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BLOCH VECTOR PROJECTION NOISE

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Abstract

In the optical measurement of the Bloch vector components describing a system of N two-level atoms, the quantum fluctuations in these components are coupled into the measuring optical field. This paper develops the quantum theory of optical measurement of Bloch vector projection noise. The preparation and probing of coherence in an effective two-level system consisting of the two ground states in an atomic three-level Λ-scheme are analyzed.

1 Introduction

The properties and generation of an optical squeezed state have been interesting subjects of study for a number of years. The Bloch vector model of a two-level atomic system interacting with a laser field, and the use of angular momentum components J_j to represent the N two-level atoms [1], have also been widely investigated. It is known that quantum systems with dynamical variables in the form of nonlinear products of the position and momentum operators are different from those involving only the position and momentum operators. For instance, in a system of N two-level atoms, described by a Bloch vector spin angular momentum $\hat{\mathbf{J}}$, the uncertainty relation, $\Delta J_1 \cdot \Delta J_2 \geq \frac{1}{2}|\langle \hat{J}_3 \rangle|$, depends on the quantum state of the system, as opposed to that of $\Delta x \cdot \Delta p \geq \frac{\hbar}{2}$ for the two quadrature phase components of an optical field [2]. The quantum fluctuations in atoms hence provide an interesting system for the study of uncertainty relations, and are of practical importance. For example, the fluctuations in an atomic system contribute to noise that can in principle, limit the accuracy of atomic frequency standards [3].

As a simple example, let us consider the spin model for a single two-level atom with a ground state $|1\rangle$ and an excited state $|2\rangle$. The Bloch vector operators are

$$\begin{aligned} \hat{s}_1 &\equiv \frac{1}{2}(\hat{\sigma}_{12} + \hat{\sigma}_{21}), \\ \hat{s}_2 &\equiv \frac{i}{2}(\hat{\sigma}_{12} - \hat{\sigma}_{21}), \\ \hat{s}_3 &\equiv \frac{1}{2}(\hat{\sigma}_{22} + \hat{\sigma}_{11}), \end{aligned} \tag{1}$$

where

$$\hat{\sigma}_{ij} \equiv |i\rangle\langle j|, \quad i,j = 1,2.$$

These operators obey the usual commutation relation for angular momentum operators,

$$[\hat{s}_i, \hat{s}_j] = i\epsilon_{ijk}\hat{s}_k,$$

where ϵ_{ijk} is the Levi-Civita symbol. It is easy to show that $\hat{\mathbf{s}}$ describes a spin-$\frac{1}{2}$ system.

For a superposition state

$$|\varphi\rangle = \cos\frac{\theta}{2}|1\rangle + e^{i\phi}\sin\frac{\theta}{2}|2\rangle, \tag{2}$$

where θ, and ϕ are some angles, we have

$$\begin{aligned} \langle\hat{s}_1\rangle &= \frac{1}{2}\sin\theta\cos\phi, \\ \langle\hat{s}_2\rangle &= -\frac{1}{2}\sin\theta\sin\phi, \\ \langle\hat{s}_3\rangle &= -\frac{1}{2}\cos\theta. \end{aligned} \tag{3}$$

Clearly, the vector $\mathbf{r} = (\langle\hat{s}_1\rangle, \langle\hat{s}_2\rangle, \langle\hat{s}_3\rangle)$ falls onto the surface of a sphere of radius $\frac{1}{2}$. The fluctuations in the components of $\mathbf{s}$ are,

$$\langle\Delta\hat{s}_j^2\rangle = \langle\hat{s}_j^2\rangle - \langle\hat{s}_j\rangle^2 = \frac{1}{4} - \langle\hat{s}_j\rangle^2, \quad j = 1,2,3, \tag{4}$$

with the fluctuation in the total Bloch vector

$$\langle\Delta\hat{\mathbf{s}^2}\rangle = \langle\hat{\mathbf{s}^2}\rangle - \langle\hat{\mathbf{s}}\rangle^2 = \frac{1}{2}.$$

It is also easy to show that

$$\langle\Delta\hat{\mathbf{s}}_i^2\rangle \cdot \langle\Delta\hat{\mathbf{s}}_j^2\rangle \geq \frac{1}{4}|\epsilon_{ijk}\langle\hat{\mathbf{s}}_k\rangle|^2. \tag{5}$$

