Strain Gage Selection in Loads Equations Using a Genetic Algorithm

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ABSTRACT

Traditionally, structural loads are measured using strain gages. A loads calibration test must be done before loads can be accurately measured. In one measurement method, a series of point loads is applied to the structure, and loads equations are derived via the least squares curve fitting algorithm using the strain gage responses to the applied point loads. However, many research structures are highly instrumented with strain gages, and the number and selection of gages used in a loads equation can be problematic. This paper presents an improved technique using a genetic algorithm to choose the strain gages used in the loads equations. Also presented are a comparison of the genetic algorithm performance with the current T-value technique and a variant known as the Best Step-down technique. Examples are shown using aerospace vehicle wings of high and low aspect ratio. In addition, a significant limitation in the current methods is revealed. The genetic algorithm arrived at a comparable or superior set of gages with significantly less human effort, and could be applied in instances when the current methods could not.

NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_j$</td>
<td>calibration load (shear) applied to structure</td>
</tr>
<tr>
<td>$[V_j]$</td>
<td>column vector of applied loads</td>
</tr>
<tr>
<td>$\beta_i$</td>
<td>coefficient of the $i$th strain gage in loads equation</td>
</tr>
<tr>
<td>$\beta_0$</td>
<td>constant in loads equation</td>
</tr>
<tr>
<td>$[\beta_i]$</td>
<td>column vector of equation coefficients</td>
</tr>
<tr>
<td>$\mu_{ij}$</td>
<td>$j$th reading of the $i$th strain gage</td>
</tr>
<tr>
<td>$[\mu_{ij}]$</td>
<td>matrix of strain gage readings, $j$ is row, $i$ is column index</td>
</tr>
<tr>
<td>$k$</td>
<td>number of available gages on a structure</td>
</tr>
<tr>
<td>$m$</td>
<td>number of data points taken during loads calibration</td>
</tr>
<tr>
<td>$i$</td>
<td>index used for gages in loads equation</td>
</tr>
<tr>
<td>$j$</td>
<td>index used for data points</td>
</tr>
<tr>
<td>$T_i$</td>
<td>T-value of the $i$th gage</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>standard error for a loads equation over all calibration points</td>
</tr>
<tr>
<td>$l_{ij}$</td>
<td>influence coefficient for the $i$th gage and loading condition $j$</td>
</tr>
<tr>
<td>$[\Sigma]^T$</td>
<td>transpose of a matrix</td>
</tr>
<tr>
<td>$%\text{rms}$</td>
<td>percent root-mean-square</td>
</tr>
<tr>
<td>GA</td>
<td>genetic algorithm</td>
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</tbody>
</table>

INTRODUCTION

In experimental flight tests it is frequently necessary to measure the shear, torsion, and bending moments on various portions of a vehicle's structure, such as an aircraft wing. Strain gages are placed strategically throughout the structure to ensure that all load paths are covered, to build in redundancy in case of gage failure, and because once placed in an aircraft the gages become
relatively inaccessible. The recorded outputs of a few selected strain gages are then used to determine applied loads in either real-time or post-flight analysis. Selected gages are generally used because there is often a limited number of data channels available. Consequently, the questions the structural engineer must answer prior to flight are

- how many gages should be used
- which gages should be used
- what kind of error is acceptable and,
- how small of an error is attainable

Before these questions can be answered, a ground loads calibration test must be performed. A typical method of performing this ground calibration test (ref. 1) consists of applying known loads to specific locations on the structure, and recording the output from all the strain gages. Curve fitting the ground test data from selected gages determines a calibration equation. This equation is then used to measure the in-flight loads using the recorded gage readings. Accurately measuring these loads is important, so it follows that finding good calibration equations is important. Therefore it is critical to select a good (but limited) set of gages.

