

11N-63
16012
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Robust Stabilization of Marginally Stable Positive-Real Systems

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(NASA-TM-109136) ROBUST
STABILIZATION OF MARGINALLY STABLE
POSITIVE-REAL SYSTEMS (NASA.
Langley Research Center) 15 p

N94-37058

Unclas

G3/63 0016012

July 1994

National Aeronautics and
Space Administration
Langley Research Center
Hampton, Virginia 23681-0001

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SUMMARY

This paper investigates the stability of the negative feedback interconnection of two positive-real systems which have poles in the closed left-half of the complex plane. A new definition of marginally strict positive real systems is introduced, and sufficient conditions are obtained for the stability of the feedback interconnection of such systems, using the Lyapunov method. The conditions obtained have direct applications to dynamic dissipative controllers for flexible spacecraft, and are the least restrictive ones published to date.

I. INTRODUCTION

A well known result in passivity theory [1] is the Passivity Theorem, which states that a stable *passive* dynamic system can be stabilized by a stable *strictly passive* controller. For linear, time-invariant (LTI) systems, passivity is equivalent to positive realness. However, the relationship between *strict* passivity and *strict* positive realness is somewhat more complicated because there are several definitions of *strict* positive realness. Strict passivity is equivalent to the strongest definition of strict positive realness (SPR) [2] for LTI systems. (We shall refer to this definition simply as "strict passivity", even for the LTI case). The Passivity Theorem, which is for general (possibly nonlinear) systems, can be directly applied to LTI systems; i.e., the feedback interconnection of a stable PR system and a strictly passive system is stable. However, the requirement of strict passivity is too stringent, as it includes only systems with a relative degree of zero. Another definition of strict positive realness was introduced in [2] for scalar systems, and was further investigated in [3, 4, 5] for multivariable systems. This definition (referred to as "strong SPR") is weaker than strict passivity.

In [6], an even less stringent definition of SPR, termed "weak SPR", was investigated, and it was stated that the feedback interconnection of a stable PR system and a weak SPR system is stable. That is, the strict passivity requirement was replaced by the "weak SPR" requirement. This was a significant improvement,

since weak SPR systems can include strictly proper systems, while strict passivity requires the relative degree to be zero. However, in all the studies in the published literature, both weak and strong definitions of SPR require the systems to be *stable* (i.e., all poles are required to be in the open left-half of the complex plane). In this paper, we shall remove this restriction.

Let $G(s)$ denote an $m \times m$ matrix whose elements are proper rational functions of the complex variable "s". Let the conjugate-transpose of a complex matrix T be denoted by T^* .

Definition 1: An $m \times m$ rational matrix $G(s)$ is said to be **positive real (PR)** if

- (i) all elements of $G(s)$ are analytic in $\text{Re}[s] > 0$;
- (ii) $G(s) + G^*(s) \geq 0$ in $\text{Re}[s] > 0$, or equivalently,
 - (iia) poles on the imaginary axis are simple and have Hermitian, nonnegative- definite residues, and
 - (iib) $G(j\omega) + G^*(j\omega) \geq 0$ for $\omega \in (-\infty, \infty)$

Given below are some definitions of strictly positive real systems. Definition 2, which represents the specialization to LTI systems of the general definition of strict passivity, is the strongest definition of strict positive realness.

Definition 2: An $m \times m$ rational matrix $G(s)$ is said to be **strictly passive** if

- (i) all elements of $G(s)$ are analytic in $\text{Re}[s] \geq 0$;
- (ii) there exists an $\epsilon > 0$ such that

$$G(j\omega) + G^*(j\omega) \geq \epsilon I \text{ for } \omega \in (-\infty, \infty)$$

Definition 3: An $m \times m$ rational matrix $G(s)$ is said to be **strictly positive real in the strong sense** (strong SPR, or SSPR) if $G(s-\epsilon)$ is PR for some $\epsilon > 0$; that is, if

- (i) all elements of $G(s)$ are analytic in $\text{Re}[s] \geq 0$;
- (ii) $G(j\omega) + G^*(j\omega) > 0$ for $\omega \in (-\infty, \infty)$
- (iii) $\mathcal{Z} = G(\infty) + G^T(\infty) \geq 0$
- (iv) $\lim_{\omega \rightarrow \infty} \omega^2 [G(j\omega) + G^*(j\omega)] > 0$ if \mathcal{Z} is singular

