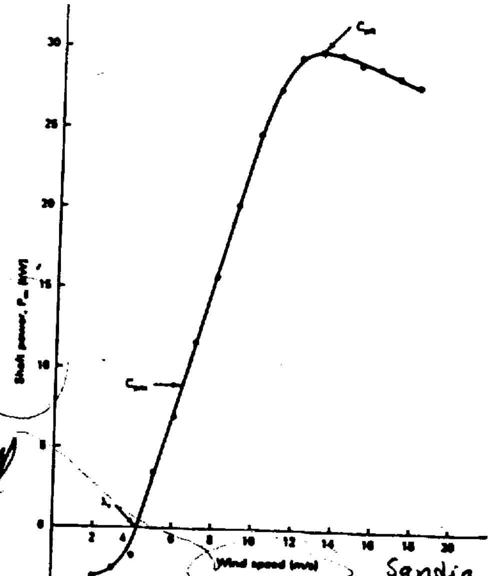
where:

r_{max} = maximum horizontal distance from shaft to blades ('b' in troposkien notation)
ω = angular velocity of shaft
V_∞ = free stream wind velocity



Sandia 17-m Darrieus

Too Small

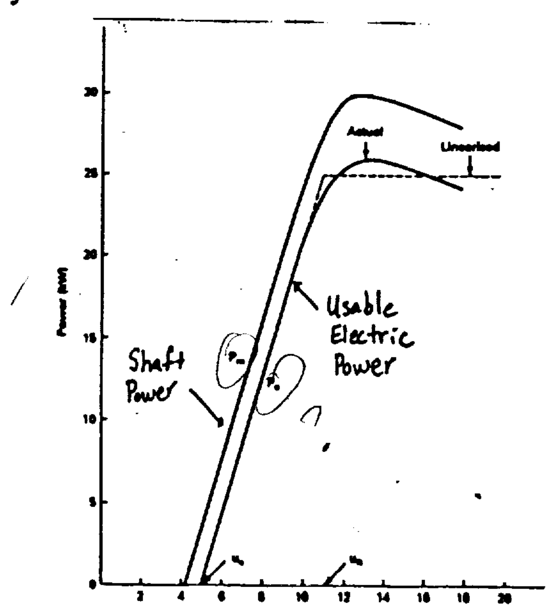


Sandia 17-m Darrieus

Although the above curves are for a particular Darrieus, the curves for other Darrieus turbines all look nearly identical. One important thing to notice is that all the curves have a maximum C_p of about 0.35 at a tip-speed ratio of just less than 6. The other major point of interest is that the maximum power generated occurs when the wind speed is about twice the wind speed for the maximum coefficient of power (C_{pm}).

In order to determine what speed to operate our turbine at, we need to analyze the wind speed probability. Data for wind speed on Mars is very difficult to find, and what is found is only for two particular locations. Since our wind speed probability function depends not only on time, but also on location where we land, the global mean wind speed was used. A probability analysis was then done using the global wind speed as our base.

The most important item to determine is the average power we are able to generate over time. This can be estimated with a probability study.



Sandia 17-m Darrieus

The previous curves show the relationship between the power and the wind speed. At first it seems odd to have the relationship between power and wind speed be nearly linear, since power depends on wind speed cubed. However, we must remember that efficiency goes down with increasing wind speed. For the previous curve, a line appears to be a good fit for the curve; however, for other turbines it has been found[7] that a better fit can be made with the relationship:

$$P_e = 0 \qquad u<u_C$$
$$P_e = a + bu^k \qquad u_C<u<u_R$$
$$P_e = P_{eR} \qquad u>u_R$$

where:

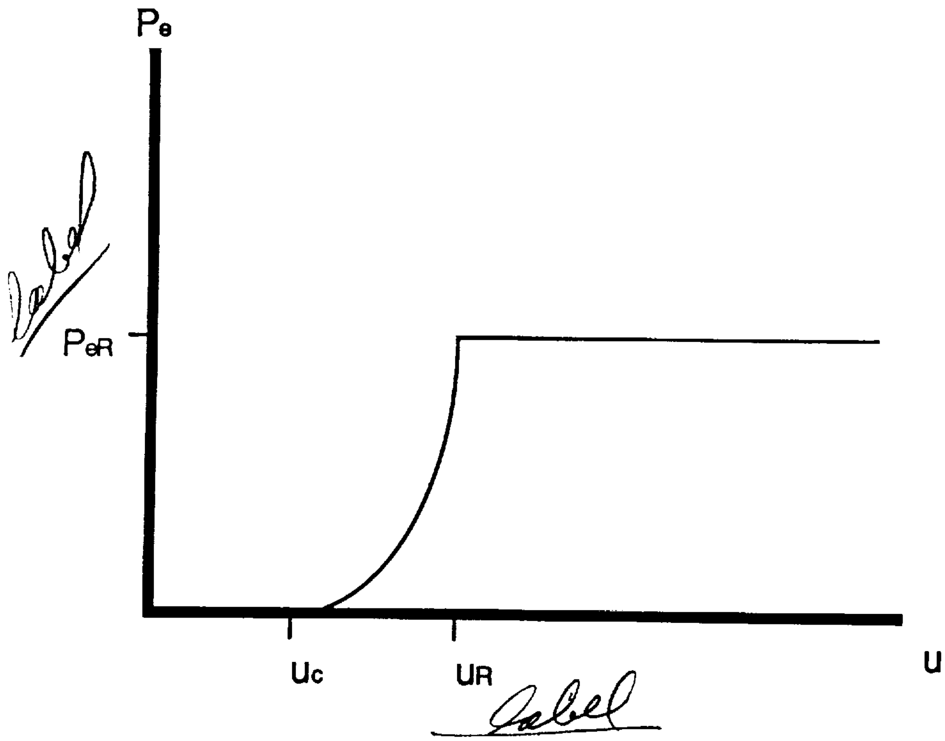
P_e = usable electric power generated
P_{eR}= the rated (maximum) electrical power
u = wind speed
u_R = wind speed at which maximum power is generated
u_C = wind speed when mechanical and electrical losses are equal to shaft power
a,b= constants used to fit curve
k = the Weibull shape parameter, a probability term

It is suggested [6] that if the wind speed probability is not well known that a value of k=2 should be used. This is the value we will use. The values of a and b that give the best curve fit are OK

$$a = \frac{P_{eR} u_C^k}{u_C^k - u_R^k}$$

$$b = \frac{P_{eR}}{u_R^k - u_C^k}$$

With this relationship, the power is now approximated with the curve on the following page.



Now that we have an equation for power as a function of wind speed, we are able to calculate our average power from the relationship:

$$P_{e,avg} = \int_{u=0}^{\infty} P_e f(u)du$$

where:

f(u) is the probability density function of wind speeds; it has the property $\int_{u=0}^{\infty} f(u)du = 1$

A recommended probability density function is [6]:

$$f(u) = \frac{k}{c}\left(\frac{u}{c}\right)^{k-1} \exp\left[-\left(\frac{u}{c}\right)^{k}\right]$$

where c is a constant; $c \approx 1.12u_{mean}$

Substituting this into the above equation for $P_{e,avg}$, yields:

$$P_{e,avg} = \int_{u_C}^{u_R} (a+bu^k)f(u)\,du \quad + \quad P_{eR}\int_{u_R}^{\infty} f(u)du$$

This equation can be integrated if the change of variables is made so that

$$x = \left(\frac{u}{c}\right)^k$$

$$dx = k\left(\frac{u}{c}\right)^{k-1} d\left(\frac{u}{c}\right)$$

Then, in terms of x, we have

$$\int f(u)du = \int e^{-x}\,dx \;=\; -e^{-x}$$

$$\int u^k f(u)du = \int c^k\left(\frac{u^k}{c^k}\right)f(u)du \;=\; \int c^k x e^{-x} dx$$

$$= -c^k(x+1)e^{-x}$$

Substituting in the limits of integration yields

$$P_{e,avg} = P_{eR}\left\{\frac{e^{-(u_C/c)^k} - e^{-(u_R/c)^k}}{(u_R/c)^k - (u_C/c)^k}\right\}$$

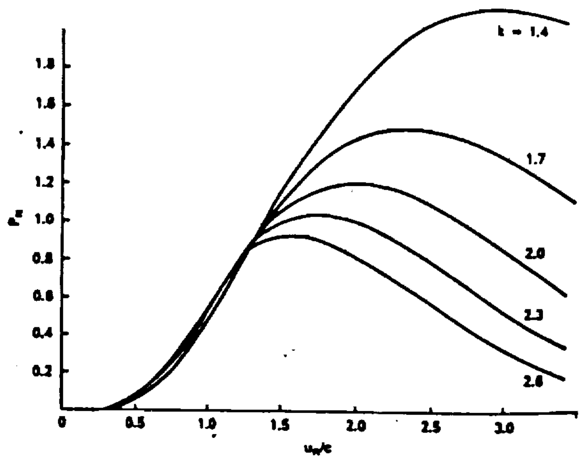
The quantity in brackets is often referred to as the plant factor (PF)

The value of u_C is nearly always in the range $0.4u_R < u_C < 0.5u_R$.

The normalized power is defined as

$$P_N = (PF)\left(\frac{u_R}{c}\right)^3$$

The following curve shows P_N for u_C=$0.4u_R$. Since we chose a value of k = 2.0, we see that we have a maximum normalized power if u_R/c equals 2.0. By designing our turbine for this situation we will produce the most usable energy over time.



Since u_R/c = 2.0 and $c=1.12u_{mean}$, we can deduce that $\mathbf{u_R} = \mathbf{1.8u_{mean}}$ will be the optimum selection.

Reviewing the curve of C_p vs. λ and using the above expression (AKA) bw yields for the optimum case: also know as

$$\lambda_R \approx 2.9 = \frac{r_{max}\omega}{1.8u_{mean}}$$ u_R ok

6 m/sec ?

Solving this equation tells us that the optimum $(r_{max}\omega)$ = **31.3 m/s**

With this information, we are now able to to size our Darrieus. We decided to get at least 1W of power from wind speeds greater than 6 m/s. Returning to our power equation, and an assuming overall efficiency of η=0.8 to cover all mechanical and electrical losses, we have:

ρ V^3

$$1 \text{ Watt} = 0.8\{ C_{p(u=6m/s)}(0.5)(0.01665 \text{ kg/m}^3)A_{swept}(6\text{m/s})^3\}$$

where
$C_{p(u=6m/s)}$ is taken off the chart for our calculated $\lambda_{(u=6m/s)}$=5.21
We find $C_{p(u=6m/s)}$ = 0.35.

Substituting this in yields:

$$\mathbf{A_{swept} = 2.00\ m^2}$$

List of Symbols

a	One-half the total height of our blade
A_{swept}	The area swept out by both blades
A/S	The ratio of the swept area to the arclength
b	The maximum horizontal position of our blade
C_f	Centrifugal force
D	A constant concerning the varying density
g	acceleration of gravity, 3.70 $\frac{m}{s^2}$ on Mars
G	Gravitational force acting on a section of the blade
H	the step size used for numerical integration
K	a parameter used in integration
KHI	a variable used in bisection method to solve for K
KLO	a variable used in bisection method to solve for K
λ	tip-speed ratio
P	an arbitrary point used in the derivation, see Figure 1
R	Horizontal coordinate
$\overline{R}$	The average horizontal coordinate for the blade
ρ	The mass density **per unit length** of our blade
ρ_{af}	The mass per unit length of a thin airfoil "skin"
ρ_c	The portion of the density (per unit length) that is constant
ρ_{supp}	The density (per unit length) of the internal blade support @ z=0
ρ_v	The portion of the density (per unit length) that is varied
s	Length of blade between point of maximum horizontal deflection and point P, see Figure 1
S	Total arclength of one blade
T	Tension at arbitrary point P
T_o	Tension at z=0
θ	Slope of blade at point P, see Figure 1
ω	The angular velocity of our blade
Ω_c, Ω_v	Rotational parameters, see Eq. (11) & (12) of derivation
ζ	A ratio of densities

Troposkien Notation

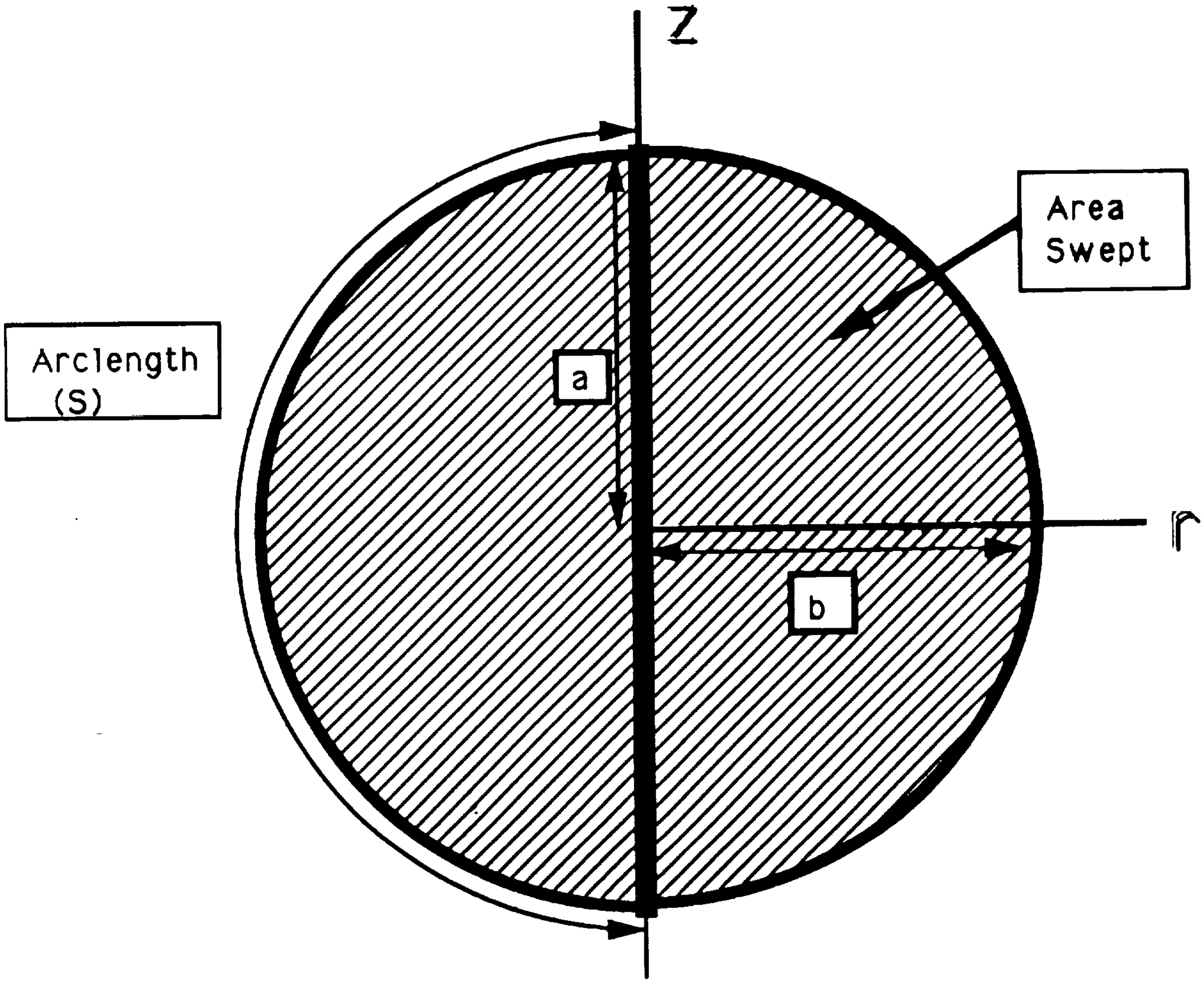


Figure 10

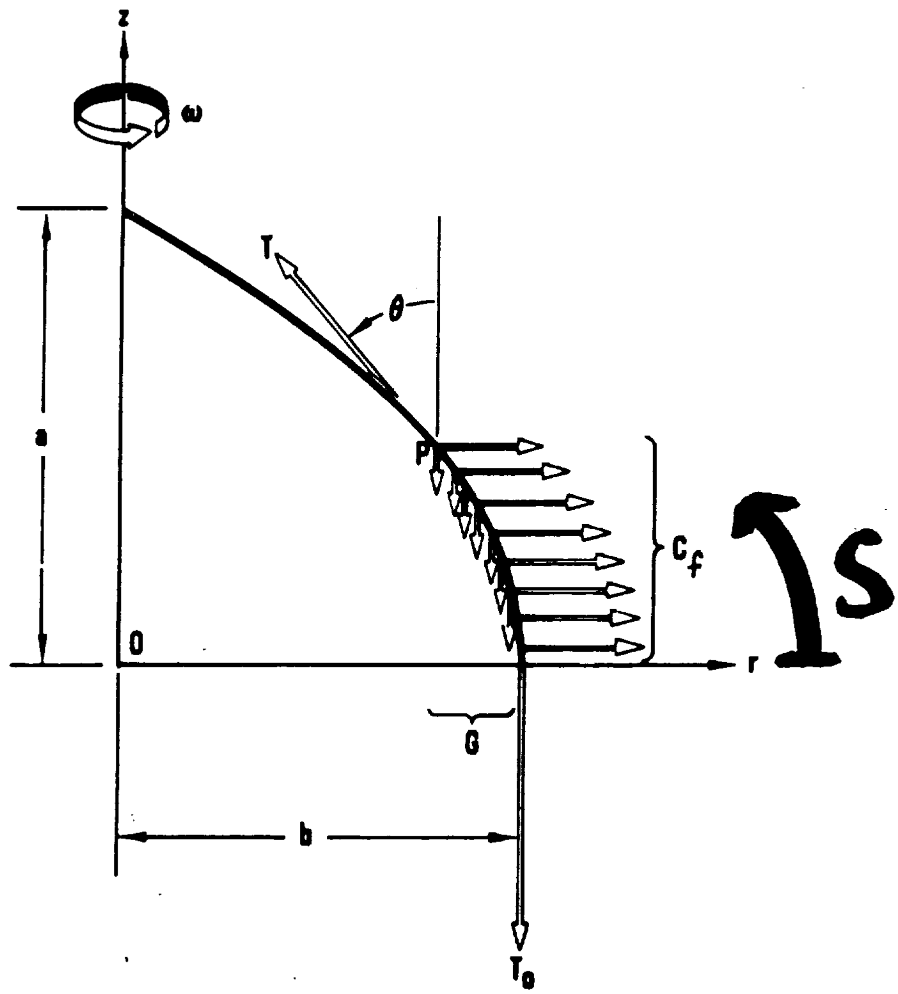


Figure 1. Schematic of a Perfectly Flexible Cable Rotating About a Vertical Axis

Referring to Figure 1, we obtain two equations that must be satisfied for equilibrium.

$$\Sigma F_z = 0$$

$$\Sigma F_r = 0$$

where ΣF_z = sum of all forces in z direction
ΣF_r = sum of all forces in r direction

For our situation these equations reduce to

$$T\sin\theta = C_f \tag{1}$$

$$T\cos\theta = T_o + G \tag{2}$$

where T = tension in member
θ = angle in Figure 1
T_o = tension at vertical midpoint
G = gravity force = $\int_0^s \rho g \, ds$
C_f = centrifugal force = $\int_0^s \rho \omega^2 r \, ds$

where ρ = mass per unit length
ω = angular velocity
s = arc length
g = acceleration of gravity

Taking the ratio of equation (1) and (2) and noting that $\tan\theta = -\dfrac{dr}{dz}$

$$\tan\theta = \frac{C_f}{T_o + G} = -\frac{dr}{dz} \tag{3}$$

Substituting in to equation 3 yields

$$\text{(4)} \qquad \frac{dr}{dz} = - \frac{\int_0^s \rho \omega^2 r \, ds}{T_o + G}$$

Equation (4) is subject to the boundary conditions

$$r = 0 \quad \text{at} \quad z = a$$
$$\frac{dr}{dz} = 0 \quad \text{at} \quad z = 0$$

Assuming a rotational speed of about 40 rad/s and considering any point with radial position of greater than 0.1 meters, (our blade's average radius is 0.677 meters)

$$\text{Centrifugal acceleration} = \omega^2 r > (40\text{rad/s})^2(0.1\text{m}) = 160 \frac{\text{m}}{\text{sec}^2}$$

$$\text{Gravitational acceleration} = 3.70 \frac{\text{m}}{\text{sec}^2}$$

Clearly, the gravitational acceleration can be neglected. After we select our material, we will also show that the aerodynamic forces are negligible in determining the shape.

For constant rotational speed, equation (4) reduces to

$$\text{(5)} \qquad \frac{dr}{dz} = - \frac{\omega^2}{T_o} \int_0^s \rho r \, ds$$

When I observed a conventional troposkein solution from Sandia[1], ~~I~~ We noticed that the tension varied according to the equation

$$\text{(6)} \qquad \frac{T}{T_o} = 1 - C \left(\frac{r^2}{b^2} - 1\right)$$

where C was some constant

In observing equation (6), it was ~~observed~~ apparent that since the tension varied along the length of the blade, it might be a good idea to vary some of our mass density so that we may reduce mass where it is not needed. This will not only reduce the mass in that location, but will lower the stresses throughout the blade. Remember that density is density per unit length, so what is really being varied is the cross-sectional area.

?

Therefore, it was decided as though we might have some portion of our blade cross section with a constant cross-section(i.e. a thin airfoil "skin"), and then an internal support with a varying cross-section. Remember that density is defined as mass per unit length, so a varying density really means a varying cross-sectional area.

you just said this

The total density would then be in the form

$$\rho = \rho_{af} + \rho_{supp}(1 - D(\frac{r^2}{b^2} - 1))$$

where

ρ_{af} = the density of the airfoil skin; a constant
ρ_{supp} = the density of the internal support at z=0
D = a constant to be optimized

Good

Eventually we decided to make our airfoil as one unit, and for our case ρ_{af} is equal to zero. The derivation is for the more general case and may be applied to our case by just setting $\rho_{af}=0$.

The density equation may be rewritten as

$$\rho = \rho_{af} + \rho_{supp}(1+D) - \rho_{supp}(D\frac{r^2}{b^2})$$

For simplicity these will be grouped in to two terms, one which does not depend on r, and one that does. The density terms will be:

ρ_c = the constant density portion

ρ_v = the varying density portion

Our density now will be written as

$$\rho = \rho_c - \rho_v r^2 \tag{7}$$

Putting equation (7) into equation (5) yields

$$(8) \qquad \frac{dr}{dz} = -\frac{\omega^2\rho_c}{T_o}\int_0^s r\,ds + \frac{\omega^2\rho_v}{T_o}\int_0^s r^3\,ds$$

We can rewrite equation (8) by noticing that

$$(9) \qquad ds = \sqrt{1 + \left(\frac{dr}{dz}\right)^2}\,dz$$

$$(10) \qquad \frac{dr}{dz} = -\Omega_c^2\int_0^z r\sqrt{1+\left(\frac{dr}{dz}\right)^2}\,dz + \Omega_v^2\int_0^z r^3\sqrt{1+\left(\frac{dr}{dz}\right)^2}\,dz$$

where

$$(11) \qquad \Omega_c^2 = \frac{\omega^2\rho_c}{T_o}$$

what a bear

$$(12) \qquad \Omega_v^2 = \frac{\omega^2\rho_v}{T_o}$$

Now, we can change the integro-differential equation (10) to an ordinary differential equation

$$(13) \qquad \frac{d^2r}{dz^2} = -\Omega_c^2 r\sqrt{1+\left(\frac{dr}{dz}\right)^2} + \Omega_v^2 r^3\sqrt{1+\left(\frac{dr}{dz}\right)^2}$$

After some algebraic manipulation, equation (13) can be written in the form of exact differentials.

$$(14) \qquad \frac{d}{dz}\sqrt{1+\left(\frac{dr}{dz}\right)^2} = -\frac{\Omega_c^2}{2}\frac{d}{dz}(r^2) + \frac{\Omega_v^2}{4}\frac{d}{dz}(r^4)$$

Integrating and substituting in the boundary condition that $\frac{dr}{dz} = 0$ at r=b ✓

$$(15) \qquad \sqrt{1+(\frac{dr}{dz})^2} = 1 - \frac{\Omega_c^2}{2}(r^2-b^2) + \frac{\Omega_v^2}{4}(r^4-b^4)$$

Squaring both sides yields

$$(16) \qquad 1+(\frac{dr}{dz})^2 = [1 - \frac{\Omega_c^2}{2}(r^2-b^2) + \frac{\Omega_v^2}{4}(r^4-b^4)\,]^2$$

Evaluating the left hand side (L.H.S.) yields

$$\text{L.H.S.} = 1 + \left(\frac{\Omega_c^2 b^2}{2}(\frac{r^2}{b^2}-1) - \frac{\Omega_v^2 b^4}{4}(\frac{r^4}{b^4}-1)\right)^2$$
$$-2\left(\frac{\Omega_c^2 b^2}{2}(\frac{r^2}{b^2}-1) - \frac{\Omega_v^2 b^4}{4}(\frac{r^4}{b^4}-1)\right)$$

This may be rewritten into equation (16) as

$$(\frac{dr}{dz})^2 = \left(\frac{\Omega_c^2 b^2}{2}(\frac{r^2}{b^2}-1) - \frac{\Omega_v^2 b^4}{4}(\frac{r^4}{b^4}-1)\right)\left(\frac{\Omega_c^2 b^2}{2}(\frac{r^2}{b^2}-1) - \frac{\Omega_v^2 b^4}{4}(\frac{r^4}{b^4}-1) -2\right)$$

This equation may now be rewritten as

$$(\frac{dr}{dz})^2 = \frac{\Omega_v^4 b^8}{16}\left(\frac{2\rho_c}{\rho_v b^2}(\frac{r^2}{b^2}-1) - (\frac{r^4}{b^4}-1)\right)\left(\frac{2\rho_c}{\rho_v b^2}(\frac{r^2}{b^2}-1) - (\frac{r^4}{b^4}-1) - \frac{8}{\Omega_v b^4}\right)$$

$$(\frac{dr}{dz})^2 = \frac{\Omega_v^4 b^8}{16}\frac{8}{\Omega_v^2 b^4}\left(\frac{2\rho_c}{\rho_v b^2}(\frac{r^2}{b^2}-1) - (\frac{r^4}{b^4}-1)\right)\left(\frac{\frac{2\rho_c}{\rho_v b^2}(\frac{r^2}{b^2}-1) - (\frac{r^4}{b^4}-1)}{\frac{8}{\Omega_v b^4}} - 1\right)$$

The above equation was obtained by noting that

$$\Omega_c^2 = \Omega_v^2 \frac{\rho_c}{\rho_v}$$

Now we will define two new parameters

$$\frac{\rho_c}{\rho_v} = \zeta \qquad \text{at } \rho_c = 0 \qquad \xi = \infty$$

$$\frac{\Omega_v^2 b^4}{8} = K^2$$

Now we may write

$$\left(\frac{dr}{dz}\right)^2 = 4K^2\left(\frac{2\zeta}{b^2}\left(\frac{r^2}{b^2}-1\right) - \left(\frac{r^4}{b^4}-1\right)\right)\left(K^2\left(\frac{2\zeta}{b^2}\left(\frac{r^2}{b^2}-1\right) - \left(\frac{r^4}{b^4}-1\right)\right) - 1\right)$$

Taking the square root of both sides yields

$$\frac{dr}{dz} = \pm 2K\sqrt{\left(\frac{2\zeta}{b^2}\left(\frac{r^2}{b^2}-1\right) - \left(\frac{r^4}{b^4}-1\right)\right)\left(K^2\left(\frac{2\zeta}{b^2}\left(\frac{r^2}{b^2}-1\right) - \left(\frac{r^4}{b^4}-1\right)\right) - 1\right)}$$

If we observe Figure 1, we can see that $\frac{dr}{dz}$ is going to be negative.