Now let us consider the situation for N two-level atoms. If there is no mutual interaction between the atoms, the system can be described by the total spin angular momentum operator

$$\mathbf{S} = \sum_{n=1}^{N}\mathbf{s}_n.$$

We assume that all atoms are in the ground state $|1\rangle$ initially or, equivalently, the system is in the angular momentum eigenstate $|S = \frac{N}{2}, S_3 = -\frac{N}{2}\rangle$ (Fig.1a). Now we have

$$\langle\hat{S}_3\rangle = \sum_{n=1}^{N}\langle\hat{\mathbf{s}}_{3,n}\rangle = -\frac{N}{2},$$

and

$$\langle\hat{\mathbf{S}}^2\rangle = \frac{N}{2}(\frac{N}{2}+1).$$

We hence obtain

$$\langle\Delta\hat{\mathbf{S}}^2\rangle = \langle\hat{S}_1^2\rangle + \langle\hat{S}_2^2\rangle = \langle\hat{\mathbf{S}}_\perp^2\rangle = \frac{N}{2}. \tag{6}$$

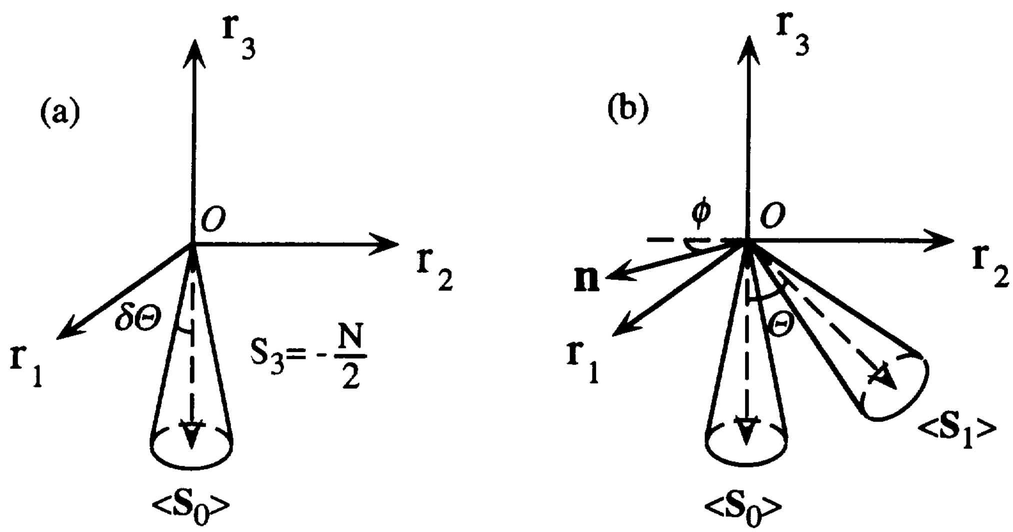

FIG. 1. Bloch vector for an N-atom system. In a), all atoms are in state $|1\rangle$, and the mean total Bloch vector $\langle \mathbf{S}_0 \rangle$ points down. b), $\langle \mathbf{S}_0 \rangle$ is rotated about an axis **n** to $\langle \mathbf{S}_1 \rangle$. The cones represent the fluctuations in Bloch vectors.

As is shown in Fig.1a, the uncertainty in **S** forms a cone centered on $\langle \mathbf{S}_0 \rangle$, pointing inversely along axis r_3, with a conic angle $\delta\theta \approx \sqrt{\frac{2}{N}}$. When a resonant laser field is applied, the Bloch vector **S** is rotated from $\langle \mathbf{S}_0 \rangle$ to $\langle \mathbf{S}_1 \rangle$, by angle θ about an axis **n** in the Or_1r_2 plane (Fig.1b). Now all atoms are in a superposition state as given in Eq.(2), and one can show that the mean square fluctuations in the components of the Bloch vector **S** are N times of that given by Eq.(4). Now let us take a closer look at Fig.1b. When the Bloch vector **S** is rotated from $\langle \mathbf{S}_0 \rangle$ to $\langle \mathbf{S}_1 \rangle$, the cone representing the fluctuations in **S** is also rotated. The projection of the base of the cone, which represents the fluctuation, onto an axis, say r_1, is merely

$$\begin{aligned}\Delta r_1^2 &= \frac{1}{2}\mathbf{S}_\perp^2 \left[1 - \left|\mathbf{r}_1 \cdot \frac{\langle \hat{\mathbf{S}}_1 \rangle}{|\langle \hat{\mathbf{S}}_1 \rangle|}\right|^2\right] \\ &= \frac{N}{4}\left(1 - \sin^2\theta\cos^2\phi\right). \qquad (7)\end{aligned}$$