Definition 4: An $m \times m$ rational matrix $G(s)$ is said to be **strictly positive real in the weak sense** (weak SPR, or WSPR) if

- (i) all elements of $G(s)$ are analytic in $\text{Re}[s] \geq 0$;
- (ii) $G(j\omega) + G^*(j\omega) > 0$ for $\omega \in (-\infty, \infty)$

Note that Definition 2 requires that $\mathcal{Z} = G(\infty) + G^T(\infty)$ to be positive definite; i.e., the system must have a relative degree of zero. This requirement makes the definition of strictly passive systems too restrictive. Definition 3 (SSPR) can include certain strictly proper systems which satisfy additional conditions (iii) and (iv). Definition 4 (WSPR) does not require these additional conditions, and is therefore less restrictive than Definition 3. However, all the definitions (2-4) of SPR require the system to be *stable*.

In this paper, we go one step further, and allow the system to have poles *on* the imaginary axis. The significance of this is that many physical systems can now be included in this (much larger) class of systems.

Definition 5: An $m \times m$ rational matrix $G(s)$ is said to be **marginally strictly positive real (MSPR)** if it is positive real, and

$$G(j\omega) + G^*(j\omega) > 0 \text{ for } \omega \in (-\infty, \infty)$$

Definition 5 of MSPR differs from Definition 1 (PR) because the frequency domain inequality (\geq) has been replaced by the strict inequality ($>$). The difference between Definitions 4 and 5 is that Definition 5 allows $G(s)$ to have poles on the imaginary axis. This is an *important* difference because many real-life systems contain pure integrators and oscillators, which are permitted under Definition 5, but not under Definitions 2, 3, and 4. For example, let $G(s) = \frac{\gamma}{s} + \frac{\delta s}{s^2 + \omega_0^2} + H(s)$, where γ and δ are real non-negative scalars and $H(s)$ is weak SPR. Then $G(s)$ is marginally SPR.

Suppose $[A,B,C,D]$ is a minimal realization of a rational matrix $M(s)$. $M(s)$ (or $[A,B,C,D]$) is said to be minimum-phase if its transmission zeros are confined to the open left-half plane (OLHP); i.e., $\text{rank} \begin{bmatrix} sI-A & B \\ C & D \end{bmatrix}$ can drop below its normal value only for values of s in the OLHP.

II. PROPERTIES OF MSPR SYSTEMS

Suppose $G(s)$ is positive real and has all poles (i.e., the eigenvalues of the system matrix of its minimal realization) in the closed left-half plane (CLHP).

Following [7], $G(s)$ can be written as:

$$G(s) = G_1(s) + G_2(s) \quad (1)$$

where $G_1(s)$ has purely imaginary poles, and $G_2(s)$ has poles only in the open left-half plane (OLHP). Furthermore, $G_1(s)$ is of the form:

$$G_1(s) = \frac{\alpha_0}{s} + \sum_{i=1}^p \frac{\alpha_i s + \beta_i}{s^2 + \omega_i^2} \quad (2)$$

where α_i , and β_i are $m \times m$ real matrices, and $\omega_i > 0$, $i=1,2,\dots,p$ ($\omega_i \neq \omega_j$ for $i \neq j$).

Some remarks regarding the nature of the poles on the imaginary axis are in order. The poles and zeros considered here are in the Smith-McMillan sense [8]; i.e., there can be more than one pole at a given location, without it being considered a "repeated" pole. In particular, using standard results in matrix fraction descriptions [8], it can be shown that the McMillan degree (i.e., the minimal order of a state space representation) of the term: $[\alpha_0/s]$ is equal to $\rho(\alpha_0)$, where $\rho(\cdot)$ denotes the rank. That is, there are $v=\rho(\alpha_0)$ simple poles at $s=0$.

Suppose the McMillan degree of the term: $[\alpha_i s + \beta_i]/(s^2 + \omega_i^2)$ is $2k_i$, where $k_i \leq m$. Then this term has k_i simple poles (each) at $s=j\omega_i$ and $s=-j\omega_i$.

The following results state that $G(s)$ is PR (respectively, MSPR) iff its stable part is PR (resp. WSPR).

