Influence of Temperature and Impact Velocity on the Coefficient of Restitution

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THE COEFFICIENT OF RESTITUTION

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SUMMARY

Tests were performed on a variety of material combinations to understand the effects of temperature and impact velocity on the coefficient of restitution (COR). The tests, performed in a vacuum at room and liquid nitrogen temperatures, consisted of dropping a ball onto a target plate and recording the impact time history of the ball's bounce-down. Time intervals between successive impacts were measured from the time history and used to calculate the coefficient of restitution and impact velocity for each impact. Maximum impact velocities ranged from approximately 33 (0.84) to 52 in/s (1.32 m/s). Five ball-target plate combinations were evaluated: type 316 stainless steel (316 SS) on 316 SS; M50 tool steel on Armalon; M50 on 4340 steel; 410C steel on Armalon; and copper on copper. The coefficient of restitution for the 316 SS-316 SS, M50-Armalon, 410C-Armalon, and copper-copper combinations increased as the temperature and impact velocity decreased. The coefficient for the hard steel combination, M50-4340, was not greatly influenced by temperature or variations in impact velocity.

INTRODUCTION

This study was performed in support of research to develop a cryogenic impact damper. The following paragraphs describe an impact damper and explain why the coefficient of restitution is an important design parameter.

An impact damper is a passive vibration control device that consists of an impactor (usually a ball) that travels freely between the walls of a hollow cavity. Damping occurs when the impactor strikes a cavity wall. Some of the relative energy of impact is converted into heat, noise, and elastic waves in the impactor and wall. The remaining energy causes the subsequent relative motion of the bodies.

This type of damper is appealing since it produces a substantial amount of damping for its size and operates effectively over a wide frequency range (ref. 1). Also, its simple design results in inherent reliability and thus, low maintenance.

A main factor in determining this damper's effectiveness is the coefficient of restitution (COR) between the impactor and a cavity wall. The COR is a measure of energy loss that occurs with the collision of two masses and varies with temperature, impact velocity, size, material, and body shape. The COR is defined as the ratio of relative velocity between two bodies after impact, divided by the relative velocity before impact. Its value always ranges from 0 to 1; 0 represents a perfectly plastic
impact with complete energy dissipation relative to the bodies; and 1, a perfectly elastic impact with no energy loss.

To be effective, the impact damper must use a combination of impactor and cavity wall materials that produce a fairly low COR in the system's frequency range. Improper material choices can result in either very little damping (high coefficient) or, for relatively high amplitudes of vibration, impact failure (low coefficient). Impact failure occurs when the impactor does not possess enough energy to traverse the cavity gap in one half-cycle. The ball and cavity wall materials have an important role in the effectiveness of an impact damper, dictating design parameter considerations such as the cavity gap, optimum operating range, and the onset of impact failure.

The intent of this work was not to propose certain materials for a cryogenic impact damper but rather to investigate the degree to which temperature and impact velocity affects the COR and to present a possible test method to calculate this value.

PREVIOUS WORK

Barber (ref. 2) and Groper (ref. 3) conducted experiments by dropping a ball (impactor), initially suspended by a suction tube, onto a target plate. A camera recorded the ball's maximum rebound height after each impact. The COR was calculated by taking the square root of the ratio of the height after impact to the height before impact.

Groper used a 0.63-in- (16-mm-) diameter AISI 1060 ball and steel plate in hardened and non-hardened conditions. He was concerned with the effects of temperature on the elastic properties of hard and mild steels and also with the effects of impact velocity represented by a change in the COR. In Groper's work, the temperature ranged from -90 to 210 °F (−67.7 to 98.8 °C) and the maximum impact velocity was limited by the drop height, which caused the onset of plastic deformation in the plate. This height limitation resulted in impact velocities for the hard steel that reached approximately 118 in/s (3 m/s) and only 55 in/s (1.4 m/s) for the mild steel. The tests showed that the COR for the mild steel at 210 °F (98.8 °C) was 0.6 and increased linearly up to 0.8 at −90 °F (−67.7 °C). Increasing the impact velocity on the mild steel while holding the temperature constant resulted in a nonlinear decrease in its coefficient from approximately 0.72 to 0.64. The coefficient for the hard steel remained at about 0.95 for the entire temperature and impact velocity range.

Barber's experiments also concentrated on the effects of temperature and impact velocity on the COR, but he limited testing to two temperatures and impact velocity conditions. The studies were conducted at room (79 °F) (26.1 °C) and cryogenic (−317 °F) (−194.4 °C) temperatures. Impact velocities of 60 in/s (1.5 m/s) and 96 in/s (2.4 m/s) were chosen because they best resembled conditions encountered in the Space Shuttle Main Engine (SSME) turbopump bearings. The material combination consisted of a 0.50-in. (13-mm) 440C steel bearing ball dropped onto a plate made of Armalon. Armalon, a Teflon-impregnated fiberglass, is used in the bearing cage material in the SSME turbopump. The combination of 440C steel on M50 tool steel was also studied but only at room temperature. Results were similar to Groper's. Cryogenic temperatures were achieved for the Armalon by submerging the steel backing plate in a bath of liquid nitrogen. The COR at an impact velocity of 60 in/s (1.5 m/s) and at 79 °F (26.1 °C) was 0.57 but increased 12 percent to 0.64 when the test was performed at −317 °F (−194.4 °C). When the impact velocity was raised to 96 in/s (2.4 m/s), the coefficient at 79 °F (26.1 °C) dropped to 0.47 but increased 28 percent to 0.60 at −317 °F (−194.4 °C).
TEST APPARATUS

Tests were based on the concept of dropping a ball onto a target plate, as were Barber's and Groper's. However, their test designs were not duplicated because the designs could not be incorporated into a vacuum-sealed containment vessel cooled to cryogenic temperatures. A vacuum was necessary to prevent frost formation on the ball and plate, a condition that could invalidate the data. Also, to replicate their use of high-speed photographic equipment to measure the COR would be very difficult in such a vessel. A different approach was taken, one which relied on the time intervals between impacts of a ball undergoing bounce-down. This new test method yields COR values for various impact velocities. Target plate-ball impacts were sensed by an accelerometer mounted on the plate. With the time intervals measured from the accelerometer output records, the COR values and impact velocities could be calculated. The correlation between time and the COR is derived in the appendix.

Figure 1 is a schematic of the unit that was designed for this test. The total height, including the vacuum and pressure gauge lines, is 17 in. (432 mm). The main components of the test apparatus are the containment vessel, which is a hollow 4-in. (102-mm-) diameter cylinder with one end sealed, and the suspending unit. The vacuum was used to seal the two components together. The suspending unit fits inside the cylinder and suspends a ball over a target plate that is bolted to the cylinder’s bottom.

For the target plate, cryogenic temperature was achieved by conduction through the cylinder; and for the ball, though threaded rods and a drop plate. A special silicon diode sensor (not shown) measured the ball’s temperature and the signal was carried from the cylinder’s chamber with electrical feed-throughs designed to transmit data without the losing the containment vacuum.

The initial drop of the ball was controlled by a unique method that incorporated a solenoid and monofilament line. The drop plate was machined with a center hole diameter of a few mils greater than the ball diameter. Placing the ball in the hole wedged one end of the string between the ball and the hole edge; the other end was attached to the solenoid. Activating the solenoid pulls on the string and releases the ball with very little spin and initial velocity. Output signals from the piezoelectric accelerometer mounted on the target plate were transferred to the waveform analyzers, where they were recorded and displayed. Hardcopies of the impact signal plots were printed.

Table I lists the ball-target plate combinations that were evaluated as well as each material’s hardness expressed in the Rockwell scale. The initial drop height for the 0.50-in. (13-mm) balls set at 1.38 in. (35 mm) for the steel target plates resulted in an initial impact velocity of 32.6 in/s (0.8 m/s). Because the thickness of the Aralon was 0.19 in. (5 mm), the drop height was 1.44 in. (37 mm), and the initial impact velocity was 33.3 in/s (0.8 m/s). These heights produced clear signals without dimpling the plate and/or ball. Because the copper ball was so small, the drop height was increased to 3.5 in. (89 mm) to enhance the impact signals.

TEST PROCEDURE

Before testing each combination, the correct sampling rate for the analyzers was determined to capture the entire impact history and to minimize errors caused by aliasing. This was done by dropping the ball manually and, by trial-and-error, adjusting the two analyzers until they captured the entire bounce-down event. A sampling rate of 1 to 2 msec to read 4096 points produced acceptable results for all combinations except copper, which had to be lowered to 600 to 700 µsec because of its short bounce-down time. Each analyzer was programmed with two different sampling periods as a way of verifying the output.
Initial tests were conducted on all specimens at room temperature to establish baseline data; cryogenic tests followed. The procedure was identical for both test conditions except for the addition of LN$_2$. Because of the similarities, only the procedure using LN$_2$ will be explained.

With the suspending unit outside the cylinder, the ball was wedged in the drop plate hole, and the sensor was placed in contact with the top of the ball. After assembling the suspending unit and containment vessel, the vacuum line was connected, and the accelerometer cable was connected to the analyzers. The vacuum pump was activated; the fixture was lifted, careful not to dislodge the ball, seated in the dewar, and leveled. The temperature gauge was connected to the sensor using the electrical feed-through wires. Liquid nitrogen was poured into the dewar until it reached the top of the cylinder. The test was begun when the ball temperature read approximately $-290$ °F ($-180.3$ °C). Equilibrium between the surrounding air and the assembly prevented any further decrease in temperature. To eliminate motor vibrations, the vacuum pump was turned off before dropping the ball. The analyzers were armed and began to take data when triggered by vibrations resulting from activating the solenoid. After the test, the pump was reactivated and the fixture was removed from the LN$_2$ bath. A plot of the time trace was made and the impact times were documented.

RESULTS AND DISCUSSION

Each test generated impact time traces similar to the one for the M50-4340 steel combination at room temperature (fig. 2). The large spikes represent the impulses caused by the ball striking the target plate. The reason for the difference in direction of these spikes was not investigated here because it did not interfere with the test objectives. The time of each impact was taken to begin with a spike. A finite number of successive spikes was recorded until an impulse signal became unclear because of insufficient time between impacts for the plate to dampen the previous signal. Two analyzers set at different sampling rates produced small differences in results, usually beginning at the third significant digit. Instead of choosing one of the two data sets, the sets were averaged and plotted against impact velocity.

The COR results are shown in figure 3 for the five material combinations. Two constant temperature curves, representing room and cryogenic temperatures, are plotted to show the actual data points. By studying these graphs, some conclusions about the effects of temperature and impact velocity on the COR can be made. These variables had little or no effect on the hard steels; this is in agreement with the results of Barber and Groper. The material properties of the mild steels and Armalon, however, are greatly affected, the Armalon exhibiting the most sensitivity. It can be seen that a decreasing (or increasing) temperature and impact velocity increases (or decreases) the coefficient.

A comparison of this study’s COR values for Armalon and those obtained by Barber show that the values measured in this study are greater. The two tests differ in the choice of impactor materials and impact velocities. The graphs for 410C-Armalon and M50-Armalon are very similar, indicating that variations in impactor materials would not cause discrepancies in COR values. However, if impact velocities are compared, the maximum impact velocity in these tests occurred at initial impact and measured approximately 33 in/s (0.8 m/s). The lowest velocity used in Barber’s experiment was 60 in/s (1.5 m/s). The findings show agreement that increasing the impact velocity decreases the COR. For the range of impact velocities in these tests, small dimples were observed on the surface of the Armalon plate. Hence, at higher velocities, it is assumed that a greater percentage of the impactor’s kinetic energy was used to cause plastic deformations in the Armalon, which effectively lowered its COR.
Copper's behavior at cryogenic temperature was unexpected; its coefficient increased only slightly, even though it is softer than 316 SS. One possible explanation for this is that the impactor's trajectory after a bounce may have been influenced by a plate or impactor deformity because of the softness of the material. Minute deformations in the form of flattening of the surface were visible on the copper balls that were used.

In order to obtain accurate hardness values for the impactors, two parallel 0.19-in-diameter flats were ground on the balls. Hardness values were then taken at various locations on the flats. It was found that the different locations did not have a significant effect on the values obtained. The 410C balls were unavailable for hardness measurements.