As an example of the difficulty in the gage selection process, suppose that a particular airplane wing is instrumented with 25 strain gages. If only 5 strain gages were used to formulate each load calibration equation, known as a 5-gage equation, there are 53,130 possible combinations. If the number of gages had been 40 there would be 658,008 possible 5-gage load equations. Furthermore, if there were 130 strain gages, as in the case of the Space Shuttle orbiter, there would be 286,243,776 possible load equations. Clearly this case presents a vast number of selections. If an exhaustive search were used to solve and evaluate each equation as a possible candidate, at the rate of 100 equations per second the time required would be in excess of one month. Furthermore, it cannot be known in advance how much error is implied in using a 5-gage equation as opposed to a 6-gage equation or an 8-gage equation. To know how many gages are needed to establish a certain accuracy, this analysis would have to be repeated for 6-gage equations, 8-gage equations, and so on. Clearly, an exhaustive search routine is not a realistic approach to determine calibration equations.

Figure 1 is a generic flow chart which shows how to instrument the structure to determine experimental loads. A gage selection method is chosen with the strain gage output data from the calibration loads test, and a load equation is generated. With the calibration equation applied to experimental test results, appropriate flight loads results can then be determined. This paper reviews the currently employed T-value technique (refs. 2 and 3), presents a Best Step-down method, and compares the results from these two techniques with those from a unique new technique using a genetic algorithm (GA) (refs. 4 and 5). It will be demonstrated how the GA improves the strain gage selection process to determine loads equations. In addition, it will be demonstrated how several applications of the GA can help determine how many gages are needed to establish a given accuracy. The analysis compares the two current techniques and the GA using the loads calibration test data from the high aspect ratio wing of the experimental Advanced Fighter Technology Integration (AFTI) F-111 Mission Adaptive Wing (MAW) and low aspect ratio wing of the Space Shuttle orbiter vehicles. These examples show how using the new GA provides consistently better results in determining strain gage calibration equations than current methods.
This work was done at the NASA Dryden Flight Research Facility in 1993 and in general reflects the typical methods currently in place at this facility.

**DETERMINATION OF THE CALIBRATION EQUATION**

This section presents an overview of linear algebra and least squares curve fitting used to determine the loads equations.

**Data Organization and Least Squares**

To describe the different methods clearly, an overview of the mathematics and the organization of the data is necessary. After the calibration loads have been applied and the strain gage responses recorded, the data are tabulated as follows.

Table: Generic representation of strain gage data taken during the ground loads calibration test.

<table>
<thead>
<tr>
<th>Applied load</th>
<th>Gage A</th>
<th>Gage B</th>
<th>Gage C</th>
<th>Gage D</th>
</tr>
</thead>
<tbody>
<tr>
<td>V1</td>
<td>μA,1</td>
<td>μB,1</td>
<td>μC,1</td>
<td>μD,1</td>
</tr>
<tr>
<td>V2</td>
<td>μA,2</td>
<td>μB,2</td>
<td>μC,2</td>
<td>μD,2</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>Vm</td>
<td>μA,m</td>
<td>μB,m</td>
<td>μC,m</td>
<td>μD,m</td>
</tr>
</tbody>
</table>

In structural calibration, the relation between the load applied and the strain measured by the gages is assumed linear. If, for example, a 3-gage equation that uses gage A, gage C, and gage D were desired, a system of \( m \) equations and 4 unknowns could be written in matrix form as

\[
\begin{bmatrix}
1 & \mu_{A,1} & \mu_{C,1} & \mu_{D,1} \\
1 & \mu_{A,2} & \mu_{C,2} & \mu_{D,2} \\
\vdots & \vdots & \vdots & \vdots \\
1 & \mu_{A,m} & \mu_{C,m} & \mu_{D,m}
\end{bmatrix}
\begin{bmatrix}
\beta_0 \\
\beta_1 \\
\beta_2 \\
\beta_3
\end{bmatrix}
= 
\begin{bmatrix}
V_1 \\
V_2 \\
\vdots \\
V_m
\end{bmatrix}
\]

(1)

\( \beta_0 \) is understood to be a constant in this equation. Two equivalent ways of writing this relationship are

\[
\beta_0 + \sum_{i=1}^{3} \beta_i \mu_{ij} = V_j
\]

(2)

and

\[
[\mu_{ij}] [\beta_i] = [V_j]
\]

(3)
where $M_{0ij}$ is understood to be 1. There is some ambiguity in indexing the gages here. As written
in equations (2) and (3), if several equations are made the indices “$i$” in each equation may refer to
different gages. They are re-indexed for every equation. This is important because the GA that will
be presented later refers to the columns of data as they are indexed in the previous table.