Lemma 1. $G(s)$ is PR (resp. MSPR) iff all of the following hold;

- (i) $G_2(s)$ is PR (resp. WSPR);
- (ii) $\alpha_i = \alpha_i^T \geq 0$, $i = 0, 1, 2, \dots, p$;
- (iii) $\beta_i = -\beta_i^T$, $i = 1, 2, \dots, p$.

Proof. From the requirement that the residues at the imaginary-axis poles be nonnegative-definite, we get (ii) and (iii) (See [9]). Therefore, we have:

$G_1(j\omega) + G_1^*(j\omega) = 0$, and $G(j\omega) + G^*(j\omega) = G_2(j\omega) + G_2^*(j\omega)$, which is positive semi-definite (resp. positive definite) for all real ω iff $G_2(s)$ is PR (resp. WSPR). ■

If $G(s)$ is MSPR, the degree of $[G(s)+G^*(s)]$ is generally less than the degree of $G(s)$; also, $G_2(s)$ is stable, and both $G(s)$ and $G_2(s)$ are minimum-phase.

We shall next consider a minimal realization of MSPR transfer functions.

III. MINIMAL REALIZATION OF MSPR SYSTEMS

Consider the realization of $G_1(s)$. For the term $[\alpha_0/s]$, since α_0 is symmetric and non-negative definite (Lemma 1), there exists an $m \times m$ real orthogonal transformation matrix T which diagonalizes it, i.e.,

$$T^T \alpha_0 T = \text{diag}[\lambda_1, \lambda_2, \dots, \lambda_v, 0, \dots, 0] \quad (3)$$

where λ_i are positive scalars. Let Λ denote $\text{diag}[\lambda_1, \lambda_2, \dots, \lambda_v]$. A minimal realization (of order v) of $T^T[\alpha_0/s]T$ is: $[\mathcal{A}_0, \bar{\mathcal{B}}_0, \bar{\mathcal{C}}_0, 0]$, where

$$\mathcal{A}_0 = [0_v]; \quad \bar{\mathcal{B}}_0 = [I_v \ 0]; \quad \bar{\mathcal{C}}_0 = \begin{bmatrix} \Lambda \\ 0 \end{bmatrix} \quad (4)$$

where 0_v and I_v denote $v \times v$ null and identity matrices. Therefore, a minimal (v^{th} -order) realization of $[\alpha_0/s]$ is given by $[\mathcal{A}_0, \bar{\mathcal{B}}_0, \bar{\mathcal{C}}_0, 0]$, where

$$\mathcal{B}_0 = \bar{\mathcal{B}}_0^T T^T; \quad \mathcal{C}_0 = T \bar{\mathcal{C}}_0 \quad (5)$$

Considering the term: $[\alpha_i s + \beta_i]/(s^2 + \omega_i^2)$, if its McMillan degree is $2k_i$ ($k_i \leq m$), a minimal realization is given by [7]: $[\mathcal{A}_i, \mathcal{B}_i, \mathcal{C}_i^T, 0]$, where $\mathcal{A}_i \in \mathbb{R}^{2k_i \times 2k_i}$,

$$\mathcal{A}_i = \text{diag}(A_{i1}, A_{i2}, \dots, A_{ik_i}) \quad (6)$$

where

$$A_j = \begin{bmatrix} 0 & -\omega_j \\ \omega_j & 0 \end{bmatrix} \quad (7)$$

and $\mathcal{B}_i \in \mathbb{R}^{2k_i \times m}$. Then a minimal realization of $G_1(s)$ is given by: $[A_1, B_1, C_1, 0]$, where $A_1 = \text{diag}(\mathcal{A}_0, \mathcal{A}_1, \mathcal{A}_2, \dots, \mathcal{A}_p)$; $B_1 = [\mathcal{B}_0^T, \mathcal{B}_1^T, \dots, \mathcal{B}_p^T]^T$; $C_1 = [\mathcal{C}_0, \mathcal{C}_1^T, \dots, \mathcal{C}_p^T]$; $A_1 \in \mathbb{R}^{n_1 \times n_1}$, $B_1 \in \mathbb{R}^{n_1 \times m}$, $C_1 \in \mathbb{R}^{m \times n_1}$ ($n_1 = v + 2 \sum_{i=1}^p k_i$)