$\therefore$

$$\frac{dr}{dz} = -2K\sqrt{\left(\frac{2\zeta}{b^2}\left(\frac{r^2}{b^2}-1\right) - \left(\frac{r^4}{b^4}-1\right)\right)\left(K^2\left(\frac{2\zeta}{b^2}\left(\frac{r^2}{b^2}-1\right) - \left(\frac{r^4}{b^4}-1\right)\right) - 1\right)} \qquad (17)$$

We may now solve equation (17) numerically with its two boundary conditions for both the shape r(z) and the unknown K

```
****************** This program will generate solutions for a varying density troposkien **************

      PROGRAM TROPOSKEIN

****************** TOOHIGH and TOOLOW will be used to determine the correct value for K **************

      LOGICAL TOOHIGH,TOOLOW

****************** A is one-half the height of the blade *********************************************
****************** Z and R contain the coordinates of the blade **************************************
****************** H is a step size used for numerical integration ***********************************
****************** K is what I called "K squared" in the derivation **********************************
****************** KLO and KHI will be used to solve for K using a bisection method ******************

      REAL RPRIME,A,Z(1000),R(1000),H,CRCY(1000)
     & ,CRCX(1000),K,KLO,KHI,MAXTEN,TENSARRAY(1000)
     & ,RATIO(31),ADIVB(31),ZDIVA(1000),STRESSARRAY(1000)
      REAL DERIVATIVE(1000),F1,F2,F3,F4,HTIMESD
      EXTERNAL RPRIME,AREASWEPT,RAVG,ARCLENGTH

****************** Write the output to a file ********************************************************

      OPEN (18,file='tropout1')
      WRITE(18,55)
      WRITE(18,*)
      WRITE(18,*)

***************** This main loop will carry out the entire calculations for varying values of A and B **
***************** so that the area swept is nearly constant. ******************************************

       LOOPSTEPS = 31
C      DO 1000 LOOPER = 1,LOOPSTEPS
C      A= 0.55+ (REAL(LOOPER-1))*0.35/30.0

***************** Set B so the area swept is nearly a constant for comparison's sake ******************
C      B= 1.31/(2*A)

****************** If we want to work with a particular a and b, we can use the following *************
****************** two lines and comment out the "DO 1000 loop" ***************************************
      A= 0.737
      B= 0.889

***************** Our boundary condition requires the next two lines to be true ***********************
      Z(1)=A
      R(1)=0.0

***************** Next I set a step size and a "guess" for the value of K *****************************
      H=A/800.0
      K=1.4
```

T-18

```
***************** Set KLO and KHI to the upper and lower bounds for K ********************************
      KLO=0.0
      KHI=45.0

***************** NPTS is the number of points I will use to approximate the integral ****************
      NPTS=800
***************** The next line is required so that our boundary condition is met. *******************
      DERIVATIVE(NPTS)=0.0

***************** MAXSTEPS is the maximum # of times I will carry out the bisection method ***********
      MAXSTEPS = 30

***************** This following loop will carry out the bisection method to solve for K ************
      DO 33 MMM = 1,MAXSTEPS
        TOOHIGH=.FALSE.
        TOOLOW =.FALSE.

***************** This loop will carry out the integration using the fourth order Runge-Kutta *********
***************** Method and keeps the values of r and z in the arrays R and Z ***********************
      DO 100 I=2,NPTS
          Z(I) = A - (REAL(I))*A/REAL(NPTS)
          F1=H*RPRIME(R(I-1),K,B)
          F2=H*RPRIME(R(I-1)+0.5*F1,K,B)
          F3=H*RPRIME(R(I-1)+0.5*F2,K,B)
          F4=H*RPRIME(R(I-1)+F3,K,B)
***************** HTIMESD is the step size times the derivative ****************************************
          HTIMESD = (F1+(2.0*(F2+F3))+F4)/6.0
          R(I) = R(I-1) + HTIMESD
          DERIVATIVE(I-1) = HTIMESD/H
100   CONTINUE
****************  End of integration loop ***********************************************************

**************** If our slope at the end does not match our boundary condition, make K higher ******
      IF (DERIVATIVE(NPTS-1).GT.0.0001) THEN
          KLO=K
          K = (K+KHI)/2.0
          TOOLOW = .TRUE.
      ENDIF
**************** End of slope at boundary equation check *******************************************

**************** If we select too high a value for K, we will get an imaginary derivative, which ***
**************** is not a solution to our problem. The follwing check prevents these solutions ****
**************** from appearing, and corrects K so that the next attempt will be better.  **********
      IF (.NOT.(TOOLOW)) THEN
        IF(DERIVATIVE(NPTS-2).EQ.0.0) THEN
          KHI=K
          K=(K+KLO)/2.0
          TOOHIGH = .TRUE.
          ENDIF
      ENDIF
```

```
****************  End of the check for K too high  **************************************************

****************  If our K is now the solution, stop performing the bisection method  ****************
      IF ((.NOT.(TOOHIGH)).AND.(.NOT.
     &     (TOOLOW))) THEN
            GOTO 444
      ENDIF
****************  End of check for correct K  ********************************************************

****************  If K is incorrect, go back with an updated value for K   ***************************
33    CONTINUE
****************  End of bisection method loop  ******************************************************
444   CONTINUE

***************  Now that we have the solution, calculate the desired parameters  *******************
***************  and print out the output  ***********************************************************

      ASPT = AREASWEPT(R,H,NPTS)
      ARCLT = ARCLENGTH(R,DERIVATIVE,NPTS,H,K,B)
      MAXTEN = SQRT( 1.0 + (DERIVATIVE(1))**2)
      WRITE(18,999) A,B,K,ASPT,ARCLT,RAVG(R,NPTS),MAXTEN,ASPT/ARCLT
      RATIO(LOOPER) = ASPT/ARCLT
      ADIVB(LOOPER) = A/B
1000  CONTINUE

******************** End of main loop for different values of a and b ********************************



********************  For a given a and b, the following loop will produce an ***********************
********************  array with the values of the tension at every point  **************************
      DO 93 LFT = 1,NPTS
        TENSARRAY(LFT) = SQRT(1.0 + (DERIVATIVE(LFT))**2)
        ZDIVA(LFT) = Z(LFT)/A
93    CONTINUE
********************  End of tension calculating loop ***********************************************

********************  This loop gives the tension and stress ratios at any given point as well as giving  ***
********************  us the total mass required with the parameter rhosum  *********************************
      RHOSUM=0.0
      DO 94 LFT2 = 1,NPTS
        VARY=1.0+1.55*(1-(R(LFT2)/B)**2)
        STRESSARRAY(LFT2)=TENSARRAY(LFT2)/VARY
        RHOSUM=RHOSUM+VARY*(1+SQRT(1+(DERIVATIVE(LFT2))**2))*H
94    CONTINUE
C      WRITE(*,*) RHOSUM

********************  The following loop will enable us to plot a circle of the same  *************
********************  arclength as our troposkien for the purpose of comparison  ******************
      MMPTS = 51
```

```
        PI = ACOS(-1.0)
        CRCY(MMPTS) = ARCLT/PI
        CRCX(MMPTS) = 0.0
      DO 9432 LCC = 1,MMPTS-1
        PHI = REAL(LCC-1)*PI/(2.0*(REAL(MMPTS)))
        CRCY(LCC) = (ARCLT*SIN(PHI))/PI
        CRCX(LCC) = (ARCLT*COS(PHI))/PI
9432  CONTINUE
********************** End of loop to calculate circle coordinates ******************************

***************  End of the entire calculation process  **********************************************

***************  These lines can be used to plot various parameters  *********************************
c      CALL XYUNIT('meters','meters')
c      CALL CURV(CRCX,CRCY,MMPTS,'CIRCLE',' ',1,.TRUE.)
c      CALL CURV(R,Z,NPTS,'TROPOSKIEN',' ',3,.FALSE.)
c       CALL CURV(ADIVB,RATIO,LOOPSTEPS,'Aswept/S',' ',2,.FALSE.)
c       CALL SPLOT('ratio.ps','POWER-TO-MASS','a/b',
c     &  'AREA/LENGTH','t',5.0,5.0,.TRUE.,.TRUE.,4,4,
c     &   5,5,0.6,1.5,0.69,0.75)
c      CALL SPLOT('tskein2.ps','COMPARISON','R',
c     &      'Z','t',5.0,5.0,.TRUE.,.TRUE.,4,4,5,5,
c     &      0.0,1.0,0.0,1.0)
c       CALL CURV(ZDIVA,TENSARRAY,NPTS,'TENSION RATIO','',2,.FALSE.)
c       CALL CURV(ZDIVA,STRESSARRAY,NPTS,'STRESS RATIO','',3,.FALSE.)
c       CALL SPLOT('tens.data','TENSION AND STRESS RATIOS','Z/A',
c     &  'RATIOS','t',5.0,5.0,.TRUE.,.TRUE.,5,10,7,10,0.0,1.0,0.5,4.0)
55    FORMAT(//,9X,'A',12X,'B',12X,'K',11X,'Aswept',10X,'S',
     &       13X,'R',7X,'(T/To)max',6X,'Aswept/S')
999   FORMAT(5X,E9.3,4X,E8.3,4X,E10.4,4X,E10.4,4X,E10.4,4X,E9.3,
     &       4X,E10.4,4X,E10.4)
      STOP
      END
***************************************  END OF THE MAIN PROGRAM  ***********************************



******************  Function RPRIME calculates dr/dz at a given point  ******************************
******************  The variable X in this function is what I call r in the rest of the program *****
      FUNCTION RPRIME(X,K,B)
      REAL X,B,Q,L,M,K,ZETA,PRODUCT,ROOT

******************  Zeta will be changed manually to optimize  **************************************
      ZETA = 1.3

******************  The following are parameters to save calculation time  **************************
      Q = 2.0*ZETA/(B**2)
      L = ((X/B)**4)-1.0d0
      M = ((X/B)**2) - 1.0d0
```

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```
      PRODUCT = (Q*M) - L
      ROOT = PRODUCT*((K*PRODUCT)-1.0)

******************  If K is too high, root will be negative, and this will lead to imaginary  ******
******************  solutions, which make no sense for this problem.  ******************************
      IF (ROOT.GE.0.0) THEN
            RPRIME =  SQRT(K/4.0) * SQRT(ROOT)

*****************  If K is too high then ROOT will be negative.  Too avoid computer error, set *****
*****************  RPRIME = 0.  Then in the main program if RPRIME = 0, I change K so that it  ****
*****************  will be lower for the next iteration.  ******************************************
      ELSE
           RPRIME = 0.0
      ENDIF
      RETURN
      END
*********************************  END OF FUNCTION RPRIME  ************************************************



****************  Function AREASWEPT will calculate the area swept by both blades *********************
      FUNCTION AREASWEPT(R,H,NPTS)
      REAL R(NPTS),H
      AREASUM = 0.0
      DO 5 III=1,NPTS
        AREASUM = AREASUM +R(III)*H
5     CONTINUE
      AREASWEPT = 4.0*AREASUM
      RETURN
      END
*************************************  END OF FUNCTION AREASWEPT  ******************************************



*****************  Function RAVG will calculate the mean value of r for our blade  ***********************
      FUNCTION RAVG(R,NPTS)
      REAL R(NPTS)
      SUMAVG = 0.0
      DO 6 JJJ=1,NPTS
        SUMAVG=SUMAVG + R(JJJ)
6     CONTINUE
      RAVG = SUMAVG/REAL(NPTS)
      RETURN
      END
*****************************************  END OF FUNCTION RAVG ********************************************
```