Because there are more equations than unknowns, an exact solution is not expected. The common
practice is to curve fit using least squares. The least squares solution for the coefficients $\beta_j$ is found
by solving the relation

$$[\mu_{ij}]^T [\mu_{ij}] [\beta_i] = [\mu_{ij}]^T [V_j]$$

(4)

In this study, the coefficients of a calibration equation for a linear structure are always found by
solving equation (4) using Gaussian elimination and back-substitution. However, before these
coefficients can be determined, a decision must be made as to which gages to use. This problem is
equivalent to asking which columns of the table to use.

**Standard Deviation**

To help determine how many and which gages to use, standard deviation is used to quantify the
error associated with an equation. A measure of how well an equation matches the actual load
applied is the standard deviation, denoted $\sigma$. For an n-gage equation, $\sigma$ is defined as

$$\sigma = \sqrt{\frac{1}{m-n-1} \sum_{j=1}^{m} \left[ V_j - \left( \beta_0 + \sum_{i=1}^{n} \beta_i \mu_{ij} \right) \right]^2}$$

(5)

The smaller the standard deviation, $\sigma$, the more closely the equation matches the calibration data,
and therefore the better the equation. In deciding how many and which gages to use, $\sigma$ is used to
quantify the error associated with an equation.

**Percent Root-Mean-Square Error**

Standard deviation works well, but it does not relate the magnitude of the error to the size of the
load. The results presented later are in percent root-mean-square (rms) error, which is the standard
deviation divided by the sum of the square of the applied load and is defined as

$$\%\text{rms} = \frac{100\sigma}{\sqrt{\sum_{j=1}^{m} V_j^2}}$$

(6)

With the foregoing established, gage reduction techniques can be applied to determine the final
load equations.

**CURRENT GAGE SELECTION METHODS**

Current gage selection techniques remove gages, one at a time, from the set of all gages. Methods
differ only when deciding which gages to select for removal. The process stops when the percent
rms error becomes unacceptably large or the desired number of gages is reached. If the methods are strictly adhered to, the removed gages never need to be considered again. These methods hold the implicit assumption that the best 3-gage equation only contains gages found in the best 4-gage equation, and that the best 4-gage equation only contains gages in the best 5-gage equation, etc. Later, this assumption will be shown to be invalid.

**The T-Value Method of Gage Reduction**

The first method of gage selection is referred to as the T-value method. A T-value (ref. 2) is a statistical value assigned to each coefficient in an equation that relates how much error is involved with that coefficient and the magnitude of the coefficient. It is defined as follows

\[
T_i = \frac{\beta_i}{\text{Standard Error of the Coefficient } \beta_i} = \frac{\beta_i}{\sigma} \sqrt{\sum_{j=1}^{m} \mu_{ij}^2}
\]  

(7)

For signals of comparable size, coefficients with smaller T-values would produce larger uncertainty in the predicted value. Thus, gages with smaller corresponding T-values are subsequently removed.

To compare the T-value method with other methods, a strict set of rules is needed:

1. Start with all relevant gages in the structure. In most cases, this will be all of the operational gages at or near the wing or fuselage station of interest.
2. Solve the equation using all n gages using equation (4).
3. Calculate the T-value for each gage.
4. Remove the gage with the smallest absolute T-value.
5. Repeat steps 2 through 4 until the desired number of gages is attained.

Figure 2(a) is a pseudo-flow chart of how this process works. In practice, this is usually not strictly observed. There are instances where there may not be enough test conditions to solve an equation with all of the gages (barfing techniques like singular value decomposition). Additionally, some exploration, random guesswork, and engineering intuition may be involved. However, this algorithm was strictly adhered to in this study to compare methods.