Let $[A_2, B_2, C_2, D]$ be a minimal realization $G_2(s)$, where $A_2 \in \mathbb{R}^{n_2 \times n_2}$, $B_2 \in \mathbb{R}^{n_2 \times m}$, $C_2 \in \mathbb{R}^{m \times n_2}$, and $D \in \mathbb{R}^{m \times m}$ ($n_2 = n - n_1$). Let $A = \text{diag}(A_1, A_2)$; $B = [B_1^T, B_2^T]^T$; $C = [C_1, C_2]$. Then $A \in \mathbb{R}^{n \times n}$, $B \in \mathbb{R}^{n \times m}$, $C \in \mathbb{R}^{m \times n}$, and $D \in \mathbb{R}^{m \times m}$. Since $G_1(s)$ and $G_2(s)$ have no poles in common, $[A, B, C, D]$ is a minimal realization of $G(s)$.

IV. CHARACTERIZATION OF MSPR SYSTEMS

We next present the state-space characterization of MSPR systems, which is an extension of the Kalman-Yacubovich lemma for WSPR systems, that was proved in [6].

Lemma 2. If $G(s)$ is MSPR, there exist real matrices: $P = P^T > 0$, $P \in \mathbb{R}^{n \times n}$, $L \in \mathbb{R}^{k \times n}$, $W \in \mathbb{R}^{k \times m}$ (where $k \geq m$), such that

$$A^T P + P A = -L^T L \quad (8)$$

$$C = B^T P + W^T L \quad (9)$$

$$W^T W = D + D^T \quad (10)$$

$$L = [0_{k \times n_1}, \mathcal{L}_{k \times n_2}] \quad (11)$$

where $[A_2, B_2, \mathcal{L}, W]$ is minimal and minimum-phase.

Proof. $G(s)$ has the form given by Eqs. (1) and (2). Since $G(s)$ is MSPR, conditions (i)-(iii) of Lemma 1 hold. Considering $G_1(s)$ in Eq. (2), it consists of $(p+1)$ transfer functions in parallel. A minimal realization (Eqs. 4 and 5) of the first term, $[\alpha_0/s]$, is: $[\mathcal{A}_0, \mathcal{B}_0, \mathcal{C}_0, 0]$. Letting $\Pi_0 = \Lambda$, it can be verified that the equations are satisfied:

$$\mathcal{A}_0^T \Pi_0 + \Pi_0 \mathcal{A}_0 = 0 \quad (12)$$

$$\Pi_0 \mathcal{B}_0 = \mathcal{C}_0^T \quad (13)$$

A minimal realization (Eqs. 6, 7) of the i th component of the second term, $[\alpha_i s + \beta_i] / (s^2 + \omega_i^2)$, is given by $[\mathcal{A}_i, \mathcal{B}_i, \mathcal{B}_i^T, 0]$. Letting $\Pi_i = I_{2k_i}$, it can be seen that this realization satisfies:

$$\mathcal{A}_i^T \Pi_i + \Pi_i \mathcal{A}_i = 0 \quad (14)$$

$$\Pi_i \mathcal{B}_i = \mathcal{B}_i \quad (15)$$

Finally, following [6,7], since $G_2(s)$ is WSPR, there exist $P_2 = P_2^T > 0$, $P_2 \in \mathbb{R}^{n_2 \times n_2}$, $W \in \mathbb{R}^{k \times m}$, $\mathcal{L} \in \mathbb{R}^{k \times n_2}$ (where $k \geq m$) such that

$$A_2 P_2 + P_2 A_2 = -\mathcal{L}^T \mathcal{L} \quad (16)$$

$$C_2 = B_2^T P_2 + W^T \mathcal{L} \quad (17)$$

$$W^T W = D + D^T \quad (18)$$

where $[A_2, B_2, \mathcal{L}, W]$ is minimal and minimum-phase.

Defining

$$P_1 = \text{diag}(\Pi_0, \Pi_1, \dots, \Pi_p) \quad (19)$$

and

$$P = \text{diag}(P_1, P_2) \quad (20)$$

we have the required result. ■

V. STABILITY OF FEEDBACK INTERCONNECTION

Consider the system in Figure 1, where $G(s)$ and $H(s)$ are $m \times m$ proper rational matrices. This system is said to be stable if its state-space realization consisting of individual minimal realizations of $G(s)$ and $H(s)$, is asymptotically stable. We have the following stability result.