```
******************  Function ARCLENGTH will calculate the arclength required of each blade  ************
      FUNCTION ARCLENGTH(R,DERIVATIVE,NPTS,H,K,B)
      REAL ARCSUM,R(NPTS),DERIVATIVE(NPTS),H,K
      ARCSUM = 0.0
      DO 7 LLL=1,NPTS
       ARCSUM = ARCSUM+(SQRT(1+(DERIVATIVE(LLL))
     &          **2))*H
7     CONTINUE
      ARCLENGTH = 2.0*ARCSUM
      RETURN
      END
****************************************  END OF FUNCTION ARCLENGTH   ************************************
```

I'm impressed with the whole thing.

Discussion on computer solution to troposkien equations

We begin with the problem of trying to solve equation (17) together with its two boundary equations. Since it is a first order differential equation with two boundary conditions and one additional unknown, we see that we have the right number of equations in order to solve for both the shape (r(z)) and the unknown K. The Runge-Kutta method of order 4 and the bisection method are applied simultaneously to equation (17). The procedure is outlined below.

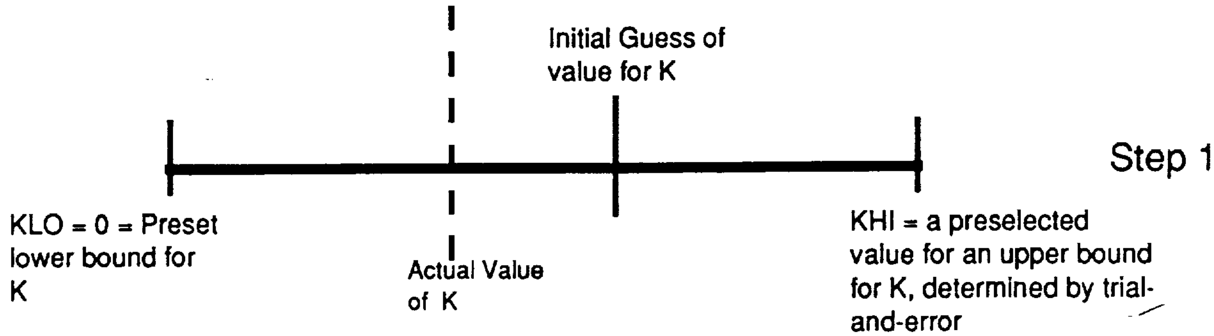
First, several values of a and b are computed by the computer so that all of these values sweep out roughly 2 m^2 (the required size for 1 Watt of power). The solution is then generated for all of these values of a and b so that the optimal height-to-width ratio will be obtained. Also a value of ζ is selected and this value will be changed manually after observation of results in order to optimize the value of this parameter. Now, for each combination of a,b, and z the only unknown left in equation (17) is K.

First we make a guess value for K, and set upper and lower bounds for K. The lower bound is 0. The upper bound and the guess value are found after trying the solution a couple of times. Then the equation is numerically integrated with this value of K.

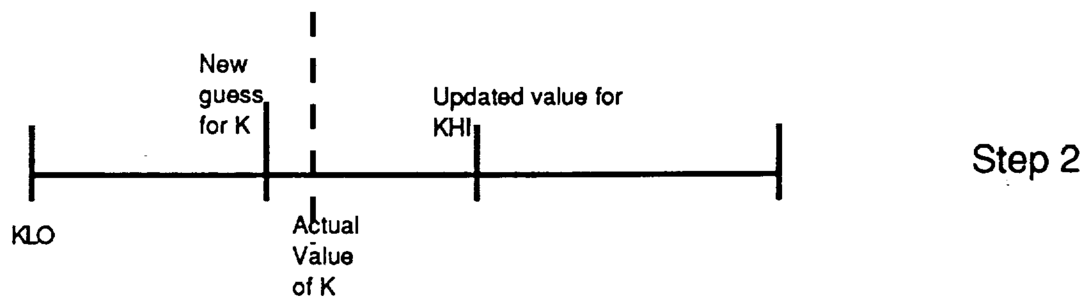
The numerical integration is done with the fourth order Runge-Kutta method. The method can be found in any numerical mathematics textbook and may be seen clearly in the included computer program. The step size was continually decreased until no change in the results resulted from a further decrease in the step size. The required step size was found to be $\frac{a}{800}$.

In order to solve for K, we must ensure that our solution satisfies the boundary conditions of no slope at z=0 and r=b at z=0. If K is selected too low, we run into two problems. First, the solution does not reach b when z=0. Second, there is a finite (non-zero) slope at z=0. Clearly, too low a value for K is not a solution to the equation. Also, if K is selected too high, the solution "steps" past b and the product under the radical in equation (17) will be negative. This violates our second boundary condition and is also not a solution. This brings us to the bisection method used.

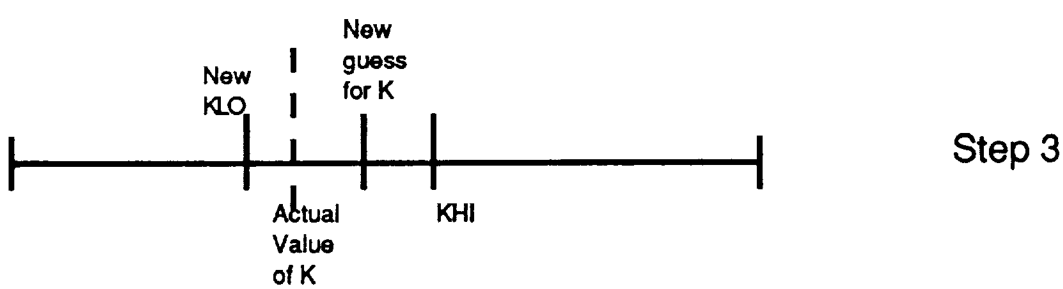
After attempting the calculations with the guess value for K, we could determine whether K was too low or too high from the results. After making this determination, K was updated using a bisection method. The method is outlined below.



We found that K was too high, so update its value and the value of KHI



Now, we find that K is too low, so update K and KLO



The procedure continues and usually converges after about 8 steps, since our initial interval is quite small.

The whole procedure is carried out for 30 different values of a and b for each selected ζ. Different solutions are then obtained for different values of ζ.

The area swept is then calculated by the formula :

$$A_{swept} = 4\int_{0}^{a} r\,dz$$

This is done numerically by noting that

$$A_{swept} \approx 4\Delta z \sum_{i=1}^{NPTS} r_i$$

where $NPTS = \frac{a}{\Delta z}$

The arclength is calculated from the formula

$$S = 2\int_{0}^{a} \sqrt{1 + \left(\frac{dr}{dz}\right)^2}\,dz$$

This is solved numerically with the approximation

$$S \approx 2\Delta z \sum_{i=1}^{NPTS} \sqrt{1 + \left(\frac{dr}{dz}\right)^2}$$

Also calculated in the computer program is the average horizontal location along the blade; it is called RAVG or $\bar{R}$.

The most important parameter generated is A_{swept}/S, which is the ratio of the area swept to the arclength. It is effectively a power-to-mass parameter; optimization of this parameter is vital.

Everything has now been solved for except for the tension. This is done by recalling equations (1) and (2) from the derivation. If these two equations are squared and added together, we obtain:

$$T^2 = C_f^2 + (T_o+G)^2$$

Dividing all terms by $(T_o+G)^2$ we obtain

$$\frac{T^2}{(T_o+G)^2} = \frac{C_f^2}{(T_o+G)^2} + 1$$

Also, from equation (3) in the derivation,

$$\frac{T^2}{(T_o+G)^2} = (\frac{dr}{dz})^2 + 1$$

R/6 HT

Now, gravity can be neglected in the equation, and we obtain:

$$\frac{T}{T_o} = \sqrt{1 + (\frac{dr}{dz})^2}$$

This can be evaluated numerically at every point.

Reading Tabular Data

Excellent!

The following is a brief outline of what all the terms on the tabular data mean and how to interpret the results

A	one-half the height of the blades
B	the maximum horizontal position of the blades
K^2	a rotational parameter, no obvious physical significance
A_{swept}	the area swept by both blades, power generated is directly proportional to A_{swept} m^2?
S	the arclength of one blade, effectively a measure of required mass
$\bar{R}$	the average horizontal coordinate of our blade
$(T/T_o)_{max}$	the maximum normalized tension (T_o is where tension is minimum)
A_{swept}/S	the most important column in the tabular data; a power-to-mass ratio; maximizing this parameter is crucial to our design

$\gamma = 1.3$

A	B	$\bar{K}^2$	Aswept	S	$\bar{R}$	(T/To)max	Aswept/S
0.550E+00	.119E+01	0.3138E+02	0.2054E+01	0.2795E+01	0.933E+00	0.1336E+02	0.7346E+00
0.562E+00	.117E+01	0.3904E+02	0.2215E+01	0.2915E+01	0.986E+00	0.1807E+02	0.7598E+00
0.573E+00	.114E+01	0.4500E+02	0.2336E+01	0.2999E+01	0.102E+01	0.2259E+02	0.7789E+00
0.585E+00	.112E+01	0.3521E+02	0.2339E+01	0.2979E+01	0.100E+01	0.1918E+02	0.7852E+00
0.597E+00	.110E+01	0.2256E+02	0.2266E+01	0.2896E+01	0.950E+00	0.1334E+02	0.7826E+00
0.608E+00	.108E+01	0.1666E+02	0.2218E+01	0.2838E+01	0.911E+00	0.1065E+02	0.7816E+00
0.620E+00	.106E+01	0.1311E+02	0.2181E+01	0.2791E+01	0.879E+00	0.9017E+01	0.7814E+00
0.632E+00	.104E+01	0.1068E+02	0.2151E+01	0.2753E+01	0.852E+00	0.7883E+01	0.7816E+00
0.643E+00	.102E+01	0.8915E+01	0.2127E+01	0.2719E+01	0.826E+00	0.7040E+01	0.7822E+00
0.655E+00	.100E+01	0.7574E+01	0.2106E+01	0.2690E+01	0.804E+00	0.6383E+01	0.7830E+00
0.667E+00	.982E+00	0.6509E+01	0.2087E+01	0.2663E+01	0.783E+00	0.5842E+01	0.7837E+00
0.678E+00	.966E+00	0.5658E+01	0.2070E+01	0.2639E+01	0.763E+00	0.5397E+01	0.7845E+00
0.690E+00	.949E+00	0.4955E+01	0.2055E+01	0.2618E+01	0.745E+00	0.5015E+01	0.7851E+00
0.702E+00	.933E+00	0.4375E+01	0.2042E+01	0.2599E+01	0.728E+00	0.4690E+01	0.7857E+00
0.713E+00	.918E+00	0.3885E+01	0.2030E+01	0.2582E+01	0.711E+00	0.4406E+01	0.7861E+00
0.725E+00	.903E+00	0.3465E+01	0.2018E+01	0.2566E+01	0.696E+00	0.4151E+01	0.7863E+00
0.737E+00	.889E+00	0.3108E+01	0.2007E+01	0.2552E+01	0.681E+00	0.3928E+01	0.7865E+00
0.748E+00	.875E+00	0.2800E+01	0.1998E+01	0.2540E+01	0.667E+00	0.3729E+01	0.7864E+00
0.760E+00	.862E+00	0.2528E+01	0.1988E+01	0.2529E+01	0.654E+00	0.3546E+01	0.7861E+00
0.772E+00	.849E+00	0.2291E+01	0.1979E+01	0.2519E+01	0.641E+00	0.3380E+01	0.7856E+00
0.783E+00	.836E+00	0.2081E+01	0.1970E+01	0.2511E+01	0.629E+00	0.3228E+01	0.7848E+00
0.795E+00	.824E+00	0.1898E+01	0.1962E+01	0.2504E+01	0.617E+00	0.3091E+01	0.7839E+00
0.807E+00	.812E+00	0.1735E+01	0.1955E+01	0.2498E+01	0.606E+00	0.2966E+01	0.7828E+00
0.818E+00	.800E+00	0.1589E+01	0.1948E+01	0.2493E+01	0.595E+00	0.2849E+01	0.7815E+00
0.830E+00	.789E+00	0.1459E+01	0.1941E+01	0.2489E+01	0.585E+00	0.2742E+01	0.7800E+00
0.842E+00	.778E+00	0.1343E+01	0.1935E+01	0.2486E+01	0.575E+00	0.2644E+01	0.7783E+00
0.853E+00	.768E+00	0.1236E+01	0.1928E+01	0.2483E+01	0.565E+00	0.2550E+01	0.7763E+00
0.865E+00	.757E+00	0.1142E+01	0.1922E+01	0.2483E+01	0.556E+00	0.2466E+01	0.7743E+00
0.877E+00	.747E+00	0.1055E+01	0.1916E+01	0.2482E+01	0.546E+00	0.2386E+01	0.7720E+00
0.888E+00	.737E+00	0.9775E+00	0.1911E+01	0.2483E+01	0.538E+00	0.2311E+01	0.7696E+00
0.900E+00	.728E+00	0.9064E+00	0.1905E+01	0.2484E+01	0.529E+00	0.2241E+01	0.7670E+00

$\gamma = 5.0$

A	B	$\bar{K}^2$	Aswept	S	$\bar{R}$	(T/To)max	Aswept/S
0.550E+00	.119E+01	0.2044E+01	0.2004E+01	0.2847E+01	0.911E+00	0.6506E+01	0.7040E+00
0.562E+00	.117E+01	0.1836E+01	0.1998E+01	0.2812E+01	0.889E+00	0.6160E+01	0.7105E+00
0.573E+00	.114E+01	0.1650E+01	0.1990E+01	0.2778E+01	0.868E+00	0.5827E+01	0.7164E+00
0.585E+00	.112E+01	0.1490E+01	0.1984E+01	0.2748E+01	0.848E+00	0.5535E+01	0.7222E+00
0.597E+00	.110E+01	0.1351E+01	0.1979E+01	0.2720E+01	0.829E+00	0.5269E+01	0.7277E+00
0.608E+00	.108E+01	0.1225E+01	0.1973E+01	0.2693E+01	0.811E+00	0.5015E+01	0.7327E+00
0.620E+00	.106E+01	0.1113E+01	0.1967E+01	0.2668E+01	0.793E+00	0.4778E+01	0.7373E+00
0.632E+00	.104E+01	0.1014E+01	0.1962E+01	0.2645E+01	0.777E+00	0.4564E+01	0.7417E+00
0.643E+00	.102E+01	0.9242E+00	0.1955E+01	0.2623E+01	0.760E+00	0.4355E+01	0.7455E+00
0.655E+00	.100E+01	0.8463E+00	0.1951E+01	0.2604E+01	0.745E+00	0.4172E+01	0.7493E+00
0.667E+00	.982E+00	0.7752E+00	0.1946E+01	0.2586E+01	0.730E+00	0.3997E+01	0.7525E+00
0.678E+00	.966E+00	0.7109E+00	0.1941E+01	0.2569E+01	0.715E+00	0.3831E+01	0.7554E+00
0.690E+00	.949E+00	0.6535E+00	0.1936E+01	0.2554E+01	0.701E+00	0.3679E+01	0.7581E+00
0.702E+00	.933E+00	0.6016E+00	0.1931E+01	0.2540E+01	0.688E+00	0.3536E+01	0.7604E+00
0.713E+00	.918E+00	0.5537E+00	0.1926E+01	0.2527E+01	0.675E+00	0.3398E+01	0.7622E+00
0.725E+00	.903E+00	0.5113E+00	0.1921E+01	0.2515E+01	0.663E+00	0.3273E+01	0.7639E+00
0.737E+00	.889E+00	0.4730E+00	0.1917E+01	0.2505E+01	0.651E+00	0.3157E+01	0.7653E+00
0.748E+00	.875E+00	0.4375E+00	0.1913E+01	0.2496E+01	0.639E+00	0.3045E+01	0.7662E+00
0.760E+00	.862E+00	0.4054E+00	0.1908E+01	0.2488E+01	0.628E+00	0.2940E+01	0.7670E+00
0.772E+00	.849E+00	0.3760E+00	0.1904E+01	0.2481E+01	0.617E+00	0.2841E+01	0.7674E+00
0.783E+00	.836E+00	0.3486E+00	0.1899E+01	0.2475E+01	0.606E+00	0.2745E+01	0.7673E+00
0.795E+00	.824E+00	0.3247E+00	0.1896E+01	0.2470E+01	0.596E+00	0.2662E+01	0.7674E+00
0.807E+00	.812E+00	0.3021E+00	0.1892E+01	0.2466E+01	0.586E+00	0.2579E+01	0.7670E+00
0.818E+00	.800E+00	0.2816E+00	0.1888E+01	0.2463E+01	0.577E+00	0.2502E+01	0.7664E+00
0.830E+00	.789E+00	0.2625E+00	0.1884E+01	0.2461E+01	0.567E+00	0.2428E+01	0.7655E+00
0.842E+00	.778E+00	0.2447E+00	0.1879E+01	0.2459E+01	0.558E+00	0.2356E+01	0.7642E+00
0.853E+00	.768E+00	0.2290E+00	0.1876E+01	0.2459E+01	0.550E+00	0.2293E+01	0.7630E+00
0.865E+00	.757E+00	0.2140E+00	0.1872E+01	0.2459E+01	0.541E+00	0.2230E+01	0.7613E+00
0.877E+00	.747E+00	0.2003E+00	0.1869E+01	0.2460E+01	0.533E+00	0.2172E+01	0.7596E+00
0.888E+00	.737E+00	0.1873E+00	0.1864E+01	0.2461E+01	0.525E+00	0.2114E+01	0.7573E+00
0.900E+00	.728E+00	0.1757E+00	0.1861E+01	0.2464E+01	0.517E+00	0.2062E+01	0.7552E+00

$\rho=20$

A	B	$\bar{K}^2$	Aswept	S	$\bar{R}$	(T/To)max	Aswept/S
0.550E+00	.119E+01	0.4184E+00	0.1976E+01	0.2835E+01	0.898E+00	0.6019E+01	0.6970E+00
0.562E+00	.117E+01	0.3794E+00	0.1970E+01	0.2800E+01	0.877E+00	0.5722E+01	0.7038E+00
0.573E+00	.114E+01	0.3445E+00	0.1965E+01	0.2767E+01	0.857E+00	0.5444E+01	0.7101E+00
0.585E+00	.112E+01	0.3138E+00	0.1960E+01	0.2737E+01	0.838E+00	0.5190E+01	0.7162E+00
0.597E+00	.110E+01	0.2864E+00	0.1956E+01	0.2709E+01	0.819E+00	0.4956E+01	0.7219E+00
0.608E+00	.108E+01	0.2618E+00	0.1951E+01	0.2683E+01	0.802E+00	0.4736E+01	0.7272E+00
0.620E+00	.106E+01	0.2393E+00	0.1945E+01	0.2658E+01	0.784E+00	0.4523E+01	0.7319E+00
0.632E+00	.104E+01	0.2194E+00	0.1941E+01	0.2635E+01	0.768E+00	0.4331E+01	0.7365E+00
0.643E+00	.102E+01	0.2017E+00	0.1937E+01	0.2615E+01	0.753E+00	0.4154E+01	0.7408E+00
0.655E+00	.100E+01	0.1853E+00	0.1932E+01	0.2595E+01	0.737E+00	0.3982E+01	0.7445E+00
0.667E+00	.982E+00	0.1709E+00	0.1929E+01	0.2578E+01	0.723E+00	0.3830E+01	0.7482E+00
0.678E+00	.966E+00	0.1576E+00	0.1924E+01	0.2561E+01	0.709E+00	0.3681E+01	0.7513E+00
0.690E+00	.949E+00	0.1453E+00	0.1919E+01	0.2546E+01	0.695E+00	0.3537E+01	0.7539E+00
0.702E+00	.933E+00	0.1343E+00	0.1915E+01	0.2532E+01	0.682E+00	0.3406E+01	0.7564E+00
0.713E+00	.918E+00	0.1244E+00	0.1911E+01	0.2520E+01	0.670E+00	0.3285E+01	0.7586E+00
0.725E+00	.903E+00	0.1152E+00	0.1907E+01	0.2508E+01	0.658E+00	0.3166E+01	0.7602E+00
0.737E+00	.889E+00	0.1070E+00	0.1904E+01	0.2499E+01	0.646E+00	0.3060E+01	0.7618E+00
0.748E+00	.875E+00	0.9929E-01	0.1899E+01	0.2490E+01	0.635E+00	0.2956E+01	0.7629E+00
0.760E+00	.862E+00	0.9229E-01	0.1895E+01	0.2482E+01	0.623E+00	0.2858E+01	0.7637E+00
0.772E+00	.849E+00	0.8579E-01	0.1891E+01	0.2475E+01	0.613E+00	0.2764E+01	0.7641E+00
0.783E+00	.836E+00	0.7998E-01	0.1888E+01	0.2469E+01	0.602E+00	0.2679E+01	0.7644E+00
0.795E+00	.824E+00	0.7451E-01	0.1884E+01	0.2464E+01	0.592E+00	0.2595E+01	0.7643E+00
0.807E+00	.812E+00	0.6956E-01	0.1880E+01	0.2461E+01	0.583E+00	0.2519E+01	0.7640E+00
0.818E+00	.800E+00	0.6494E-01	0.1876E+01	0.2458E+01	0.573E+00	0.2445E+01	0.7634E+00
0.830E+00	.789E+00	0.6067E-01	0.1872E+01	0.2456E+01	0.564E+00	0.2374E+01	0.7625E+00
0.842E+00	.778E+00	0.5682E-01	0.1870E+01	0.2455E+01	0.555E+00	0.2311E+01	0.7616E+00
0.853E+00	.768E+00	0.5315E-01	0.1865E+01	0.2454E+01	0.546E+00	0.2247E+01	0.7601E+00
0.865E+00	.757E+00	0.4990E-01	0.1863E+01	0.2455E+01	0.539E+00	0.2191E+01	0.7589E+00
0.877E+00	.747E+00	0.4674E-01	0.1859E+01	0.2456E+01	0.530E+00	0.2134E+01	0.7570E+00
0.888E+00	.737E+00	0.4392E-01	0.1857E+01	0.2458E+01	0.522E+00	0.2083E+01	0.7553E+00
0.900E+00	.728E+00	0.4119E-01	0.1852E+01	0.2460E+01	0.515E+00	0.2030E+01	0.7529E+00

Evaluation of Parameters from troposkien solution