**The Best Step-down Method of Gage Reduction**

A second method of gage reduction is the Best Step-down method. As previously stated, a T-value can be thought of as a measure of the relative error in an equation associated with a particular gage. A T-value can also be thought of as a number that tells which gage to remove. Modifications have been proposed (ref. 3) to improve the forecast of which gage to remove, but the underlying concept is similar to the T-value method. If the intent is to remove a gage, there can be no better method than to test all of the possible resulting equations that could be formulated by removing each gage. This is the concept behind the Best Step-down method.

If there are n gages in an equation, and it is desired to remove one, then there are n candidates, and therefore n possible resulting (n-1)-gage equations. In the Best Step-down method, each of the (n-1)-gage equations are solved and evaluated. The best (n-1)-gage equation is the one with the smallest standard deviation, so the gages that comprise that equation are kept.
The Best Step-down method is defined by the following steps:

1. Start with all relevant gages in the structure.

2. If there are \( n \) possible gages, it is possible to formulate \( n \) different \((n-1)\)-gage equations. Solve each one.

3. Calculate the standard error for each equation in step 2.

4. Choose the set of gages producing the equation with the smallest standard error. This is equivalent to removing the gage that this set doesn't contain.

5. Repeat steps 2 through 4 until the desired number of gages is attained.

Figure 2(b) is a pseudo-flow chart of how this process works. When starting with \( n \) gages, the T-value method removes one gage and arrives at the \((n-1)\)-gage equation with the lowest standard deviation. From a specific \( n \)-gage equation, the Best Step-down method automatically arrives at the \((n-1)\)-gage equation with the lowest standard deviation because the equations are actually solved and evaluated.

**DETERMINATION OF STRAIN GAGE LOAD EQUATIONS BY THE APPLICATION OF A GENETIC ALGORITHM**

For the past few years GAs (refs. 4 and 5) have been used to solve problems dealing with optimization and artificial intelligence. GAs are search techniques based on the mechanics of natural selection, and they differ from classical optimization techniques in a number of ways that suit the determination of strain gage load equations.

Genetic algorithms work well when the desired solution is a set of characteristics. This use of characteristics is what originally inspired this work; a characteristic of a strain gage equation is the use (or nonuse) of a particular gage. However, there are other features of the GA that differ from more classical techniques.

Genetic algorithms do not work with a single point in a large space. They work with a population, which is a sampling of several points. This means that not one but many calibration equations are considered at the same time. Additionally, most classical techniques move from one point to another based on gradient information (or something similar). In a genetic algorithm each point that is sampled is assigned a fitness. Fitness is simply a number that portrays how useful that point (or set of characteristics) is in solving the problem at hand. In this application fitness is dependent on the standard deviation and the number of gages used. The next generation is then made using the characteristics of the more fit points of the previous generation, known as the parents. In the application presented here, each generation consists of several equations, each of which use a set of gages. In subsequent generations, equations are formulated from gages that were found in the more fit equations of the previous generation.

Subsequent generations are always created based on the points in previous generations. The mechanism behind the GA can produce interesting and complex behavior. Equations that use good gages (or good combinations of gages) will have a higher fitness, and they will pass these characteristics on to the next generation. In a similar fashion, gages that have problems (i.e., nonlinear effects or hysteresis) are rejected, because any equation that uses them will have a higher standard deviation and therefore a lower fitness. Eventually solutions are found with good sets of
characteristics (in other words, a good set of gages) even though only a fraction of the possible sets of characteristics has been sampled.

**General Implementation of the Genetic Algorithm**

The specific algorithm used in this study is outlined in the following.

1. Each equation is encoded into a “string” as shown in figures 3(a) and 3(b). A string is a series of bits, with each bit holding information about a particular characteristic. If the bit holds the value 1, then the gage corresponding to that bit’s position on the string will be used. If the bit holds the value 0, then that gage will not be used. These bits are ordered to represent specific gages, reflecting the columns of the previous table. Thus string #1 in figure 1 represents an equation that will use gage A, gage C, and gage F.