Theorem 1. The negative feedback interconnection of $G(s)$ and $H(s)$ is stable if all of the following conditions are satisfied:

- (i) $G(s)$ is MSPR;
- (ii) $H(s)$ is PR;
- (iii) None of the $j\omega$ -axis poles of $G(s)$ is a transmission zero of $H(s)$.

Proof. Let $[A, B, C, D]$ denote the minimal realization of $G(s)$ described in Section III, and let x be the corresponding state vector of order n . Let n_2 denote the number of poles of the stable part $G_2(s)$ of $G(s)$, and let $[A_2, B_2, C_2, D]$ denote its minimal realization. Let $[\hat{A}, \hat{B}, \hat{C}, \hat{D}]$ denote a minimal realization of $H(s)$, and let \hat{x} denote the corresponding \hat{n} th order state vector. Since $G(s)$ is MSPR, from Lemma 2, there exist matrices $P = P^T > 0$, $P \in \mathbb{R}^{n \times n}$, $\mathcal{L} \in \mathbb{R}^{m \times n}$, $W \in \mathbb{R}^{m \times m}$, such that Eqs. (8)-(11) are satisfied, and $[A_2, B_2, \mathcal{L}, W]$ is minimal and minimum-phase.

Since $H(s)$ is PR, there exist matrices $\hat{P} = \hat{P}^T > 0$, $\hat{P} \in \mathbb{R}^{\hat{n} \times \hat{n}}$, $\hat{L} \in \mathbb{R}^{\hat{k} \times \hat{n}}$, $\hat{W} \in \mathbb{R}^{\hat{k} \times \hat{m}}$, such that [7]

$$\hat{A}^T \hat{P} + \hat{P} \hat{A} = -\hat{L}^T \hat{L} \quad (21)$$

$$\hat{C} = \hat{B}^T \hat{P} + \hat{W}^T \hat{L} \quad (22)$$

$$\hat{W}^T \hat{W} = \hat{D} + \hat{D}^T \quad (23)$$

Consider the candidate Lyapunov function:

$$V(x, \hat{x}) = x^T P x + \hat{x}^T \hat{P} \hat{x} \quad (24)$$

Proceeding as in the proof of Theorem 1 in [6] we have:

$$\dot{V} = 2u^T y - z^T z + 2\hat{u}^T \hat{y} - \hat{z}^T \hat{z} \quad (25)$$

where

$$z = Lx + Wu = \mathcal{L}x_2 + Wu \quad (26)$$

$$\hat{z} = \hat{L}\hat{x} + \hat{W}\hat{u} \quad (27)$$

Since $\hat{u} = y$ and $\hat{y} = -u$,

$$\dot{V} = -z^T z - \hat{z}^T \hat{z} \leq -z^T z \quad (28)$$

i.e., \dot{V} is negative semi-definite. $\dot{V}=0$ implies $z=0$. However, $z(t)$ is the output of the system: $F(s) = W+L(sI-A)^{-1}B = W+ \mathcal{L}(sI-A_2)^{-1}B_2$, which is minimum-phase. Every input $u(t)$ that results in $z(t) = 0$ must have the form: $u(t) = \sum_j \bar{u}_j t^k e^{\zeta_j t}$, where ζ_j 's are zeros of $F(s)$, and $\text{Re}[\zeta_j] < 0$. Therefore, $u(t) \rightarrow 0$ exponentially; i.e., $\hat{y}(t) \rightarrow 0$ exponentially. As a result, $y(t)$ will consist of i) exponentially decaying terms corresponding to the zeros of $F(s)$ as well as stable poles of $G(s)$, and ii) persistent terms such as $\mu e^{j\omega_i t}$, corresponding to unstable poles at $s=j\omega_i$ (including at $s=0$) of $G(s)$. Since $j\omega_i$ are not the transmission zeros of $H(s)$, this would imply that $\hat{y}(t) [= -u(t)]$ will contain persistent terms such as $e^{j\omega_i t}$. However, this contradicts the fact proved previously that $u(t)$ decays exponentially. Therefore, $y(t)$ can consist only of exponentially decaying terms, i.e., $y(t) \rightarrow 0$ exponentially. Because of the minimality of $[A,B,C,D]$ and $[\hat{A}, \hat{B}, \hat{C}, \hat{D}]$, this implies $x(t) \rightarrow 0$, $\hat{x}(t) \rightarrow 0$ exponentially. Using LaSalle's invariance theorem [10], the system is asymptotically stable. ■

It should be noted that, in Theorem 1, $G(s)$ and $H(s)$ are completely interchangeable.