We have solved for the troposkien shape in terms of the parameter K. However, this parameter gives us little insight as far as the magnitudes of the stresses. In order to obtain these figures we must go back and determine the physically meaningful terms in terms of K. Recall that

$$K^2 = \frac{\Omega_v{}^2 b^4}{8}$$

$$\Omega_v{}^2 = \frac{\omega^2 \rho_v}{T_o}$$

$$\therefore \quad T_o = \frac{\omega^2 \rho_v b^4}{8K^2} \qquad (1)$$

We also know that

$$b\omega = 31.3 \text{ m/s}$$

(from our tip-speed ratio)

$$\rho_v = \frac{\rho_{supp} D}{b^2} \qquad \text{(by definition)}$$

Substituting these into Equation (1) yields

$$T_o = \frac{122.46 \rho_{supp} D}{K^2} \qquad (2)$$

We are able to obtain an expression for D also,

$$\zeta = \frac{\rho_{af} + \rho_{supp}(1+D)}{\frac{1}{b^2}\rho_{supp} D} \qquad (3)$$

This relationship is also by definition (see derivation)

A third relationship is that for the maximum tension

$$T_a = T_{max} = T_o(T/To)_{max}$$

The optimal case would be if the stress at all points was the same. Therefore, we will try to find a relationship between the tension and the stress. T_{max} is also referred to as T_a because it occurs when z=a.

The tensile stress is simply the tension divided by the cross-sectional area. Since our density terms are density per unit length, they are really the mass density of the material times the cross-sectional area.

$$\therefore \frac{\sigma_a}{\sigma_o} = \frac{T_a/A_a}{T_o/A_o} \propto \frac{\rho_o}{\rho_a}(T/T_o)_{max}$$

<= GOOD STYLE

where:

σ = stress
A = cross-sectional area

$\propto$ means "is proportional to"

Since our airfoil has no separate "skin", $\rho_{af}=0$ (see derivation) and we may replace "$\propto$" by "=".

$$\frac{\sigma_a}{\sigma_o} = \frac{1}{1+D} (T/T_o)_{max} \qquad (4)$$

Immediately we see that the stress ratio is always less than the ratio of the tensions which is what was desired.

We now have 4 equations to work with which enable us to solve for the tensile forces, the constant D, and the densities per unit length. The equation for ζ has been simplified for the case where $\rho_{af}=0$. They are presented below as a summary

(1) $$T_o = \frac{122.46\rho_{supp}D}{K^2}$$

(2) $$\zeta = b^2\left(1 + \frac{1}{D}\right)$$

(3) $$T_a = T_{max} = T_o(T/To)_{max}$$

(4) $$\frac{\sigma_a}{\sigma_o} = \frac{1}{1+D} \ (T/T_o)_{max}$$

We see that equation (2) can be solved for D once ζ has been picked. In order to determine ζ, the computer program was run several times for various choices of ζ, until we could obtain a ratio in equation (4) as close to unity as feasible. If ζ was selected too low, the tension ratio became very large and the ratio would not approach 1. If ζ was selected very high, we lose the benefits of varying the cross-sectional area and can not approach a ratio of 1. Therefore, when trying to obtain a ratio in equation (4) close to unity while also minimizing arclength, the optimal value of ζ was found to be ζ=1.3. There is no closed form solution to show that this is the optimum, however iterative computer solutions reveal that this is the case.

With ζ=1.3, the computer program was run, and the output of interest here was:

b= 0.889 meters
$(T/T_o)_{max} = 3.928$
$K^2 = 3.108$

With these values known, we are able to solve for D and our stress ratio.

$$D = 1.55$$
$$\frac{\sigma_a}{\sigma_o} = 1.52$$

We see that we have taken a loading ratio of nearly 4 and reduced it to a stress ratio of approximately 1.5. This enables us to reduce our support mass significantly.

Next, we would like to approximate how much better our solution is than the traditional constant cross-sectional area troposkien solution. To do this, we estimate the mass required for the blades of both machines. Remember that our solution provides benefits in three separate ways. First, we reduce weight by varying the cross-section along the blades. Second, we obtain a better shape than the constant cross-section solution, therefore requiring less weight. Finally, since the stresses are mass dependent, the mass we removed where it was not needed helps to reduce stresses throughout the blade. Since there is no benefit in completely developing a constant cross-section solution, we will consider each of the three parts separately, and add their contributions to approximate our total mass and stress reduction.

Step (I)

First, we generated a computer solution for the case of ζ=1.3 and for the case ζ=20 (nearly constant cross-section). The arclength for the z=20 solution was longer than the z=1.3 case because of the shape benefits of varying the cross-section. The constant cross-section solution would need to have a large enough cross-section to support the maximum tensile load. The maximum tensile load would also be greater for the constant cross-section solution due to more mass present, but this topic will be discussed in section (II). Therefore its cross-section will be approximated with the cross-section of the varying cross-section solution at r=0.

$$\rho(r) = \rho_{supp}(1+D) \qquad \text{constant cross-section solution}$$

$$\rho(r) = \rho_{supp}(1-D(\frac{r^2}{b^2} - 1)) \qquad \text{varying cross-section solution}$$

To find the total mass of each blade, we integrate the density times the differential arclength.

$$\text{Mass} = \int_o^s \rho(r)ds$$

We know that $ds = \sqrt{1 + (\frac{dr}{dz})^2}\ dz$

Now the mass equation can be integrated numerically from z=0 to z=a, and the ratio of the constant cross-section to the varying cross-section can be obtained. This was done and the results are

$$\frac{MASS_{\text{constant cross-section}}}{MASS_{\text{varying cross-section}}} = 1.538$$

We see that varying the cross-section allows us to reduce mass by 34.2%.

Step (II)

The maximum tensile loads were compared for the constant cross-section and varying solutions and we found that the constant cross-section troposkein had a 54% greater maximum tensile load. Since in step (I), we assumed that the two solutions had the same cross-section at the point of maximum tension, we see that the stress is 54% greater in the constant cross-section solution with this assumption.

Conclusion

Therefore, we find that by varying the cross-section we are able to reduce blade mass by **over 34%**. We also have a maximum stress 54% less than that of a constant cross-section solution. We conclude that varying the cross-section has some remarkable advantages and should be implemented in our design. Neat!

POWER-TO-MASS

Scale:

H. inch = 0.1900

V. inch = 0.0180

Legend:

(1) ________ **ZETA =** 1.3

(2) ________ **ZETA =** 5.0

(3) ________ **ZETA =** 20.0

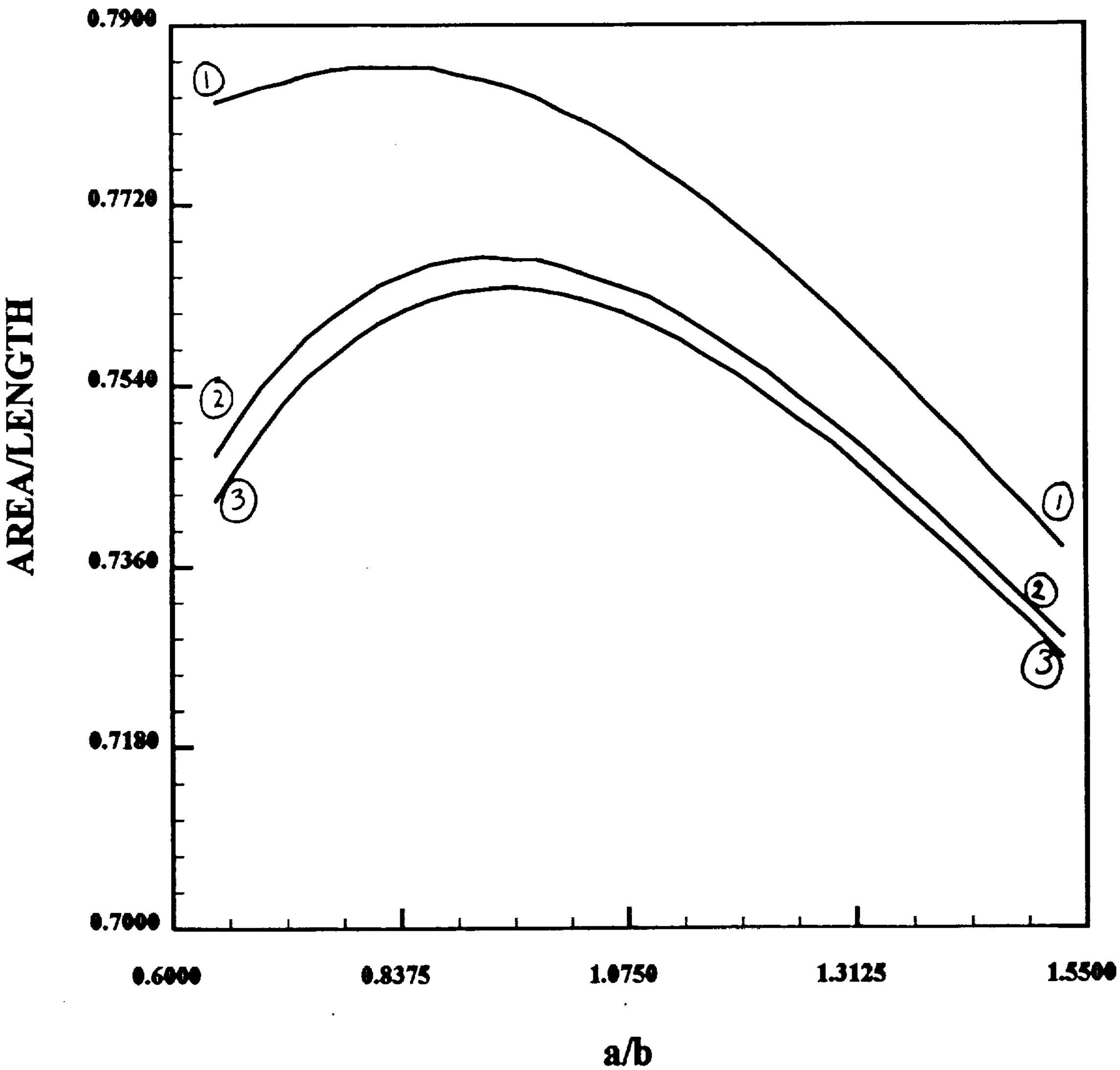


Figure 2

COMPARISON

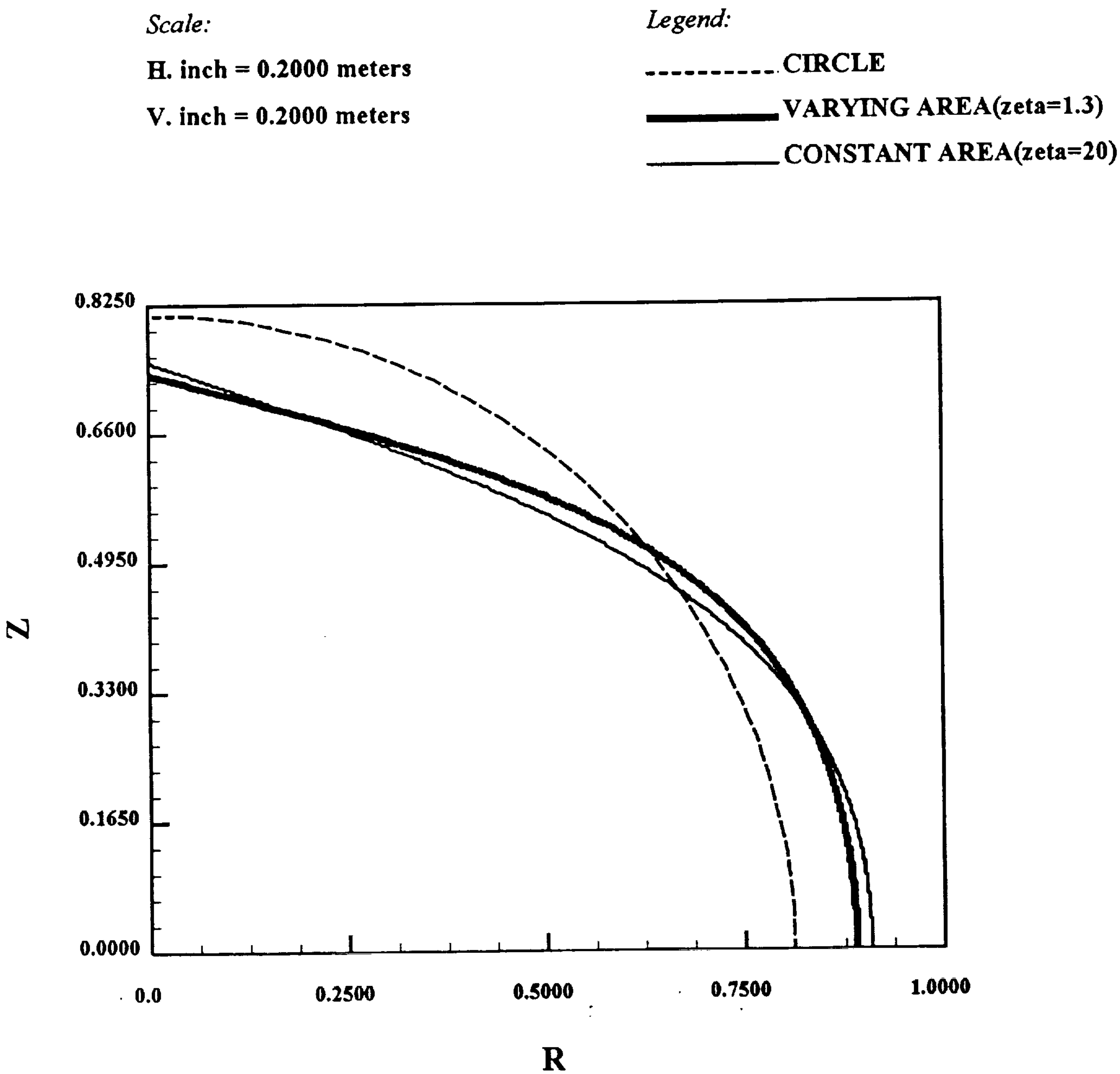


Figure 3

TENSION AND STRESS RATIOS

Scale:

H. inch = 0.2000

V. inch = 0.7000

Legend:

TENSION RATIO

STRESS RATIO

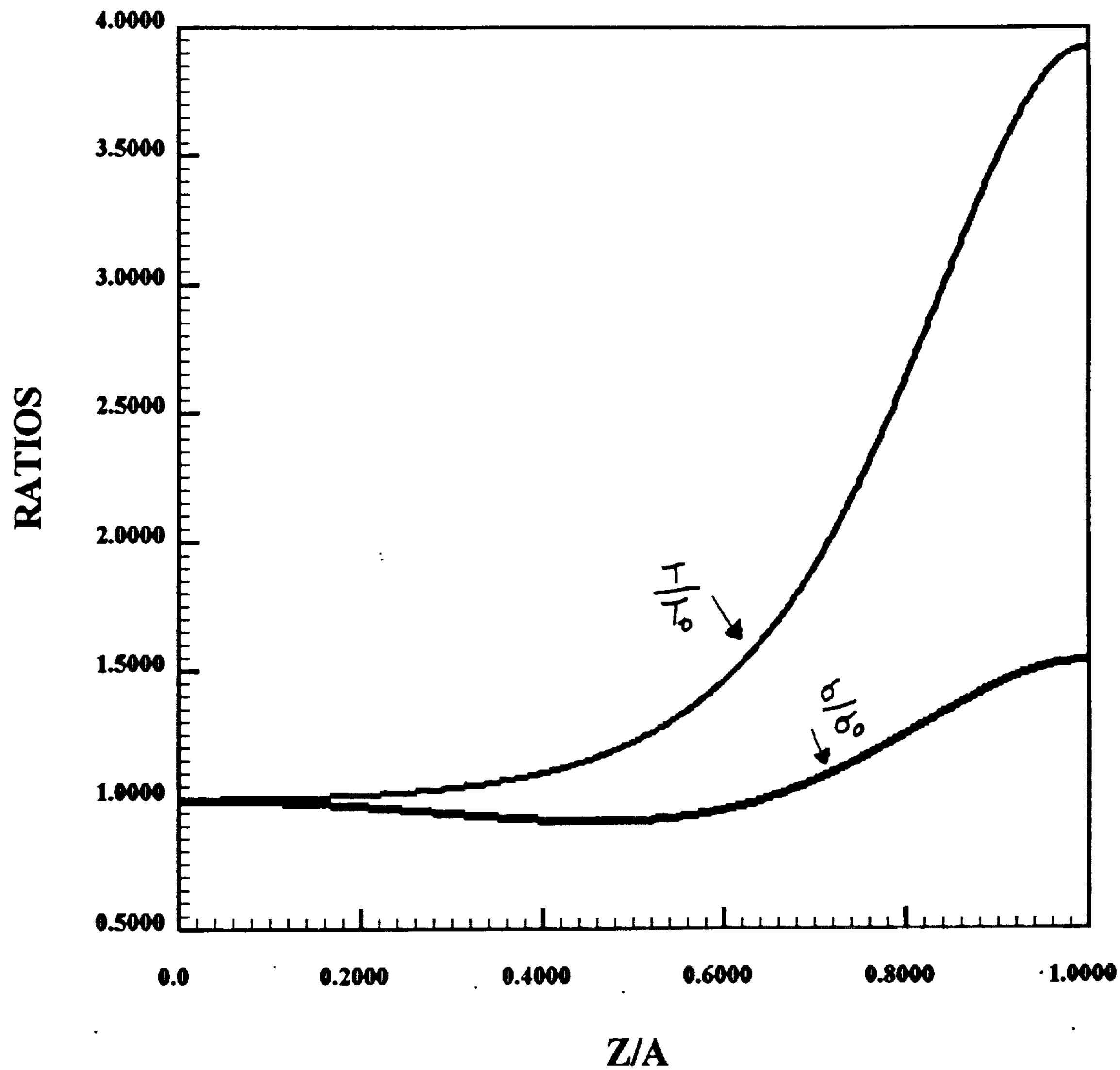
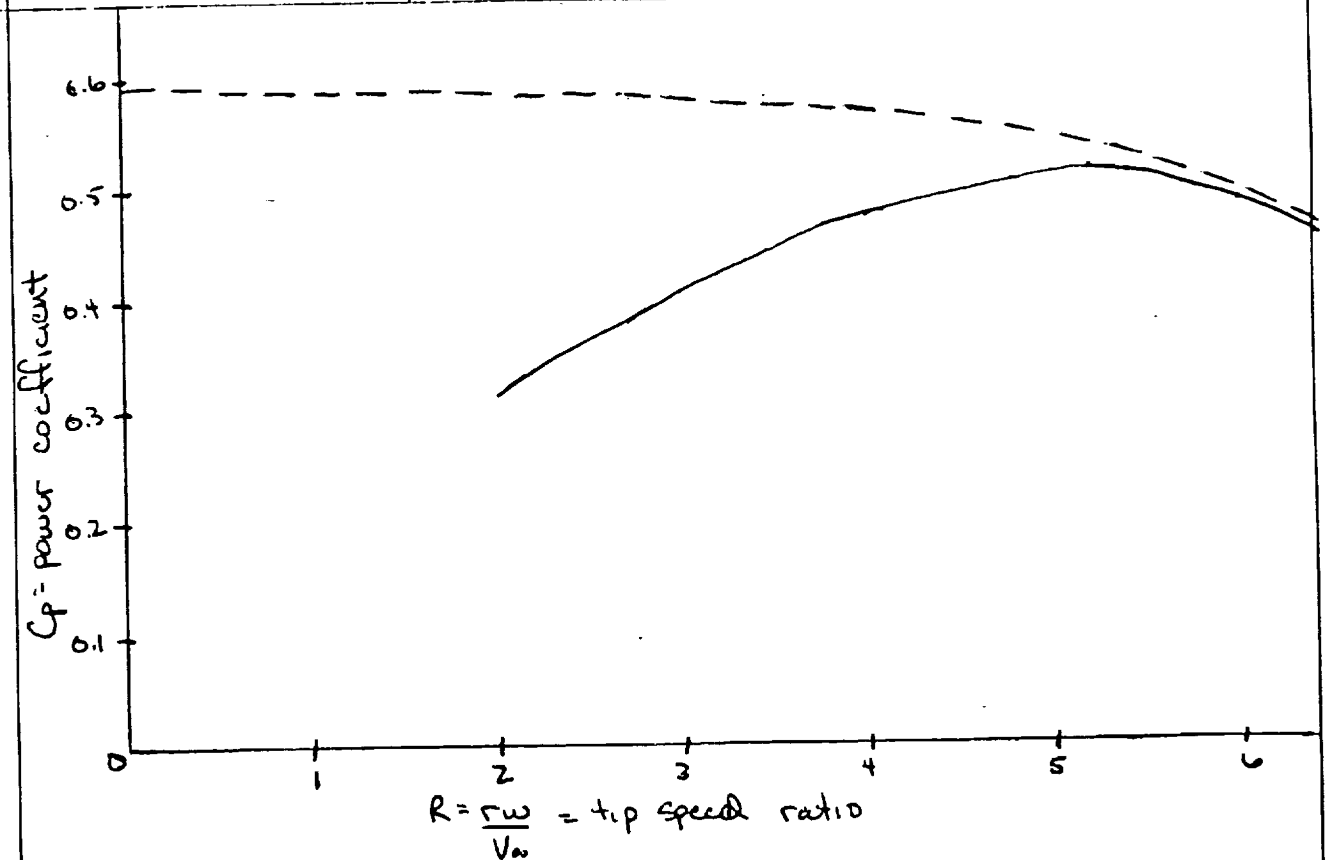
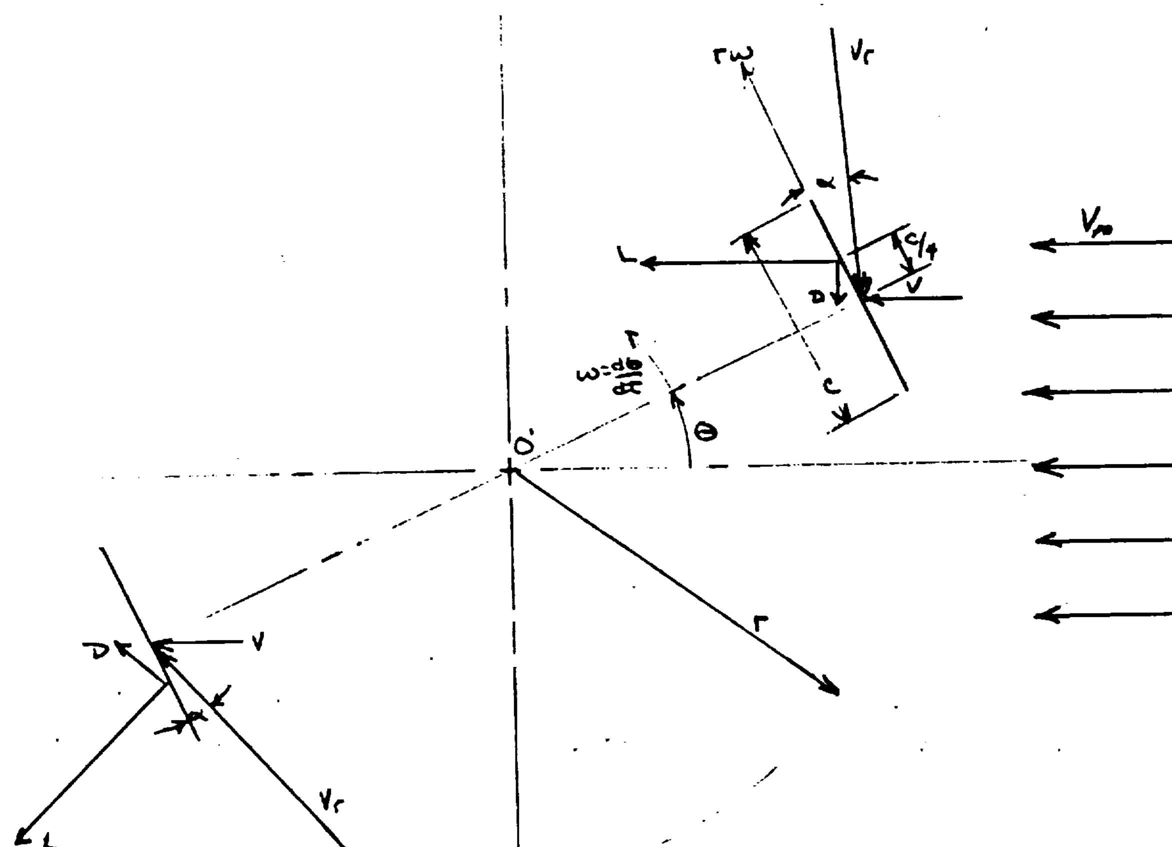


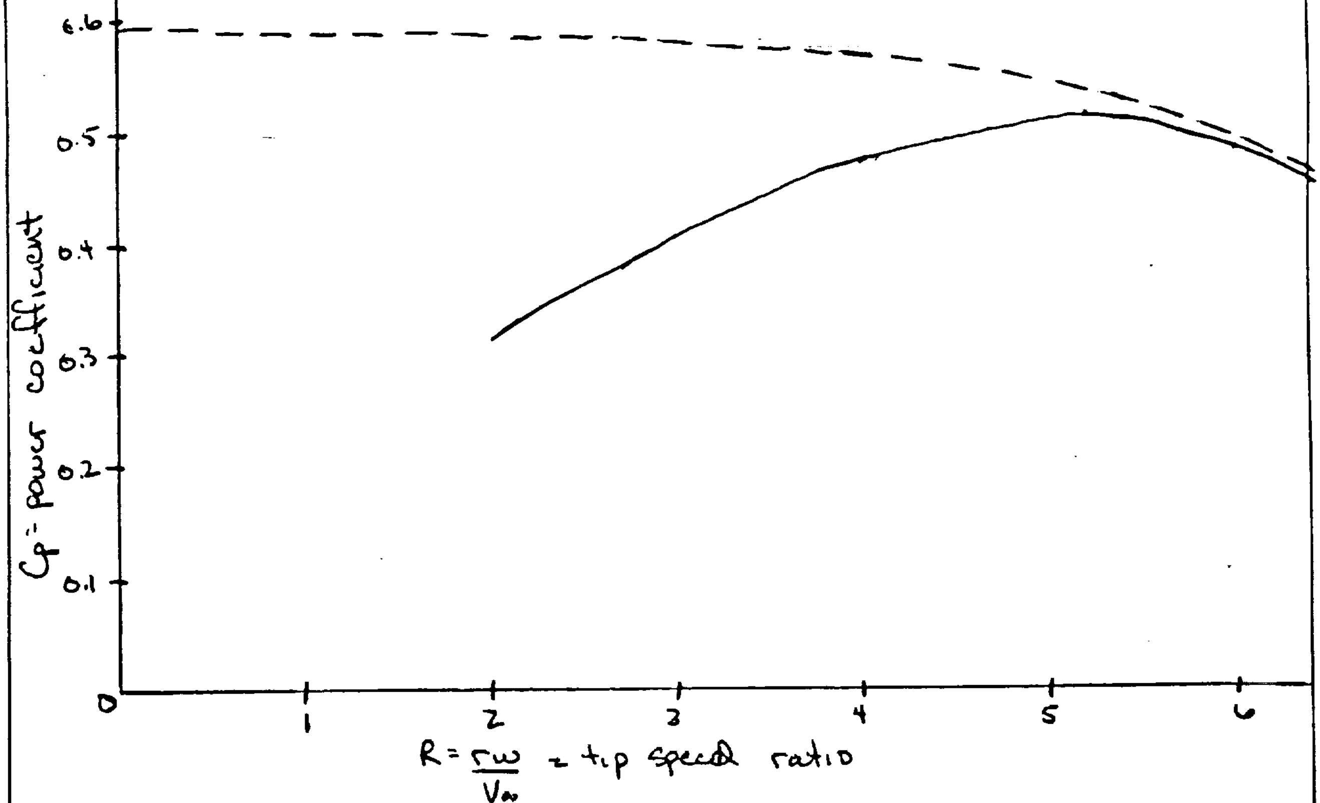
Figure 4

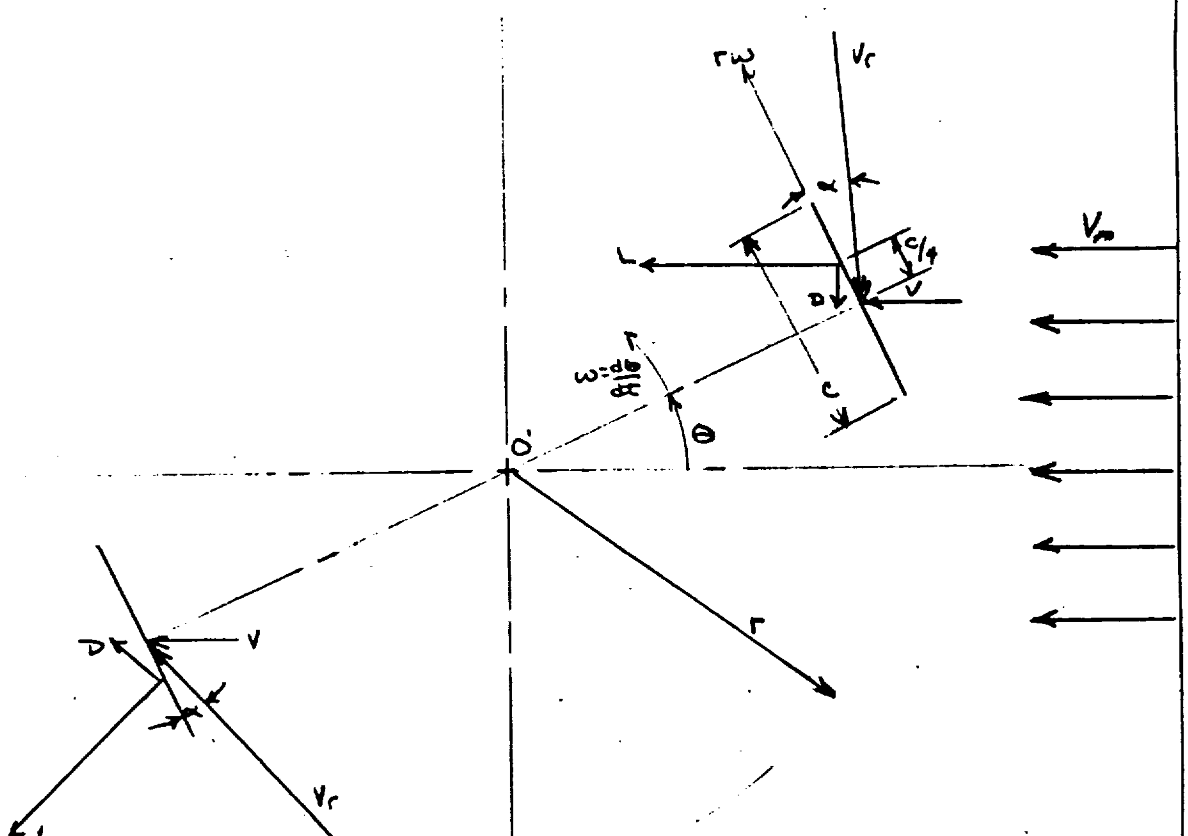


C_P = power coefficient
0.6
0.5
0.4
0.3
0.2
0.1
0
1
2
3
4
5
6
$R = \frac{r\omega}{V_\infty}$ = tip speed ratio



$r\omega$
V_r
α
V_∞
L
c/4
D
V
$\omega = \frac{d\theta}{dt}$
c
θ
O'
r
D
V
L
V_r


0.6
0.5
0.4
0.3
0.2
0.1
0
C_p = power coefficient
1
2
3
4
5
6
$R = \frac{r\omega}{V_\infty}$ = tip speed ratio


rω
V_r
α
c/4
V_∞
L
D
V
c
$\omega = \frac{d\theta}{dt}$
θ
O'
r
D
V
L
V_r

T-40

Darrieus $r = 0 \rightarrow 0.839\,m$ $\omega = 37.31\,rad/s$ $\lambda = 5.217 @ r_{max}$

Using results obtained from giromill calculations

$$\lambda = \frac{16}{3n\bar{c}K} \qquad v = 2/3 \qquad v = 1 - \frac{2.0}{V_\infty} \qquad v^2 = 1 - \frac{4.0}{V_\infty} + \frac{4.0}{V_\infty^2}$$

$$c = \frac{8V_\infty}{3\omega K} = \frac{8.6}{3(37.31)2\pi} = 0.0683\,m = 6.83\,cm$$

$$C_p = \frac{1}{4} n\bar{c}K\lambda v^2 - \frac{1}{2} n\bar{c} C_{do}\lambda^3$$

Substituting for $n, \bar{c}, \lambda, v^2$ gives

$$C_p = \frac{Kc\omega}{2V_\infty} - \frac{4c\omega K}{2V_\infty^2} + \frac{4c\omega K}{2V_\infty^3} - \frac{cC_{do}r^2\omega^3}{V_\infty^3}$$

$$C_p = \frac{8.01}{V_\infty} - \frac{32.02}{V_\infty^2} + \frac{32.02 - 26.96r^2}{V_\infty^3}$$

$$C_t(\theta) = \frac{\bar{c}}{2}\left[K\left(v^2\cos^2\theta + \bar{c}v\cos\theta(\lambda - v\sin\theta)\right) - C_{do}(\lambda - v\sin\theta)\{(\lambda - v\sin\theta) - \bar{c}v\cos\theta\}\right]$$

$$C_{max} = \frac{1}{4} n\bar{c}Kv^2 - \frac{1}{2} n\bar{c}C_{do}\lambda^2 = \frac{cKv^2}{2r} - \frac{cC_{do}r\omega^2}{V_\infty^2}$$

$$M(\theta) = C_n(\theta)\frac{\rho}{2}V_\infty^2 r A_s \qquad A_s \approx 0.18\,m^2$$

$$C_n(\theta) = \frac{\bar{c}}{2}\left[K\left(v^2\cos^2\theta + \bar{c}v\cos\theta(\lambda - v\sin\theta)\right) - C_o(\lambda - v\sin\theta)\{(\lambda - v\sin\theta) - \bar{c}v\cos\theta\}\right]$$

$$F_L = \frac{\rho}{2} V_r^2 C_L S \qquad S = A_S \approx 0.18m$$

$$V_r^2 = (\lambda^2 + \nu^2 - 2\lambda\nu \sin\theta) V_\infty^2$$

$$= \left[\frac{r^2\omega^2}{V_\infty^2} + \left(1 - \frac{2}{V_\infty}\right)^2 - \frac{2r\omega}{V_\infty}\left(1 - \frac{2}{V_\infty}\right)\sin\theta\right] V_\infty^2$$

$$= \left[\frac{r^2\omega^2}{V_\infty^2} + 1 - \frac{4}{V_\infty} + \frac{4}{V_\infty^2} - \frac{2r\omega \sin\theta}{V_\infty} + \frac{4r\omega}{V_\infty^2}\sin\theta\right] V_\infty^2$$

$$= r^2\omega^2 + V_\infty^2 - 4V_\infty + 4 - 2r\omega V_\infty \sin\theta + 4r\omega \sin\theta$$

$$V_r^2 = (r^2\omega^2 + 4 + 4r\omega\sin\theta) - (4 + 2r\omega\sin\theta)V_\infty + V_\infty^2$$

$$F_L = 6.79\times10^{-3} \text{Arctan}\left[\frac{(1 - 2/V_\infty)\cos\theta}{r\omega/V_\infty - (1 - 2/V_\infty)\sin\theta}\right]\left[4 + 4r\omega + r^2\omega^2 - (4 + 2r\omega\sin\theta)V_\infty + V_\infty^2\right]$$

$$= 6.79\times10^{-3} \text{Arctan}\left[\frac{(1 - 2/V_\infty)\cos\theta}{r\omega/V_\infty - (1 - 2/V_\infty)\sin\theta}\right]\left[4 + 149.24 r\sin\theta + 1392.04r^2 - (4 + 74.62 r\sin\theta)V_\infty + V_\infty^2\right]$$

$V_\infty = 6 \quad r = 0.839 \quad \alpha_{max}$ @ $\theta = 9.59°$

$$F_L = 6.79\times10^{-3}\left(\frac{6.37\pi}{180}\right)\left[4 + 149.24(0.839)(\sin 9.59) + 1392.04(0.839)^2 - (4 + 74.62(0.839)(\sin 9.59)6 + 6^2\right]$$

$$F_L = 7.55\times10^{-4}(954.16)$$

$$= 0.72N.$$

The outer shell of the troposkien blade is a NACA 0009 airfoil shape. Generally, the shape of a NACA four digit series airfoil is given by the equation:

$$\pm y = \frac{t}{0.20}\left[0.2969\sqrt{x} - 0.1260x - 0.35160x^2 + 0.2843x^3 - 0.1015x^4\right]$$

where t = % cord thickness

x = % cord

The cross sectional area of the blade can be computed, as follows

$$A_{cs} = 0.68503\,tC$$

for a hollow airfoil things more complex. The following proceedure is done

$$A_{cs} = 0.68503\left[tC - (t-2w)(C-2w)\right]$$

where $w \equiv$ wall thickness

At the equator of the troposkien blade we have the following

$$t = 0.09c$$

$$C = 0.074\ m$$

$$w = 0.0005\ m$$

so the cross sectional area is

$$(A_{cs})_o = 5.4569(10)^{-5}\ m^2$$

an additional parameter of the troposkien solution is

$$(A_{cs})_r = (A_{cs})_o\left[1+D\right] - (A_{cs})_o\left[D\frac{r^2}{b^2}\right]$$

this equation determines the cross sectional area at any location from the center line.

The troposkien blade attaches to its

bracket at $r = 6\,cm$. the cross sectional area at this point is.

$$(A_{cs})_{r=6cm} = 13.88(10)^{-5}\,m^2$$

we have specified maximum cord and a wall thickness at this location:

$$\text{cord}_{r=6cm} = 9.0\,cm$$

$$w_{r=6cm} = 1.0\,mm$$

We have specified a linear decrease in the cord length from the root to the equator.

$$C(r) = -1.799775\,r + 9.0$$

The centrifugal loading is determined by the governing equations for the troposkien solution.

$$T_0 = \frac{122.46 \, \rho_{supp} \, D}{K^2}$$

For our solution

$D = 1.551$

$K^2 = 3.108$

ρ_{supp} = density of blade material (δ) ✓

cross sectional area at equator (A_{cs_0})

For a $\delta = 1380 \frac{kg}{m^3}$

$T_0 = 84,279.9 \, (A_{cs_0})$

so the uniaxial stress is defined to be

$\sigma = \frac{T_0}{A_{cs_0}} = 84.3 \, kPa$ at equator we also have a cross sectional area

at the equator.

$$A_{cs_0} = 5.46(10)^{-5}\ m^2$$

which yields,

$$T_0 = 4.60\ N$$

another relationship determined by the

moeskien solution is:

$$A_{cs}(r) = A_{cs_0}[D+1] - A_{cs_0}\left[D\frac{r^2}{b^2}\right]$$

where

$$b = r_{max} = 0.889$$

The location of the pin is at $r = 0.06\,m$

so evaluating the stress at this

position we obtain

$$A_{cs}(r=0.06) = 13.88(10)^{-5}\ m^2$$

the tension at the root is given by

$$\left(\frac{T}{T_0}\right)_{max} = 3.928$$

=

so

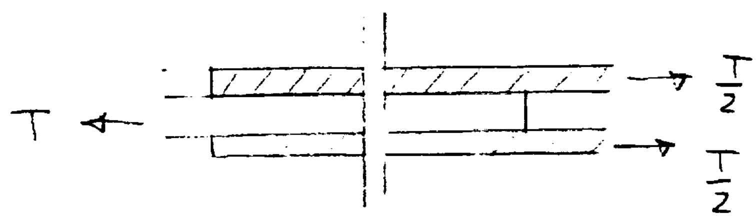
$$T_{max} = 18.1\,N.$$

the tension at r = 0.06 m is approximately the same as that at the root (conservative estimate). so the stress at the r = 0.06 location is

$$\sigma_{r=0.06} = \frac{T_{max}}{A_{cs_{r=0.06}}} = 130.1336\ kPa$$

The σ_{ULT} for Epoxy 60%, Kevlar 49, u/d is 1400 $\frac{MN}{m^2}$, so we have a factor of safety of over 10,000.

The airfoil will be attached to the bracket by means of a pin ($\Phi = 0.05$ mm). We may analyse the stress intensity due to this pin by investigating modeling the airfoil and bracket as plates.



the stress states near the hole will generally be observed to be similar to below

An empirical equation for the stress concentration at a lightly loaded bolt hole is

$$k = 2 + \left(\frac{w}{d} - 1\right) - \frac{1.5\left(\frac{w}{d} - 1\right)}{\left(\frac{w}{d} + 1\right)}$$

Reference!

we have at the pin location

pin dia $= d = 0.05\ mm$ $\qquad t = 0.001$

$w = 8.628\ cm$

$k = 1{,}725.1$

$$\sigma_{max} = \frac{P}{t(w-d)} k \qquad P = \frac{T}{2}$$

$= 181.\ MPa$

$\sigma_{max} < \sigma_{ult}$

$F.S. = 7.73$