CONCLUSIONS

Temperature and impact velocity have a major influence on the coefficient of restitution between two materials and should be considered when designing an impact damper. A test method using a sealed vacuum chamber has been developed and enables coefficient of restitution values to be determined for materials at cryogenic temperatures.
APPENDIX

The ball can be modeled as a free-falling body that encounters no air resistance because it is dropped in a vacuum as shown in figure 4. Let \( i \) represent the \( i \)th impact, \( i+1 \) the following impact, and so on. It is well known that the impact velocity acquired by a free-falling body starting at a height \( h \) is

\[
v = \sqrt{2gh}
\]  

(1)

where \( g \) is the Earth’s gravity. The time \( t \) necessary for the ball to travel the distance \( h \) is

\[
t = \sqrt{\frac{2h}{g}}
\]  

(2)

The impact between the ball and target plate is modeled using the definition of the COR, \( e \), stated in its most general form (ref. 4):

\[
e = \frac{(v_p' - v_b')}{(v_b - v_p)}
\]  

(3)

where \( v_p \) and \( v_b \) are the velocities of the plate and ball before impact; the prime denotes the velocities afterward. Because a ball will impact an assumed stationary target plate, equation (3) reduces to

\[
e = \frac{-v_b'}{v_b}
\]  

(4)

where the negative sign accounts for the direction change. Equation (4) states that the COR for a mass colliding with a stationary object is equal to the ratio of the velocity after impact to the velocity before impact.

The time between impacts is just twice the time necessary for the ball to travel from its maximum rebound height to the plate surface:

\[
t_i - t_{i-1} = \sqrt{\frac{2h_i}{g}}
\]  

(5)

Solving equation (1) for \( h_i \) and substituting into equation (5) results in a relation between the time difference and the \( i \)th impact velocity \( v_{b,i} \):

\[
v_{b,i} = \frac{g(t_i - t_{i-1})}{2}
\]  

(6)
Equation (6) is valid for any time difference between impacts and can be written for the difference between times $t_{i+1}$ and $t_i$,

$$v_{b,i+1} = \frac{g(t_{i+1} - t_i)}{2}$$

(7)

The velocity at impact $i$ and impact $i+1$ are related by equation (4); the negative was dropped because only the magnitude is of interest here:

$$e_i = \frac{v_{b,i+1}}{v_{b,i}}$$

(8)

Substituting equations (6) and (7) into equation (8) and canceling terms gives

$$e_i = \frac{(t_{i+1} - t_i)}{(t_i - t_{i-1})}$$

(9)

Equation (9) calculates the COR for an impact $i$ if the impact times at $i-1$ and $i+1$ are known. Because a previous impact time is needed, this equation is valid beginning with the second impact. The coefficient for the first impact can be found by calculating the initial impact velocity $v_o$ from equation (1) since the initial drop height $h_o$ is known. The resulting velocity after the impact can be determined from equation (6). Substituting these two values into equation (4) yields the COR.
REFERENCES


TABLE I—MATERIAL COMBINATIONS FOR THE COEFFICIENT OF RESTITUTION (COR) TESTS.

<table>
<thead>
<tr>
<th>Impactor hardness</th>
<th>Plate hardness</th>
<th>Impactor diameter, in/mm</th>
<th>Plate thickness, in/mm</th>
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</thead>
<tbody>
<tr>
<td>316 SS (Rb 77)</td>
<td>316 SS (Rb 80)</td>
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<td>0.50/13</td>
</tr>
<tr>
<td>Copper (a)</td>
<td>Copper (Rb 14)</td>
<td>.19/5</td>
<td>.50/13</td>
</tr>
<tr>
<td>M50 (Re 62)</td>
<td>4340 (Re 52)</td>
<td>.50/13</td>
<td>.25/6</td>
</tr>
<tr>
<td>M50 (Re 62)</td>
<td>Armalon (b)</td>
<td>.50/13</td>
<td>.19/5</td>
</tr>
<tr>
<td>410C (c)</td>
<td>Armalon</td>
<td>.50/13</td>
<td>.19/5</td>
</tr>
</tbody>
</table>

*aCopper balls were too small to take hardness measurements.*

*bMaterial was unsuitable for hardness measurements.*

*cBalls were unavailable for hardness measurements.*
Figure 1.—Coefficient of restitution test apparatus.

Figure 2.—Impact time trace for the M50-4340 steel combination at room temperature.
Figure 3.—Coefficient of restitution (COR) results for material combinations. (a) 316 SS on 316 SS. (b) M50 on 4340 steel. (c) Copper on copper. (d) 410C on Armalor. (e) M50 on Armalon.
Figure 4.—Impactor modeled as a free-falling body during bounce-down.
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