One can obtain similar results for the fluctuations along other axes. The fluctuations in the Bloch vector components are hence the projections of the Bloch vector uncertainty onto the corresponding axes. It has been pointed out that the shape of the cone (Fig.1b) can be altered, and turned to an ellipse, by introducing a non-uniform interaction between the external field with the atoms [4], or by mutual interaction between the atoms. We see from Eq.(7) that the noise in the Bloch vector component along r_1 reaches a minimum point at $\theta = \frac{\pi}{2}$ and $\phi = 0$ when the component is maximized. Yet the shot noise in the radiation field from the atomic medium is proportional to $\sqrt{r_1}$ and also maximized. It is predicted that the total noise consisting of the shot noise and the Bloch vector projection noise would reach its minimum value at $\theta = \frac{\pi}{3}$ [2].

2 Theory

In this section, we develop the quantum theory of optical measurement of atomic Bloch vector projection noise. We will consider the experimental situation of Λ-three-level atoms in a beam interacting with spatially separated laser fields. In a Λ-scheme three-level atom (Fig.2), we assume that dipole transitions between states $|1\rangle$ and $|0\rangle$, and $|2\rangle$ and $|0\rangle$ are both allowed, with resonant frequencies, ω_{01}, ω_{02}, and transition dipole moments $\mathbf{d}_{01}, \mathbf{d}_{02}$, respectively. We assume for simplicity that $\mathbf{d}_{01}$ and $\mathbf{d}_{02}$ are orthogonal. First, a resonant optical pumping field is applied to pump all atoms into state $|1\rangle$. Two co-propagating off-resonant laser fields of frequencies ω_1, ω_2, polarizations $\mathbf{e}_1, \mathbf{e}_2$, respectively, are applied downstream to prepare the atoms into a superposition state of $|1\rangle$ and $|2\rangle$. Level $|0\rangle$ adiabatically follows the ground state amplitudes and can be eliminated. Hence, we are left with an effective two-level system consisting only of ground states, which do not spontaneously decay. At a later point in the atomic beam, a probe field of frequency ω_2, and polarization $\mathbf{e}_2$ is applied. This induces an $\mathbf{e}_1$-polarized radiation field, oscillating between states $|0\rangle$ and $|1\rangle$ which is homodyned with an external local oscillator field of frequency ω_1. We will show that the homodyne output is proportional to the atomic Bloch vector components and carries its noise characteristics.

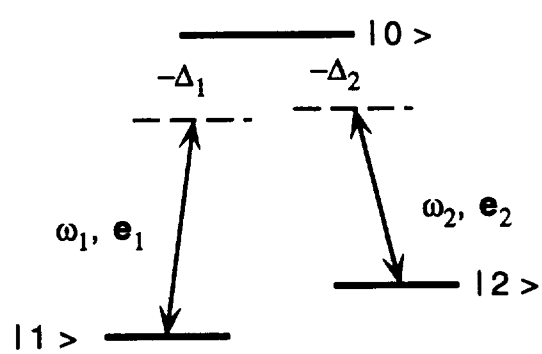

FIG. 2. Level diagram of a Λ-scheme three level atom.

We first treat the preparation process of the atomic ground state superposition. As illustrated in Fig.3.a, in the lab frame, atoms in the beam moving along axis x with speed v, enter the coherence preparation region I, between $x = 0$ and $x = x_0$. It is more convenient, however, to calculate the atomic state in the atomic center-of-mass (CM) frame. Let us consider an atom that appears at an arbitrary position x in the probe region II, at time t'. Referring to Fig.3.b, we see that the atom entered region I at a previous time $t' - x/v$ and exits region at time $t' - (x - x_0)/v$.