The pressure distribution acting over an airfoil can be closely approximated by the term $\left(\frac{v}{V}\right)^2$ which is termed the low speed pressure distribution.

Define $\left(\frac{v}{V}\right)^2 \equiv S$

$$S = \frac{H - p}{\frac{1}{2} \rho V_\infty^2}$$

where

$H - p$ = pressure acting on surface of airfoil

ρ = density of the atmosphere

V_∞ = the apparent wind speed.

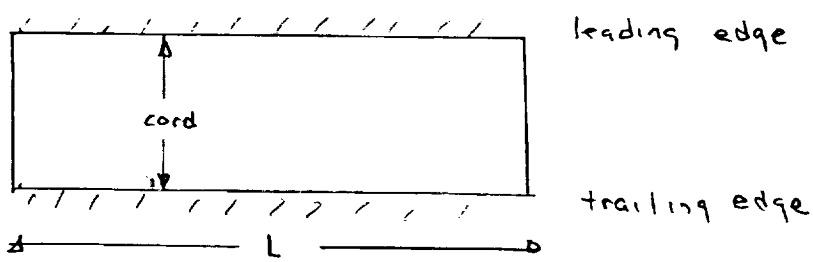
The values of S are tabulated for different locations along the airfoil.

The pressure distribution for the NACA 0009 airfoil has been computed and graphed. the Values used were:

$$\rho = 0.01556 \ \frac{kg}{m^3}$$

$$V_\infty = 12.59 \ \frac{m}{s}$$

To determine the effect this pressure has on the blade we approximate the blade to be a plate with an initial deflection y.



the maximum deflection (at the center) is

$$y_{max} = \frac{-\alpha q b^4}{E t^3}$$

we have the following

b = cord = 0.074 m

t = thickness = 0.0005 m

E = Flexural Modulus = 80 GN/m²

$y = \dfrac{\text{maximum thickness of airfoil}}{2}$

= 0.00333 m

L = length of airfoil = 2.549 m

note :

L/b = large $\cong \infty$

B = 0.0285 for $L/b = \infty$ (table)

Solve for q :

q = 38.96 kPa

$Q = q - P =$ 38.94 kPa

$$y_{max} = \frac{\alpha Q b^4}{E t^3} = 0.003328 \text{ units}$$

Δy = 0.054 %

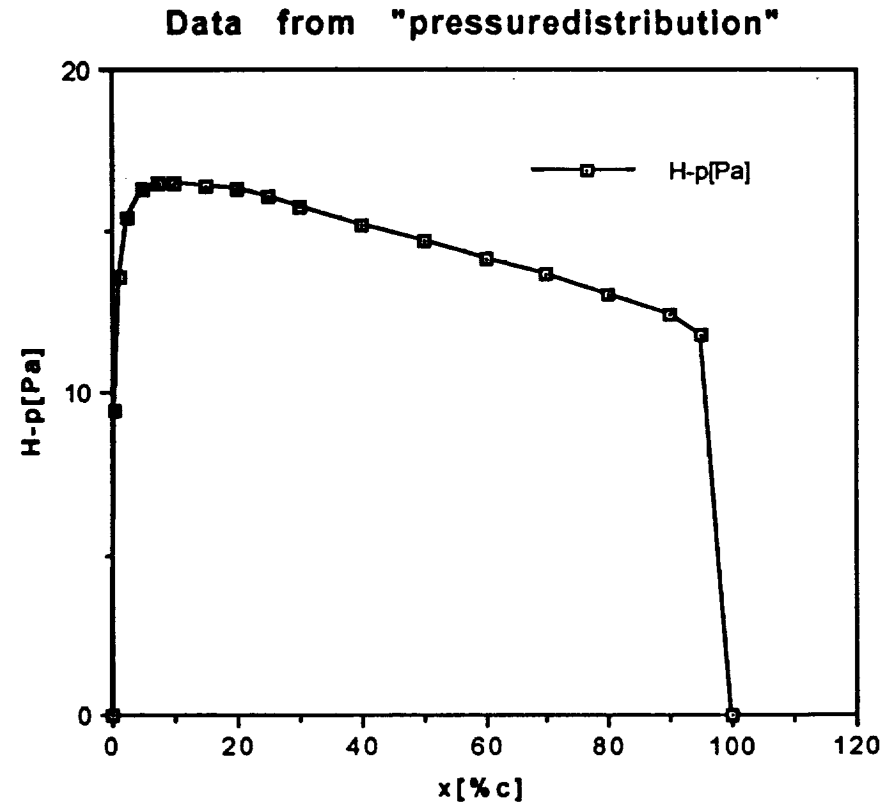


Figure 15

what does this tell me

PRESSURE DIST. ?

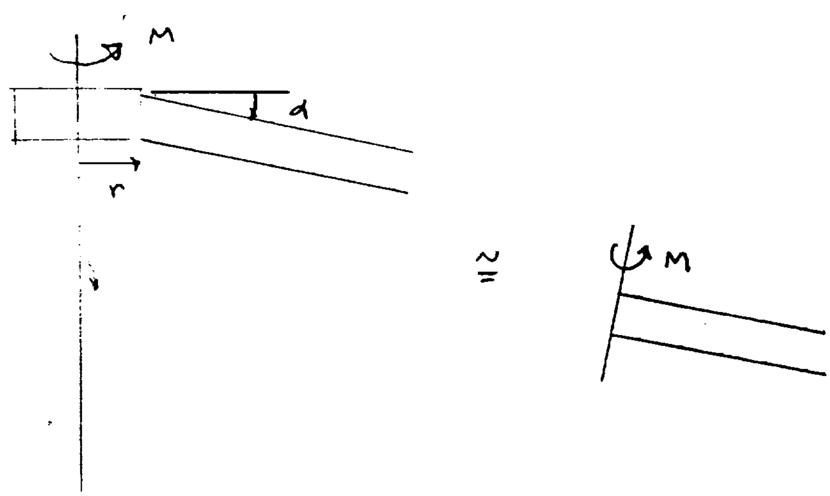
	x[%c]	S[(v/V)2]	H-p[Pa]
1	0.000	0.000	0.000
2	0.500	0.750	9.444
3	1.250	1.083	13.637
4	2.500	1.229	15.476
5	5.000	1.299	16.357
6	7.500	1.310	16.496
7	10.000	1.309	16.483
8	15.000	1.304	16.420
9	20.000	1.293	16.281
10	25.000	1.275	16.055
11	30.000	1.252	15.765
12	40.000	1.209	15.224
13	50.000	1.170	14.733
14	60.000	1.126	14.179
15	70.000	1.087	13.688
16	80.000	1.037	13.058
17	90.000	0.984	12.391
18	95.000	0.933	11.748
19	100.000	0.000	0.000

or [illegible]?

(DITTO)

The aerodynamic loading of the troposkien creates a maximum moment of 0.06 Nm about the shaft.

Assuming that the moment near the axis of rotation will be approximately the same as that acting about the center of rotation, we have the following situation:



the moment of inertia at r = 0.06m

is $I_{yy} = 314.08(10)^{-9}\ m^4$.

the distance to the farthest edge from $\bar{y}$ is:

$$y = 0.0522\,m$$

$$\sigma_{max} = \frac{My}{I}$$

$$= 6{,}092.4\,Pa = 6.1\,kPa$$

The stress acting at this location due to the centrifugal loading is

$$\sigma = 130.1\,kPa$$

$\sigma_{max} << \sigma$ so its effects are negligible.

Epoxy 60%, Kevlar 49 u/d has the following thermodynamic characteristics.

Thermal expansion

axially $\alpha = -2.0\,(10^{-6})\,\frac{1}{K}$

transverse $\alpha = 60\,(10^{-6})\,\frac{1}{K}$

The total length of the blade is 2.549 m. The diurnal temperature change can be 100° K.

<u>axially</u>

$$\epsilon_z = \frac{1}{E_z}\sigma_z + \alpha_z T$$

$$= \frac{84.3\,kPa}{90\,GPa} + {}^{-}0.2(10)^{-6}\frac{1}{K}\,100\,K$$

$$= 9.367(10)^{-7} + 2.00(10)^{-5}$$

$$= 2.09(10)^{-5}$$

over a total length of 2.54 m

$$\delta = 2.05(10)^{-5}\ 2.55\,m$$

$$= 53.4\ \mu m$$

<u>transverse</u>

$$\varepsilon_1 = 0 + 60(10)^{-6}\frac{1}{K}\ 100\,K$$

$$\varepsilon_1 = 6.00(10)^{-3}$$

length in the transverse direction can be approximated by the cord length = 0.074m

$$\delta = 6.00(10)^{-3}(0.074\,m)$$

$$= 444\ \mu m$$

The deflection in the axial direction is negligibly small.

The deflection in the transverse direction is not a factor as it is unrestrained.

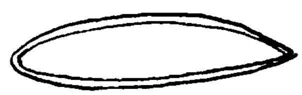
11-90 - 2:ACOS

Brain storm.

Basis: From troposkien equations, want max density @ roots and min density @ equator. This will reduce aerodynamic and centrifugally induced stresses.

To facilitate construction want constant shell, this leaves the interior to optimize.

(a)



variable wall thickness

pro
no internal components

con
difficult forming / molding

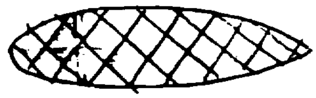
(b)

shell, central core + filler

pro

con
construction
mass
multi material

(c)



Honeycomb

pro

con

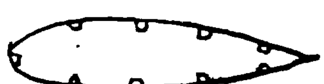
(d)

multi ~~cord~~ tube, shell, filler
have the wall thickness of the ~~tubes~~ tubes increase from ~~the~~ equator

pro

con

(e)



thin ribbed shell

NACA 0009 Airfoil

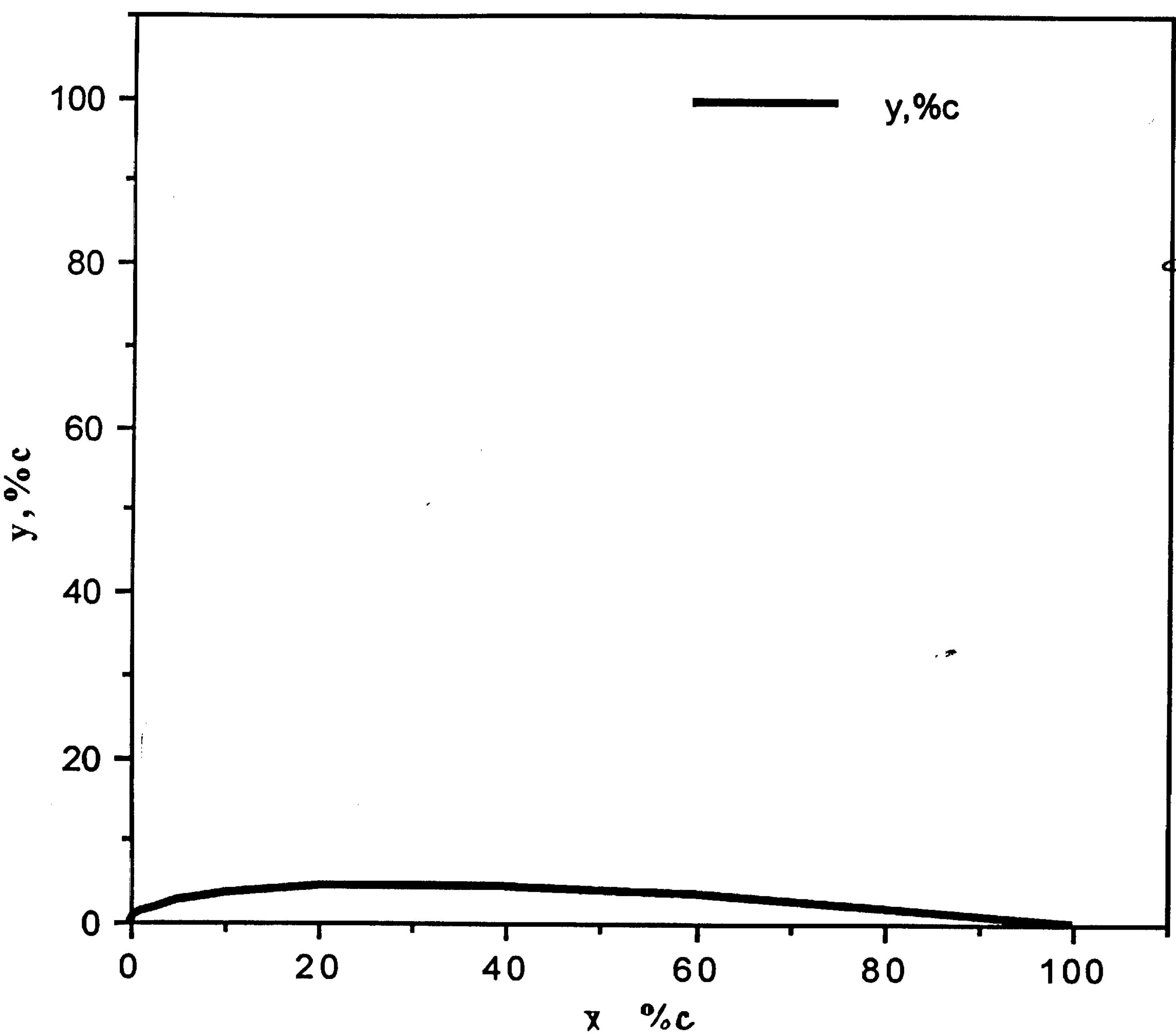


Figure 16

This last part of Appendix Could have been presented better!

(ROTATE, FOR EXAMPLE)

T-62

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Giro mill Dimensions

From Wind Energy Theory, it has been determined that the maximum power that can be extracted is.

① $P_{mech} = C_p\,(0.5\,\rho\,A_{sw}\,V_o^3)$ *(See Pg T-8)

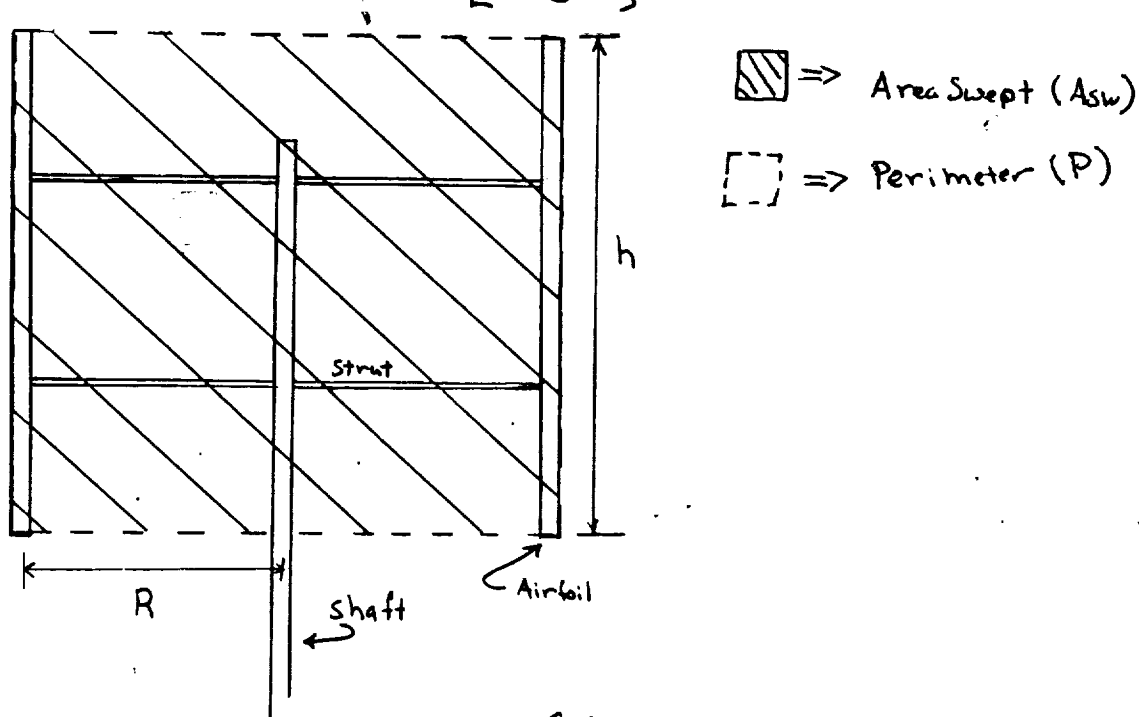
C_p => Coefficient of Power

ρ => Density of Martian air [= 0.01665 kg/m³]

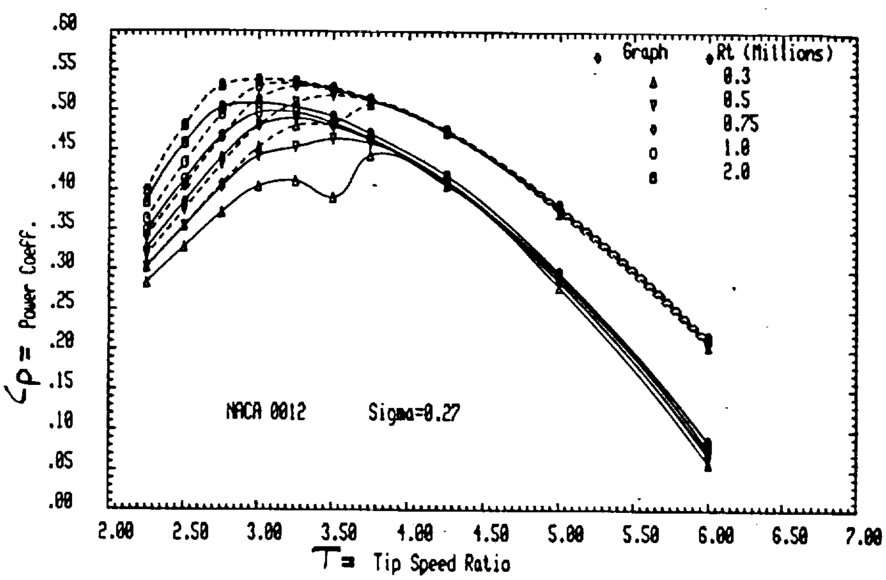
A_{sw} => Area Swept

V_o => Free Stream velocity

Giro mill Drawing [Fig 1]



Determining from graph below $C_{p\,max} \approx .5$ at a $T \approx 3.0$ for Giromill. Using an average wind speed on the surface of 6 m/s and $\rho = 0.01665\ kg/m^3$



* From Refer [9]

$$P_{mech} = (.5)\left[.5\left(.01665\ \tfrac{kg}{m^3}\right) A_{sw}\left(6\ \tfrac{m}{s}\right)^3\right]$$

$$= 0.89991\ A_{sw}$$

Assuming efficiency $\eta = .80$ why?

② $P_{mech} = (.80).89991\ A_{sw}$

$= 0.719928\ A_{sw}$

Setting Eq ② equal to the required 1 kWh power, and solving for Asw.

$$② \; A_{SW} = \frac{(1.0\,\text{kWH})}{0.719928} = 1.389\,m^2$$ * see Fig I for Asw representation.

<u>Determining h and R for gimill See Fig I.</u>

Looking at Fig I

$$A_{SW} = 2Rh = 1.389\,m^2$$

which is equal to Eq ②

Looking at Fig I again

$$③ \; P = 4R + 2h$$

Minimize Eq ③ in conjunction with Eq ② to obtain R and h

Should have been separated a little better

From Eq ②

$$Rh = 0.6945$$

Setting $h = x$

$$R = \frac{0.6945}{x}$$

Plugging h and R into Eq ③

$$f(x) = P = 4\left(\frac{0.6945}{x}\right) + 2x$$

$$= 2.778\,(1/x) + 2x$$

$$\frac{df(x)}{dx} = f'(x) = -2.778\,(1/x^2) + 2$$

Setting $f'(x) = 0$ Min

$$\frac{1}{x^2} = 0.71993$$

$$x = 1.17857\,m$$

Plugging x back into h and R

$$\boxed{\begin{aligned} h &= 1.17857\,m \\ R &= 0.5546\,m \end{aligned}}$$

Tip Speed Ratio and Angular Velocity

Know from graph in Giromill Dimension $T \approx 3.0$

Equation for T is:

$$① \quad T = \frac{\text{Rotational velocity}}{\text{Free Stream velocity}} = \frac{\omega R}{V}$$

$\omega \Rightarrow$ Angular velocity of Giromill in operation

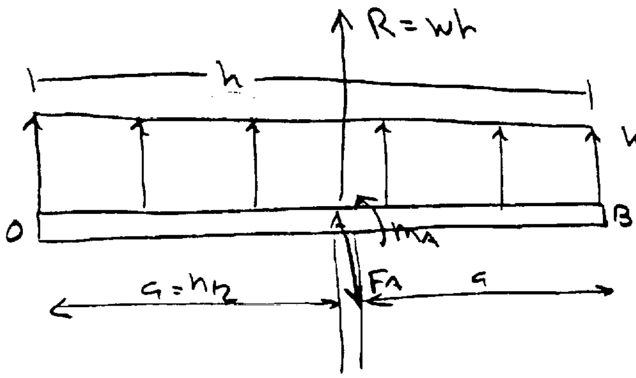
$R \Rightarrow$ Radius of Giromill

$V \Rightarrow$ free Stream velocity

Using $R = .5546\,m$ determined in dimensions, and assuming $V = 6\,m/s$ on Martian surface. Setting $T = 3.0$ and solving Eq ① for ω

$$\boxed{\omega = \frac{(3.0)(6\,m/s)}{0.5546\,m} = \underline{32.45\ rad/s}}$$

SHEAR and BENDING MOMENT DIAGRAMS for ONE STRUT CASE



Assumptions

- Uniform Disturbuted load
- Beam represents an air foil
- Strut is fastened.

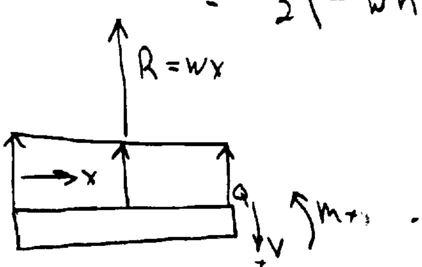
THE "ONE-STRUT DESIGN" SEEMS TO HAVE MANY DISADVANTAGES
- COULD HAVE BEEN "SHELVED" EARLY

$\uparrow^{+} \sum F_y = R - F_A \qquad F_A = R = wh$

$\sum M_O = R(h/2) - F_A(h/2) + M_A$

$M_A = -R(h/2) + F_A(h/2)$

$= \frac{h}{2}(-wh + wh) = 0$



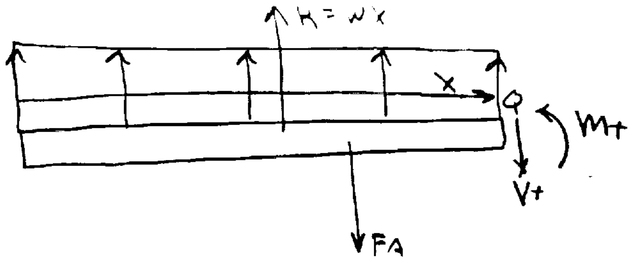
FBD I

$\sum F_y = R - V$

$V = R = wx \qquad (0 < x < h/2)$

$\sum M_Q = -R(x/2) + M$

$M = R(x/2) \quad (0 < x < h/2)$



$+\uparrow \sum F_y = R - F_A - V$

$$V = R - F_A = wx - wh = \underline{w(x-h)} \quad (h/2 < x < h)$$

$$\overset{\curvearrowleft +}{\sum M_Q} = M - R\,(x/2) + F_A\,(x - h/2)$$

$$M = \frac{wx^2}{2} - wh(x - h/2)$$

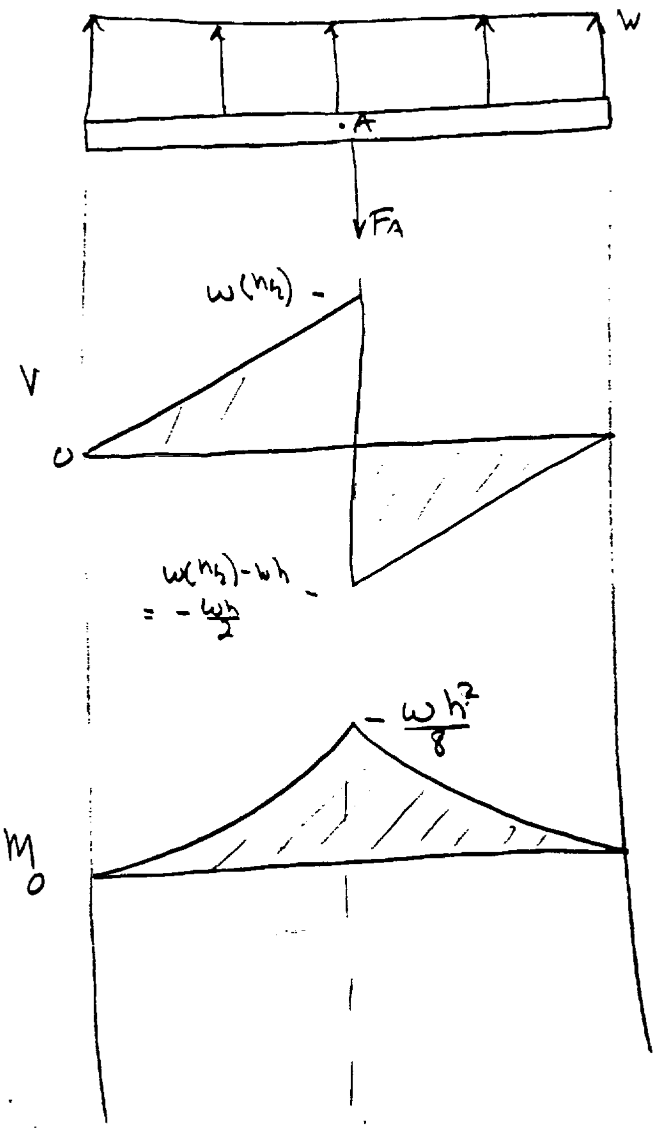
$$M = \underline{w\left(\frac{x^2}{2} - h(x - h/2)\right)} \quad \left(\frac{h}{2} < x < h\right)$$

Summary (Symmetric about Point A)

$O \to A$ $A \to B$

$V = wx$ $V = w(x-h)$

$M = w\frac{x^2}{2}$ $M = w\left(\frac{x^2}{2} - h\left(x - \frac{h}{2}\right)\right)$

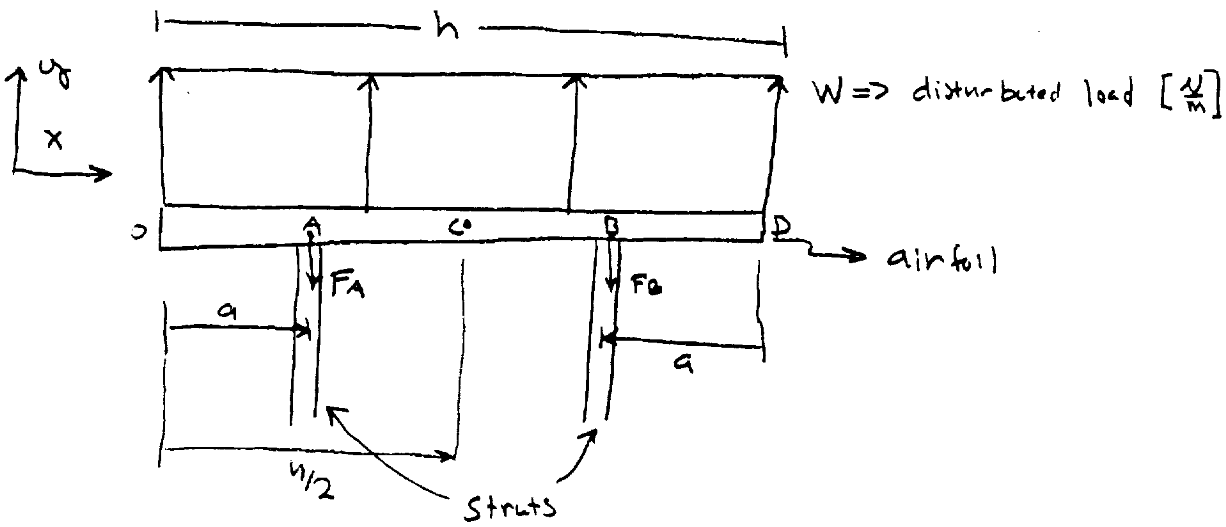


The largest moment occurs at A

$$\boxed{M_A = \frac{wh^2}{8}}$$

$$\boxed{M_A)_1 = \frac{wh^2}{8}}$$

Shear and Bending Moment Diagrams and Strut Position for Two Strut Case



Assumptions

- Uniform Disturbed load
- Beam repesents an airfoil
- Struts are pin connected

Force and Moment Balance

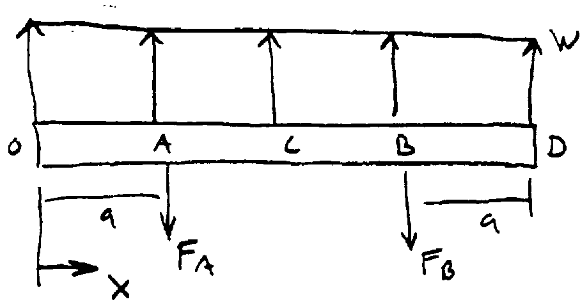
$$\circlearrowleft + \; \Sigma M_y = 0 = wh(h/2 - a) - F_B(h-2a) \quad [\text{about pt } A]$$

$$F_B = \frac{wh}{2}$$

$$\uparrow \Sigma F_y = 0 = wh - F_A - F_B$$

$$\boxed{F_A = F_B = \frac{wh}{2} \;(-y\,\text{dir})}$$

Free Body diagram of airfoil



Force and Moment Balance for $0 \leq x \leq A$

$$+\uparrow \sum F_y = wx - V = 0$$

$$V = wx \; [N]$$

$$\sum M = 0 = M - wx(x/2)$$

$$M = \frac{w}{2}x^2 \; [N \cdot m]$$

Force and Moment Balance for $A \leq x \leq C$

$$+\uparrow \sum F_y = wx - F_a - V \qquad \boxed{F_a = \frac{wh}{2}}$$

$$V = wx - \frac{wh}{2} = \underline{w\left(x - \frac{h}{2}\right)} \; [N]$$

$$\curvearrowleft \sum M = 0 = M + F_a(x-a) - wx(x/2) \qquad \left[F_a = \frac{wh}{2}\right]$$

$$M = -\frac{wh}{2}(x-a) + w\frac{x^2}{2} = \underline{\frac{w}{2}(x^2 - h(x-a))} \quad [N \cdot m]$$

<u>In summary</u>

<u>From O to A: $(0 < x < a)$</u>

$V = wx$

① $M = \frac{w}{2}x^2$

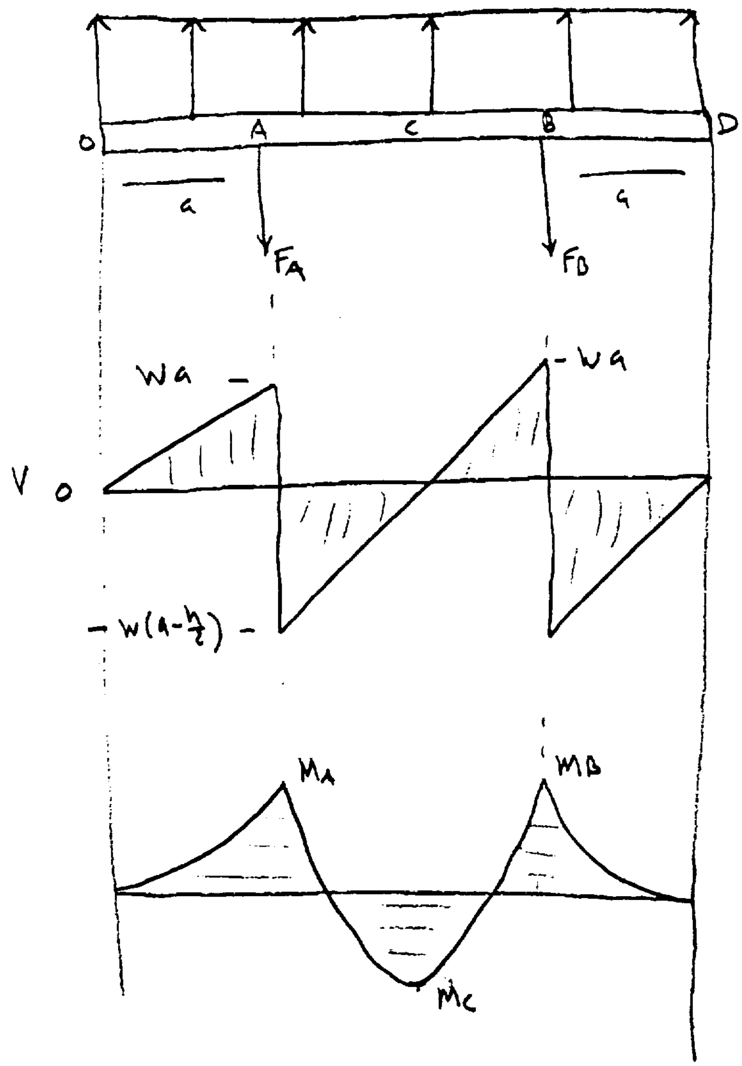
<u>From A to C : $(a < x < \frac{h}{2})$</u>

$V = w(x - h/2)$

② $M = \frac{w}{2}(x^2 - h(x-a))$

Due to symmetry abt C load is a mirror image beyond C.

Shear and Moment Diagrams



AGREE

The largest positive moment occurs at A

$$\therefore M_A = \frac{W}{2} x^2 \Big|_{x=a} = \frac{W}{2} a^2 \qquad \text{Eq } ①$$

$$\boxed{M_A = M_B = \frac{W}{2} a^2}$$

The largest negative moment occurs at C

$$\therefore M_C = \frac{W}{2}\left(x^2 - h(x-a)\right)\Big|_{x=\frac{h}{2}} = \frac{W}{2}\left(\frac{h^2}{4} - h\left(\frac{h}{2} - a\right)\right) \quad \text{Eq } 2$$

$$\boxed{M_C = \frac{wh}{8}(-h + 4a)}$$

Notice as $M_A \uparrow$ $M_C \downarrow$ and $M_A \downarrow$ $M_C \uparrow$

Setting $|M_A| = |M_C|$ to determine a and minimize

Moments.

$$\left|\frac{W}{2}a^2\right| = \left|\frac{wh}{8}(-h + 4a)\right|$$

$$\left|\frac{W}{2}a^2\right| = \left|-\frac{wh^2}{8} + \frac{wha}{2}\right|$$

because: $\frac{wh^2}{8} = \frac{wha}{2}$

$$a = \frac{\cancel{w}h^{\cancel{2}}}{8}\frac{2}{\cancel{w}\cancel{h}} = \frac{1}{4}h$$

$\therefore$ if $a < \frac{1}{4}h$ or $.250h$ M_C is negative

COMPARING Max. MOMENT for ONE and TWO STRUT CASE

Max Moment

two Struts $(M_A)_2 = \frac{w}{2}a^2$ with $a = .207h$

Pg 12 of Appendix

$= \underline{0.02142\ wh^2}$

one strut $(M_A)_1 = \frac{wh^2}{8} = \underline{0.125\ wh^2}$

Pg 8 of Appendix

$$\boxed{\begin{array}{l} \therefore\ (M_A)_2 < (M_A)_1 \\ (M_A)_2 = 5.84\ (M_A)_1 \end{array}}$$

$$\boxed{(M_A)_2 = 17.1\%\ (M_A)_1}$$

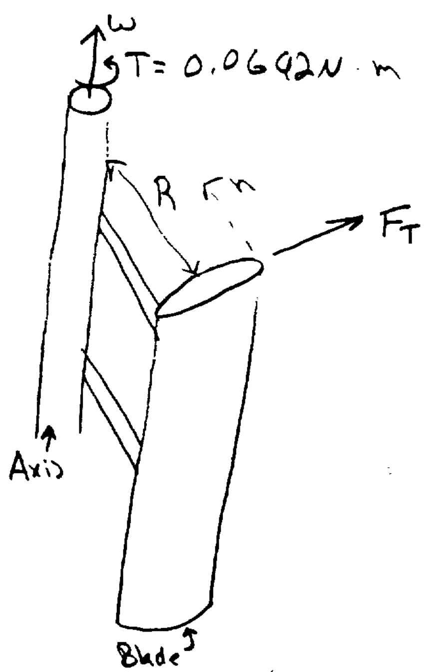
JUSTIFICATION for NEGLIGIBLE

AERODYNAMIC LOADS

Aerodynamic Load

It has been determined the maximum resulting Torque on the gimmill shaft is:

$T_{max} \simeq 0.0642 N \cdot m$ * (Determined for Aero. loads)



We know the components of Aerodynamic loads in the tangential direction multiplied by the radius of gimmill result in the Torque about the shaft.

F_T (Aero load in tangential direction)

It's also safe to say F_T is the largest resultant Aero. load otherwise the max. Torque (... Power) would be reduced.

Hence

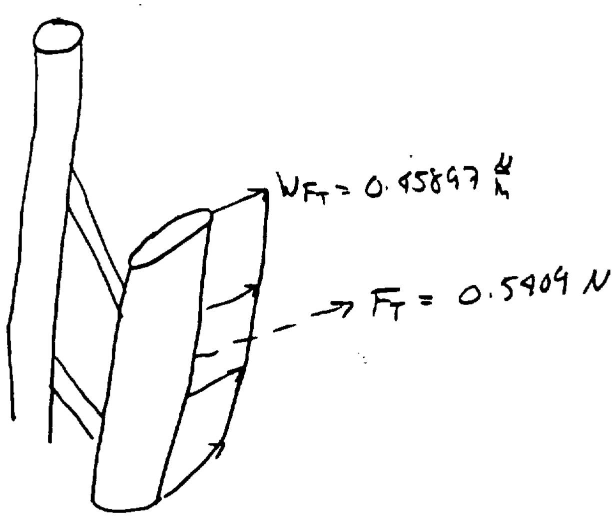
Total $T = .0642\ N\cdot m$ = twice the $F_T \times$ radius R

$$T = .0642\ N\cdot m = 2\,(F_T \times R)$$

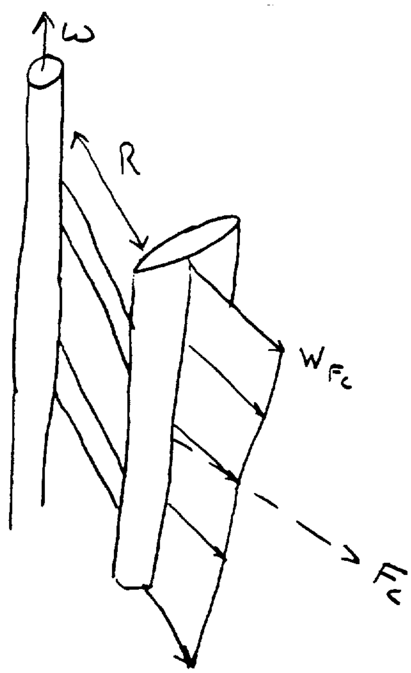
Solving for $F_T = \dfrac{T}{2R} = \dfrac{.0642}{2\times(.5546m)}$

$$F_T = 0.05788\ N$$

or $W_{F_T} = \dfrac{F_T}{h} = 0.0491\ \frac{N}{m}$



Centrifugal load



Know centrifugal load (F_c) is:

$$F_c = \omega^2 R \,\text{Mass}$$

Mass of Blade = .19269 kg

$$F_c = (32.45)^2 (.5546)(.19269)$$

$$= 112.53\,N$$

$$W_{Fc} = 9548\ \frac{N}{m}$$

Therefore

$$\boxed{\frac{F_T}{F_c} = \frac{0.05788}{112.53N} = 5.144\times10^{-4} \qquad \text{or} \qquad F_T = .0514\% \ F_c}$$

This is negligible, and allow design to be driving by the centrifugal loads.

STRESS CALCULATION For σ_x in x-dir

Due to CENTRIFUGAL LOADS on BLADES

⓪ $\sigma_x = \dfrac{My}{I_x}$

$M \Rightarrow$ Moment

$y \Rightarrow$ Distance from Neutral Axis

$I_x \Rightarrow$ Moment of Inertia

① $M = \dfrac{W}{2} a^2$ * From Eq ① in Strut Position for two strut case

② $W = \dfrac{\omega^2 R \text{ Mass}}{h}$

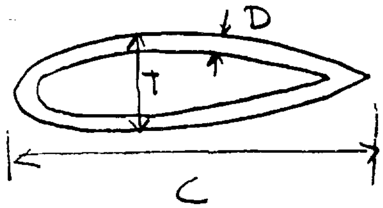
Mass $= h \, \rho_{Al\text{-}B} \, A_{cs}$

$A_{cs} \Rightarrow$ Cross Section of Airfoil (Blade)

* $A_{cs} \approx \underbrace{.6850833333}_{\phi} \, tC$

*(For derivation See Table of Context Appendix (G))

③ $\text{Mass} = h\, \rho_{Al-B}\, \underbrace{\phi\left(TC - (T-2D)(C-2D)\right)}_{A_{cs}}$



Plug ③ into ②, and ② into ①

④ $M = \dfrac{\omega^2 R a^2}{2h}\, h\, \rho_{Al-B}\, \phi\left(TC - (T-2D)(C-2D)\right)$

⑤ $y = \dfrac{T}{2}$

⑥ $I_x \approx \underbrace{.03946745767}_{\beta}\left(T^3 C - (T-2D)^3 (C-2D)\right)$

* Subtracting inner airfoil shape from out airfoil shape

* For derivation for I_x see Table of Content for Appendix G

$\therefore\ \sigma_x = \dfrac{M y}{I_x}$

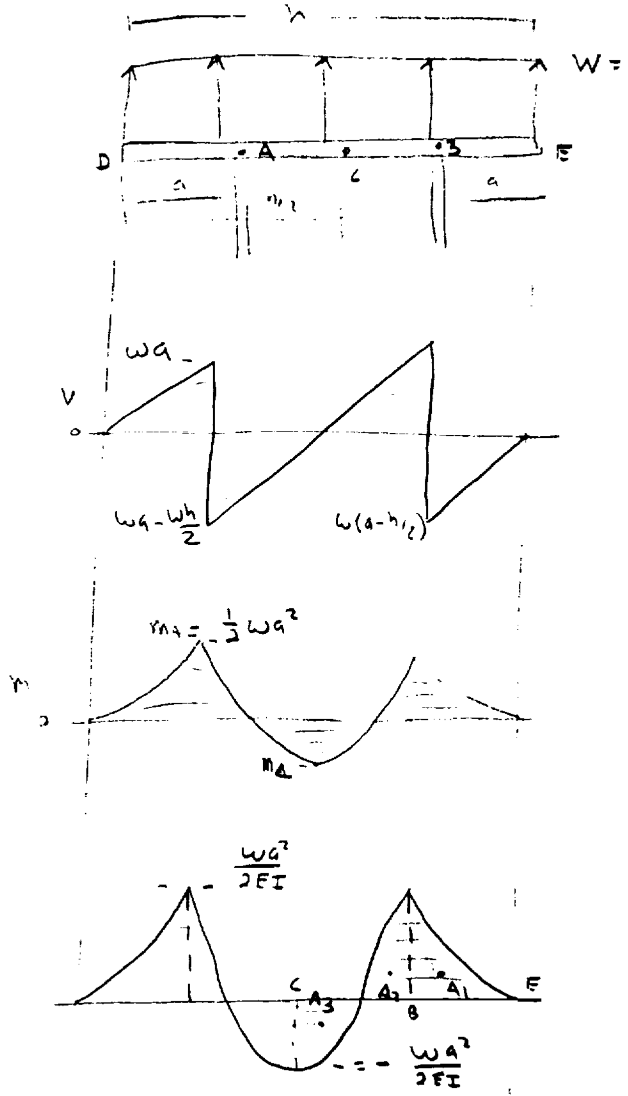
$$\sigma_x = \underbrace{\frac{\omega^2 R a^2 \phi\, h\, \rho_{Al-B}}{2 h \beta\, 2}}_{\Delta}\ \frac{\left(TC - (T-2D)(C-2D)\right) \cancel{x} T}{\left(T^3 C - (T-2D)^3 (C-2D)\right)}$$

$\Delta = 4.004421526 \times 10^5$

7 $$\boxed{\nabla x = \Delta \frac{(TC - (T-2D)(C-2D)) \times T}{(T^3C - (T-2D)^3(C-2D))}}$$

DEFLECTION of Blade End

(M/EI) Diagram



Shear and Bending Diagram identical from strut position case

Pg G 12

Reference Tangent

Point C is a point of symmetry (midpoint)

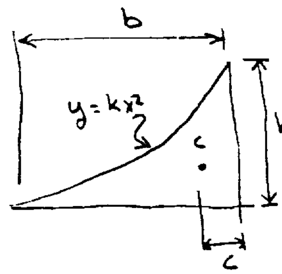
$\theta_C = 0$

$\theta_E = \theta_C + \theta_{E/C} = \theta_{E/C}$

$y_E = t_{E/C} - t_{B/C}$