The following corollaries are an immediate consequence of Theorem 1.

Corollary 1.1 The negative feedback interconnection of $G(s)$ and $H(s)$ is stable if $G(s)$ is WSPR and $H(s)$ is PR.

Corollary 1.1 is the same stability result which was given in [6].

Corollary 1.2 The negative feedback interconnection of $G(s)$ and $H(s)$ is stable if both $G(s)$ and $H(s)$ are MSPR.

Example: Consider the rotational motion of a flexible spacecraft with m torque actuators and m collocated attitude sensors ($m \geq 3$). Assume that there is at least one torque actuator for each (orthogonal) axis of rotation. The transfer function from the torque input to the attitude (position) output, y_p , is given by:

$$G(s) = G'(s)/s \quad (29)$$

where

$$G'(s) = \frac{\alpha_0}{s} + \sum_{i=1}^p \frac{\alpha_i s}{s^2 + 2\rho_i \omega_i s + \omega_i^2} \quad (30)$$

where $G'(s)$ is the transfer function from the torque input to the attitude rate $y_r (= \dot{y}_p)$; $\alpha_i = \alpha_i^T \geq 0$ ($i = 0, 1, 2, \dots, p$), and α_0 is a rank-3 matrix; $\omega_i (> 0)$ represent the natural frequencies, and $\rho_i \geq 0$ represents the inherent damping ratio, for the i^{th} elastic mode ($i=1,2,\dots,p$). It can be easily verified that $G'(s)$ is PR, and therefore, from Theorem 1, it can be stabilized by any MSPR controller. Let $\mathcal{C}(s)$ denote an $m \times m$ stable transfer function which has no transmission zeros on the imaginary axis, and suppose $H(s) = [\mathcal{C}(s)/s]$ is MSPR. Then $H(s)$ stabilizes $G'(s)$. Examining the block diagram in Figure 2, $\mathcal{C}(s)$ stabilizes $G(s)$. In other words, a flexible spacecraft, which has zero-frequency rigid-body modes as well as damped or undamped elastic modes, is stabilized by the controller $\mathcal{C}(s)$ which has the above properties. The stability does not depend on the number of elastic modes, or the parameter values, and is therefore *robust*.

VI. CONCLUDING REMARKS

The concept of marginally strictly positive real (MSPR) systems was introduced, which allows poles on the imaginary axis, and is therefore less restrictive than the

previous definitions of strict positive realness. A state-space characterization of MSPR systems was obtained, and it was proved that the negative feedback interconnection of an MSPR system and a positive real (PR) system, is asymptotically stable. The result significantly extends the previous passivity-based stability results for linear time-invariant systems.

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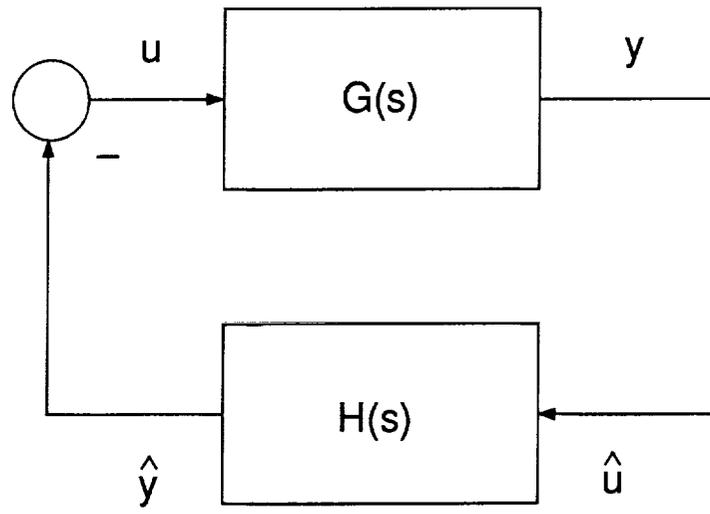


Figure 1. Negative Feedback Loop.