We start from the effective Hamiltonian

$$\hat{H} = -\hbar\omega_{01}|1\rangle\langle 1| - \hbar\omega_{02}|2\rangle\langle 2| + \hat{V}. \tag{8}$$

and the interaction

$$\hat{V} = -\frac{\hbar\Omega_{01}}{2}e^{-i\omega_1 t}|0\rangle\langle 1| - \frac{\hbar\Omega_{02}}{2}e^{-i\omega_2 t}|0\rangle\langle 2| + H.c., \tag{9}$$

where we take

$$\begin{aligned}\Omega_{01} &= \Omega^*_{10} = \frac{(\mathbf{e}_1\cdot\mathbf{d}_{01})E_1}{\hbar},\\ \Omega_{02} &= \Omega^*_{20} = \frac{(\mathbf{e}_2\cdot\mathbf{d}_{02})E_2}{\hbar},\end{aligned}$$

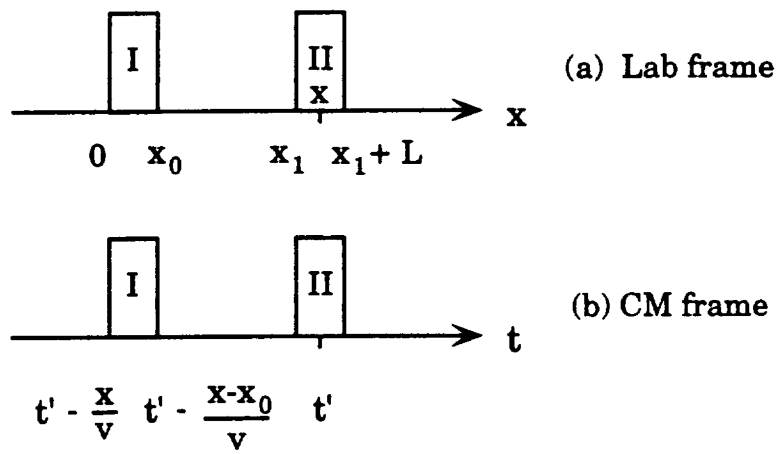

FIG.3, Schematic illustration of the experimental situation in a) the Lab frame, and b), in the atomic center-of-mass frame. Regions I and II are the coherence preparation and the probe region, respectively.

as the Rabi frequencies for the applied laser field given by

$$E(t) = \frac{\mathbf{e}_1 E_1}{2} e^{-i\omega_1 t} + \frac{\mathbf{e}_2 E_2}{2} e^{-i\omega_2 t} + c.c..$$

Taking the energy of state $|0\rangle$, $\tilde{E}_0 = 0$, the atomic state takes the form

$$|\psi(t)\rangle = a_0(t)\,|0\rangle + a_1(t)\,e^{i\omega_{01}t}\,|1\rangle + a_2(t)\,e^{i\omega_{02}t}\,|2\rangle. \tag{10}$$

We obtain from the Schrödinger equation,

$$\begin{aligned}\dot{a}_0(t) &= i\frac{\Omega_{01}}{2}\,e^{-i\Delta_1 t}\,a_1(t) + i\frac{\Omega_{02}}{2}\,e^{-i\Delta_2 t}\,a_2(t),\\ \dot{a}_1(t) &= i\frac{\Omega^*_{01}}{2}\,e^{i\Delta_1 t}\,a_0(t),\\ \dot{a}_2(t) &= i\frac{\Omega^*_{02}}{2}\,e^{i\Delta_2 t}\,a_0(t),\end{aligned} \tag{11}$$

where

$$\Delta_j \equiv \omega_j - \omega_{0j}, \qquad \text{j=1,2}$$

are the detunings. When $\Omega_1, \Omega_2 \ll \Delta_1, \Delta_2$, we may adiabatically eliminate level $|0\rangle$ by defining

$$a_0(t) = B_1(t)\,e^{-i\Delta_1 t} + B_2(t)\,e^{-i\Delta_2 t}, \tag{12}$$

where

$$B_1(t) = -\frac{\Omega_{01}}{2\Delta_1}\, a_1(t),$$
$$B_2(t) = -\frac{\Omega_{02}}{2\Delta_2}\, a_2(t).$$

Eq.(11) then becomes

$$\begin{aligned}
\dot{a}_1(t) &= -i\frac{|\Omega_{01}|^2}{4\Delta_1} a_1(t) - i\frac{\Omega_{10}\Omega_{02}}{4\Delta_2}\, e^{-i(\Delta_2-\Delta_1)t}\, a_2(t), \\
\dot{a}_1(t) &= -i\frac{\Omega_{20}\Omega_{01}}{4\Delta_1}\, e^{i(\Delta_2-\Delta_1)t}\, a_1(t) - i\frac{|\Omega_{02}|^2}{4\Delta_2} a_2(t).
\end{aligned} \tag{13}$$

The initial condition of the atomic state as given in Eq.(10) is

$$\begin{aligned}
a_1(t'-x/v) &= 1, \\
a_2(t'-x/v) &= 0,
\end{aligned} \tag{14}$$

or that the atom is in state $|1\rangle$ when entering region I at time $t'-x/v$.

