Development of Nonintrusive, Scatter-Independent
Techniques for Measurement of Liquid Density
Inside Dense Sprays

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Interim Report

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Abstract

A nonintrusive optical technique for measuring the liquid density in sprays used to simulate LOX injector flows is under development. This manuscript is a report on work toward that development which is currently in progress. The technique is a scatter-independent, absorption-based approach which depends on the numerical inversion of a collection of absorption profiles. For the case in which visible radiation passes through liquid-gas interfaces so numerous in sprays, substantial reductions and alterations in the signal result from scattering even in the absence of absorption. To avoid these problems, X-Rays will be used as the absorbed radiation. The experimental process is simulated by integrating the absorption spectrum for a known distribution, adding instrument noise to this 'measurement', creating a projection from the 'measurement', filtering the projection, inverting the projection and comparing the results with the original prescribed distribution.

Introduction

The liquid density distribution in liquid spray injectors has been studied extensively using probe-based grid paternators and other probe sampling techniques. While these techniques have provided valuable insight into the mixing mechanisms for two-phase flows, they are severely limited by poor spatial resolution and low data rates. To date, quantitative nonintrusive measurement approaches have been developed for sparse sprays; however, in dense sprays, nonintrusive measurements have been used primarily for visualization. The inversion of laser-induced fluorescence signals has been employed in the measurement of density distributions in sprays with limited success; however, problems associated with scattering at optical wavelengths limits the theoretical accuracy of this approach. At the X-Ray wavelength, scattering is not problematic and X-Ray absorption has been used for the visualization of sprays. While X-Ray tomography has been applied with considerable success to flows involving medical and industrial applications, no quantitative measurements of density distributions in sprays have been reported using X-Ray tomography. This work attempts to advance the use of electromagnetic radiation for quantitative measurements in dense injector sprays by investigating the application of standard inversion techniques to measurements obtained at X-Ray wavelengths.

A straightforward application of transform-based tomographic imaging involves the convolution of signals obtained using a uniform collimated source. For optical wavelengths, the collimation of the source can be accomplished with geometric optics without substantial difficulty. Indeed, reconstruction methods based on optical wavelength absorption from a collimated source have been used to determine concentrations in combustion applications. Most X-Ray sources are point sources; hence, the reconstruction techniques must be based on a fan-beam analysis. This experimental approach is employed on modern medical imaging equipment with considerable success. The particular scheme employed for this investigation will be a filtered backprojection convolution for fan-beam absorption measurements.

This work is designed to demonstrate the theoretical practicality of the measurement approach by simulating the experimental process. The absorption measurement process is simulated by numerically tracing light rays through a variable density spray and integrating the absorption. To accurately simulate the measurement, noise is to be added to this data using a random number generator. It should be noted that this paper is an initial report on a project in progress; hence only some initial noise-free signal calculations are included herein. However, before the project is
completed, the effects of noise, signal enhancement using dyes, detector sensitivity and a non-monoenergetic source of specified power will be incorporated in a final recommendation of setup requirements to complete an accuracy measurement at a specified level of accuracy.

Flow Fields of Interest

The motivation for this technique development arises from the need for accurate experimental measurements of the liquid phase distribution in LOX injector sprays being investigated as part of the development of advanced liquid rocket engines such as the Space Transport Main Engine (STME). Currently, a dominant area of emphasis for improving the mixing induced by the LOX injector posts is the inclusion of swirl in the interior liquid flow. The intention here is to induce mixing by forcing the interior LOX flow outward into the coflowing annular gaseous hydrogen stream in the spray region. The swirl is induced using tangential entrance ports at the head end of the injector and is intensified in a neck-down region near the injector exit. To illustrate the geometry, a cutaway view of a proposed STME LOX post injector configuration is shown in Fig. 1. A more detailed discussion of the spray flow field for this geometry can be found in Ref. 1.

The flow field generated by this injector geometry is symmetric when only a single injector is considered and much can be learned about the mixing mechanisms by studying a single injector flow field using axisymmetric techniques. However, the performance of the combustor composed of an array of swirl coaxial injectors is largely dependent on the interaction between multiple sprays. To investigate such flow fields, a method of determining density distributions of arbitrary shape is needed. This was the basis for the selection of a general form of computer-aided tomography as a measurement approach.

Measurement Simulation and Tomographic Calculation

The ray-tracing algorithm used to simulate the measurements for this investigation applies Beer's law along a prescribed path to generate the signal which the detector is expected to see after a fan of X-Rays has passed through a spray of arbitrary cross-section. Before the conclusion of this project, small amounts of expected experimental noise will be added to this data to fully simulate the data collection process. Figure 2 is a plot showing a prescribed distribution of the spray density and the predicted noise-free absorption signal. The prescribed distribution is an arbitrary irregular shape in which the density increases toward the center of the distribution. The shape is represented using a contour plot in which the contours represent increasing magnitudes as the radius (measured from the center of the distribution) is decreased. The function \( A(t, \Theta) \) is the normalized profile of the absorption signal where \( t \) is the distance along the flat screen detector and \( \Theta \) is the angle at which the central ray of the fan beam passes through the spray.