2. Using this coding, make several initial random strings. This original population constitutes the first generation.

3. Evaluate each string. That is, solve the equation that the string implies using equation (4), and calculate its fitness.

4. Sort all of the equations according to their fitness.

5. Create another generation of strings based on the last generation. This is done by picking two strings from the old generation, favoring the better strings, and creating a new string based on the contents of the two old strings. This is repeated until the next generation is complete.

6. For each string, randomly decide whether or not one of the bits should be changed to enhance the search process. This is known as mutation, and it is performed every generation.

7. Repeat steps 3 through 6 a specific number of generations.

Strings in successive generations improve their fitness by following these steps. Therefore, because fitness is related to how well a particular set of gages can be used to calibrate the structure, the strings in later generations represent successively better calibrations. The steps outlined above are expanded upon in the next few sections.

**Evaluating Loads Equations**

The fitness of each equation needs to address two key issues. First, there must be a penalty if a string uses too many gages. Second, the equation formed must fit the calibration data to some degree of accuracy. The fitness function used has the form

\[
\text{fitness} = \exp \left\{ - [A + B]^2 \right\}
\]

(8)

The exponential function was only used to normalize the fitness to have a value between 1 (a perfect equation) and 0 (a very poor equation). The coefficient A would penalize the string if too many gages were used. Coefficient A was defined by
\[ A = \frac{\max(\text{No. of gages used, target}) - \text{target}}{\text{bandwidth}} \]  

(9)

Coefficient A would then have a nonzero value if a string did not use more gages than the desired number (the target). The actual size of the penalty is weighted by the bandwidth.

The coefficient B would penalize the string if the resulting equation had a large standard deviation.

\[ B = \frac{\sigma_{\text{this string}} \times \text{(number of strings)}}{\sum_{\text{all strings}} \sigma_i} \]  

(10)

The standard deviation of the equation was normalized by the average standard deviation of all of the equations in the current generation.

Although somewhat heuristic, a bandwidth of one would then penalize an equation that uses an extra gage approximately the same amount as an equation whose standard deviation is twice that of the average.

**Attempts at Overcoming a GA-Hard Function**

When formulating equations to fit calibration data, using more gages will automatically decrease the standard deviation. Therefore the GA would have a tendency to find equations that used more gages. However, the fitness function previously defined penalizes strings that use too many gages. This type of problem can be intrinsically difficult for genetic algorithms to solve (ref. 4). The GA must be allowed to explore equations that use more gages, but this exploration must be accomplished within reasonable limits.

The purpose of the bandwidth parameter is to establish the reasonable limits, so that the two competing qualities in each equation “using few gages” and “having a low standard deviation” are weighted against each other. Generally the largest bandwidth possible is desirable, so that exploration is encouraged. If the GA failed to find any acceptable equations for the number of gages requested, a smaller bandwidth was found to solve this dilemma.

**Use of Crossover Operator vs. Similar Trait Operator**

The classical GA uses a single-point crossover operator to create new strings based on strings from the previous generation. This involves picking a random crossover point, and copying everything from the first string up to the crossover point into the new string as shown in figure 3(b). After the crossover point, all of the information from the second string is used. This operation favors keeping combinations whose locations are close together on the string.

To avoid this favoritism, the similar trait operator shown in figure 3(a) performs the same function, keeping common material and leaving to chance material that was different between the two strings, without favoring position within the string. If two strings had the same value for a specific bit, that value was copied to the new string. If the value differed between the two strings, the new value was chosen randomly from the two old strings. The performance of this operator in relation to the single-point crossover operator has not been studied at this time.
Choosing Parents

When choosing parents, two special features of the GA presented here should be noted. First, no duplicate strings were allowed in a single generation. Second, when the parents were chosen, a normalized roulette wheel approach along with an ordered population (ref. 1) was used. That is, if there were $k$ strings in one generation, the probability of choosing the $n$th best string as one of the parents is

$$
\text{probability of choosing nth best} = \frac{k - n + 1}{\binom{k + 1}{2}}
$$

(11)

Every string that was created had to have two unique parents, as opposed to using the same parent twice. This is known as preventing asexual reproduction. Additionally, an elitist strategy was used: the top 5 percent of the strings from each generation were directly copied into the next generation of strings, and this top 5 percent were safe from any mutation.