When the atomic ground splitting $\omega_{21} \ll \Delta_1, \Delta_2$, and $\Delta_1 \approx \Delta_2, |\Omega_{01}| \approx |\Omega_{02}|$, we may define the light shift frequency

$$\tilde{\Delta} \equiv \frac{|\Omega_{01}|^2}{2\Delta_1} \approx \frac{|\Omega_{02}|^2}{2\Delta_2},$$

the Raman Rabi frequency

$$\beta_R \equiv \frac{\Omega_{10}\Omega_{02}}{2\Delta_2} \approx \frac{\Omega_{10}\Omega_{02}}{2\Delta_1} = |\beta_R|\, e^{i\phi},$$

and the net detuning

$$\Delta \equiv \Delta_1 - \Delta_2 = (\omega_1 - \omega_2) - \omega_{21}.$$

Eq.(13) can be readily solved by changing variables

$$\begin{aligned}
a_1(t) &= A_1(t)\, e^{i\Delta t/2}\, e^{-i\int^t dt_1 \tilde{\Delta}(t_1)/2}, \\
a_2(t) &= A_2(t)\, e^{-i\Delta t/2}\, e^{-i\int^t dt_1 \tilde{\Delta}(t_1)/2},
\end{aligned} \tag{15}$$

which yields when $\Delta \ll |\beta_R|$.

$$\begin{aligned}
A_1(t) &= \cos\left[\frac{1}{2}\,|\beta_R|(t-t'+x/v)\right], \\
A_2(t) &= e^{i\phi}\sin\left[\frac{1}{2}|\beta_R|(t-t'+x/v)\right].
\end{aligned} \tag{16}$$

Hence we obtain the atomic state for an atom which leaves region I at time $t'-(x-x_0)/v$ and will arrive at position x at time t', as

$$\begin{aligned}
|\psi(t'-\frac{x-x_0}{v})\rangle &= A_1(t'-\frac{x-x_0}{v})\, e^{i\frac{\Delta}{2}(t'-\frac{x-x_0}{v})}\, e^{i\omega_{01}(t'-\frac{x-x_0}{v})}\,|1\rangle \\
&\quad + A_2(t'-\frac{x-x_0}{v})\, e^{-i\frac{\Delta}{2}(t'-\frac{x-x_0}{v})}\, e^{i\omega_{02}(t'-\frac{x-x_0}{v})}\,|2\rangle \\
&= \cos\frac{\theta}{2}\,|1\rangle + e^{i\phi}\sin\frac{\theta}{2}\, e^{-i(\omega_1-\omega_2)(t'-\frac{x-x_0}{v})}\,|2\rangle,
\end{aligned} \tag{17}$$

where

$$\theta = |\beta_R| \frac{x_0}{v},$$

is the Raman Rabi area. All common time dependent phase factors in Eq.(17) are left out, as these will not affect the atomic coherence $\langle\hat{\sigma}_{12}\rangle$.

Now let us closely examine the probe region. As illustrated in Fig.4, a probe field

$$\mathcal{E}_p(t) = \mathbf{e}_2 \frac{\mathcal{E}_p}{2} e^{-i\omega_2 t} + c.c.$$

is incident onto the atomic beam at region II, and introduces an interaction Hamiltonian

$$\hat{V}^{(j)}(t) = -\frac{\hbar\Omega_2}{2} e^{-i\omega_2 t} \hat{\sigma}_{02}^{(j)} + H.c., \tag{18}$$

for the j-th atom at position x at time t. Here $\Omega_2 = (\mathbf{d}_{02}\cdot\mathbf{e}_2)\mathcal{E}_p/\hbar$ is again the Rabi frequency for the probe field, and $\hat{\sigma}_{02}^{(j)} = |0\rangle\langle 2|$ is the Schrödinger picture operator for the j-th atom.