Formally, \( A(t, \Theta) \) can be represented as

\[
A(t, \Theta) = \frac{I}{I_0} \exp \left[ -\int_{\epsilon} \mu(x,y) ds \right]
\]

where \( I_0 \) is the signal with no absorption, \( I \) is the measured signal, \( \mu(x,y) \) is the product of the absorption cross section (taken to be constant for
the assumed monoenergetic source and uniform liquid phase composition) and the number density at the prescribed location, s is the path taken by the ray being traced and c is the path length through the spray. The projection of $\mu$ (essentially the density field) is the parameter from which the density distribution can be reconstructed provided $A$ is obtained for a sufficient number of angles, $\Theta$. The projection is simply the argument of the exponential in eq.(1); hence, the profile of the projection of the density distribution can be obtained from

$$P(t, \theta) = \ln \left[ \frac{1}{A(t, \theta)} \right]. \quad (2)$$

In practice, the projection will be calculated from the absorption profile. For this analysis, the measurement simulation algorithm is used to produce the projection and, for clarity, the projection of the arbitrarily prescribed distribution is shown in Fig. 3 for the same angle $\Theta$ as that used to produce the measurement data shown in Fig. 2.

There are two broad categories of techniques for reconstructing a distribution from a series of projections of that distribution. These categories are series expansion methods and transform methods. The series expansion methods involve the discretization of the reconstruction region and the solution of a set of linear equations developed for that region based on the assumption that the function is constant within each discrete element. The transform methods are more complex but they are also much faster. The theoretical basis for the transform approach lies in the Fourier Slice Theorem. The fundamental principle set forth by this theorem is that a one-dimensional Fourier transform of the projection is equal to a slice of the two-dimensional Fourier transform of the projected distribution. This slice passes through the center of transform space and oriented at the angle at which the projection was taken. The transform reconstruction process involves constructing radial profiles of the Fourier transform of two-dimensional density distribution by taking the Fourier transforms of the projections and then taking the inverse Fourier transform of the resulting two-dimensional function.

For this work, a filtered backprojection scheme using the transform approach is under development. The fundamental justification for this approach is described in Ref.9. In this approach, a
filter is applied to each of the radial profiles developed from the transform of the projections. The reason for this filter is that, in order to fill the transform space, what is actually needed is data for a pie-shaped region rather than data for a straight line. The filter converts the data for the line into data for a pie-shaped region by weighting the line data with a linearly increasing function of radius. After the filtering is completed, the inverse transform is applied to obtain a more accurate representation for the density distribution.

At the moment, the final mathematical step for the technique has not been fully developed. Completion of this step is anticipated in the immediate future and at that point an exercise to evaluate ultimate achievable uncertainties will be conducted by comparing prescribed distributions and reconstructions.

Projected Experimental Equipment and Preliminary Uncertainty Estimates

The primary driving factors for determining both an appropriate X-Ray source and the appropriate detection system is, of course, the desired accuracy of the measurement after the mathematical transformations. A combination of the power of the X-Ray source and the absorption cross section of the absorbing species will set the dynamic range of the signal produced. Equally important is the dynamic range and noise level of the detector. Since the calculation of the density at a prescribed location requires information from many different projections, the uncertainty of the reconstructed signal is a complicated function of the number of measurements taken and the uncertainties of the measurements. The direct calculation of the uncertainty for each reconstruction site is not practical; however, a limiting factor is clearly the uncertainty of the projection which is, of course, directly dependent on the measurement uncertainty. It can be argued that, because of the linearity of the transform process, if a sufficient number of projections is taken, a uniform uncertainty in the projection data would lead to approximately the same uniform uncertainty in the reconstruction. The uncertainty of the projection measurement can be obtained by taking the logarithmic derivative of eq. (2) which yields

\[ \frac{\Delta P}{P} = \frac{1}{\ln(A)} \frac{\Delta A}{A}. \]  

Since the value of A will always be less than unity, the right side of eq. (3) will always be negative. However, referring to uncertainties as negative entities is not useful. For that reason, only the magnitude of the uncertainty in P will be addressed. To gain some insight into the behavior of the magnitude of the projection error, a plot of this function is included in Fig. 4.

![Figure 4: Projection uncertainties as a function of measurement accuracy.](image)

While this calculation clearly does not directly estimate the uncertainty of the ultimate measurement, it does prescribe an estimate of the lower limit of the uncertainty. Note that the function A is the measured signal and not the part of the signal that is absorbed. Hence, as the amount of absorption decreases, the value of A increases and the uncertainty of the projection increases. Also noteworthy is the fact that for a maximum absorption of 63% of the signal, (A = 0.37) the proportionality between the projection uncertainty and the measurement uncertainty is unity. In that case, a 1%-5% uncertainty in the measurement would lead ultimately to a limiting uncertainty for reconstruction of the same range.

It is useful, particularly in the absence of a completed simulation, to note experimental uncertainties claimed by researchers who have applied X-Ray tomography to other problems. A preliminary survey of previous work has indicated
that an X-Ray source of 100 kW magnitude used in combination with gas-type detectors can result in experimental uncertainties as low as 5% for appropriate absorbing species. While these findings are preliminary, it does appear that using a roughly 100 kW source (currently available from GE) and state-of-the-art detectors along with absorption enhancing dyes, uncertainties of 5% can be achieved with this approach.

Clearly, adding absorbing dye to the water used as LOX simulate in a spray rig measurement will decrease the measured value of the function A and improve the results. The absorption characteristics of some appropriate dyes are reviewed in Ref. 15 and this data will used to recommend a selection of possible dyes for the experiment.

Summary and Future Work

This report has discussed a measurement approach currently under development. To date, the mathematical basis for the procedure has been mapped out, the process for simulating the experimental process has been principally developed and part of the data reduction scheme has been completed. Additionally, some preliminary estimates of expected uncertainty have been completed and some general requirements for equipment have been identified. The completion of this on-going project as described above is anticipated in November 1994.

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References