Different Performance Parameters

When executing a GA, there are some parameters that define how the algorithm operates that have nothing to do with the physical problem (i.e., the gages or the structure). These parameters are (1) how many strings are in each generation, (2) how many generations are run, (3) how many strings are directly copied from generation to generation, and (4) the mutation rate. Setting these parameters can be confusing, and making the GA perform optimally may depend more on the topology of the fitness space (which isn't known before analysis) than the physical problem.

Running the GA with several different parameter values showed this method to be robust. Several graphs similar to figure 4 were made for various parameter values, and little statistical significance could be found. The number of generations was set to 50 because almost all activity seemed to diminish by then. However, this number is expected to change from problem to problem. Also, the use of an elitist strategy seemed to diminish activity quicker, but the specific optimal number of strings to be kept between generations could not be determined. Different values for the mutation rate did not significantly affect the problem. Most parameters were simply set by the use of these graphs and then ignored.

This approach can be justified in that these parameters have little to do with the physical problem. The person performing a calibration only need be concerned with the structure and decisions regarding which gages, how many, and an acceptable level of error.

PERFORMANCE COMPARISONS

The choice of where to place the calibration loads and how many to use to represent in-flight loads is an important question that is outside the scope of this paper. It is therefore vital that the calibration loads be representative of in-flight loads to the best available knowledge. Comparisons are made by examining the percent rms error of the equations found by each of the three methods for specific numbers of gages.
The remainder of this paper presents the outcome of these three methods for a high aspect ratio structure - the AFTI/F-111 MAW wing, and a low aspect ratio structure - the Space Shuttle orbiter wing.

Discussion of the F-111 Structure and Data

The first data set used in comparing different gage selection methods was a calibration performed on the AFTI/F-111 Mission Adaptive Wing, shown in figure 5. The calibration was performed at the Dryden Flight Research Facility Thermostructural Laboratory in June 1985. This is a fairly high aspect ratio structure with few spars, which implies that there are few load-paths, and therefore few gages are needed to calibrate the wing. Eight separate loads were applied to various parts of the wing. Each load consisted of one to three point loads that were applied by hydraulic jacks. The total shear load was between 8,000 and 15,000 lb each. A total of approximately 400 data points were taken at regular intervals between the onset of the load (approximately 10 percent of the maximum) and the maximum load for each calibration load. These calibration loads are typical for the purposes of representing in-flight loads. The location of the applied loads is shown in figure 5. There were 25 strain gages that were mounted along the strain gage axis.

When running the GA, there were 80 strings per generation, and a total of 50 generations were run. The four best strings of each generation were copied directly into the next generation, and each string had a 1 in 2 chance of being mutated. A bandwidth of 2 was used in all runs except for the search for 3-gage equations. This particular run returned many 4-gage equations, so the bandwidth parameter was reset to 1 and the run was performed again.

Results from the F-111 Calibrations

Figure 6 graphs the percent rms error for equations found using the T-value method, the Best Step-down method, and the GA. Because the GA has random elements, ten different runs using the same parameters are shown. Genetic algorithms do not necessarily return at the optimal solution, but consistently return near optimal solutions. On average, each run lasted approximately 15 min on a SUN 690 MP®, so a certain degree of confidence in the solution is added without a great investment in time. A variety of answers like this is not necessarily unfavorable, given that there may be circumstances when the best set found is not usable. Perhaps a gage may fail in the middle of a testing program. Multiple answers may also help when the sets of gages are combined to calculate more than one load – that is when shear, bending moment, and torque are all calculated in-flight from the same gages.