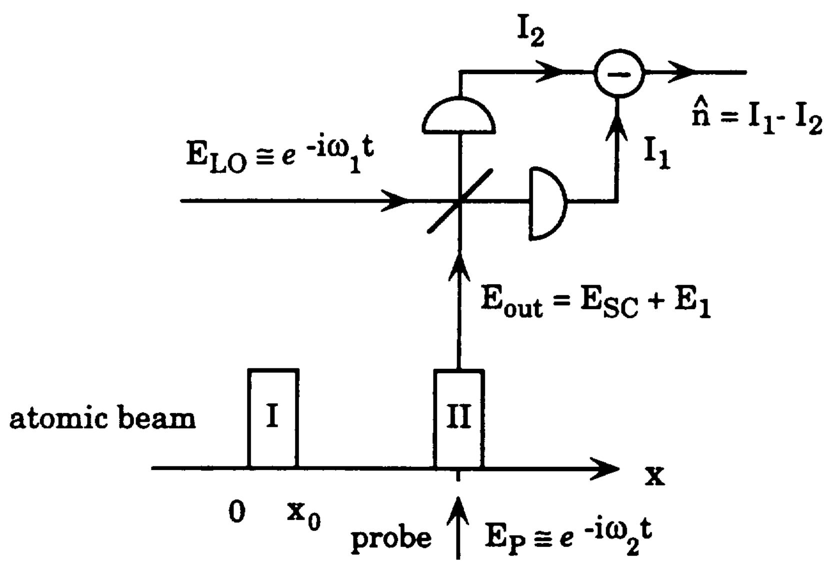

FIG. 4. Schematic diagram of the homodyne detection of the probe-field-induced Raman transition field.

It is convenient to use Heisenberg equations in the atomic CM frame for operators $\hat{\sigma}_{02}^{(j)}(x,t)$, etc., which are

$$\left(\frac{\partial}{\partial t} - i\,\omega_{21}\right) \hat{\sigma}_{21}^{(j)}(t) = 0, \tag{19}$$

and

$$\left(\frac{\partial}{\partial t} + \frac{\gamma}{2} - i\,\omega_{01}\right) \hat{\sigma}_{01}^{(j)}(t) = -i\,\frac{\Omega_2^*}{2}\, e^{i\omega_2 t}\, \hat{\sigma}_{21}^{(j)}(t), \tag{20}$$

where γ is the spontaneous decay rate of state $|0\rangle$. γ is small in the experiment so that noise terms in Eq.(20) will be neglected. Eq.(19) can be readily solved for the evolution between times

$t' - (x - x_0)/v$ when the atom leaves the preparation region I, and t' in the atom frame:

$$\hat{\sigma}_{21}^{(j)}(t) = \hat{\sigma}_{21}^{(j)}(t' - \frac{x - x_0}{v})\, e^{i\omega_{21}[t - t' + \frac{x - x_0}{v}]}. \tag{21}$$

$\hat{\sigma}_{21}^{(j)}(t)$ is now determined from $\hat{\sigma}_{21}^{(j)}(t' - (x - x_0)/v)$ whose expectation value will be evaluated for the atomic state $|\psi(t' - (x - x_0)/v)\rangle$ in Eq.(17). Using Eq.(21), Eq.(20) is solved

$$\begin{aligned} \hat{\sigma}_{01}^{(j)}(t') &= -i\frac{\Omega_2^*}{2}\int_{t'-\frac{x-x_0}{v}}^{t'} dt_1\, e^{i\omega_2 t_1}\, e^{-(\frac{\gamma}{2} - i\omega_{01})(t'-t_1)}\, \hat{\sigma}_{21}^{(j)}(t_1) \\ &\approx -i\frac{\Omega_2^*}{2}\, e^{i\omega_2 t'}\, e^{i\omega_{21}\frac{(x-x_0)}{v}}\, \frac{1}{\frac{\gamma}{2} + i\Delta_2}\, \hat{\sigma}_{21}^{(j)}(t' - \frac{x - x_0}{v}), \end{aligned} \tag{22}$$

where we assumed that $\gamma\,(x - x_0)/v \gg 1$.