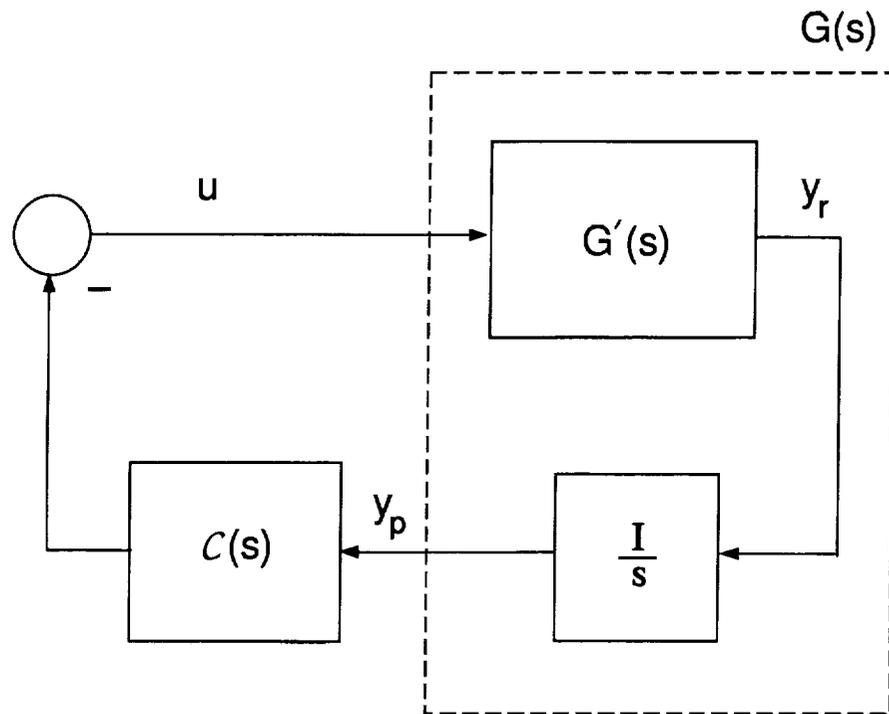


Figure 2. Flexible Structure Example.

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302, and to the Office of Management and Budget, Paperwork Reduction Project (0704-0188), Washington, DC 20503.

1. AGENCY USE ONLY (Leave blank)		2. REPORT DATE <p style="text-align: center;">July 1994</p>	3. REPORT TYPE AND DATES COVERED <p style="text-align: center;">Technical Memorandum</p>	
4. TITLE AND SUBTITLE <p style="text-align: center;">Robust Stabilization of Marginally Stable Positive-Real Systems</p>			5. FUNDING NUMBERS <p style="text-align: center;">WU 233-01-01-05</p>	
6. AUTHOR(S) <p style="text-align: center;">Suresh M. Joshi and Sandeep Gupta</p>				
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) NASA Langley Research Center Hampton, VA 23681-0001			8. PERFORMING ORGANIZATION REPORT NUMBER	
9. SPONSORING / MONITORING AGENCY NAME(S) AND ADDRESS(ES) National Aeronautics and Space Administration Washington, DC 20546-0001			10. SPONSORING / MONITORING AGENCY REPORT NUMBER <p style="text-align: center;">NASA TM-109136</p>	
11. SUPPLEMENTARY NOTES S. Gupta - Vignyan Research Associates, Hampton, Virginia				
12a. DISTRIBUTION / AVAILABILITY STATEMENT Unclassified - Unlimited Subject Category 63			12b. DISTRIBUTION CODE	
13. ABSTRACT (Maximum 200 words) This paper investigates the stability of the negative feedback interconnection to two positive-real systems which have poles in the closed left-half of the complex plane. A new definition of marginally strict positive real systems is introduced, and sufficient conditions are obtained for the stability of the feedback interconnection of such systems, using the Lyapunov method. The conditions obtained have direct applications to dynamic dissipative controllers for flexible spacecraft and are the least restrictive ones published to date.				
14. SUBJECT TERMS <p style="text-align: center;">Passive systems, Positive-real systems, Robust stabilization</p>			15. NUMBER OF PAGES <p style="text-align: center;">14</p>	
			16. PRICE CODE <p style="text-align: center;">A03</p>	
17. SECURITY CLASSIFICATION OF REPORT <p style="text-align: center;">Unclassified</p>	18. SECURITY CLASSIFICATION OF THIS PAGE <p style="text-align: center;">Unclassified</p>	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT	