Extended Cooperative Control Synthesis

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Abstract

This paper reports on research for extending the Cooperative Control Synthesis methodology to include a more accurate modeling of the pilot's controller dynamics. Cooperative Control Synthesis (CCS) is a methodology that addresses the problem of how to design control laws for piloted, high-order, multivariate systems and/or nonconventional dynamic configurations in the absence of flying qualities specifications. This is accomplished by emphasizing the parallel structure inherent in any pilot-controlled, augmented vehicle. The original CCS methodology is extended to include the modified optimal control model (MOCM), which is based upon the optimal control model of the human operator developed by Kleinman, Baron, and Levison in 1970. This model provides a modeling of the pilot's compensation dynamics that is more accurate than the simplified pilot dynamic representation currently in the CCS methodology. Inclusion of the MOCM into the CCS also enables the modeling of pilot-observation perception thresholds and pilot-observation attention allocation effects. This Extended Cooperative Control Synthesis (ECCS) allows for the direct calculation of pilot and system open- and closed-loop transfer functions in pole/zero form and is readily implemented in current software capable of analysis and design for dynamic systems. Example results based upon synthesizing an augmentation control law for an acceleration command system in a compensatory tracking task using the ECCS are compared with a similar synthesis performed by using the original CCS methodology. The ECCS is shown to provide augmentation control laws that yield more favorable, predicted closed-loop flying qualities and tracking performance than those synthesized using the original CCS methodology.

Introduction

Increasing aircraft agility, maneuvering at high angles of attack, and exploring radical flight vehicle geometries to obtain low observability are areas of current research that show promise of greatly increasing aircraft mission performance. To fully exploit these and other possible new capabilities, future aircraft may require high-order, multivariate flight control systems or demand designs dealing with nonconventional flight dynamics. Although design guidance is available in the form of flying qualities specifications for aircraft exhibiting conventional dynamics, very little design guidance is available to the flight control designer for synthesizing control laws to achieve both good piloted performance and good flying qualities for high-order and/or nonconventional configurations.

In 1979, Schmidt proposed a synthesis methodology that addresses the problem of how to design control laws for piloted, high-order, multivariate and/or nonconventional dynamics configurations in the absence of flying qualities specifications. This methodology, referred to as "Cooperative Control Synthesis (CCS)," emphasizes the parallel structure inherent in any pilot-controlled augmented vehicle. (See fig. 1.) The CCS methodology is applicable to high-order systems and leads to control laws for good piloted performance and subjective evaluation. In this method, optimal control theory is utilized for both the control law synthesis and a simplified modeling of the pilot's compensation dynamics. By including this simplified model, the CCS methodology explicitly includes design objectives based upon pilot acceptability. The original CCS methodology was extended by Innocenti and Schmidt (1984) to include state estimation in
the pilot model and to allow a measurement feedback control law. Garg and Schmidt (1989) have shown how the CCS methodology can be used to synthesize pilot-acceptable display dynamics.

This paper reports on research for extending the CCS methodology to include a more accurate modeling of the pilot's controller dynamics than the simplified pilot dynamic representation currently used in the CCS. This is accomplished by replacing the simplified model of the pilot's controller dynamics currently in the CCS with the modified optimal control model (MOCM) (Davidson and Schmidt, 1992). The MOCM is based upon the optimal control model (OCM) of the human operator developed by Kleinman, Baron, and Levison (1970) and Baron, Kleinman, and Levison (1970). Inclusion of the MOCM into the CCS also enables the modeling of pilot-observation perception thresholds and pilot-observation attention allocation effects. This extended CCS allows for the direct calculation of pilot and system transfer functions in pole/zero form and is designed for easy implementation in current software capable of analysis and design for dynamic systems.

A theoretical development of the Extended Cooperative Control Synthesis (ECCS) methodology is provided, and this methodology is used to synthesize augmentation control laws for an acceleration command system in a compensatory tracking task. This analysis is compared with similar designs performed by using the original CCS methodology, and conclusions are presented.

Symbols and Abbreviations

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<thead>
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<th>Symbol</th>
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<td>B</td>
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<td>cost function control-rate weighting</td>
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<td>$G$</td>
<td>gain matrix</td>
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<td>$g_i$</td>
<td>$i$th pilot regulator gain</td>
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<td>objective function</td>
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<td>system measurement matrix</td>
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<td>$x_d$</td>
<td>Padé delay state vector</td>
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<td>$y$</td>
<td>pilot observation vector</td>
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<tr>
<td>$z$</td>
<td>measurement vector</td>
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</table>
\[ \delta \] input to plant

\[ \theta \] pitch attitude

\[ \rho \] signal-to-noise ratio

\[ \Sigma \] filter Ricatti solution matrix

\[ \tau \] effective time delay

\[ \tau_\eta \] "neuromotor" lag (neuro-lag)

\[ \chi \] augmented state vector

Abbreviations:

CCS Cooperative Control Synthesis

E_{\infty} expected value

ECCS Extended Cooperative Control Synthesis

LQG linear quadratic Gaussian

MOCM modified optimal control model

OCM optimal control model

PR pilot rating

rms root mean square

Subscripts:

\[ a \] augmented

\[ c \] pilot-commanded control

\[ \text{cmd} \] command signal

\[ d \] delay

\[ obs \] pilot observed

\[ p \] pilot

\[ s \] plant and delay augmented system

\[ u \] control

\[ \delta \] plant input

\[ 0 \] control-rate augmented system

\[ 1 \] plant and pilot augmented system

Operators and superscripts:

\[ T \] transpose

\[ -1 \] inverse

\[ \ast \] optimal

\[ (\cdot) \] derivative with respect to time

\[ (\hat{\cdot}) \] estimate
Theoretical Development

The Cooperative Control Synthesis (CCS) methodology involves the simultaneous solution of two coupled optimal control problems (Papavassilopoulos, Medanic, and Cruz, 1979). One optimal controller can be thought of as representing a pilot's control dynamics, whereas the other represents the augmentation control law dynamics. (See fig. 1.) A simultaneous solution is required because the pilot's control strategy is a function of the augmented vehicle dynamics, and the vehicle augmentation control law is not known a priori. This section presents a development of the Extended Cooperative Control Synthesis (ECCS) methodology incorporating the modified optimal control model (MOCM) to represent both the pilot's control dynamics and a direct-output-feedback linear quadratic controller for the augmentation control law dynamics.

Control Solution

The plant dynamics to be acted upon by the two optimal controllers acting in parallel, augmented with the system disturbance dynamics, are given by the state space, time-invariant linear form

\[ \begin{align*}
\dot{x} &= Ax + B\delta + Ew \\
\delta &= \delta_p + \delta_a \\
y &= Cx \\
y_{\text{obs}} &= y + v_y \\
z &= Mx
\end{align*} \]

(1)

where \( x \) is an \( n_x \)-dimensional state vector, \( \delta \) is an \( n_u \)-dimensional vector equal to the sum of the pilot's control input \( (\delta_p) \) and augmentation controller input \( (\delta_a) \), and \( w \) is an \( n_w \)-dimensional disturbance vector modeled as zero-mean Gaussian white noise with an intensity \( W \). The vector \( y \) of dimension \( n_y \) represents variables that the pilot can perceive, either by observation or feel. The perceptual model observed by the pilot \( (y_{\text{obs}}) \) is assumed to be corrupted by an observation noise \( (v_y) \), i.e., a zero-mean Gaussian white noise process with intensity \( V_y \). The vector \( z \) of dimension \( n_z \) denotes system measurements available for feedback. For this development, the assumption is made that the system is stabilizable and detectable and that the measurements are noise free. The basic structural components of these two controllers acting on the dynamic system are represented in the block diagram in figure 1.

Solution for pilot dynamics. This formulation of the CCS methodology incorporates the MOCM (Davidson and Schmidt, 1992) to model the pilot's compensation dynamics. A block diagram of the model components of the MOCM is given in figure 2. The MOCM is based upon the optimal control model (OCM) of the human operator developed by Kleinman, Baron, and Levison (1970) and is a variation of simplified optimal pilot models developed by Hess (1976), Broussard and Stengel (1977), and Schmidt (1979). This model of the human operator is input compatible with the OCM and retains other key aspects of the OCM, such as the modeling of pilot-observation perception thresholds and pilot-observation attention allocation effects. Unlike the OCM, however, the structure allows for the direct calculation of pilot and system transfer functions in pole-zero form.

In the MOCM, the pilot's effective time delay is modeled by a second-order Padé approximation. This time delay is placed at each of the pilot's outputs and is treated as part of the plant dynamics for the determination of the pilot's regulation and filter gains. To simplify the notation, this development considers the case of a single control input, although the algorithm can easily
be extended to account for multiple pilot and/or augmentation control inputs. A second-order Padé approximation is given by

\[ \frac{u_d}{u_p} = \frac{1 - 1/2(\tau s) + 1/8(\tau s)^2}{1 + 1/2(\tau s) + 1/8(\tau s)^2} \]  

(2)

where \( \tau \) is the delay interval, \( u_p \) is the pilot's output, and \( u_d \) is the delayed pilot's output. In state space form, this can be expressed as

\[
\begin{align*}
\dot{x}_d &= A_d x_d + B_d u_p \\
\delta_p &= u_d = C_d x_d + u_p
\end{align*}
\]

(3)

where \( x_d \) is a two-element vector of Padé delay states.

The plant dynamics augmented with the pilot's effective time delay are given by

\[
\frac{d}{dt} \begin{bmatrix} x \\ x_d \end{bmatrix} = \begin{bmatrix} A & BC_d \\ 0 & A_d \end{bmatrix} \begin{bmatrix} x \\ x_d \end{bmatrix} + \begin{bmatrix} B \\ B_d \end{bmatrix} u_p + \begin{bmatrix} B \\ 0 \end{bmatrix} \delta_p + \begin{bmatrix} E \\ 0 \end{bmatrix} w
\]

(4)

\[
y = \begin{bmatrix} C \\ 0 \end{bmatrix} \begin{bmatrix} x \\ x_d \end{bmatrix}
\]

(5)

\[
z = \begin{bmatrix} M \\ 0 \end{bmatrix} \begin{bmatrix} x \\ x_d \end{bmatrix}
\]

The pilot's observation vector is given by

\[
y_{obs} = \begin{bmatrix} C \\ 0 \end{bmatrix} \begin{bmatrix} x \\ x_d \end{bmatrix} + v_y
\]

This model makes the assumption that the pilot's control task can be defined by the minimization of the quadratic performance index \( (J_p) \) given by

\[
J_p = E_{\infty} \left\{ y^T Q_p y + u_p^T r_p u_p + \hat{u}_p^T f_p \hat{u}_p \right\}
\]

(5)
subject to pilot observations ($y_{obs}$) with cost-functional weightings $Q_p \geq 0$, $r_p \geq 0$, and $f_p > 0$.

The system given by equation (4) can be expressed in a control-rate formulation as

$$\frac{d}{dt} \begin{bmatrix} x \\ x_d \\ u_p \end{bmatrix} = \begin{bmatrix} A & BC_d & B \\ 0 & A_d & B_d \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ x_d \\ u_p \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u_p + \begin{bmatrix} B \\ 0 \\ 0 \end{bmatrix} \delta_a + \begin{bmatrix} E \\ 0 \\ 0 \end{bmatrix} w$$

$$y_{obs} = [C \ 0 \ 0] \begin{bmatrix} x \\ x_d \\ u_p \end{bmatrix} + v_y$$

$$z = [M \ 0 \ 0] \begin{bmatrix} x \\ x_d \\ u_p \end{bmatrix}$$

or by defining a new state vector as

$$\chi = \begin{bmatrix} x^T \\ x_d^T \\ u_p \end{bmatrix}^T$$

leads to

$$\dot{\chi} = A_0 \chi + B_p \delta_a + E_0 w$$

$$y_{obs} = C_1 \chi + v_y$$

$$z = M_1 \chi$$

The pilot controls the augmented aircraft. The augmentation control law is given by

$$\delta_a = G_{a} z = G_{a} M_1 \chi$$

The system with the augmentation loop closed is given by

$$\dot{\chi} = (A_0 + B_a G_{a} M_1) \chi + B_p \delta_a + E_0 w$$

$$y_{obs} = C_1 \chi + v_y$$

$$z = M_1 \chi$$

The performance index can be rewritten in terms of the augmented state vector ($\chi$) as

$$J_p = E_{\infty} \left\{ \chi^T Q \chi + \dot{u}_p^T f_p \dot{u}_p \right\}$$

where

$$Q = \text{diag} \left[ C_1^T Q_p C_1, r_p \right]$$

The minimizing control law is obtained by application of standard LQG solution techniques (Kwakernaak and Sivan, 1972) to the delayed augmented system (eq. (8)). This leads to the linear feedback law (Kleinman, Baron, and Levison, 1970)

$$\dot{u}_p^* = -g_{n+1} u_p^* - \sum_{i=1}^{n} g_i \dot{x}_i$$

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where \( n = n_x + 2 \) (system states plus two Padé states) and \( \hat{X} \) is the estimate of the state \( X \). By letting
\[
\tau_\eta = \frac{1}{g_{n+1}}
\]
and
\[
l_p = \tau_\eta [g_1, \ldots, g_n]
\]
one obtains
\[
\tau_\eta \hat{u}_p + u_p = u_c
\]
where the pilot's commanded control \( u_c \) is given by
\[
u_c = -l_p \hat{x}_s
\]
where \( x_s = [x^T \ x_d^T]^T \).

The \( n + 1 \) feedback gains \( G_p \) are obtained from
\[
G_p = [g_1, \ldots, g_n, g_{n+1}] = f_p^{-1}B_p^T K
\]
where \( K \) is the unique positive semidefinite solution of the Ricatti equation
\[
0 = (A_0 + B_a G_a M_1)^T K + K (A_0 + B_a G_a M_1) + Q - KB_p f_p^{-1} B_p^T K
\]
To account for the uncertainty of the human operator's control input, motor noise \( (v_u) \) is added to the commanded control \( (u_c) \). Thus,
\[
\tau_\eta \hat{u}_p + u_p = u_c + v_u
\]
where \( v_u \) is a zero-mean Gaussian white noise process with intensity \( V_u \). Solving for \( \hat{u}_p \) gives
\[
\hat{u}_p = -\frac{1}{\tau_\eta} u_p + \frac{1}{\tau_\eta} u_c + \frac{1}{\tau_\eta} v_u
\]
The controller gains are assumed not to be affected by the inclusion of the motor noise. This reduces the solution of the human operator model to a suboptimal control law.

Therefore, the augmented system combined with equation (16) is given by
\[
\dot{\hat{x}} = \begin{bmatrix} A & BC_d & B \\ 0 & A_d & B_d \\ 0 & 0 & -1/\tau_\eta \end{bmatrix} \hat{x} + \begin{bmatrix} B \\ 0 \\ 0 \end{bmatrix} G_a M_1 \hat{x} + \begin{bmatrix} 0 \\ 0 \\ 1/\tau_\eta \end{bmatrix} u_c + \begin{bmatrix} E \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} w \\ v_u \end{bmatrix}
\]
or
\[
\dot{\hat{x}} = (A_1 + B_a G_a M_1) \hat{x} + B_1 u_c + E_1 w_1
\]
The current estimate of the state \( \hat{X} \) is given by a Kalman filter
\[
\dot{\hat{x}} = (A_1 + B_a G_a M_1) \hat{x} + B_1 u_c + F (y_{obs} - \hat{y})
\]
\[
\dot{\hat{x}} = (A_1 + B_a G_a M_1 - FC_1) \hat{x} + FC_1 x + B_1 u_c + F v_y
\]
where
\[
F = \Sigma C_1^T V_y^{-1}
\]
The covariance matrix of the estimation error ($\Sigma$) is the unique positive semidefinite solution of the Ricatti equation

$$0 = (A_1 + B_aG_aM_1)\Sigma + \Sigma(A_1 + B_aG_aM_1)^T + E_1W_1E_1^T - \Sigma C_1^T V_y^{-1} C_1 \Sigma$$  \hspace{1cm} \text{(19)}$$

where $W_1 = \text{diag}[W, V_u]$ with $W \geq 0$, $V_u \geq 0$, and $V_y > 0$.

**Solution for augmentation controller.** As in the original Cooperative Control Synthesis methodology, this formulation employs a direct-output-feedback linear quadratic controller (Levine and Athans, 1970) for the augmentation controller. This augmentation control law is chosen to be optimal with respect to the objective function ($J_a$) given by

$$J_a = E\infty \left\{ y^T Q_a y + u_p^T r_{a1} u_p + \delta_a^T r_{a2} \delta_a + \dot{u}_p^T f_a \dot{u}_p \right\}$$  \hspace{1cm} \text{(20)}$$

where $Q_a \geq 0$, $r_{a1} \geq 0$, $r_{a2} > 0$, and $f_a > 0$. This approach hypothesizes that the pilot’s index of performance is correlated with the pilot’s subjective rating of the vehicle handling qualities in the design task (Schmidt, 1981). This can be accomplished by letting $Q_a = Q_p$, $r_{a1} = r_p$, and $f_a = f_p$; then, $J_a$ can be expressed by

$$J_a = J_p + E\infty \left\{ \delta_a^T r_{a2} \delta_a \right\}$$  \hspace{1cm} \text{(21)}$$

The solution for the augmentation controller is carried out on the system in the presence of the pilot’s control compensation. The system with the augmentation loop open is given by

$$\dot{x} = \begin{bmatrix} A & BC_d & B \\ 0 & A_d & B_d \\ 0 & 0 & -1/\tau_\eta \end{bmatrix} \begin{bmatrix} x \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} B \\ 0 \\ 0 \end{bmatrix} \delta_a + \begin{bmatrix} 0 \\ 0 \\ 1/\tau_\eta \end{bmatrix} u_c + \begin{bmatrix} E & 0 \\ 0 & 0 \\ 0 & 1/\tau_\eta \end{bmatrix} \begin{bmatrix} w \\ v_u \end{bmatrix}$$

or

$$\dot{x} = A_1 x + B_a \delta_a + B_1 u_c + E_1 w_1$$  \hspace{1cm} \text{(22)}$$

The pilot’s compensation dynamics are given by

$$\dot{x} = (A_1 + B_aG_aM_1)\hat{x} + B_1 u_c + F(y_{obs} - \hat{y})$$

$$u_c = -t_1 \hat{x}$$

where $t_1 = [t_p \ 0]$. Therefore, the pilot-augmented system is given by

$$\frac{d}{dt} \begin{bmatrix} x \\ \hat{x} \end{bmatrix} = \begin{bmatrix} A_1 & -B_1 t_1 \\ FC_1 & A_1 + B_aG_aM_1 - B_1 t_1 - FC_1 \end{bmatrix} \begin{bmatrix} x \\ \hat{x} \end{bmatrix} + \begin{bmatrix} B_a \\ 0 \end{bmatrix} \delta_a + \begin{bmatrix} E_1 & 0 \\ 0 & F \end{bmatrix} \begin{bmatrix} w_1 \\ v_y \end{bmatrix}$$

$$z = [M_1 \ 0] \begin{bmatrix} x \\ \hat{x} \end{bmatrix}$$

or, by letting

$$q = [x^T \ \hat{x}^T]^T$$

then

$$\dot{q} = \bar{A} q + \bar{B} \delta_a + \bar{E} w$$

$$z = M q$$  \hspace{1cm} \text{(23)}$$
The augmentation control objective function written in terms of the augmented state vector \( q \) is given as

\[
J_a = E_{\infty} \left\{ q^T \overline{Q} q + \delta_a^T r_a \delta_a \right\}
\]  
(24)

where

\[
\overline{Q} = \begin{bmatrix}
    C_1 Q_p C_1^T & 0 & 0 \\
    0 & r_p & 0 \\
    0 & 0 & G_p^T f_a G_p
\end{bmatrix}
\]

The minimizing control law is given by

\[
\delta_a = G_a z = G_a \bar{M} q
\]

where the minimizing gains \( G_a \) are given by

\[
G_a = -r_{a2}^{-1} B^T H L M^T (MLM^T)^{-1}
\]

The matrices \( H \) and \( L \) are obtained from the simultaneous solution of two coupled steady-state Lyapunov equations:

\[
\begin{align*}
(\overline{A} + \overline{B} G_a \overline{M}) L + L (\overline{A} + \overline{B} G_a \overline{M})^T + \bar{E} \bar{W} \bar{E}^T &= 0 \\
(\overline{A} + \overline{B} G_a \overline{M})^T H + H (\overline{A} + \overline{B} G_a \overline{M}) + (\overline{Q} + \overline{M}^T G_a^T r_a G_a \overline{M}) &= 0
\end{align*}
\]  
(27)

**System closed-loop and pilot state space models.** A state space representation of the closed-loop pilot-vehicle system is given by

\[
\frac{d}{dt} \begin{bmatrix}
    \chi \\
    \dot{x}
\end{bmatrix} = \begin{bmatrix}
    A_1 + B_a G_a M_1 & -B_1 t_1 \\
    FC_1 & A_1 + B_a G_a M_1 - B_1 t_1 - FC_1
\end{bmatrix} \begin{bmatrix}
    \chi \\
    \dot{x}
\end{bmatrix} + \begin{bmatrix}
    E_1 & 0 \\
    0 & F
\end{bmatrix} \begin{bmatrix}
    w_1 \\
    v_y
\end{bmatrix}
\]

\[
\begin{bmatrix}
    y \\
    \delta
\end{bmatrix} = \begin{bmatrix}
    C_1 & 0 \\
    C_6 & 0
\end{bmatrix} \begin{bmatrix}
    \chi \\
    \dot{x}
\end{bmatrix}
\]

where

\[
C_6 = [G_a M \ C_d \ 1 \ 0 \ 0 \ 0]
\]

A block diagram of the model components of the pilot's dynamics is given in figure 2, and a state space representation of the pilot's dynamics is given by

\[
\frac{d}{dt} \begin{bmatrix}
    \hat{\chi} \\
    u_p \\
    x_d
\end{bmatrix} = \begin{bmatrix}
    A_1 + B_a G_a M_1 - FC_1 - B_1 t_1 & 0 & 0 \\
    -l_1/\tau_1 & -1/\tau_1 & 0 \\
    0 & -l_1/\tau_1 & 0
\end{bmatrix} \begin{bmatrix}
    \hat{\chi} \\
    u_p \\
    x_d
\end{bmatrix} + \begin{bmatrix}
    F & 0 \\
    0 & y \\
    0 & 0
\end{bmatrix} \begin{bmatrix}
    v_y \\
    v_{u}
\end{bmatrix}
\]

\[
\delta_p = [0 \ 1 \ C_d] \begin{bmatrix}
    \hat{\chi} \\
    u_p \\
    x_d
\end{bmatrix}
\]

or

\[
\begin{bmatrix}
    \dot{x}_p = A_p x_p + B_p y + E_p v_p \\
    \delta_p = C_p x_p
\end{bmatrix}
\]

(29)
where
\[ x_p = \begin{bmatrix} \hat{X}^T & u_p & x_d^T \end{bmatrix}^T \]

**Algorithm Implementation**

The solution to the ECCS problem requires the simultaneous solution for the pilot’s control dynamics (by solution of the MOCM) and the augmentation controller (by solving a direct-output-feedback control synthesis problem). The solution of the MOCM involves solving two algebraic Ricatti equations for the pilot’s control and estimation gains. Solution of the direct-output-feedback, linear quadratic control synthesis problem requires the simultaneous solution of two Lyapunov equations. The optimal augmentation gains can be obtained by numerical iteration by employing a conjugate gradient search technique (Fletcher and Powell, 1963). A conceptual flowchart of the ECCS algorithm is given in the appendix. The implementation of this algorithm requires the capability of solving steady-state Ricatti and Lyapunov equations.

The structure of the extended CCS methodology allows implementation in current software capable of analysis and design for dynamic systems. Implementation in this type of environment allows for rapid calculation of pilot and system transfer function descriptions from state space models and rapid determination of system frequency responses. Also, this environment allows users to interactively modify various pilot and plant parameters and quickly ascertain the impact of these changes on the pilot/closed-loop performance.

Values of effective time delay, “neuromotor” lag (neuro-lag), observation, and motor noise intensities are chosen in the same manner as for the OCM. Like in the OCM, desired values of neuromotor lag (τ_ν) can be obtained by an appropriate choice of cost-function control-rate weighting (f_p). Manual control experiments have shown that the effective time delay of the pilot (τ) is typically 0.1 to 0.2 sec (Kleinman, Baron, and Levison, 1970).

The intensity of the observation noise (V_y) is dependent upon the nature of the display, human limitations, and the pilot’s environment. Over a wide range of viewing conditions, the diagonal elements of the observation noise-intensity matrix are proportional to the variance of its associated observed variable. Single-axis manual-tracking control tasks have shown that on the average, \( \rho_y = 0.01 \), which corresponds to a normalized observation noise ratio of -20 dB. The intensity of the motor noise (V_u) is assumed to be proportional to the variance of the commanded control (u_c). An analysis of single-axis, manual-tracking control task experiments has shown that, typically, \( \rho_u = 0.003 \), which corresponds to a normalized motor noise ratio of -25 dB (Kleinman, Baron, and Levison, 1970).

The next section will present an application of the ECCS method to synthesize pilot optimal control gains for an acceleration command system and a comparison of these results with results obtained by using the original CCS methodology.

**Synthesis Example**

This section will present the application of the ECCS methodology to synthesize “pilot optimal” augmentation control laws for an acceleration command system in a compensatory tracking task. The ECCS results will also be compared with a similar synthesis performed by using the original CCS methodology.

**Example**

For an acceleration command system, the controlled system dynamics in transfer function form is given by
\[ \frac{\theta}{\delta} = \frac{k}{s^2} \]  
(30)
To be consistent with the references (Schmidt, 1979, and Innocenti and Schmidt, 1984), the plant gain is set to $k = 11.7$.

In this task, the pilot’s objective is to track a displayed command signal. The signal to be tracked is generated by a second-order, low-pass filter driven by unit intensity white noise

$$\dot{\theta}_{\text{cmd}} + 3.0\dot{\theta}_{\text{cmd}} + 2.25\theta_{\text{cmd}} = 3.67w(t) \quad (31)$$

The state space representation of the plant augmented with the disturbance dynamics is

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -2.25 & -3 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 11.7 \end{bmatrix} \delta + \begin{bmatrix} 0 \\ 3.67 \\ 0 \\ 0 \end{bmatrix} w \quad (32)$$

where

$$x = \begin{bmatrix} \theta_{\text{cmd}}, \dot{\theta}_{\text{cmd}}, \theta, \dot{\theta} \end{bmatrix}^T$$

and $\delta$ is the sum of the pilot’s and the augmentation controller inputs, $\delta = \delta_p + \delta_a$.

The pilot’s performance index is given by

$$J_p = E_\infty \left\{ (\theta_{\text{cmd}} - \theta)^2 + f_p \dot{\theta}_p^2 \right\} \quad (33)$$

and the augmentation control performance index is given by

$$J_a = J_p + E_\infty \left\{ r_a \delta_a^2 \right\} \quad (34)$$

In this development, the common assumption is made that the human controller is able to perceive both position error and error rate from the display. Therefore, the pilot’s observation vector is given by

$$y_{\text{obs}} = \begin{bmatrix} \theta_{\text{cmd}} - \theta \\ \dot{\theta}_{\text{cmd}} - \dot{\theta} \end{bmatrix} + v_y = \begin{bmatrix} e \\ \dot{e} \end{bmatrix} + v_y = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix} x + v_y \quad (35)$$

The measurements available for feedback are $z = \{\theta, \dot{\theta}\}^T$. Therefore, the augmentation control law is

$$\delta_a = G_a z = G_a \begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix} = G_a M x = G_a \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} x \quad (36)$$

where

$$G_a = \begin{bmatrix} G_\theta & G_\dot{\theta} \end{bmatrix}$$

Values of effective time delay, neuromuscular time constant, and observation and motor noise intensities were chosen to be consistent with single-axis manual-tracking control tasks. The control-rate weighting on $\dot{\theta}_p$ in $J_p$, which is $f_p$, was adjusted to obtain a neuromuscular time constant of $\tau_n = 0.1$ sec. The pilot’s effective time delay ($\tau$) was chosen to be 0.1 sec, and the pilot’s observation noise-to-signal ratio and motor noise-to-signal ratio were chosen to be $-20$ dB and $-25$ dB, respectively. Values of input parameters for the MOCM are summarized in table 1.