Note that in the lab frame, the j-th atom is at position x at time t'. Hence we obtain from Eq.(22) the atomic polarization

$$\hat{\mathbf{P}}(x, t') \propto \sum_j \mathbf{d}_{10}\, \hat{\sigma}_{10}^{(j)}(x, t') + H.c., \tag{23}$$

where

$$\hat{\sigma}_{10}^{(j)}(x, t') = -\frac{\Omega_2}{2\Delta_2}\, e^{-i\omega_2 t'}\, e^{-i\omega_{21}\frac{(x-x_0)}{v}}\, \hat{\sigma}_{12}^{(j)}(t' - \frac{x - x_0}{v}), \tag{24}$$

under the assumption that the detuning $\Delta_2 \gg \gamma$, the spontaneous decay rate.

Now if the atomic dipole moment $\mathbf{d}_{10}$ and hence $\mathbf{P}$ are orthogonal to the polarization of the probe field $\mathbf{E}_p$, the optical radiation field due to the atomic polarization $\mathbf{P}$

$$\mathbf{E}_s = -2\pi i k l\, \mathbf{P}, \tag{25}$$

can be separated using a polarizer from $\mathbf{E}_p$. Here kl is the optical thickness of the atomic beam.

Adding the vacuum field $\hat{E}_1$ of the same polarization as $\hat{E}_s$, we obtain for the positive frequency part of the total output field from the atomic medium

$$\hat{E}_{out}^{(+)}(t) = \hat{E}_1^{(+)}(t) + \hat{E}_s^{(+)}(t) \tag{26}$$

where

$$\hat{E}_s^{(+)}(t) = i\mathcal{T}\mathcal{E}_p(t) \sum_j e^{-i\omega_{21}\frac{(x-x_0)}{v}}\, \hat{\sigma}_{12}^{(j)}(x, t - \frac{x - x_0}{v}). \tag{27}$$

$$\mathcal{T} \propto \pi k l\, \rho\, \frac{(\mathbf{d}_{02}\cdot\mathbf{e}_2)(\mathbf{d}_{01}\cdot\mathbf{e}_1)}{\hbar\Delta_2}$$

is a dimensionless scattering coefficient, where ρ is the number density of the atomic beam, and $\mathbf{e}_1, \mathbf{e}_2$ are the polarization unit vectors of the probe field $\mathcal{E}_p$ and the scattered field $\hat{E}_{out}$, respectively.

$\hat{E}_{out}$ is mixed with a local oscillator field $E_{LO}(t) \sim E_{LO}\, e^{-i\omega_1 t}$ in the homodyne detection scheme illustrated in Fig.4. The output signal n can be written in the operator form as the difference of photo-currents I_1 and I_2,

$$\begin{aligned} \hat{n}(t) &= \hat{I}_1(t) - \hat{I}_2(t) \\ &\propto E_{LO}^*(t)\cdot\hat{E}_{out}^{(+)}(t) + E_{LO}(t)\cdot\hat{E}_{out}^{(-)}(t). \end{aligned} \tag{28}$$

Using Eq.(27), and (17) for the atomic state, we obtain

$$\langle \hat{n}\,(t)\rangle = |\mathcal{T} E_{LO}^{*}\mathcal{E}_p| \sum_{j}^{M} e^{i(\omega_1-\omega_2)t}\, e^{-i\omega_{21}\frac{(x-x_0)}{v}}\, \langle \hat{\sigma}_{12}^{(j)}\,(t-\frac{x-x_0}{v})\rangle + c.c., \tag{29}$$

where $\langle \hat{\sigma}_{12}^{(j)}\,(t-\frac{x-x_0}{v})\rangle$ is evaluated for the atomic state $|\psi(t-\frac{x-x_0}{v})\rangle$ given in Eq.(17). The photocurrent difference $\langle \hat{n}\,(t)\rangle$ can be written in terms of the Bloch vector components $\langle s_1\rangle$, and $\langle s_2\rangle$ of Eq.(1). Then Eq.(29) can be further simplified

$$\langle \hat{n}\,(t)\rangle = M|\mathcal{T} E_{LO}^{*}\mathcal{E}_p|\, \sin\theta\, \cos[(\omega_1-\omega_2-\omega_{21})\cdot \frac{(x-x_0)}{v} + \phi_0], \tag{30}$$

where M is the number of atoms in the probe region, and ϕ_0 some reference phase. Eq.(30) gives the usual Ramsey fringe pattern [5].

