THE STRONG BELL INEQUALITIES
A PROPOSED EXPERIMENTAL TEST

Edward S. Fry
Physics Department, Texas A&M University
College Station, TX 77843

Abstract

All previous experimental tests of Bell inequalities have required additional assumptions. The strong Bell inequalities (i.e., those requiring no additional assumptions) have never been tested. An experiment has been designed that can, for the first time, provide a definitive test of the strong Bell inequalities. Not only will the detector efficiency loophole be closed; but, the locality condition will also be rigorously enforced. The experiment involves producing two $^{199}$Hg atoms by a resonant Raman dissociation of a mercury dimer ($^{199}$Hg$_2$) that is in an electronic and nuclear spin singlet state. Bell inequalities can be tested by measuring angular momentum correlations between the spin one-half nuclei of the two $^{199}$Hg atoms. The method used to make these latter measurements will be described.

1 Introduction

Due to low detector efficiencies, previous experimental tests of Bell inequalities have required an auxiliary assumption in order to derive an experimentally testable inequality. [1-5] A new experimental concept has been developed that makes it possible to test the strong Bell inequalities (Bell inequalities without any auxiliary assumptions). It also leads directly to an extension that rigorously enforces locality.

This approach is an experimental realization of Bohm's version of the Einstein-Podolsky-Rosen Gedankenexperiment.[3, 6] Instead of photon pairs, it involves measurements of the correlations between angular momentum components of two atoms (actually spin one-half nuclei) of the isotope $^{199}$Hg. A brief description of the proposed experiment was recently published,[7] and a very detailed discussion has also been provided.[8]

In general, the measurement of components of spin suggests the use of Stern-Gerlach magnets. For spin one-half particles, there are two possible projections, + ("spin-up") and − ("spin-down"), of the spin on the axes of the magnets. As indicated in Figure 1, the magnetic field gradient deflects each spin one-half particle into one of two detectors depending on the projection of its spin on the magnet axis. To measure a spin component of a particle in any specified direction, one simply rotates the axis of the corresponding magnet to that direction. However, such an approach is unsuitable in the present work.
for two reasons. First, to enforce Einstein locality, it would be necessary to rotate a magnet to an arbitrary direction in a time short compared to the time it takes light to travel from one magnet to the other - a few nanoseconds. This appears to be physically impossible. Second, based on available magnetic fields and the magnitude of the nuclear magnetic moment, the deflection produced by $^{199}$Hg nuclear spins is so small as to preclude this approach.

![Diagram of a Stern-Gerlach apparatus](image)

**FIG. 1.** Schematic of a Stern-Gerlach apparatus for measuring spin correlations. The diagram assumes spin one-half particles and the magnets are oriented for measurements of the spin components of both particles in the Z direction.

Other approaches to the measurement of spin components are possible. For example, a Bell inequality involving proton spin correlations has been tested using proton-proton scattering to determine the spin components. However, this type of approach requires several auxiliary assumptions.

Efficient detection of atoms is combined with spin analysis in the approach to be described here. This approach takes advantage of the hyperfine structure interaction in $^{199}$Hg atoms in order to select nuclear spin components via electric dipole transitions. Detection efficiency is sufficiently high to close that loophole. And finally, this approach permits a very rapid change in the direction of the measurement of spin components; hence Einstein locality can be enforced.

## Overview of the Proposed Experiment

The $^{199}$Hg atoms are produced by a stimulated Raman excitation to a dissociating state of the $X^1\Sigma_g^+$ ground state of a $^{199}$Hg dimer. The total electron and the total nuclear spin angular momenta are both zero in the initial rotational state of the Hg dimers, and are not changed in the dissociation process. The two mercury atoms resulting from the dissociation are both in $^1S_0$ ground states. Since the electronic
angular momentum is $J=0$ and the nuclear spin of $^{199}\text{Hg}$ is $I=1/2$, each ground state atom therefore has a total angular momentum $F=1/2$. The component of total angular momentum (which is therefore the component of nuclear spin) in any given direction is measured by orienting excitation laser beams at 253.7 nm in that direction and using polarization selective excitation of one of the Zeeman sublevels. For left circularly polarized excitation, only atoms whose component of angular momentum, $M_F=+1/2$, in the direction of laser beam propagation are excited. Similarly, right circularly polarized light only excites atoms with $M_F=-1/2$. Excited atoms are photoionized via an auto-ionizing state using a laser at 197.3 nm. The resulting photoelectrons and ions are detected using Channeltron electron multipliers. Assuming a mercury atom has been detected if EITHER an electron OR an ion is observed, then the overall detection efficiency for the Hg atoms is greater than 96%. Figure 2 shows a schematic of the experiment indicating the relative directions of the dimer, atom, and laser beams.