Often the separate genetic runs found the same solution; even when this didn’t occur all of the solutions found were comparable or superior to the equations found by the T-value method or the Best Step-down method.

From figure 6 it is obvious that better equations exist than were found by the Best Step-down method. If it were possible to remove gages one by one to find the best calibration equation, the Best Step-down method would accomplish this. The existence of better equations proves that this class of methods does not work for the general case.

®The Sun 690 MP is a registered trademark of Sun Microsystems, Mountainview, CA.
The Shuttle Orbiter Structure and Data

Figure 7 shows the wing of the Space Shuttle orbiter and the locations of the calibration loads. This is a very low aspect structure with highly redundant load paths, so many gages and loads will be needed to calibrate the structure. The strain gage loads calibration was performed by Rockwell International in Palmdale, California. The analysis used the 130 possible gages that were installed in the left wing and listed in reference 6 as not having any data acquisition problems or any inconsistencies with a previous calibration. The actual data points (gage readings and applied loads) were not available, but estimated through the use of influence coefficients supplied by Rockwell.

An influence coefficient \( I_{ij} \) is the linear rate of response of a particular gage to a particular load, so there is one influence coefficient per gage per loading condition. The original data points were then estimated by

\[
\mu_{ij} = V_j I_{ij}
\]  

Any nonlinear effects such as hysteresis or local loadings were therefore masked. This is a common practice because it is a more compact form of data. Influence coefficients are often studied to see the effects of loading different points on the structure. Only 2 data points for each of the 42 loadings were approximated, corresponding to loads of 17,500 and 35,000 lb.

When running the GA, there were 320 strings per generation, and a total of 50 generations were run. The 16 best strings of each generation were copied directly into the next generation, and each string had a 1 in 2 chance of being mutated. A bandwidth of 2 was used in all runs.

The T-value method or Best Step-down method could not be applied directly because there were more gages (130) than data points (84) and many of the data points were linear multiples of each other. To address this problem, the GA was run first, looking for equations using 3 to 12 gages. A list of 31 gages was then compiled using the best equations that the GA provided. The other two algorithms were performed with this filtered subset.

Results from the Shuttle Orbiter Calibration

A plot of the percent rms error versus the number of gages used in the equation is given in figure 8. There are two major points to be made about these particular results:

1. The GA could be applied even when there were not enough data points to use the T-value or the Best Step-down methods.

2. The GA was able to search through a set of 130 gages and find better equations than either of the other methods were able to find using a set of 31 gages. This is in spite of the fact that it was possible to construct each of the equations that the GA found using these 31 gages.

CONCLUDING REMARKS

It was found that, in general, the genetic algorithm was able to determine comparable or superior strain gage load equations than either the T-value or Best Step-down methods. In addition, the genetic algorithm was able to find loads equations significantly faster than an exhaustive search. It
was also determined that in the presence of a large number of strain gages and fewer calibration load points, when the T-value or Best Step-down methods could not be applied directly, the genetic algorithm was used to assist the other methods. In this situation the genetic algorithm determined better loads equations significantly faster than the currently used methods.

Because of the random nature of the genetic algorithm, only an exhaustive search will absolutely find the best equation for a given number of gages. As was stated, this is not a practical method of solution. The genetic algorithm presented in this paper completed 50 generations in under 15 min on a SUN 690 MP. However, performing a number of runs can give the engineer a feeling of confidence for a particular solution, or an idea that there may be several sets of gages that do not differ appreciably.

In the case of current methods, if the assumption can be made that the best set of gages can be arrived at by removing one gage at a time, then by construction the Best Step-down method would accomplish this goal. However, the existence of equations with less error shows a fundamental flaw in the current methods.

No special qualities with regard to structural phenomena are expected for the gages that are chosen by the genetic algorithm. The only properties that they would necessarily have are that the gages work well together in predicting the applied loads.

The method outlined here could easily be adapted for use in any field where the needs of accuracy and the limitation of data channels compete.