Results obtained from applying the ECCS methodology to this system are given in table 2. Synthesis results from applying the original CCS methodology to this system are given in table 3. Model-based predictions of pilot root-mean-square (rms) performance, augmentation gains, and
augmented plant poles are shown as a function of augmentation control weighting. The predicted rms tracking error and pilot control are determined from the steady-state covariance of the closed-loop pilot-vehicle system. Also shown is a prediction of the subjective pilot rating (PR), based upon the value of the pilot’s index of performance (Schmidt, 1979), calculated using the relation

\[
PR = 25 \ln(10J_p) + 0.3
\]  

(37)

These predicted subjective pilot ratings are based upon values of the pilot’s index of performance calculated using both the ECCS methodology in table 1 and the original CCS methodology in table 2.

Table 1. MOCM Input Parameters

<table>
<thead>
<tr>
<th>Control weighting</th>
<th>rms tracking error</th>
<th>rms pilot control</th>
<th>( J_p )</th>
<th>Predicted PR</th>
<th>( G_{\theta} )</th>
<th>( G_{\hat{\theta}} )</th>
<th>Augmented poles</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unaugmented</td>
<td>0.8222</td>
<td>0.8474</td>
<td>0.9363</td>
<td>5.9</td>
<td>0</td>
<td>0</td>
<td>(0), (0)</td>
</tr>
<tr>
<td>100.0</td>
<td>.7807</td>
<td>.7991</td>
<td>.8368</td>
<td>5.6</td>
<td>-.0073</td>
<td>-.0043</td>
<td>[0.29, 0.09]</td>
</tr>
<tr>
<td>10.0</td>
<td>.6104</td>
<td>.6181</td>
<td>.4894</td>
<td>4.3</td>
<td>-.0464</td>
<td>-.0346</td>
<td>[0.74, 0.27]</td>
</tr>
<tr>
<td>1.0</td>
<td>.4938</td>
<td>.5636</td>
<td>.3095</td>
<td>3.1</td>
<td>-.1148</td>
<td>-.1179</td>
<td>[1.16, 0.59]</td>
</tr>
<tr>
<td>.1</td>
<td>.4463</td>
<td>.7089</td>
<td>.2529</td>
<td>2.6</td>
<td>-.2890</td>
<td>-.2680</td>
<td>[1.76, 0.92]</td>
</tr>
<tr>
<td>.01</td>
<td>.4301</td>
<td>.8736</td>
<td>.2379</td>
<td>2.4</td>
<td>-.4102</td>
<td>-.3877</td>
<td>(2.20), (2.86)</td>
</tr>
</tbody>
</table>

\( \alpha \) = Stable real pole; \( \omega, \xi \) = Frequency and damping, respectively, of complex pole pair.

Table 2. ECCS Augmentation Results for Acceleration Command System

<table>
<thead>
<tr>
<th>Data from Innocenti and Schmidt (1984)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Control weighting</td>
</tr>
<tr>
<td>-------------------</td>
</tr>
<tr>
<td>Unaugmented</td>
</tr>
<tr>
<td>100.0</td>
</tr>
<tr>
<td>1.0</td>
</tr>
<tr>
<td>.1</td>
</tr>
<tr>
<td>.01</td>
</tr>
</tbody>
</table>

\( \alpha \) = Stable real pole; \( \omega, \xi \) = Frequency and damping, respectively, of complex pole pair.

\( * \) No data available.
A plot of predicted pilot rating versus augmentation gain magnitude for various values of augmentation control weighting ($r_{a2}$) is given in figure 3. Figure 4 shows a locus of closed-loop system eigenvalues as a function of augmentation control weighting. Frequency responses of the unaugmented and augmented plant, the pilot compensation dynamics, and the system loop transfer functions as a function of augmentation control weighting ($r_{a2}$) are given in figures 5-7.

**Discussion of Example**

An analysis of the results obtained with CCS incorporating the MOCM (table 2) shows an improvement both in terms of predicted rms tracking performance and in predicted subjective rating over those obtained from the original CCS formulation (table 3). For example, a synthesis design that achieves a predicted pilot rating of Cooper-Harper level I (Cooper and Harper, 1969) at a minimum of gain magnitude is at a control weighting of 1.0. (See fig. 3.) At this design point, the predicted average Cooper-Harper pilot rating is a 3 and the rms tracking performance is 0.4938, which compares with a Cooper-Harper rating of 4 and an rms tracking performance of 0.63 with the original CCS methodology.
As can be seen from the augmented system frequency responses (fig. 5), the augmented plant dynamics are type 0. The pilot compensation required to achieve the \( k/s \) loop shape in the region of crossover (approximately 3 rad/sec) is given in figure 6.

Research in pilot-in-the-loop systems by McRuer (1980) has shown that in a compensatory tracking task, a human will adjust his dynamic compensation such that the system loop dynamics will approach \( k/s \) in the region of frequency crossover. When this is achieved, the system has good closed-loop stability characteristics. The system loop frequency responses (fig. 7) show that this has been achieved in all the designs for the values of control augmentation weight chosen.