Now let us evaluate the fluctuations in the atomic Bloch vector components. We calculate the power spectrum of the homodyne output signal $n\,(t)$

$$S(\omega) = \frac{1}{2\pi}\int d\tau\, \langle \hat{n}\,(t)\,\hat{n}\,(t+\tau)\rangle\, e^{-i\omega\tau}. \tag{31}$$

After some algebra, we obtain

$$\begin{aligned} S(\omega) \;=\; & |E_{LO}|^2 - M|\mathcal{T}|^2\,|E_{LO}|^2\,|\mathcal{E}_p|^2\,A\,(\omega)\,\langle \hat{\sigma}_{11}-\hat{\sigma}_{22}\rangle_\varphi \\ & + M^2|\mathcal{T}|^2|E_{LO}|^2|\mathcal{E}_p|^2\,\langle \hat{\sigma}_{12}+\hat{\sigma}_{21}\rangle_\varphi^2\,\delta(\omega) \\ & + M\,|\mathcal{T}|^2\,|E_{LO}|^2\,|\mathcal{E}_p|^2 A\,(\omega)\,\left[1-\langle \hat{\sigma}_{12}+\hat{\sigma}_{21}\rangle_\varphi^2\right], \end{aligned} \tag{32}$$

under the assumption that the net detuning Δ is small so that $\Delta\cdot x_1/v \ll 1$, where x_1 is the distance between the preparation region I and the probe region II. $|E_{LO}|^2$ is the intensity of the local oscillator field, and $|\mathcal{E}_p|^2$ the intensity of the probe field in units of photon number per second. $A(\omega)$ is a spectral function that centers at $\omega = 0$, with a spectral width of order of the transit bandwidth of the probe region. $\langle \hat{\sigma}_{11}-\hat{\sigma}_{22}\rangle_\varphi = \cos\theta$, and $\langle \hat{\sigma}_{12}+\hat{\sigma}_{21}\rangle_\varphi = \sin\theta\cos\phi$ are components of the Bloch vector expectation values evaluated for the atomic state $|\varphi\rangle$ given in Eq.(2).

Now let us closely examine the four terms in Eq.(32). The first term is clearly due to the shot noise in the homodyne process. The second term is the reduction of vacuum shot noise level due to Raman absorption. It can also be viewed as the shot noise associated with the spontaneous Raman transition. The third term is proportional to M^2, and centers at $\omega = 0$, represents the power spectrum of the stationary Ramsey fringe signal of Eq.(30). The last term is the phase ϕ-dependent Bloch vector projection noise given in Eqs.(4) and (7).

3 Summary

In this paper, we developed the quantum theory for the experimental study of the Bloch vector projection noise. Eqs.(30) and (32) are the primary results. In a Λ-three-level atomic system, when two off-resonant Raman fields are applied, the upper state adiabatically follows the ground state amplitudes and the Λ-scheme is hence reduced to an effective two-level system. Decayless ground state coherence is prepared. By probing with one optical transition, and detecting the induced transition between the other ground state and the upper level with a homodyne technique, we can measure the Bloch vector components as given in Eq.(30). The photo-current difference in the homodyne scheme also yields the noise characteristics of the Bloch vector components as given in Eq.(32).

An experimental study is currently being conducted using a wide-angled supersonic ytterbium (Yb) atomic beam. The 556 nm transition of ^{171}Yb $^1S \rightarrow ^3P$ transition is used. In a 2.6 kG magnetic field, the ^{171}Yb ground states of nuclear spin $I = \frac{1}{2}$ are split by 2 MHz and form a Λ-system with the upper state $|F = \frac{3}{2}, F_3 = \frac{1}{2}\rangle$. Doppler shifts in the corresponding σ^+ and π transition in the wide angle atomic beam are Zeeman compensated [6] simultaneously by a quadrupole magnetic field. With this technique, the transition linewidth is narrowed to a few MHz for an interaction path length l of 2.5 cm. The Ramsey fringe pattern of Eq.(30) has been observed.

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References

[1] R. H. Dicke, Phys. Rev. **93**, 99 (1954).

[2] K. Wodkiewicz and J. H. Eberly, J.O.S.A. **B 2**, 458 (1984).

[3] W. M. Itano et al., Phys. Rev. **A 47**, 3554 (1993).

[4] D. J. Wineland et al., Phys. Rev. **A 46**, R6797 (1992).

[5] N. F. Ramsey, *Molecular Beams* , (Oxford University Press, London, 1956).

[6] K. D. Stokes et al. Optics Lett. **14**, 1324 (1989).