Slope at E

Referring to the (M/EI) diagram and using the first moment-area theorem, we will



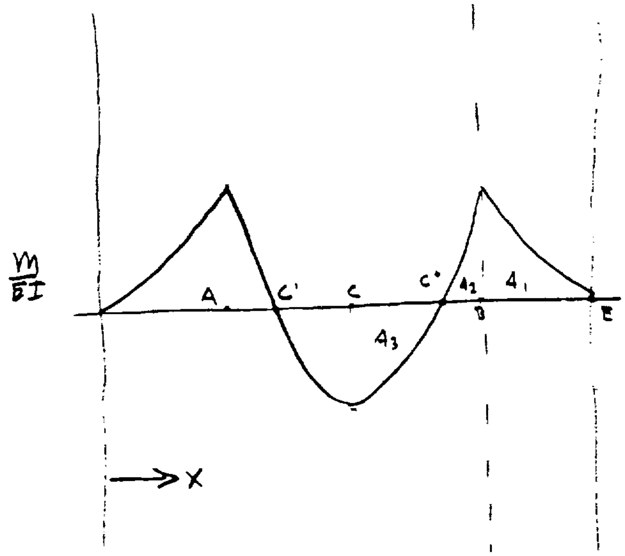
Area $= \frac{bh}{3}$ $\quad c = \frac{b}{4}$

A_1: $b = a$

$h = \frac{wa^2}{2EI}$

$$A_1 = +\frac{1}{3}(a)\left(\frac{wa^2}{2EI}\right) = +\frac{wa^3}{6EI}$$

$c = \frac{1}{4}(a)$



A_2: $\quad h = \dfrac{wa^2}{2EI}$

b: $C'' \to B = \cancel{C'' + (h - a - C'')} = h - C'' - a$

from A & B

$$M = \frac{w}{2}\left(x^2 - h(x-a)\right)$$

when does $M = 0$ at C' ad C''

$$0 = x^2 - hx + ha$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{h \pm \sqrt{h^2 - 4(1)(ha)}}{2(1)}$$

$$= \frac{h \pm \sqrt{h^2 - 4ha}}{2} = \frac{h \pm \sqrt{h(h-4a)}}{2}$$

$$= \frac{1}{2}\left(h \pm \sqrt{h(h-4a)}\right)$$

$$C' = \frac{1}{2}\left(h - \sqrt{h(h-4a)}\right)$$

$$C'' = \frac{1}{2}\left(h + \sqrt{h(h-4a)}\right)$$

Example

$h = 1.0\,m$

$a = .207\,(1.0\,m) = .207\,m$

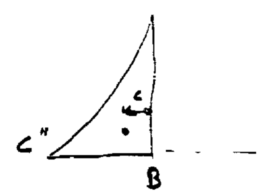
$$x = \tfrac{1}{2}\left(1 \pm \sqrt{(1)(1-4(.207))}\right) \qquad 0.207 < x < \overset{0.793}{\cancel{0.586}}$$

$$x = \tfrac{1}{2}\left(1 \pm \sqrt{1-4(.207)}\right)$$

$$x = \tfrac{1}{2}\left(1 \pm 0.4197\right)$$

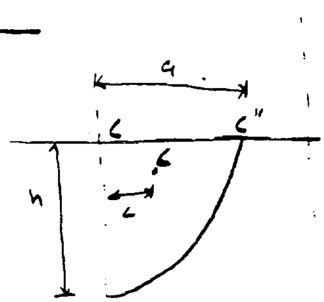
$$\boxed{x = 0.7074\,m \text{ or } 0.2926\,m}$$

$$A_2 = \frac{1}{3}\left(\frac{Wa^2}{2EI}\right)\left(h - c'' - a\right)$$



$$c = \frac{1}{4}\left(h - c'' - a\right)$$

A_3



$a = c'' - c$

$h = \dfrac{Wa^2}{2EI}$

$$A_3 = -\frac{2}{3}ah = -\frac{2}{3}\left(c''-c\right)\left(\frac{Wa^2}{2EI}\right)$$

$$c = \frac{3}{8}a = \frac{3}{8}\left(c''-c\right)$$

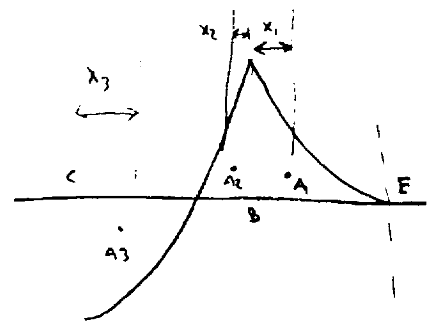
$$\theta_E = \theta_{E/C} = A_1 + A_2 + A_3$$

$$= \frac{wa^3}{6EI} + \frac{wa^2}{6EI}(h - c'' - a) - \frac{2wa^2}{6EI}(c'' - c)$$

$$= \frac{wa^3}{6EI}(1 + h - c'' - a - 2c'' + 2c)$$

$$= \frac{wa^3}{6EI}(1 + h - a - 3c'' + 2c)$$

<u>Deflection at E</u>



$$y_E = t_{E/C} - t_{B/C}$$

$$t_{E/C} = (-A_3)\left(\frac{h}{2} - x_3\right) + A_2(x_2 + a) + A_1(a - x_1)$$

$$x_1 = \frac{1}{4}a$$

$$x_2 = \frac{1}{4}(h - c'' - a)$$

$$x_3 = \frac{3}{8}(c'' - c)$$

$$t_{B/C} = (-A_3)\left(\frac{1}{2}h - a - \frac{3}{8}(c'' - c)\right) + (A_2)\left(\frac{1}{4}(h - c'' - a)\right)$$

$$y_E = t_{E/L} - t_{B/L}$$

$$= (-A_3)\left(\frac{h}{2} - \frac{3}{8}(c''-c)\right) + A_2\left(\frac{1}{4}(h-c''-a)+a\right) + A_1\left(a - \frac{1}{4}a\right)$$

$$-(-A_3)\left(\frac{h}{2} - a - \frac{3}{8}(c''-c)\right) - A_2\left(\frac{1}{4}(h-c''-a)\right)$$

$$= -A_3(a) + A_2(a) + A_1\left(\frac{3}{4}a\right)$$

$$= a\left(\frac{3}{4}A_1 + A_2 - A_3\right)$$

$$= a\left(\frac{3}{4}\,\frac{Wa^3}{6EI} + \frac{1}{3}\left(\frac{Wa^2}{2EI}\right)(h-c''-a) - \frac{2}{3}(c''-c)\left(\frac{Wa^2}{2EI}\right)\right)$$

$$= \frac{Wa^3}{6EI}\left(\frac{3}{4}a + h - c'' - a - 2c'' + 2c\right)$$

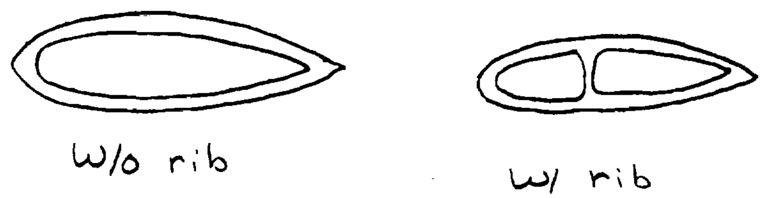
$$\boxed{y_E = \frac{Wa^3}{6EI}\left(-\frac{1}{4}a + h - 3c'' + 2c\right)} \quad \text{Eq } 1$$

INCOMPLETE ?

COMPARISON of DEFLECTION of BLADE

WITH and WITHOUT RIB

Blades



From Deflect of Blade and in Appendix know [Eq ①]

Eq ①

$$yE = \frac{wa^2}{6EI} \underbrace{\left(-\tfrac{1}{4}a + h - 3c'' + 2c\right)}_{\Delta''}$$ * see Pg G27 Eq①

$$yE|_{w/o} = \frac{W_{w/o}\, a^2}{6E\, I_{w/o}} \Delta \,, \qquad yE|_{w/} = \frac{W_{w/}\, a^2}{6E\, I_{w/o}}$$

$$② \quad \delta = \frac{yE|_{w/o}}{yE|_{w/}} = \frac{W_{w/o}\, I_{w/}}{W_{w/}\, I_{w/o}} = \frac{Mass|_{w/o}\, I_{w/}}{Mass|_{w/}\, I_{w/o}}$$

Sub in $W = \dfrac{\omega^2 R \,Mass}{h}$

$Mass|_{w/o} = .19269$ kg , $Mass|_{w/} = .20636$ kg

$I_{w/o} = 8.51 \times 10^{-10}\ m^4$, $I_{w/} = 8.79 \times 10^{-10} m^4$

$I_x \cong 0.03941\, T^3 C$ * See Appendix for calculation

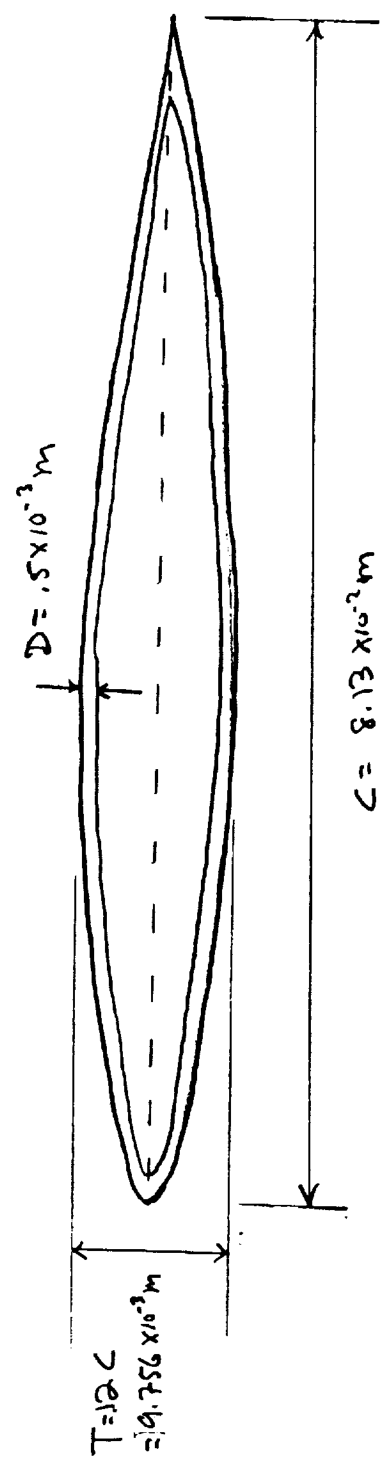
$\therefore$ Eq ②

$\delta = 0.96444$

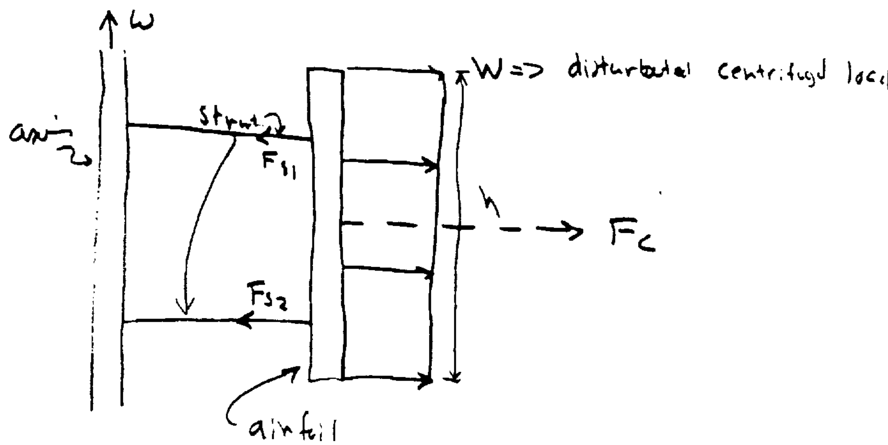
and Blade deflects slightly more with rib

no advantage

BLADE STRUCTURE DIMENSIONS



STRUT STRUCTURE



Considering only centripetal forces. Aerodynamic loads are negligible.

(1) $$W = \frac{F_c}{h} = \frac{\omega^2 R \text{ Mass (airfoil)}}{h} = \frac{(32.45\,\text{r/s})^2 (.5546\,\text{m})(.19269\,\text{kg})}{(1.17857\,\text{m})}$$

$$\underline{W = 95.4777 \tfrac{N}{m}}$$

$$\underline{F_c = 112.5\,N}$$

$$F_S = F_{S1} = F_{S2} = \frac{F_c}{2} = 56.25$$ [F_c is split between each strut]

Due to low density of Martian air a circular cross section strut was used, with negligible drag consideration

Determine radius of solid strut made of Aluminum Boron.

$$\boxed{\begin{array}{l} \text{Al-B} \\ \sigma_{ult} = 1.32\ \text{GPa} \\ \rho = 2650\ \text{kg/m}^3 \end{array}}$$

$$\sigma_x = \frac{F_s}{A_{str}} = \frac{F_s}{\pi r^2} = 1.32\ \text{GPa}$$

$$r^2 = \frac{F_s}{\pi\,(1.32\ \text{GPa})}$$

$$r = 1.165 \times 10^{-4}\ \text{m}$$

$$\boxed{\text{dia (of strut)} = 2.3293 \times 10^{-4}\ \text{m} = .23293\ \text{mm}}$$

This dia for a solid strut is extremely small. Too small

So consider we want one strut to be 5% of Total Mass of one airfoil.

Mass airfoil = 0.19269 kg

(M_S) = Mass strut = 0.05 (.19269) kg = 9.6345×10^{-3} kg

Determine for (M_S) what radius has to be

$$r^2 = \frac{M_S}{R\,\pi\,\rho_{Al-B}} = \frac{M_S}{(.5596\,m)(\pi)(2650\,\frac{kg}{m^3})}$$

$r = 1.44 \times 10^{-3}\ m \cong 1.5\ mm$

$$\boxed{\therefore\ dia = 3.0\ mm}$$

diameter of solid shaft

A diameter equal to 3.0 mm is a bit easier to deal with, and is still only a mass of

$$Mass\,(strut)\Big|_{r=1.5mm} = R\,\rho_{Al-B}\,\pi r^2 = 10.39\ g$$

or 5.39% of the air foil

Check stress for dia = 3.0 mm

① $$\boxed{\tau_x = \frac{F_3}{\pi r^2} = 7.957747 \times 10^{6}\ Pa}$$

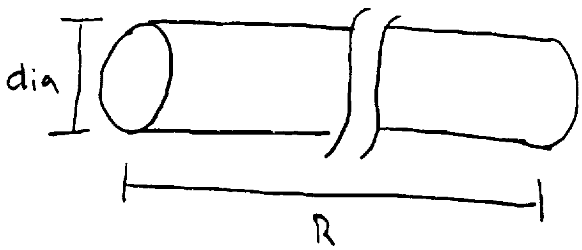
Resulting in a factor of safety of

② $$\boxed{F.S. = \frac{\tau_{ult}}{\tau_x} = 165.87}$$

Much rather have big factor of safety than dia = .233 mm

Mass is of little concern when talking about a few grams.

Find Dimension of Struts



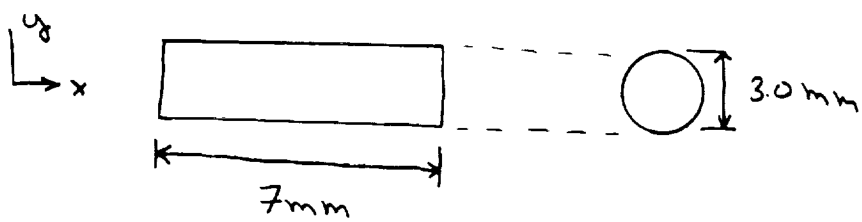
$R = .5546\ m = 554.6\,mm$

$dia = 3.0 \times 10^{-3}\ m = 3.0\,mm$

This big factor of Saftey (165.87) above is helpful, in that it is so large, stand up bending stress on strut can be confindently considered negligible

Dimensions for pin connection

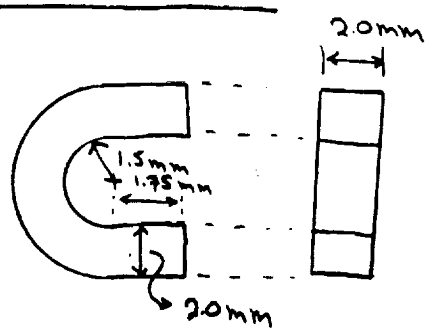
Small Cylinder



$$\text{Mass} = (7\times10^{-3})(\pi\,(1.5\times10^{-3})^2)\,\rho_{Al\text{-}B}$$

$$= 1.3112\times10^{-4}\ \text{kg}$$

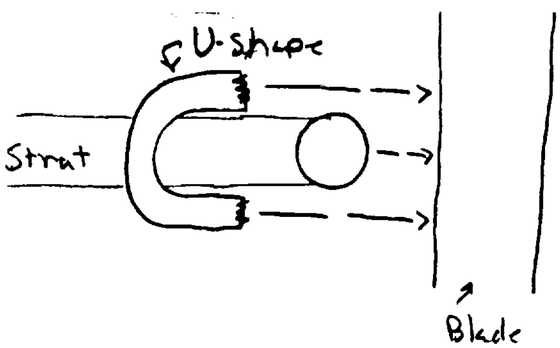
U-shaped fitting



$$\text{Mass} = (2.0\times10^{-3})\left(\pi\left[(3.5\times10^{-3})^2-(1.5\times10^{-3})^2\right]\right)\rho_{Al\text{-}B}$$
$$+\ (2.0\times10^{-3})(1.75\times10^{-3})(2.0\times10^{-3})\,\rho_{Al\text{-}B}$$

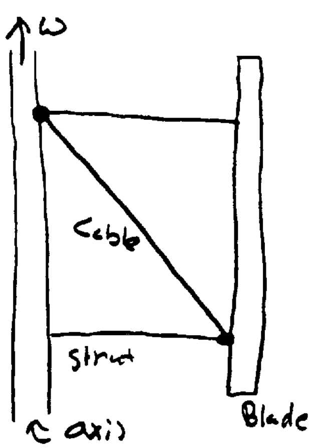
$$= 1.851\times10^{-4}\ \text{kg}$$

Method of attaching Strut

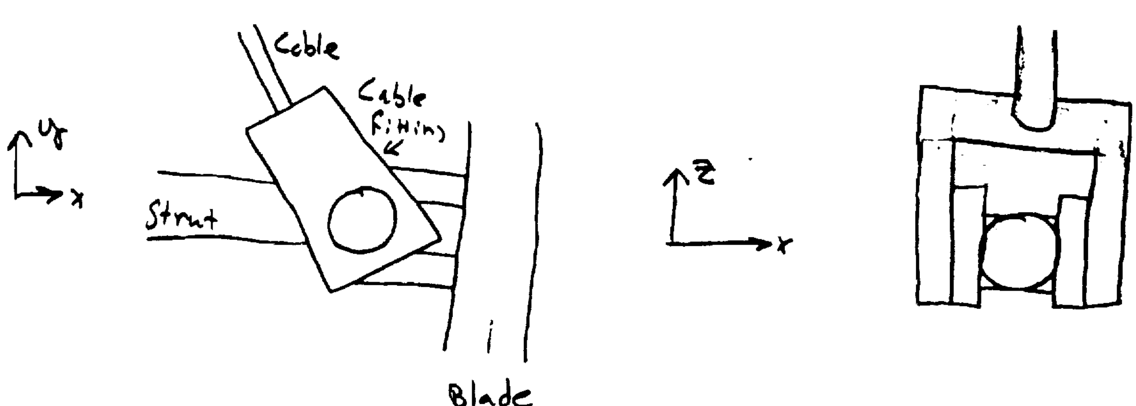


the U-shape fitting is welded to the blade areas

Special consideration for cable attachment to bottom blade-strut connection. See below in fig.



Cable is attached as so

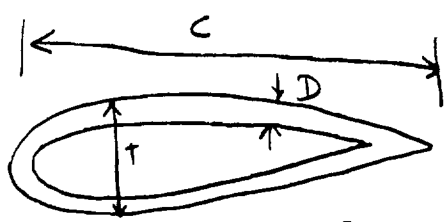


STRESS CALCULATION on BLADE

For a hollow blade the stress equation derived in Appendix Pg [ ] for σ_x is as follows

DOCUMENT

$$\sigma_x = \Delta \frac{(TC - (T-2D)(C-2D)) \times T}{(T^3 C - (T-2D)^3 (C-2D))}$$



Blade Cross Section

Sample Calculation of σ_x of different D

D (m)	σ_x (Pa)
$\frac{T}{2}$ (solid)	4.1045×10^7
30mm	2.788×10^7
1.5mm	2.009×10^7
1.0mm	1.8076×10^7
.5mm	1.629760792×10^7

* All σ_x calculated for
C= 8.13 cm
T= 9.756mm

* Actually Stress for design Blade

Table 1

Notice in sample Calculation σ_x is surprisingly decreasing with decreasing thickness D of Blade.

In other words looking at Equation for σ_x:

$$\sigma_x = \frac{M\,y}{I_x}$$

This tells us the the Moment (M) is decreasing fast then Moment of Inertia (I_x) as thickness D is decreased. See Eq ④ and ⑥ in Appendix Page G 20 for M and I_x as a function of Thickness D.

This relation between M and I_x tells us that D can be infinitely thin and stresses will decrease never reaching materials ultimate strength.

For designed thickness $D = .5 \times 10^{-3}$ m of Blade

$\sigma_x = 1.629766792 \times 10^{7}$

which results in a Factor of Safety of:

$$F.S. = \frac{1.32\ GPa}{\sigma_x} = 81.$$

We are stuck with the large factor of safety.

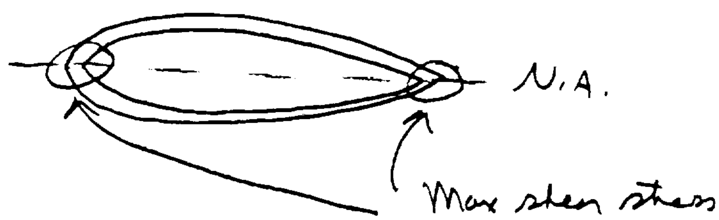
It will only increase with reduced mass,

which is what we ultimately want.

CALCULATION OF SHEAR STRESS

in BLADE

It's know that for a symmetric airfoil the maximum shear will occur were max. Shear Force and Neutral Axis meet.



① Shear stress $(\tau) = \dfrac{V Q_x}{I_x t}$

$V \Rightarrow$ Shear Force

$Q_x \Rightarrow$ 1st Moment of Area

$I_x \Rightarrow$ Moment of Inertia

$t \Rightarrow$ Thickness crossing the N.A.

$Q_x \simeq (0.068846680375)\, T^2 C$

* See Appendix G Table 6 content for Derivation

$V = wa - \frac{wL}{2}$

* See Appendix Pg G11-G12 for shear diagram

Sample Calculation for design Blade

$\tau_{shear\,(Al\text{-}B)} = 0.08\,GPa = 80\,MPa$ (UNITS)

$Q_x = 1.088945169 \times 10^{-7}\, m^3$

$V = 32.57\, N$

$I_x = 8.50 \times 10^{-10}\, m^4$

$t = 2(D) \Rightarrow D =$ thickness of Blade

$= 2(.5 \times 10^{-3})$

Plug into Eq ①

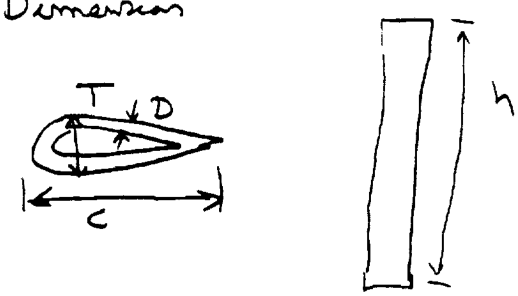
$$\boxed{\tau = \frac{VQ_x}{I_x t} = 4.1\,MPa\,, \quad F.S. = \frac{0.08\,GPa}{4.1\,MPa} = 19.5}$$

This Factor of Safety of 19.5 is large, but due to limited manufacturability of Aluminum Boron little can be done.

TOTAL MASS of BLADES & STRUTS

Mass of one blade

Dimensions



$$Mass = h \rho_{Al-B} A_{CS}$$

$$= h \rho_{Al-B} \left(.6850833337 \left(TC - (T-2D)(C-2D)\right)\right)$$

For $T = 9.756 \times 10^{-3} m$, $C = 8.13 \times 10^{-2} m$, $D = .5 \times 10^{-3} m$

$\rho_{Al-B} = 2650\ kg/m^2$

$$\boxed{M_B = Mass\Big|_{one\ blade} = 0.19269\ kg}$$

Mass of one Strut (*) See Appendix Pg [ ] M_S

$$\boxed{M_S = 10.39\,g = .01039\,kg}$$

Mass of on Connecting fastener (x) Sees Appendix Pg G 35-38

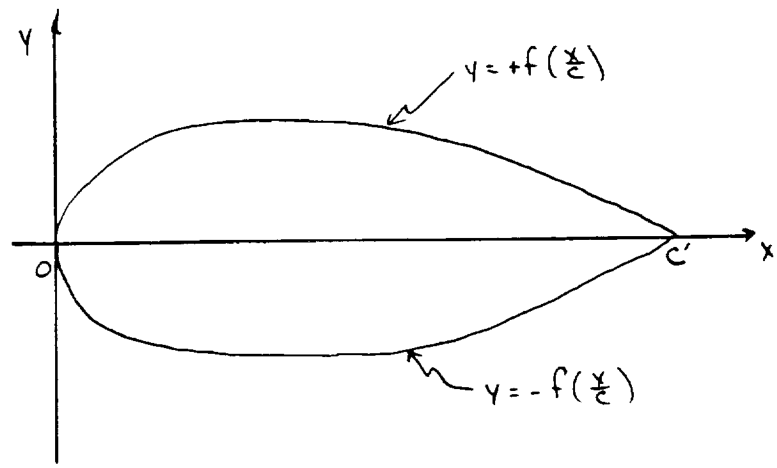
$$\boxed{M_U = Mass\Big|_{one\ U\text{-}shape} = 1.85\times10^{-4}\,kg}$$

$$\boxed{M_{cL} = mass\Big|_{on\ cylinder} = 1.13112\times10^{-4}\,kg}$$

Total mass of Blades and Struts

$$\boxed{\begin{aligned} Mass_{Total} &= 2M_B + 4M_S + 8M_U + 4M_{cL} \\ &= \underline{0.42887\ kg} \end{aligned}}$$

Moment of Inertia of a Symmetric Airfoil (I_{xx})



$$f\left(\frac{x}{c}\right) = \left(\frac{t}{0.2}\right)\left[A\sqrt{\frac{x}{c'}} - B\left(\frac{x}{c'}\right) - C\left(\frac{x}{c'}\right)^2 + D\left(\frac{x}{c'}\right)^3 - E\left(\frac{x}{c'}\right)^4\right]$$

where:

t = maximum thickness of blade

c' = chord length

$A = .2969$

$B = .126$

$C = .3516$

$D = .2843$

$E = .1015$

$$I_{xx} = \int y^2 \, dA$$

$$I_{xx} = \int_0^{c'} \int_{-f(\frac{x}{c'})}^{+f(\frac{x}{c})} y^2 \, dy \, dx$$

Moment of Inertia of a Symmetric Airfoil (I_{xx}) con't

By symmetry,

$$I_{xx} = \frac{2}{3}\int_0^{c'} f^3\left(\tfrac{x}{c}\right)dx$$

$$f^2\left(\tfrac{x}{c}\right) = \left(\frac{t^2}{0.04}\right)\Big[A^2\left(\tfrac{x}{c}\right) - 2AB\left(\tfrac{x}{c}\right)^{3/2} + B^2\left(\tfrac{x}{c}\right)^2 - 2AC\left(\tfrac{x}{c'}\right)^{5/2} + 2BC\left(\tfrac{x}{c'}\right)^3$$

$$+ 2AD\left(\tfrac{x}{c'}\right)^{7/2} + (C^2 - 2BD)\left(\tfrac{x}{c'}\right)^4 - 2AE\left(\tfrac{x}{c'}\right)^{9/2}$$

$$+ 2(BE - CD)\left(\tfrac{x}{c'}\right)^5 + (2CE + D^2)\left(\tfrac{x}{c'}\right)^6 - 2DE\left(\tfrac{x}{c'}\right)^7 + E^2\left(\tfrac{x}{c'}\right)^8\Big]$$

$$f^3\left(\tfrac{x}{c}\right) = \left(\frac{t^3}{0.008}\right)\Big[A^3\left(\tfrac{x}{c'}\right)^{3/2} - 3A^2B\left(\tfrac{x}{c'}\right)^2 + 3AB^2\left(\tfrac{x}{c'}\right)^{5/2} - (3A^2C + B^3)\left(\tfrac{x}{c'}\right)^3$$

$$+ 6ABC\left(\tfrac{x}{c'}\right)^{7/2} + 3(A^2D - B^2C)\left(\tfrac{x}{c'}\right)^4 + 3(AC^2 - 2ABD)\left(\tfrac{x}{c'}\right)^{9/2}$$

$$+ 3(B^2D - A^2E - BC^2)\left(\tfrac{x}{c'}\right)^5 + 6(ABE - ACD)\left(\tfrac{x}{c'}\right)^{11/2}$$

$$+ (6BCD - 3B^2E - C^3)\left(\tfrac{x}{c'}\right)^6 + 3(2ACE + AD^2)\left(\tfrac{x}{c'}\right)^{13/2}$$

$$+ 3(C^2D - BD^2 - 2BCE)\left(\tfrac{x}{c'}\right)^7 - 6ADE\left(\tfrac{x}{c'}\right)^{15/2}$$

$$+ 3(2BDE - C^2E - CD^2)\left(\tfrac{x}{c'}\right)^8 + 3AE^2\left(\tfrac{x}{c'}\right)^{17/2}$$

$$+ (6CDE - 3BE^2 + D^3)\left(\tfrac{x}{c'}\right)^9 - 3(CE^2 + D^2E)\left(\tfrac{x}{c'}\right)^{10}$$

$$+ 3DE^2\left(\tfrac{x}{c'}\right)^{11} - E^3\left(\tfrac{x}{c'}\right)^{12}\Big]$$

Moment of Inertia of a Symmetric Airfoil (I_{xx}) con't

So,

$$I_{xx} = \frac{2}{3}\left(\frac{t^3C'}{0.008}\right)\Big[\frac{2}{5}A^3 - A^2B + \frac{6}{7}AB^2 - \frac{1}{4}(3A^2C + B^3) + \frac{4}{3}ABC + \frac{3}{5}(A^2D - B^2C)$$

$$+ \frac{6}{11}(AC^2 - 2ABD) + \frac{1}{2}(B^2D - A^2E - BC^2) + \frac{12}{13}(ABE - ACD)$$

$$+ \frac{1}{7}(6BCD - 3B^2E - C^3) + \frac{2}{5}(2ACE + AD^2) + \frac{3}{8}(C^2D - BD^2 - 2BCE)$$

$$- \frac{12}{17}ADE + \frac{1}{3}(2BDE - C^2E - CD^2) + \frac{6}{19}AE^2 + \frac{1}{10}(6CDE - 3CE^2 + D^3$$

$$- \frac{3}{11}(CE^2 + D^2E) + \frac{1}{4}DE^2 - \frac{1}{13}E^3\Big]$$

$$\boxed{I_{xx} = (.03941)t^3C'}$$

Example:

NACA 0012 airfoil with chord length of .0813m.

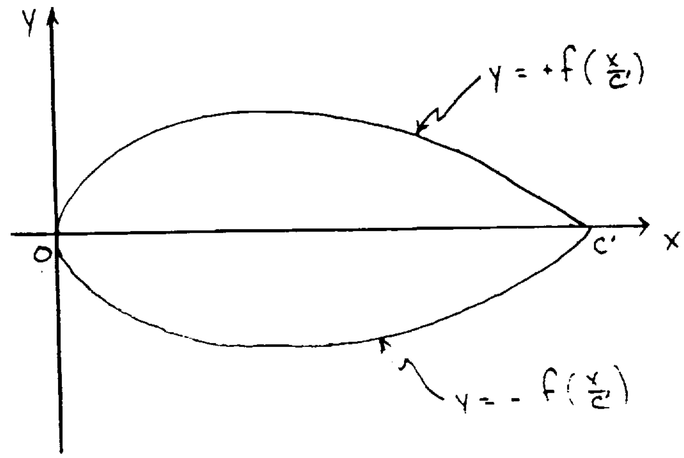
$$\Rightarrow t = 9.756\times10^{-3}\,m$$

So,

$$I_{xx} = (.03941)(9.756\times10^{-3}\,m)^3(.0813\,m)$$

$$\underline{I_{xx} = 2.975\times10^{-9}\,m^4}$$

Area of a Symmetric Airfoil



$$f\left(\frac{x}{c'}\right) = \left(\frac{t}{0.2}\right)\left[A\sqrt{\frac{x}{c'}} - B\left(\frac{x}{c'}\right) - C\left(\frac{x}{c'}\right)^2 + D\left(\frac{x}{c'}\right)^3 - E\left(\frac{x}{c'}\right)^4\right]$$

t = maximum thickness of the blade
C' = chord length

A = .2969
B = .126
C = .3516
D = .2843
E = .1015

$$\text{Area} = 2\int_0^{c'} f\left(\frac{x}{c'}\right)dx$$

