EFFECTS OF THERMOCAPILLARITY ON AN EVAPORATING EXTENDED MENISCUS IN MICROGRAVITY

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ABSTRACT

An analytical investigation of the effects of thermocapillarity on the flow field within and heat transfer from the extended meniscus region of a heated meniscus which is re-supplied by capillarity is presented. Microgravity conditions are considered. The analysis shows that even for extremely small temperature differences between the wall and the vapor (< 1 mK) thermocapillary stresses at the liquid-vapor interface due to a non-uniform interfacial temperature drastically alters the flow field. At the same time, these stresses were shown to have only a slight effect on the heat transfer from the extended meniscus but increasing with an increasing temperature difference. Additionally, thermocapillary effects were shown to be sensitive to pore size. A criterion was established from a scaling analysis identifying the conditions necessary for thermocapillarity to affect the operation of capillary-pumped heat transport devices in microgravity. A critical Marangoni number and corresponding critical temperature difference between wall and vapor were identified.

INTRODUCTION

Evaporation from curved menisci is observed in a variety of heat transport devices which rely upon nucleate boiling or capillary phase-change, such as heat pipes, sweat coolers, grooved evaporators, and other enhanced heat transfer surface devices. Capillary phase-change devices (heat pipes and capillary-pumped loops or CPL’s) have much promise for high heat transport applications in space. Reliance upon capillarity for the transport of liquid to where energy can be dissipated via evaporation offers outstanding potential for the thermal control of high heat flux devices in low gravity environments. The passive means of fluid transport also precludes the need for mechanically pumping liquid from one location to another. The heat transfer enhancement realized with phase change over pure convection also affords the potential for substantial weight and power savings.

Evaporation of a nearly perfectly wetting liquid in a pore, groove, or capillary due to heat input has received little attention at the microscale. Recent theoretical work, however, has identified the extended meniscus region for wetting liquids as where the maximal evaporation flux is present at least when the wall’s thermal conductivity is much greater than the liquid [1,2]. In fact, over distances on the order of microns, the evaporative flux goes from zero to a maximum value and then decreases rapidly as the film thickens and therefore the thermal resistance increases. As a consequence, the liquid-vapor interfacial temperature gradients (shear stresses) are likely most severe in the extended meniscus. However, the previous analytical work has neglected the resulting thermocapillary stresses. It is hypothesized that these stresses can choke the flow into and/or be responsible for destabilizing the extended meniscus. The practical implication relative to the operation of capillary-pumped heat transport devices in microgravity is that if the thermocapillary stresses at the near contact line region are important, they may result ultimately in the deprime of the evaporator wick, such as commonly occurred in practice [3].

Identification of the conditions which lead to interfacial instabilities and the subsequent observation of the evolution of instabilities in microgravity, should they arise, are vital. In space, capillary-pumped heat transport devices have not performed as expected [3]. Whereas in 1-g body forces are always present to stabilize such instabilities, their absence in low-g dictates that evaporating menisci are inherently more
unstable. If the evolution of instabilities in low-g ultimately leads to the dry-out of the pores within heat pipes and CPL’s, then an understanding of the conditions leading to their onset may allow designers of such devices the opportunity to design around such a failure mode. This present research ultimately aims to investigate the effects of the thermocapillarity on the stability of the extended meniscus.

**MODEL**

A model is developed to assess the importance of thermocapillary stresses in the thin film portion of a meniscus within a circular or grooved pore geometry. The disjoining pressure gradient theory is used to describe the flow in the thin film region of the meniscus. The governing system of equations is developed through application of conservation laws.

In developing the model, the following assumptions are used: (1) $\mu_v \ll \mu_l \Rightarrow$ viscous stresses on the vapor side are neglected; (2) the effect of the circumferential curvature is negligible; (3) a Gibbs approximation for both the liquid-vapor and liquid-solid interface is assumed; (4) small Bond number $\Rightarrow$ microgravity; (5) the liquid is perfectly wetting and non-polar; and (6) the thin film evaporative flux equation, developed by Brown et al. [4] is used.

Boundary conditions include no-slip and impermeability at the wall, as well as the conservation of mass, momentum and energy at the liquid-vapor interface.