An extension of this algorithm could be made to find sets of gages when more than one quantity is desired. For example, shear, torque, and bending moment are usually calculated from the same set of gages. Care should be taken here because the relative errors of these three quantities are usually very different, so that weighting these quantities would play a major role. However, the underlying concepts are the same.

REFERENCES


Instrument the structure

1. Application of calibration loads
2. Recording of strain gage outputs

Determination of the loads equations

1. Data organization
2. Application of gage selection method
   a. T-value method
   b. Best step-down method
   c. Genetic algorithm

Experimental data acquisition

Flight testing

Experimental test results

Calibration loads test

- 1. Application of calibration loads
- 2. Recording of strain gage outputs

Figure 1. A generic flow chart illustrating the sequence of obtaining experimental structural loads results from strain gage measurements.

Starting set of gages

Gages
A
B
C
D

Calculate equation coefficients and T-values

<table>
<thead>
<tr>
<th>Gages</th>
<th>T-values</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>23</td>
</tr>
<tr>
<td>B</td>
<td>200</td>
</tr>
<tr>
<td>C</td>
<td>13</td>
</tr>
<tr>
<td>D</td>
<td>2</td>
</tr>
</tbody>
</table>

Gage D has smallest T-value. Remove it.
Gage C has smallest T-value. Remove it.

Calculate new equation coefficients and T-values

<table>
<thead>
<tr>
<th>Gages</th>
<th>T-values</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>49</td>
</tr>
<tr>
<td>B</td>
<td>150</td>
</tr>
<tr>
<td>C</td>
<td>13</td>
</tr>
</tbody>
</table>

Continue process until desired number of gages is reached

(a) T-value.

Figure 2. A pseudo flow chart.
Starting set of gages
Gages
A
B
C
D

This set of gages had the smallest standard deviation. Use them.

Calculate equation coefficients and standard deviation for all possible 3-gage equations

Gages
A
B
C

Standard deviation
387

A
B

154

A
B
C
D

238

B
C
D

532

This set of gages had the smallest standard deviation. Use them.

Continue process until desired number of gages is reached

Calculate equation coefficients and standard deviation for all possible 2-gage equations

Gages
A
B

Standard deviation
438

A
B
D

212

B
D

745

(b) Best Step-down method.
Figure 2. Concluded

Parents

<table>
<thead>
<tr>
<th>Gages</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
</tr>
</thead>
<tbody>
<tr>
<td>String # 1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>String # 2</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>New string</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

(a) Bit by bit representation of a string and how new strings are formed based on the characteristics of the older parent strings using the similar trait operator.

Figure 3.
(b) How new strings are formed based on the characteristics of the older parent strings using the single point crossover operator.

Figure 3. Concluded.

Figure 4. Percent rms error of the best 4-gage string in successive generations for four identical runs of the genetic algorithm.
Figure 5. Calibration load placement for the AFTI/F-111 Mission Adaptive Wing.

Figure 6. Percent rms error for calibration equations of the F-111 arrived at by various methods.
Figure 7. Locations of the 42 calibration point loads applied to the space shuttle orbiter.

Figure 8. Percent rms error for calibration equations of the space shuttle orbiter arrived at by various methods.
Traditionally, structural loads are measured using strain gages. A loads calibration test must be done before loads can be accurately measured. In one measurement method, a series of point loads is applied to the structure, and loads equations are derived via the least squares curve-fitting algorithm using the strain gage responses to the applied point loads. However, many research structures are highly instrumented with strain gages, and the number and selection of gages used in a loads equation can be problematic. This paper presents an improved technique using a genetic algorithm to choose the strain gages used in the loads equations. Also presented are a comparison of the genetic algorithm performance with the current T-value technique and a variant known as the Best Step-down technique. Examples are shown using aerospace vehicle wings of high and low aspect ratio. In addition, a significant limitation in the current methods is revealed. The genetic algorithm arrived at a comparable or superior set of gages with significantly less human effort, and could be applied in instances when the current methods could not.