$$\text{Area} = 2\left(\frac{t}{0.2}\right)\int_0^{c'}\left[A\sqrt{\frac{x}{c'}} - B\left(\frac{x}{c'}\right) - C\left(\frac{x}{c'}\right)^2 + D\left(\frac{x}{c'}\right)^3 - E\left(\frac{x}{c'}\right)^4\right]dx$$

$$\text{Area} = 10t\left[\frac{2}{3}A\frac{x^{3/2}}{c'^{1/2}} - \frac{1}{2}B\frac{x^2}{c'} - \frac{1}{3}C\frac{x^3}{c'^2} + \frac{1}{4}D\frac{x^4}{c'^3} - \frac{1}{5}E\frac{x^5}{c'^4}\right]\Bigg|_0^{c'}$$

Area of a Symmetric Airfoil (con't)

$$\text{Area} = 10tC'\left[\frac{2}{3}A - \frac{1}{2}B - \frac{1}{3}C + \frac{1}{4}D - \frac{1}{5}E\right]$$

$$\boxed{\text{Area} = (.6851)\,tC'}$$

Example:

NACA 0012 Airfoil with chord length of .0813 m

$$\Rightarrow t = (.12)(.0813\,m) = 9.756\times10^{-3}\,m$$

So,

$$\text{Area} = (.6851)(9.756\times10^{-3}\,m)(.0813\,m)$$

$$\underline{\text{Area} = 5.434\times10^{-4}\,m^2}$$

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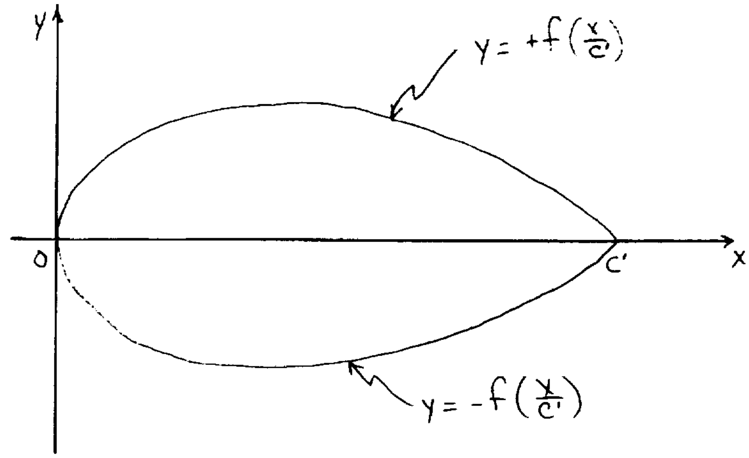
First Moment of a symmetric Airfoil (Q_x)



$$f\left(\frac{x}{c'}\right) = \left(\frac{t}{0.2}\right)\left[A\sqrt{\frac{x}{c'}} - B\left(\frac{x}{c'}\right) - C\left(\frac{x}{c'}\right)^2 + D\left(\frac{x}{c'}\right)^3 - E\left(\frac{x}{c'}\right)^4\right]$$

where t = maximum thickness of the blade

c' = chord length

A = .2969

B = .126

C = .3516

D = .2843

E = .1015

$$Q_x = \int y\,dA$$

$$Q_x = \int_0^{c'}\int_0^{f(\frac{x}{c'})} y\,dy\,dx$$

$$Q_x = \frac{1}{2}\int_0^{c'} y^2\,dx$$

First Moment of a Symmetric Airfoil (Q_x) cont

$$Q_x = \frac{1}{2}\left(\frac{t^2}{0.04}\right)\int_0^{c'}\Big[A^2\left(\frac{x}{c'}\right) - 2AB\left(\frac{x}{c'}\right)^{3/2} + B^2\left(\frac{x}{c'}\right)^2 - 2AC\left(\frac{x}{c'}\right)^{5/2} + 2BC\left(\frac{x}{c'}\right)^3$$

$$+ 2AD\left(\frac{x}{c'}\right)^{7/2} + (C^2 - 2BD)\left(\frac{x}{c'}\right)^4 - 2AE\left(\frac{x}{c'}\right)^{9/2}$$

$$+ 2(BE - CD)\left(\frac{x}{c'}\right)^5 + (2CE + D^2)\left(\frac{x}{c'}\right)^6 - 2DE\left(\frac{x}{c'}\right)^7 + E^2\left(\frac{x}{c'}\right)^8\Big]$$

$$Q_x = \frac{1}{2}\left(\frac{t^2}{0.04}\right)C'\Big[\frac{1}{2}A^2 - \frac{4}{5}AB + \frac{1}{3}B^2 - \frac{4}{7}AC + \frac{1}{2}BC + \frac{4}{9}AD + \frac{1}{5}(C^2 - 2BD)$$

$$- \frac{4}{11}AE + \frac{1}{3}(BE - CD) + \frac{1}{7}(2CE + D^2) - \frac{1}{4}DE + \frac{1}{9}E^2\Big]$$

$$\boxed{Q_x = (0.0688)\,t^2 C'}$$

Example

NACA 0012 with chord length of .0813m

1st moment of top half of blade about x-axis is,

$$Q_x = (0.0689)(9.756\times 10^{-3}\,m)^2(.0813\,m)$$

$$\underline{Q_x = 5.327\times 10^{-7}\,m^3}$$

Giromill $r = 0.575m$ $\omega = 31.30\,rad/s$

optimum performance at $C_p = 0.5$ and $\lambda = 3$

$$C_p = \frac{n\bar{c}K\lambda v^2}{4} - \frac{n\bar{c}C_{Do}\lambda^3}{2} \qquad (1)$$

$$v = 1 - \frac{n\bar{c}\lambda(K + 3C_{Do})}{16} \qquad (2)$$

Substituting (2) into (1)

$$C_p = \frac{n\bar{c}K}{4}\left[\lambda - \frac{n\bar{c}K\lambda^2}{8} + \frac{3}{256}n^2\bar{c}^2K^2\lambda^3\right] - \frac{n\bar{c}}{2}C_{Do}\lambda^3 \qquad (3)$$

taking $\frac{dC_p}{d\lambda}$ and setting $\frac{dC_p}{d\lambda} = 0$ for maximum C_p

$$\lambda = \frac{16}{3n\bar{c}K} \quad \text{and } v = 2/3 \qquad (4),(5)$$

Solving for c in (4) yields

$$c = \frac{16r}{3n\lambda K} = \frac{16(0.575)}{3 \cdot 2 \cdot 3 \cdot 2\pi} = 0.0813\,m = 8.13\,cm$$

(5) simply means that during optimum performance, the incident velocity on the wind machine is 2/3 the free stream velocity.

Solving (2) in terms of the free stream velocity results in

$$v = \frac{1}{16}\,2\,\frac{c}{r}\,\frac{r\omega}{V_\infty}\left[2\pi + 3(0.0096)\right] = 1 - \frac{c\omega}{8V_\infty}(6.306)$$

$$v = 1 - \frac{2.0}{V_\infty} \qquad v^2 = 1 - \frac{4.0}{V_\infty} + \frac{4.0}{V_\infty^2} \qquad (6)(7)$$

Solving (1) in terms of the free stream velocity yields

$$C_p = \frac{1}{2}\,2\,\frac{c}{r}\,K\frac{r\omega}{V_\infty}v^2 - \frac{1}{2}\,2\,\frac{c}{r}\,C_{Do}\frac{r^3\omega^3}{V_\infty^3} = \frac{Kc\omega}{2V_\infty}\left(1 - \frac{2.0}{V_\infty}\right)^2 - \frac{C_{Do}cr^2\omega^3}{V_\infty^3}$$

$$C_p = \frac{25.71}{V_\infty^3} - \frac{31.98}{V_\infty^2} + \frac{7.99}{V_\infty} \qquad (8)$$

$$M(\theta) = C_m(\theta) \frac{\rho}{2} V_\infty^2 r A_s \qquad A_s = 2rh = 4r^2$$

$$= C_m(\theta) \rho V_\infty^2 2r^3 \qquad (9)$$

$$C_m(\theta) = \frac{\bar{c}}{2}\left[K(V^2\cos^2\theta + \bar{c}V\cos\theta(\lambda - V\sin\theta)) - C_0(\lambda - V\sin\theta)\{(\lambda - V\sin\theta) - \bar{c}\dot{\theta}\cos\theta\}\right] \qquad (10)$$

$$C_{mave} = \frac{n}{2\pi}\int_0^{2\pi} C_m(\theta)\,d\theta = \frac{1}{4} n\bar{c}KV^2 - \frac{1}{2} n\bar{c}\, C_{00}\lambda^2$$

$$= \frac{1}{4}\, 2\, \frac{c}{r}\, 2\pi\left(1 - \frac{4.0}{V_\infty} + \frac{4.0}{V_\infty^2}\right) - \frac{1}{2}\, 2\, \frac{c}{r}\, C_{00}\frac{r^2\omega^2}{V_\infty^2}$$

$$C_{mave} = 0.44 - \frac{1.78}{V_\infty} + \frac{1.79}{V_\infty^2} - \frac{0.35}{V_\infty^2}$$

$$C_{mave} = 0.44 - \frac{1.78}{V_\infty} + \frac{1.44}{V_\infty^2} \qquad (11)$$

$$F_L = \frac{\rho}{2} V_r^2 C_L S \qquad V_r^2 = \lambda^2 + V^2 - 2\lambda V\sin\theta$$

$$= V_\infty^2 - (4.01 + 36.05\sin\theta)V_\infty + 327.93 + 72.35\sin\theta$$

$$C_L = 2\pi\alpha = 2\pi\,\mathrm{Arctan}\left[\frac{V\cos\theta}{\lambda - V_\infty\sin\theta}\right]$$

$$S = ch = (0.0813)(0.575) = 0.094\,m^2$$

$$F_L = 3.54\times10^{-3}\,\mathrm{Arctan}\left[\frac{(1 - {}^{2.0}/_{V_\infty})\cos\theta}{{}^{r\omega}/_{V_\infty} - (1 - {}^{2.0}/_{V_\infty})\sin\theta}\right](V_\infty^2 - (4.01 + 36\sin\theta)V_\infty + 327.93 + 72.35\sin\theta)$$

$V_\infty = 6$

$\alpha_{max} @ \theta = 19.47^\circ \qquad F_L = (7.88\times10^{-4})(291.99) = 0.23\,N$

Maximum Overturning Moment

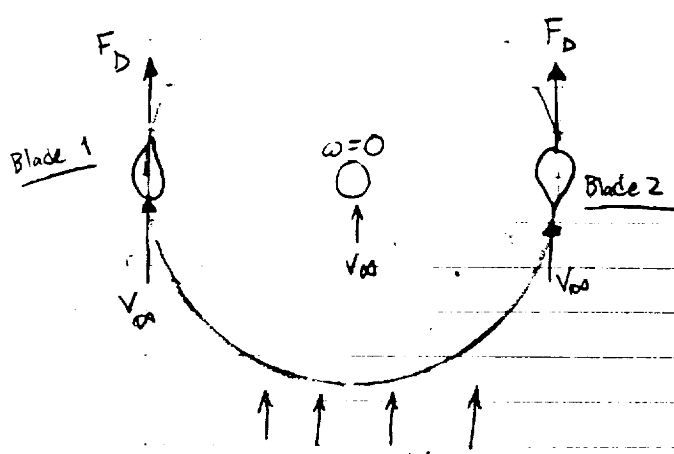
4 possible worst case scenarios

1. Blades at rest ; facing wind
2. Blades at rest; perpendicular to wind
3. Blades spinning; facing wind
4. Blades spinning; perpendicular to wind

The will all be analyzed for both the Darrieus and Giromill. The drag calculations are all quite conservative; however, the force due to dust particles hitting our blades and shaft have been neglected. We believe that our estimates will still be slightly conservative when the dust effect is added in.

Case 1: Blades at rest ; facing wind

Top view

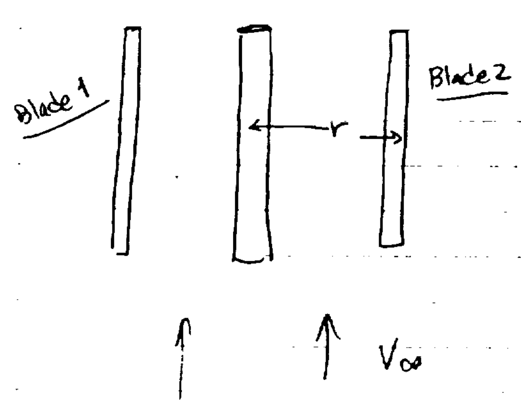


* We will design for a maximum wind speed of 150 km/hr. (= 41.7 m/s)

* Throughout analysis, Darrieus will be represented by two straight blades at the average radius of the Darrieus *

- For Blade 1, the angle of attack is 0° giving a Coefficient of Drag of ≈ 0.04
- For Blade 2, the angle of attack is 180°,

Front View



* We will assume for all cases where we cannot look up angle of attack that the airfoil will not act any worse aerodynamically than a flat plate which has a coefficient of drag of 2.0. This is a conservative estimate.

Darrieus : $r_{avg} = 0.679m$

The notation will always be

$$F_D = \tfrac{1}{2}\rho V^2 S C_D$$

$$F_L = \tfrac{1}{2}\rho V^2 S C_L$$

F_D = drag force
F_L = lift force
ρ = density of Martian air
V = effective velocity
S = planform area
C_L = Coefficient of lift
C_D = coefficient of drag

For blade 1

$$F_{D_1} = \tfrac{1}{2}(.01665 \ \text{Kg/m}^3)(41.7 \ \text{m/s})^2 \overbrace{\underset{\text{length of blade}}{(2.549 \ \text{m})}\underset{\text{average chord length}}{(.08 \ \text{m})}}^{S} (0.04)$$

$$F_{D_1} = 0.112 \ \text{N}$$

For blade 2,

$$F_{D_2} = \tfrac{1}{2}(.01665)(41.7)^2(2.549)(.08)(2.0)$$

$$F_{D_2} = 5.58 \ \text{N}$$

For the shaft

$$F_{D_{shaft}} = \tfrac{1}{2}(.01665)(41.7)^2 \overbrace{\underset{\text{estimated height of shaft}}{(1.6 \ \text{m})}\underset{\text{estimated diameter of shaft}}{(0.15 \ \text{m})}}^{S} \underset{C_{D\ shaft}}{(1)}$$

$$F_{D_{shaft}} = 3.47 \ \text{N}$$

$\boxed{F_{D_{total}} = 9.50 \ \text{N}}$ Darrieus, Case 1

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* For Giromill we must also consider struts; struts to blade 1 and struts to blade 2 are both analyzed

Giromill: r = 0.70 m

height of blades — estimated avg. chord

For blade 1: $F_{D_1} = \frac{1}{2}(.01665)(41.7)^2(1.4\ m)(0.15\ m)(0.04)$

For blade$_2$: $F_{D_2} = \frac{1}{2}\rho V^2 S(2.0)$

For shaft: $F_{D_{shaft}} = 3.47\ N$ from last part

length — estimated chord length

For Struts: $F_{D_{strut1}} = 2\cdot\frac{1}{2}(.01665)(41.7)^2(0.7\ m)(0.15\ m)(0.06)$ — estimated C_D

struts to blade 1

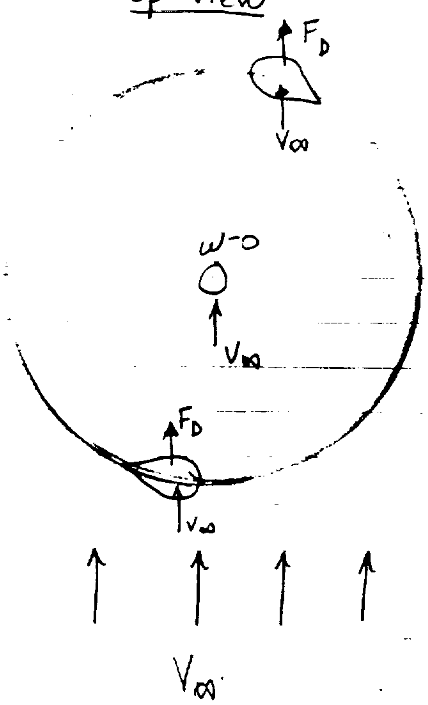
struts to blade 2 $F_{D_{strut2}} = 2\cdot\frac{1}{2}(.01665)(41.7)^2(0.7)(0.15)(2.0)$

$\boxed{F_{D,total} = 15.9\ N}$ Giromill, case 1

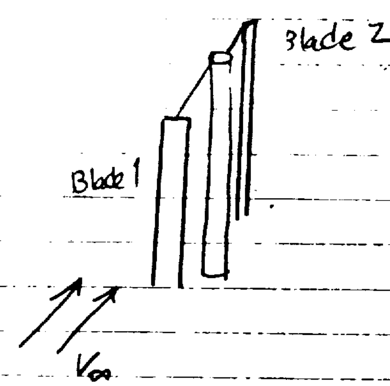
Case 2: Blades at rest; perpendicular to wind

- Now both blades at "bad" angles of attack
  - approximate C_D for both as $C_D = 2.0$ (this is conservative)

Top View



Front View



- The conservative estimate will be made that both of the blades "see" V_∞ although actually the wind will be slower at blade 2 and at the shaft

For the Darrieus

$$(F_D)_{\text{both blades}} = (.01665)(41.7)^2(2.549)(0.09)(2.0)$$

$$= 11.9 \text{ N}$$

$$F_{D_{\text{shaft}}} = 3.47 \text{ N} \quad \text{from before}$$

$$\boxed{F_{D_{\text{total}}} = 15.3 \text{ N}}$$ Darrieus : Case 2

For Giromill

$$(F_D)_{\text{both blades}} = (.01665)(41.7)^2(1.4)(0.15)(2)$$

$$= 12.1 \text{ N}$$

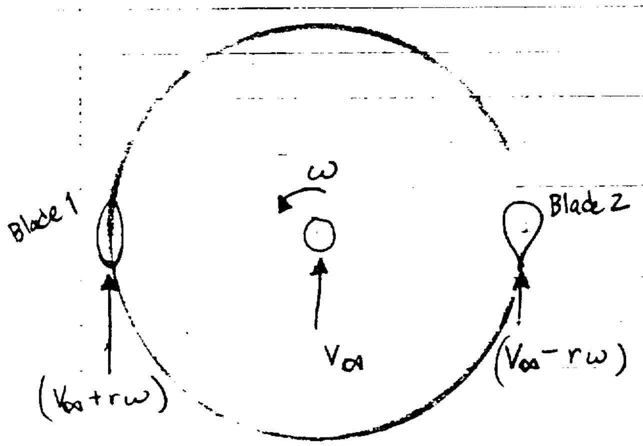
$$F_{D_{\text{shaft}}} = 3.47 \text{ N}$$

* In this position, struts do not "see" any wind

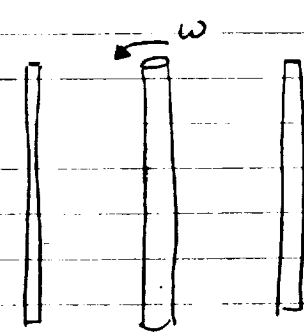
$$\boxed{F_{D_{\text{total}}} = 15.6 \text{ N}}$$ Giromill : Case 2

Case 3 : Blades Spinning ; facing wind

Top View



Front View



For Darrieus, $r_{avg}\omega = 23.4$ m/s

For Blade 1 : $F_{D_1} = \frac{1}{2}\rho(41.7 + 23.4)^2 S(.04)$

For Blade 2 : $F_{D_2} = \frac{1}{2}\rho(41.7 - 23.4)^2 S(2.0)$

For spinning shaft at this speed, $C_D \approx 1.3$ [1]

$F_{D\,shaft} = 3.47 \cdot \frac{1.3}{1}$ (from ratio of current force to previous force)

Computing these yields

$\boxed{F_{D,total} = 5.94 \text{ N}}$ Darrieus, Case 3

For Giromill, $r\omega = 18$ m/s

For blade 1 : $F_{D_1} = \frac{1}{2}\rho(41.7 + 18)^2 S(.04)$

For blade 2 : $F_{D_2} = \frac{1}{2}\rho(41.7 - 18)^2 S(2.0)$

For shaft : $F_{D_{shaft}} = (3.47 \times 1.3)$

For struts to blade 1 $F_{D_{struts\,1}} = 2 \cdot \frac{1}{2}\rho(41.7 + \underset{\text{average } r\omega \text{ for struts}}{9})^2(0.7)(0.15)(0.06)$

For struts to blade 2 $F_{D_{struts\,2}} = 2 \cdot \frac{1}{2}\rho(41.7 - 9)^2(0.7)(0.15)(2.0)$

$\boxed{F_{D,total} = 10.7 \text{ N}}$ Giromill, Case 3

[1] Engineering Fluid Mechanics, Roberson and Crowe, pg. 492

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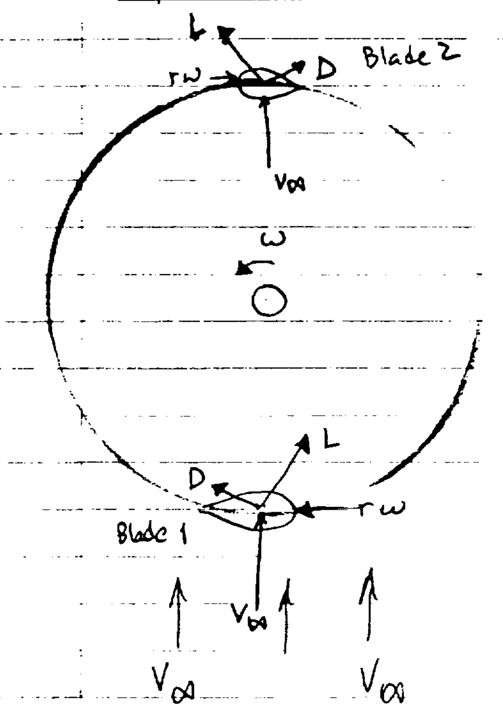
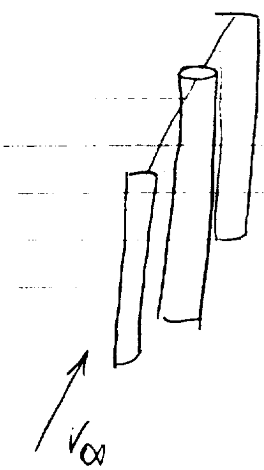
Case 4: Blades Spinning ; Perpendicular to wind

Top View

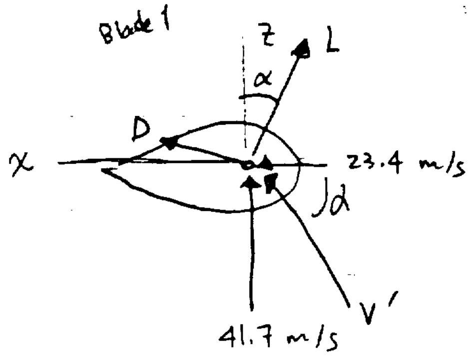


Front View



* Now, both lift and drag need to be considered

Darrieus :



Worst case if $C_L = C_{L_{max}} = 1.5$
and $C_D = 2.0$

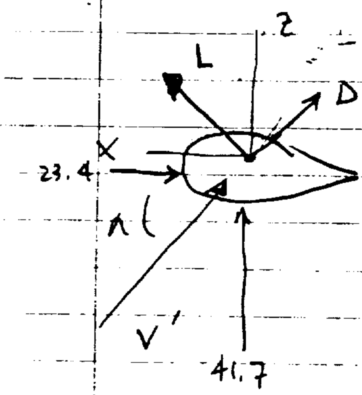
- Again this is quite conservative

$\alpha = \tan^{-1}\frac{41.7}{23.4} = 60.7°$

For blade 1
$$\begin{cases} F_z = \frac{1}{2}\rho V'^2 S\left[C_L\cos\alpha + C_D\sin\alpha\right] = \frac{1}{2}(.01665)(41.7^2+23.4^2)(2.549)(.08)\left[1.5\cos 60.7° + 2\sin 60.7°\right] \\ F_x = \frac{1}{2}\rho V'^2 S\left[C_D\cos\alpha - C_L\sin\alpha\right] = \frac{1}{2}\rho V'^2 S\left[2\cos 60.7 - 1.5\sin 60.7\right] \end{cases}$$

Blade 1
$$\begin{cases} F_z = 9.62\ N \\ F_x = -1.28\ N \end{cases}$$

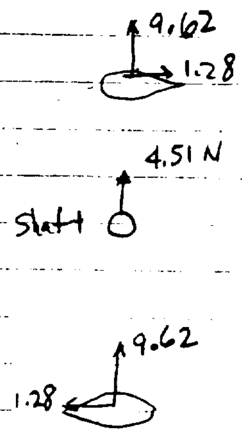
For blade 2



$3.67 = \frac{1}{2}\rho v^{*2} S$

Similar to before, now

Blade 2 $\begin{cases} F_z = 9.62 \text{ N} \\ F_x = +1.28 \text{ N} \end{cases}$



Our equivalent forces will be

$$\boxed{\begin{aligned} F_z &= 23.75 \text{ N} \\ F_x &= 0 \end{aligned}}$$

For Darrieus, case 4

* We also see we have a force couple trying to slow us down at these high windspeeds which is "good."

For Giromill, case 4

here, $(r\omega)_{blades} = 18$; $\alpha = 66.7°$

$(r_{avg}\,\omega)_{struts} = 9$ $\alpha = 77.8°$

Proceeding similarly to previous problem we see

$F_{z\,blade\,1} = \frac{1}{2}(.01665)(41.7^2+18^2)(1.4)(.15)\left[1.5\cos 77.8 + 2\sin 77.8\right]$

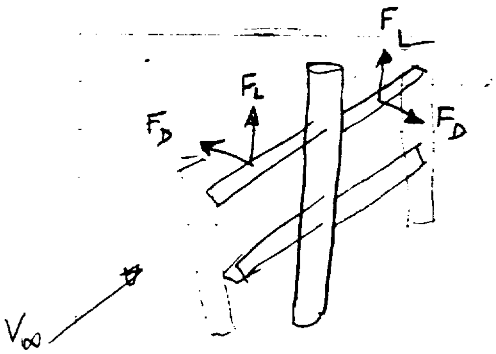
$= 8.19$ N

F_z will be same on blade 2, and F_x will again be equal and opposite so I will not calculate it

$F_{z\,blades} = 16.38$ N

* struts in this position only "see" their (rw) ; they are shielded from wind.

For Struts to blade 1



The drag forces will cancel and the only effect we get from the struts in this position is a lifting force upward and a torque to slow us down

$F_{z\,shaft} = 4.51$ N

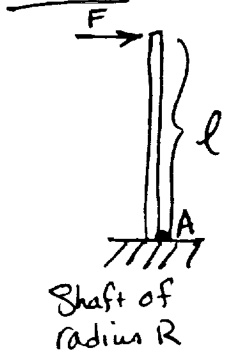
$$\boxed{F_{z_{total}} = 20.9 \text{ N}}$$

Giromill, case 4.

* Note that z direction is <u>not</u> vertical it is the axis which passes through the midpoints of the blades and the shaft.

The resultant bending moment about the base is simply the product of the force times its "lever arm". The lever arm is one-half the machine's height, or about 0.75 m.

FIND MINIMUM DIAMETER OF INSIDE SHAFT FROM BENDING STRESS:



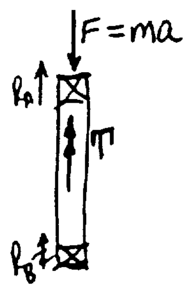
$$\sigma_A = \frac{MR}{\frac{\pi}{4}R^4} = \frac{4M}{\pi R^3} = \frac{4F\ell}{\pi R^3} \Rightarrow R = \left[\frac{4W\ell}{\sigma_{ult}}\right]^{1/3} \quad \text{— (Eq 1)}$$

[see numerical calculations on the following pages].

OUTER ROTATING SHAFT

inside radius $\equiv r_i$
outside radius $\equiv r_o$

ROARK'S HANDBOOK OF STRESS & STRAIN: $\frac{1}{r_o^2 - r_i^2} = \frac{\sigma_{max}\pi}{ma}$ for axial loading on impact



$$\Rightarrow r_o = \left[\frac{ma}{\sigma\pi} + r_i^2\right]^{1/2} \quad \text{bending}$$

$$\frac{r_o + r_i}{2} = \frac{0.3Et}{\sigma_{max}} \quad \text{for buckling}$$

plug in $E = 193\,GPa$, $\sigma_{max} = 1.2\,GPa$ for A1B:

$$\frac{t + 2r_i}{t} = 96.5 \Rightarrow t = \frac{-2r_i}{-95.5}$$