**SCALING ANALYSIS**

The governing equations and boundary conditions are scaled in a manner analogous to Burelbach, Bankoff, and Davis [5] except in the specification of length scales appropriate for the $x$ and $y$ directions (respectively the coordinates parallel and normal to the wall). A reasonable length scale in the vertical direction is the film thickness, $h_0$, which defines a balance between the capillary pressure terms and the disjoining pressure terms in the extended meniscus. The bulk meniscus curvature, $K$, which in the thin film where the slope is small is approximately equal to $h_{ss}$. This term can be scaled as $h_{0}/z_{0}$ or $1/R$. Thus, the balance between capillary pressure and disjoining pressure terms yields:

$$\frac{\bar{A}}{h_0^2} = \frac{\sigma_{lv}}{R} \quad \Rightarrow \quad h_0 = \left( \frac{\bar{A}R}{\sigma_{lv}} \right)^{1/3}$$

where $\bar{A}$ is the modified Hamaker constant, $\sigma_{lv}$ is the surface tension, and $R$ is the radius of the pore or groove. Likewise, the $x$-scale is chosen such that it effectively defines the length of the extended meniscus, obtained again by balancing the capillary pressure and disjoining pressure terms.

$$\frac{\sigma_{lv}h_0}{x_0} \sim \frac{\bar{A}}{h_0^3} \quad \Rightarrow \quad x_0 = \left( \frac{\sigma_{lv}h_0^3}{\bar{A}} \right)^{1/2}$$

Viscous scales for time, velocity, and pressure are selected to respectively be $h_0^2/\nu$, $\nu/h_0$, and $\rho\nu^2/h_0^2$, where $\nu$ is the kinematic viscosity. Temperature differences are scaled by $T_w - T_e$. Finally, the mass flux is scaled with $k\Delta T/h_0 h_{lv}$.

A regular perturbation method was utilized to simplify the analysis of the resulting non-dimensional governing system. The characteristic slope, $X = h_0/x_0$, was chosen as the perturbation parameter. The following zeroth order system is derived for the change in film thickness with time,

$$h_{0w} = -EJ_0 + \left\{ P_{0e} \frac{h_0^3}{3} - MaPr^{-1} \left[ J_0 h_0 + J_0 h_0 \right] h_0^2 \right\}$$

(1)

where

$$P_{0e} = -Sh_{0\epsilon\epsilon} + 3\Pi \frac{h_0}{h_0^3}$$

(2)
and \( E = k \Delta T/\rho v h_i \) ⇒ evaporation number, \( j_0 \) ⇒ dimensionless evaporative flux, \( Ma = \gamma \Delta T h_0^2/\rho v k x_0 \) ⇒ Marangoni number, \( Pr = \mu c_p/k \) ⇒ Prandtl number, \( S = -\kappa v/\rho v^2 X^2 \) ⇒ dimensionless curvature, and \( \Pi = A/12 \pi \rho v^2 h_0 \) ⇒ dimensionless disjoining pressure.

This equation is an extremely non-linear differential equation and can only be solved numerically.

**NUMERICAL APPROACH**

A series of numerical experiments were conducted for quasi-steady conditions, i.e., where the liquid evaporating from the extended meniscus is continually replenished and the meniscus is fixed in space (\( \partial \tau = 0 \)). The boundary conditions used for solving equation (1) were chosen in both the meniscus and adsorbed film regions. Since the capillary number, \( Ca = \mu\eta v_0/\sigma_\text{te} \), associated with the bulk meniscus flows is small, as described by Hallinan et al. [6], the apparent contact angle is assumed to be unchanged dynamically. Therefore, the bulk meniscus curvature does not change relative to the equilibrium meniscus curvature. This assumption dictates that in meniscus region, i.e. at \( \xi \to \infty \), the curvature is \( K_m = 1/R \). In the adsorbed film region, where there is no evaporation, the thin film thickness, \( h_0 \), can be calculated [4]. The slope, curvature and third derivative of the film thickness are also zero in the adsorbed film region. But the application of these boundary conditions leads to a trivial solution of equation (1) for steady state of constant film thickness. In order to get a nontrivial solution, a very small perturbation, \( \Delta h = 10^{-5} h_0 \), is applied to the adsorbed film thickness. The third derivative is set to zero, the second derivative is found by letting evaporative flux go to zero, and the slope is chosen such that the curvature of the extended meniscus matches that of bulk meniscus at \( \xi \to \infty \).

**NUMERICAL RESULTS**

The basic goals of this study are to identify the role of the thermocapillary stresses on the flow field and the heat transfer from the extended meniscus. Results are presented for pentane as the working fluid at a temperature of 20 °C for a pore size of 10 cm. Such a pore size is considered because of its likely use in a future space flight experiment. Differences in temperature between the wall and the vapor in the range from 10^-5 - 10^-2 °C are considered with the corresponding Marangoni numbers ranging from 10^-7 - 10^-8. Numerical results obtained by considering thermocapillary effects are compared to those obtained neglecting their contribution, i.e., for \( Ma=0 \).

Thermocapillary effects for the range of Marangoni numbers studied were shown to have little effect on the film profile. Of significance, however, was their effect on the velocity profile. As \( \Delta T \) increases, as shown in Fig. 1, the length of the film decreases considerably. Thus, because \( \Delta T \) characterizes the maximum temperature difference along the film, if the film shortens in length, the interfacial temperature gradients are magnified.