![Schematic of the experiment](figure2.png)

**FIG. 2.** Schematic of the experiment showing the direction of the mercury dimer beam together with a pair of the dissociated atoms and their respective detection planes. Relative directions of the various laser beams are also shown.

Rates for simultaneous detection (coincidence rates) of the two atoms are measured for components of their angular momenta in the directions $\theta_1$ and $\theta_2$, respectively (i.e. in the directions of the excitation laser beams). A set of four angles can be chosen that give a maximum violation of the strong Bell inequality.\(^7\)\(^8\) This experiment also lays the foundation for an experiment that enforces locality
since a combination of a Pockels cell and a beamsplitting polarizer can be used to stochastically change the direction of the exciting laser beams on the nanosecond time scale.

3 Measurement of the Nuclear Spin Components

Figure 3 shows the relevant energy levels of the mercury atom\cite{10} and the corresponding transitions at 253.7 nm for spin analysis and at 197.3 nm for detection via photoionization. The first transition is from the (6s\(^2\))\(^1\)S\(_0\) (\(F = \frac{1}{2}\)) ground state (level 1) to the (6s6p)\(^3\)P\(_1\) (\(F = \frac{3}{2}\)) state (level 2); the second transition is from level 2 to the (6p\(^2\))\(^3\)P\(_0\) autoionizing state (level 3). Atom detection is via both the resulting ion and the photoelectron.

Assuming a Fano profile for the autoionizing transition, a calculated value for its oscillator strength, and a measured value for its width, a theoretical value of \(\approx 2.3 \times 10^{-14} \text{ cm}^2\) is obtained for the peak cross-section\cite{7,8}. This is an unusually large cross-section, but is consistent with measured cross-sections for analogous transitions in Cd\cite{11}. Since both transitions are so strong, laser pulse energies of only 60 \(\mu\)J and 250 \(\mu\)J, respectively, are sufficient to ionize all the atoms in a few nanoseconds.

As shown in Figure 2, the laser beams for spin analysis have a wavelength of 253.7 nm and lie in planes perpendicular to the direction to the source. They are at angles \(\theta_1\), \(\theta_2\) to the +Z-axis. The angles \(\theta_1\), \(\theta_2\) of these laser beams define the directions in which each atom's angular momentum components is observed.

If we choose some arbitrary direction as the quantum axis for our system, then the quantum numbers for the two components of angular momentum in that direction are \(M_F = \pm \frac{1}{2}\) (for an \(F=\frac{1}{2}\) ground state...
If the 253.7 nm laser beam is propagating in the direction of the quantum axis and has right circular polarization (σ+), then $M_F$ must increase by one in the transition. Thus, only ground state atoms for which the projection of the angular momentum in the direction of laser beam propagation is $M_F = -1/2$ can be excited to the $6^3P_1$ ($F = 1/2$) state and ionized; see Figure 4. Similarly, for left circular polarization, only atoms with $M_F = +1/2$ are excited and ionized. In summary, by choosing an arbitrary direction and using a circularly polarized excitation laser, a Hg atom with either component of angular momentum in that direction can be selectively detected.

![Figure 4](image.png)

**FIG. 4.** Analysis scheme with right circularly polarized light. Since $M_F$ must increase by 1 in the transition, only ground state atoms with $M_F = -1/2$ can reach the $6^3P_1$ ($F = 1/2$) state.

## 4 Summary

The use of electric dipole transitions to measure the component of nuclear spin of $^{199}$Hg in an arbitrary direction has been described. Its application to a definitive experimental test of the previously untested strong Bell inequalities is discussed. This novel approach makes it feasible to rigorously enforce Einstein locality.

## 5 Acknowledgement

This research was supported by the Robert A. Welch Foundation grant No. A-1218 and by the National Science Foundation grant PHY-9221038. The author also thanks Dr. Shifang Li for many helpful discussions and suggestions.
6 References