Concluding Remarks

This paper has presented research for extending the Cooperative Control Synthesis methodology to include a more accurate modeling of the pilot's controller dynamics. Cooperative Control
Synthesis (CCS) is a methodology that addresses the problem of how to design control laws for piloted, high-order, multivariate systems and/or nonconventional dynamic configurations in the absence of flying qualities specifications by emphasizing the parallel structure inherent in any pilot-controlled augmented vehicle. The simplified model of the pilot's controller dynamics currently in the CCS is replaced by the modified optimal control model (MOCM). The MOCM (based upon the optimal control model of the human operator developed by Kleinman, Baron, and Levison in 1970) provides a modeling of the pilot's compensation dynamics that is more accurate than the simplified pilot dynamic representation currently in the CCS methodology. Inclusion of the MOCM into the CCS also enables the modeling of pilot-observation perception thresholds and pilot observation attention allocation effects.

The structure of this Extended Cooperative Control Synthesis (ECCS) methodology allows implementation in current software capable of analysis and design for dynamic systems. Implementation in this type of environment allows for the rapid calculation of pilot and system transfer function descriptions from state space models and the rapid determination of system frequency responses. Also, this environment allows users to modify interactively various pilot and plant parameters and to quickly ascertain the impact of these changes on the closed-loop pilot-vehicle performance.

The ECCS methodology was used to synthesize pilot optimal augmentation control laws for a simple dynamic example in a compensatory tracking task. This analysis is compared with similar designs using the original CCS methodology. Analysis results obtained with the CCS incorporating the MOCM show an improvement both in terms of predicted root-mean-square tracking performance and predicted subjective rating over those obtained from the original CCS formulation.

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Appendix

Methodology Flowchart of Extended Cooperative Control Synthesis

A conceptual flowchart of the ECCS algorithm is presented in this appendix. The implementation of this algorithm requires the capability of solving steady-state Ricatti and Lyapunov equations.

Load system matrices

Input pilot control and limitation parameters

Augment system matrices with delay dynamics

Create control-rate formulation

Initialize augmentation controller gains to zero

Augment system with controller

Calculate pilot gains

Adjust control-rate weighting

Achieved desired value of neuro-lag?

Yes

No

(Continued next page)
Augment system with lag dynamics

Generate initial guess for observation and control noise intensity

Calculate effects of fractions of attentions and thresholds on observation noise

Calculate estimator gains

Form closed-loop pilot system

Calculate achieved observation and control noise-to-signal ratios

Achieved desired noise-to-signal ratios?

No

Adjust values of observation and control noise intensity

Yes

(Continued next page)
Calculate closed-loop eigenvalues and rms output values

Form pilot closed-loop system

Augment system with current guess of augmentation controller

Calculate pilot and augmentation controller costs and augmentation gradient

Algorithm converged to solution for current pilot gains?

Yes

(Continued next page)

No

Calculate new augmentation controller gains

(To previous page)
Calculate new pilot model for current augmentation controller. Loop until stopping condition is met.

Form pilot matrices

Generate system and pilot frequency responses and transfer functions

Calculate pilot and vehicle output rms values

Adjust input parameters and reanalyze or end analysis
References


# Extended Cooperative Control Synthesis

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**Performing Organization:** NASA Langley Research Center

**SPONSORING/MONITORING AGENCY:** National Aeronautics and Space Administration

**Abstract:**

This paper reports on research for extending the Cooperative Control Synthesis methodology to include a more accurate modeling of the pilot's controller dynamics. Cooperative Control Synthesis (CCS) is a methodology that addresses the problem of how to design control laws for piloted, high order, multivariate systems and/or nonconventional dynamic configurations in the absence of flying qualities specifications. This extension is accomplished by emphasizing the parallel structure inherent in any pilot-controlled, augmented vehicle. The original CCS methodology is extended to include the modified optimal control model (MOCM), which is based upon the optimal control model of the human operator developed by Kleinman, Baron, and Levison in 1970. This model provides a modeling of the pilot's compensation dynamics that is more accurate than the simplified pilot dynamic representation currently in the CCS methodology. Inclusion of the MOCM into the CCS also enables the modeling of pilot-observation perception thresholds and pilot-observation attention allocation effects. This Extended Cooperative Control Synthesis (ECCS) allows for the direct calculation of pilot and system open- and closed-loop transfer functions in pole/zero form and is readily implemented in current software capable of analysis and design for dynamic systems. Example results based upon synthesizing an augmentation control law for an acceleration command system in a compensatory tracking task using the ECCS are compared with a similar synthesis performed by using the original CCS methodology. The ECCS is shown to provide augmentation control laws that yield more favorable, predicted closed-loop flying qualities and tracking performance than those synthesized using the original CCS methodology.

**Subject Terms:**
- Manual vehicular control
- Pilot modeling
- Flying qualities
- Optimal control