Thermocapillary effects on the velocity profile are shown to be important even when \( \Delta T \) or the Marangoni number is extremely small. Ignoring thermocapillary effects, the velocity profiles for a \( \Delta T \) of 0.002 \( K \) are shown in Fig. 2(a), where the maximum velocity appears at the liquid-vapor interface. Accounting for the thermocapillary effects, the velocity profile changes even for the very small \( \Delta T \) considered. Figure 2(b) demonstrates that the maximum velocity is no longer at the liquid-vapor interface after a certain distance away from the adsorbed film region. As \( \Delta T \) is increased further, reversed flows, which are counter the model's assumptions, are predicted.

Figure 3, which provides a plot of the local evaporative flux versus axial distance, shows that for small \( \Delta T \) the thermocapillary effects on heat transfer are hardly noticeable for the cases considered. But when \( \Delta T \) is increased, e.g., \( \Delta T = 0.02 K, Ma = 2.6e-5 \), the evaporative flux does decrease. The Marangoni effects therefore can affect the heat transfer primarily near the peak evaporative flux, where temperature gradients are most severe.
ESTIMATE OF THE IMPORTANCE OF THERMOCAPILLARITY

The previous numerical results showed quantitatively that the effects of thermocapillarity on the flow and temperature fields were important even for extremely small differences in temperature between the assumed isothermal wall and the vapor (\(< 1 \text{ mK}\)). In fact, for Marangoni numbers greater than \(10^{-5}\) the numerical solution was observed to lead to reversed flows or thermocapillary flows on the surface, which is inconsistent with the approximations inherent to using the lubrication approximation. That a critical Marangoni number on this order of magnitude possibly produces reversed flow in the extended meniscus is not surprising, especially upon close consideration of the equation for the film profile shown in equation (1). Apparent from inspection of this equation is that when the terms involving the Marangoni number become on the same order of magnitude as the driving pressure terms, either the meniscus will recede due to thermocapillarity forces or it will recede due to the inability of the bulk flow to replenish the evaporation from the extended meniscus.

Scaling arguments can therefore be used to estimate a critical Marangoni number. Assuming the terms associated with the driving pressure can be scaled by \(\tilde{f}_1\), and \(j_0\) scales as approximately 1 for low evaporative thermal resistance, and all terms involving \(h\) can be assumed already to be on the order of 1, then the critical Marangoni number can be estimated to be:

\[
Ma_c/Pr = \tilde{f}_1 - E
\]

In this expression for \(Ma_c\), \(\tilde{f}_1\) effectively describes the potential available in the film which must minimally overcome the thermocapillary force acting on the film, while still being capable of replenishing the flow of liquid which evaporates from the film. The significance of the evaporation number, \(E\), is that it increases as \(\Delta T\) increases. Thus it provides a measure of the total evaporation from the extended meniscus. Therefore, not surprisingly, as the evaporative flux increases the critical Marangoni number decreases. For the test conditions considered in the numerical solution, only a small \(\Delta T\) of 0.035K is required for this condition to be satisfied.

Interestingly this relationship for the critical Marangoni number reveals a slight sensitivity to pore size. Using the lengthscales described earlier for the characteristic length and thickness of the film \((x_0 \text{ and } h_0)\), for example, the critical Marangoni number and associated \(\Delta T\) for a pore size of 10 \(\mu\)m is respectively, 9.671E-4 and 0.754 K.

CONCLUSIONS

Thermocapillary stresses at the liquid-vapor interface within an evaporating extended meniscus have been shown to significantly affect the flow field within the extended meniscus even when the extended meniscus heat transport is very small. It is also clear that their effect is first felt on the velocity field near the liquid-vapor interface, and as \(\Delta T\) increases they begin to affect the heat transport as the interfacial thermocapillary stresses near the same order of magnitude as the shear stresses at the wall. When this occurs, the available potential to drive the liquid into the extended meniscus must overcome both forces, and thus, for \(T_w - T_v\) fixed, thermocapillary effects will choke the flow into the extended meniscus. Finally, a scaling analysis was used to develop an expression for the critical Marangoni number which defines the condition where the disjoining pressure potential is roughly balanced by the thermocapillary force acting over the entire film. For such a condition, it is reasonable to expect that flow into the extended meniscus would be entirely choked, and if evaporation occurs, the meniscus would recede.

ACKNOWLEDGMENTS

We would like to acknowledge the support of NASA Microgravity Science Directorate through grant number NAG3-1391.
References


Figure 1: Extended meniscus profile for pore of radius 5 cm with Ma = 0
Figure 2: Velocity profile for pore of radius 5 cm with Ma = 0

Figure 3: Axial evaporative flux for pore of radius 5 cm