[See numerical calculations on the following pages]

FIND MINIMUM DIAMETER OF INSIDE SHAFT FROM BENDING STRESS

$F \equiv 24 \cdot N$ For AlB: $\sigma ult \equiv 1.1 \cdot GPa$

$l \equiv 0.75 \cdot m$

$$Dinside := \left[\frac{4 \cdot F \cdot l}{\pi \cdot \sigma ult}\right]^{\frac{1}{3}} \cdot 2$$

$$Dinside = 5.503 \cdot mm$$

Use a rather large FS, say 1.75, because the ultimate tensile stress was used; the yield strength will be somewhat lower than this, but is unknown:

$Dinside \cdot 1.75 = 9.631 \cdot mm$ <--- use 10-mm shaft

MASS = 156g

****

OUTER ROTATING SHAFT

Inner radius of the rotating shaft is dependent on the size of the bearing between it and the stationary shaft. We chose a bearing from SKF (p.) which has an outside diameter of 19mm.

Mass of two blades: $MassBlades \equiv 2 \cdot kg$

Acceleration during landing impact: $Acc \equiv 7 \cdot 9.81 \cdot \left[\frac{m}{sec^2}\right]$

Inner radius of shaft: $Ri \equiv 9.5 \cdot mm$

BENDING: $$Ro := \sqrt{\frac{MassBlades \cdot Acc}{\sigma ult \cdot \pi} + Ri^2}$$

$$Ro = 9.502 \cdot mm$$

BUCKLING: $$thickness := \frac{2 \cdot Ri}{95.5}$$

$$Ro := Ri + thickness$$

$$Ro = 9.699 \cdot mm$$

So use an outer diameter of 19.5 mm based on critical buckling stress.

$m \equiv 1L \quad kg \equiv 1M \quad sec \equiv 1T$

$mm \equiv 0.001 \cdot m \quad N \equiv \frac{kg \cdot m}{sec^2} \quad Pa \equiv \frac{N}{m^2} \quad GPa \equiv 10^9 \cdot Pa$

GIROMILL

Radius of Giromill: $R := 0.55 \cdot m$

Length of Giromill Blades: $h := 1.0 \cdot m$

Mass of one strut from -R to +R: $MassStrut := 0.020 \cdot kg$

Mass of two blades: $MassBlades := 0.380 \cdot kg$

$$Jgiro := \frac{2 \cdot MassStrut \cdot h^2}{12} + 2 \cdot MassBlades \cdot R^2$$

$$Jgiro = 0.233 \cdot kg \cdot m^2$$

DARRIEUS [approximate blade shape as circular]

Radius of Darrieus: $R := 0.75 \cdot m$

Height of Darrieus: $h := 2 \cdot R \qquad h = 1.5 \cdot m$

Mass of two blades: $MassBlades := 0.989 \cdot kg$

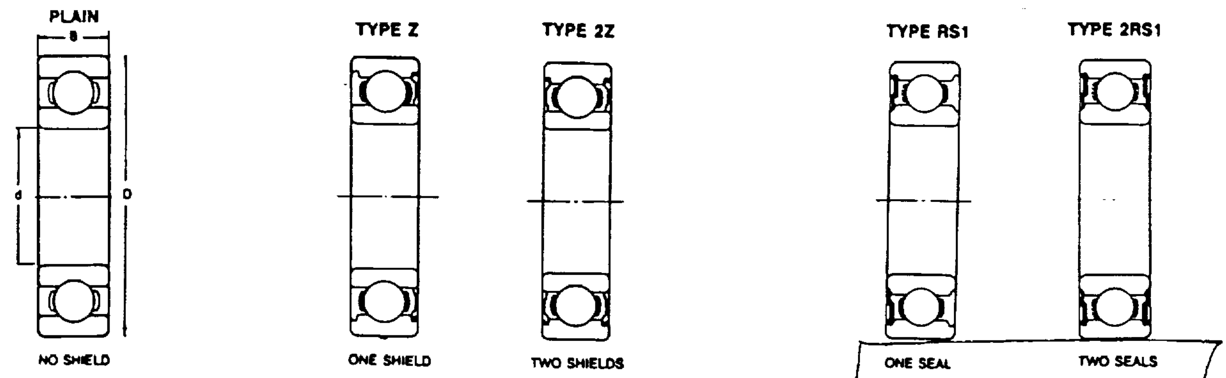
Inner radius of blades: $r1 := 0.75 \cdot m$

Outer radius of blades: $r2 := 0.755 \cdot m$

$$Jdarr := MassBlades \cdot \frac{r1^2 + r2^2}{4}$$

$$Jdarr = 0.28 \cdot kg \cdot m^2$$

Single Row Deep Groove Ball Bearings



use sealed bearings to prevent fouling by Martian dust.

Principal dimensions d mm in.	D mm in.	B mm in.	Basic load rating dynamic C N lbf.	Speed rating[1] rpm	Max. fillet radius mm in.	Mass kg lbs.	Designation Open	One shield	Two shields	One seal	Two seals
7 0.2756	14 0.5512	5 0.1969	753 169	63 000	0.15 0.006	0.0020 0.0044	—	—	AX7ZZ	—	—
	17 0.6693	5 0.1969	1170 263	56 000	0.3 0.012	0.0049 0.011	AY 7	—	AY7ZZ	—	—
	19 0.7480	6 0.2362	1 720 387	40 000	0.3 0.012	0.0075 0.016	607	607 Z	607 2Z	607 RS1	607 2RS1
	22 0.8661	7 0.2756	3 250 731	36 000	0.3 0.012	0.013 0.028	627	627 Z	627 2Z	627 RS1	627 2RS1
8 0.3150	16 0.6299	4[2] 0.1575	1 040 234	56 000	0.2 0.008	0.0031 0.0068	X8	—	X8ZZ	—	—
	19 0.7480	6 0.2362	1 460 328	50 000	0.3 0.012	0 0071 0.016	AY8	—	AY8ZZ	—	—
	22 0.8661	7 0.2756	3 250 731	38 000	0.3 0.012	0.012 0.026	608	608 Z	608 2Z	608 RS1	608 2RS1
9 0.3543	17 0.6693	4[2] 0.1575	1 110 250	53 000	0.2 0.008	0.0034 0.0075	X9	—	X9ZZ	—	—
	20 0.7874	6 0.2362	1590 357	48 000	0.3 0.012	0.0076 0.030	AY 9	—	AY9ZZ	—	—
	24 0.9449	7 0.2756	3 710 835	36 000	0.3 0.012	0.014 0.031	609	609 Z	609 2Z	609 RS1	609 2RS1
	26 1.0236	8 0.3150	4 620 1 040	32 000	0.3 0.012	0.020 0.044	629	629 Z	629 2Z	629 RS1	629 2RS1
10 0.3937	19 0.7480	5 0.1969	1 170 263	48 000	0.3 0.012	0.0054 0.012	X10	—	X10ZZ	—	—
	19 0.7480	5 0.1969	1 480 333	38 000	0.3 0.012	0.0055 0.012	61800	—	61800 2Z	—	—
	22 0.8661	6 0.2362	1 900 427	38 000	0.3 0.012	0.016 0.035	61900	—	61900 2RZ[3]	—	61900 2RS1
	26 1.0236	8 0.3150	4 620 1 040	34 000	0.3 0.012	0.0055 0.012	6000	6000 Z	6000 2Z	6000 RS1	6000 2RS1
	30 1.1811	9 0.3543	5 070 1 140	30 000	0.6 0.024	0.032 0.071	6200	6200 Z	6200 2Z	6200 RS1	6200 2RS1
	35 1.3780	11 0.4331	8 060 1 810	26 000	0.6 0.024	0.053 0.11	6300	6300 Z	6300 2Z	6300 RS1	6300 2RS1

[1] This refers to oil lubrication and moderate load. Consult SKF for lower ratings applicable to grease lubrication.
Series 16100 — 16101, 16002 — 16072, also available.
Series 6200 through 6220 and 6303 through 6317 are also available as precision bearings (ABEC 5).
[2] Seal and shield versions 1mm wider than listed.
[3] Suffix 2RZ denotes rubberized shield.

S-13

SKF

Driving Torque produced by Blades:

From momentum theory, we know;

$$\text{Moment} = \tfrac{1}{2}\rho V_\infty^2 r A\left[\tfrac{1}{4} n \bar{c} K \bar{v}^2 - \tfrac{1}{2} n \bar{c} C_{D_0} R^2\right]$$

Where:

$\rho = 0.01665\ kg/m^3$ (density)

$V_\infty = 8\ m/s$ (free stream velocity)

$r = .572m$ (Blade radius)

$A = 1.31 m^2$ (Swept Area)

$n = 2$ (number of blades)

$\bar{c} = 0.142$ (chord / radius)

$K = 2\pi$ (slope of lift curve)

$C_{D_0} = .02$ ("AVERAGE" drag coefficient)

$R = 3$ (tip-speed ratio)

$$\bar{v} = 1 - \tfrac{1}{16} n \bar{c} R\left[K + 3C_{D_0}\right]$$

So,

$$\bar{V} = 1 - \frac{1}{16}(2)(0.142)(3)\left[2\pi + 3(.02)\right]$$

$$\bar{V} = 0.664$$

AND

$$\text{Moment} = \frac{1}{2}(0.01665\,kg/m^3)(8\,m/s)^2(.572\,m)(1.31\,m^2)\left[\frac{1}{4}(2)(.142)(2\pi)(.664)^2 - \frac{1}{2}(2)(.142)(.02)(3)^2\right]$$

(Torque)
Moment = .068 N.m

Output Power (neglecting friction in bearings of shaft)

$$P = T\omega\eta$$ efficiency

Efficiency of Motor

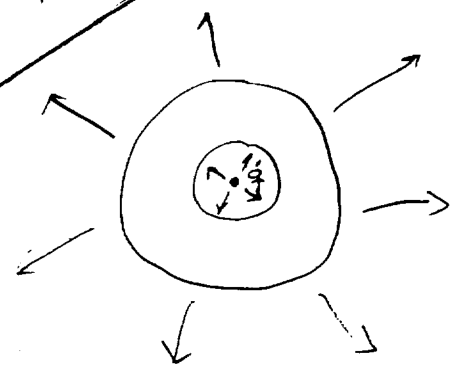
$$P = (.068\,N\cdot m)(32\,rad/s)(.85)(.8)$$

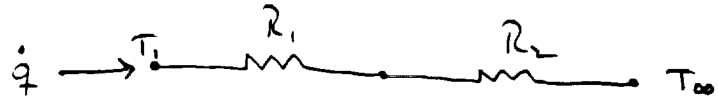
efficiency of gearhead

$$P \simeq 1.48 \text{ Watts}$$

Heat Transfer:

Not clear what you have done!





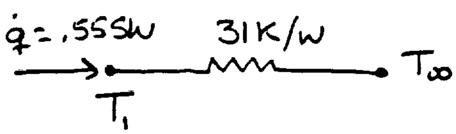
$\dot{q}$ (for 15% inefficiency) $= (.15)(3.7\,W) = .555\,W$

From [26],

$R_1 = 4\,K/W$

$R_2 = 27\,K/W$

$\Rightarrow$



$T_1 = T_\infty + (31\,K/W)(.555\,W)$

$T_1 = T_\infty + 17.21\,K$

$\Rightarrow$ Interal Steady-State Temp is 17.21 K warmer than surroundings mars materials

Fri, May 1, 1992 5:02 PM

	Material	ρ (g/cm)	Strength? Tensile (MPa)	Strength Shear (MPa)	E (GPa)	10^{-6}/K α /K	Fatigue (10^7 cycles)	Birnell Hardness	Tensile/ρ
1	Steel Structural	7.86	250	145	200	11.7	570	197-260	50.9
2									
3	Aluminum	2.71	95	55	70	23.6	70	45-185	40.6
4									
5	5083		315						107.0
6									
7	7075		510	325			240		219.0
8									
9	Titanium Alloy (6Al4V)	4.42	825	400	114	7.9	390		187.0
10									
11	Magnesium	1.80	275	100	45	26.0	60		152.0
12									
13	Boron Reinforced Aluminum	2.65	1080-1570	80-250	193	20.0	1080	190-220	500.0
14									
15	Kevlar Woven	1.35	495	200	80				336.0

WHICH KEVLAR?

SOURCES [14,15,16,17]

Table 1

M-1

unidirectional materials

Fri, May 1, 1992 6:35 PM

	material	ρ (g/cm)	Tensile (MPa)	Shear (MPa)	E (GPa)	α /k	Fatigue (10^7 cycles)	Tensile/ρ
1			long/trans					
2								
3	Eglass/Epoxy	2.100	1020/40	70.000	51.000			486.000
4								
5	Sglass/Epoxy	2.000	1680/40	80.000	51.000			840.000
6								
7	Carbon/Epoxy	1.580	1240/41	80.000	152.000			784.000
8								
9	Kevlar/Epoxy	1.380	1240/30	60.000	100.000	-2.000	1103.000	899.000
10								
11	Boron/Epoxy	2.080	1520/73	40.000	214.000	4.500	1000.000	730.000

SOURCES [14, 15, 16, 17]

Table 2

	Solid Lubricant	Kinetic Coeff. of Friction	Wear Life (cycles)
1			
2	MoS_2	0.036	103680.000
3	Graphite	0.080	8640.000
4	Cadmium iodide	0.088	4320.000
5	Boron nitride	0.148	360.000

Source [11]

Table 3

MATERIAL SELECTION

The driving properties which pertain to the Darrieus blades are found below in the criteria.

CRITERIA		B	C	D		
Tensile/ρ	A	A/4	A/3	A/3		A = 7
E		B	C/3	D/2		B = 0
α			C	D/1		C = 3
Fatigue				D		D = 3

Preference Weightings
0 - No difference.
1 - Very slightly more important.
2 - Slightly more important.
3 - Reasonably more important.
4 - Much more important.
5 - Extremely more important.

ANALYSIS MATRIX

	Tensile/ρ 7	E 0	α 3	Fatigue 3	
E Glass	2/14	—	2/6	2/6	= 26
S Glass	4/28	—	2/6	4/12	= 46
Carbon	3/21	—	3/9	3/9	= 39
Keylar	5/35	—	5/15	5/15	= 65
Boron	3/21	—	5/15	4/12	= 48

Kevlar is the best material for the Darrieus blades.

again which # ?

Material Selection

The driving properties which pertain to the Shaft are found below in the criteria.

CRITERIA		B	C	D	E	
Tensile/ρ	A	A/2	A/3	A/1	A/1	A = 7
E		B	B/2	D/2	E/1	B = 2
α			C	D/2	E/1	C = 0
Fatigue				D	D/2	D = 6
Shear					E	E = 2

Preference Weightings

0- No difference.
1- Very slightly more important.
2- Slightly more important.
3- Reasonably more important.
4- Much more important.
5- Extremely more important.

Analysis Matrix

	Tensile/ρ 7	E 2	α 0	Fatigue 6	Shear 2	
Steel	1/7	5/10	—	3/18	2/4	= 39
Aluminum	1/7	2/4	—	1/6	1/2	= 19
Titanium	3/21	3/6	—	3/18	5/10	= 55
Magnesium	2/14	1/2	—	1/6	2/4	= 26
Boron Aluminum	5/35	4/8	—	5/30	4/8	= 81

Boron Aluminum is the best material for the Shaft.

MATERIAL SELECTION

The driving properties which pertain to the Giromill blades are found below in the criteria.

CRITERIA		B	C	D	E	
Tensile/ρ	A	A/3	A/3	A/3	A/1	A= 10
E		B	B/2	B/1	0	B= 3
α			C	C/1	E/1	C= 1
Fatigue				D	0	D= 0
Shear					E	E= 1

Preference Weightings
0- No difference.
1- Very slightly more important.
2- Slightly more important.
3- Reasonably more important.
4- Much more important.
5- Extremely more important.

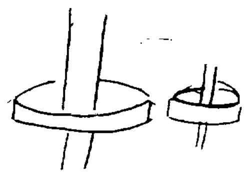
ANALYSIS MATRIX

	Tensile/ρ	E	α	Fatigue	Shear	
	10	3	1	0	1	
Steel	1/10	5/15	2/2	—	2/2	= 29
Aluminum	1/10	2/6	1/1	—	1/1	= 18
Titanium	3/30	3/9	3/3	—	5/5	= 47
Magnesium	2/20	1/3	1/1	—	2/2	= 26
Boron Aluminum	5/50	4/12	1/1	—	4/4	= 67
Kevlar Woven	4/40	2/6	5/5	—	3/3	= 54

which alloys?

Boron Aluminum is the best material for the Giromill blades.

GEARS

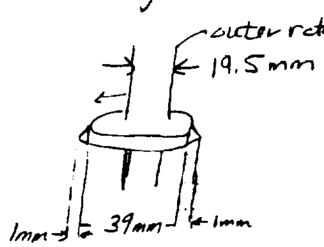


the LARGE GEAR is made of boron reinforced Aluminum and is part of the outer rotating shaft. the small gear is made of 7075 Aluminum. These materials were chosen for connection simplicity in the large gear and lightweight in the small gear. A Film of MoS_2 will be added to both gears for lubrication purposes.

the outer rotating shaft rotates at 35 rad/s thus a 1:6 gear ratio is need to accomodate the generator.

Because of the complexity of gear design only a simplistic analysis will be done to estimate size and weight.

1:6 ratio
large gear 72 teeth
small gear 12 teeth

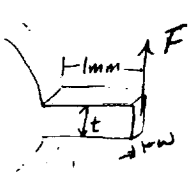


$2\pi R = 39\pi = 122.522$

Assume teeth & space are equal

$\frac{122.52}{72} = 2t = 1.7$

$t = 0.85\,mm$



$POWER = 3W = T\omega$

$T = \frac{P}{\omega} = \frac{3}{35} = 0.086\,Nm$

$RF = T$

$F = T/R = \frac{0.086\,Nm}{21 \times 10^{-3}\,m} = 4.09\,N$

Cantileaver Beam Analysis

$M = Fd = (4.09N)(1 \times 10^{-3} m) = 0.00409$

$I = \frac{1}{12} w t^3$

$c = \frac{1}{2} t$

$F.S. = 2$

$\sigma_x = \frac{2Mc}{I} = \frac{2(.00409)(\frac{1}{2}t)}{\frac{1}{12} w t^3} = \frac{0.049}{wt^2}$

$= 67820\,Pa$ where $t = 0.85mm$, $w = 1mm$ which is within the material stress of $\sigma = 95\,MPa$

M-7 (strength?

Gear & Bearing Weight

Gear

large gear

$wgt = vol(\rho)$

$= \pi R^2 L \rho$

$= \pi (20mm)^2 (1mm)(2.65\ g/cm^3)\left(\frac{cm}{10mm}\right)^3$

$= 3.33$ grams

Small gear

$wgt = vol(\rho)$

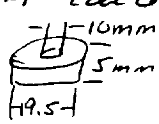
$= \pi R^2 L \rho$

$= \pi (7.5)^2 (1mm)(2.71)\left(\frac{1}{10}\right)^3$

$= 6.48$ grams

Bearings

the bearings are integrated into the shaft - rotating shaft so they will be made of boron reinforced Aluminum with MoS_2 film coated. They will utilize steel balls, and will be shielded against dust.



$wgt = vol \rho$

$= (\pi R_o^2 L - \pi R_i^2 L)(2.65\ g/cm^3)$

$= (\pi (9.75)^2(5) - \pi (5)^2 (5))(2.65)\left(\frac{1}{10}\right)^3$

$= 2.9$ grams per bearing

2 Bearings

7.3 grams

steel balls (2mm dia)

$\frac{4}{3}\pi R^3 = 4.2mm^3$

$wg = Vol \rho$

$= 4.2mm^3 \left(\frac{1}{10}\right)^3 (7.86\ g/cm^3)$

$= 0.032$ grams

23 ball per bearing - .74 grams

DARRIEUS

Pin sizing.

Titanium Yield : 825 MPa
Shear : 400 MPa
ρ : 4.42 g/cm³

$$\tau_{all} = \frac{\tau_u}{F.S.} = \frac{400\,MPa}{F.S.} = \frac{400\,MPa}{5} = 80\,MPa$$

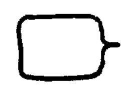
$$A_{req.} = \frac{V/2}{80\,MPa} = \frac{18.1N/2}{80\,MPa} = \frac{9.05N}{80\,MPa} = 1.13\times10^{-7}\,m^2$$

$$= 1.13\times10^{-7}\,m^2\left(\frac{100\,mm}{1\,m}\right)^2 = .00113\,mm^2$$

$$A_{req.} = \frac{\pi}{4}d^2 = .00113\,mm^2$$

$$d^2 = 1.44\times10^{-3}\,mm^2$$

$$d = 0.038\,mm$$

use $d = \underline{0.05\,mm}$ used for  and rivet.

check bending moment  :

$$\sigma = \frac{Mc}{I}\ ;\quad M = \frac{1}{2}wL^2 = \frac{1}{2}(2N)(.0021)^2 = 4.41\times10^{-6}\,Nm$$

$$c = .000025\,m$$

$$I = \frac{1}{2}mR^2 = \frac{1}{2}(1.8\times10^{-7})(.000025)^2 = 5.625\times10^{-17}$$

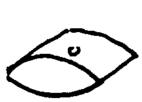
$$\sigma = 1960000 \cong 2\,MPa$$

cannot exceed 825 MPa $\therefore$ pin will hold!

Weights:

 Volume: $\pi r^2 L = \pi(.0025cm)^2(.83cm) = 1.63\times10^{-5}cm^3$

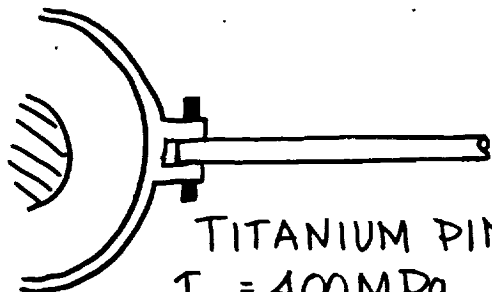
Wt: $= \rho\cdot V = 4.42\,g/cm^3\,(1.63\times10^{-5}cm^3) = \underline{7.2\times10^{-5}g}$

 Volume: $\cong LWH = (2cm)(6cm)(.4cm) = 4.8cm^3$

Wt: $= 4.42\,g/cm^3\,(4.8\,cm^3) = \underline{21.22\,g}$

Total: 21.220072 g × 4 pieces = $\underline{84.880288\,g}$

GIROMILL



TITANIUM PIN

$\tau_u = 400\,MPa$

$F.S. = 5$

$$\tau_{all} = \frac{400\,MPa}{5} = 80\,MPa$$

$$A_{req.} = \frac{S/2}{80\,MPa} = \frac{56.3\,N/2}{80\,MPa} = 3.52\times10^{-7}\,m^2 \left(\frac{100\,mm}{1\,m}\right)^2 = .0035\,mm^2$$

$$A_{req.} = \frac{\pi}{4} d^2 = .0035\,mm^2$$

$$d^2 = 4.478\times10^{-3}\,mm^2$$

$$d = 0.067\,mm$$

$$\underline{\underline{d = 0.07\,mm}}$$

ABOUT THE SIZE OF A HAIR

Weight:

Volume of pin: $\pi r^2 L = \pi(.0035\,cm)^2(1.5\,cm)$

$= 5.77\times10^{-5}\,cm^3$

$wt = \rho \cdot V = 4.42\,g/cm^3\,(5.77\times10^{-5}\,cm^3) = \underline{2.55\times10^{-4}\,g}$

Total:

4 pins @ $2.55\times10^{-4}\,g = \underline{1.02\times10^{-3}\,g}$