THE EFFECT OF AIRCRAFT SPEED ON THE PENETRATION OF SONIC BOOM NOISE INTO A FLAT OCEAN

Victor W. Sparrow
Penn State University
157 Hammond Building
University Park, PA 16802

ABSTRACT

As U.S. aircraft manufacturers now have focused their HSCT efforts on overwater supersonic flight, a great deal more must be known about sonic booms propagating overwater and interacting with the ocean. For example, it is thought that atmospheric turbulence effects are often much less severe over water than over land. Another important aspect of the overwater flight problem is the penetration of the sonic boom noise into the ocean, where there could be an environmental impact on sea life.

This talk will present a brief review on the penetration of sonic boom noise into a large body of water with a flat surface. It has been determined recently that faster supersonic speeds imply greater penetration of sonic boom noise into the ocean. The new theory is derived from the original Sawyers paper and from the knowledge that for level flight a boom’s duration is proportional to the quantity $M/(M^2 - 1)^{3/8}$ where $M$ is the Mach number. It is found that for depths of 10 m or less, the peak SPL varies less than 6 dB over a wide range of $M$. For greater depths, 100 m for example, increased Mach numbers may increase the SPL by 15 dB or more.
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INTRODUCTION

Times are certainly changing in atmospheric propagation research for the NASA High Speed Research program. During the last few years research has focused on the effects of atmospheric turbulence on the propagation and distortion of sonic boom waves. This research has yielded many important results.

However, the U.S. aircraft manufacturers who are preparing for the construction of supersonic commercial aircraft have shifted their emphasis away from overland flight. They are now centering on flying overwater to essentially eliminate the sonic boom impact on people.

This decision has important repercussions for propagation research. It is known that atmospheric turbulence effects are much less severe over water than over land. Over land the ground heats up during the day as the sun shines, creating thermals which greatly increase atmospheric turbulence. The ocean surface, in comparison, does not heat up. The ocean acts as a great heat reservoir, and the water churning over and over keeps the surface temperature nearly constant. No thermals affecting turbulence are created.

Changing Times

- Last few years, HSR focusing on atmospheric turbulence.

- U.S. aircraft manufacturers say
  - no to overland flight
  - yes to overwater flight.

- Atmospheric turbulence much less severe over water than over land.
NEW TECHNICAL CHALLENGES

Since atmospheric turbulence does not seem to be as pressing an issue for the overwater program as it did for the overland program, one may ask: What are the technical challenges that will be encountered in overwater flight?

Just as in the planning of any large project, one must be concerned with environmental impact. Thus the environmental noise impact of sonic booms on wildlife should be addressed. It turns out that the local wildlife for overwater flight consists of sea life, and particularly marine mammals such as whales.

All whales breathe air, and must surface periodically. Most species spend the great majority of their time within 100 m or so of the ocean surface, many within the top 25 m.

One may then ask: How much sound gets from the air into the water? What would one hear if just under the water’s surface?

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**Overwater brings new challenges**

- Look at environmental noise impact on wildlife.

  Overwater local wildlife: marine mammals

- How much sound gets from the air into the water?
SOUND PENETRATION THEORY

It turns out that one can make predictions with reasonable certainty, as the acoustical theory has been well understood for many years. One first notices that the characteristic impedances for water and air differ greatly, approximately 415 for air and 1,500,000 for water. This means that a plane sound wave propagating in air directly toward the surface of the water at an angle of 0 degrees, called normal incidence, would have 99.8% of its energy reflected. Very little propagating sound energy gets into water.

Further it turns out that the sound from a sonic boom is not normally incident but is incident at an angle greater than the critical angle, which for the air–water interface is 13.2 degrees. This means that 100% of the incident energy is reflected. One might think that this means no sound gets into the water, but this is not the case.

When a force is applied normally to the surface of some material, the material must push back or the surface will be moved. So obviously when a sonic boom is incident on the water

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Brief Review of Sound Penetration Theory

- Characteristic impedances differ greatly:
  - air: \( c_1\rho_1 = 415 \text{ kg/(s m}^2) \)
  - water: \( c_2\rho_2 = 1,500,000 \text{ kg/(s m}^2) \)

- For typical HSCT flight, sound will be incident at angles greater than the critical angle, 13.2°.
  - \( \Rightarrow \) all sound power reflected
  - \( \Rightarrow \) but must still match pressure boundary condition
surface, the water must push back on the surface. What this means is that the acoustic pressure on both sides on the ocean surface must be the same. This boundary condition is important because it implies that there will be substantial sound in the water although all of the energy is reflected.

If one uses typical numbers for a projected high speed civil transport, such as Mach number of 2.1 and boom duration of 0.3 second, one can straightforwardly predict what the acoustic pressures will be under the surface of the water. Mach 2.1 implies that the incident angle will be 28.4 degrees from normal, which implies that there will be a pressure wave in the water whose amplitude decays with depth. The “fundamental” frequency of a 0.3 second boom is 3.33 Hz, and the decay of this frequency is indicated in the figure. As one can see, given a relative amplitude of 1 at the ocean surface, the amplitude decays slowly with depth. There is significant noise penetration beyond a depth of 50 m. Of course a sonic boom is composed of many component frequencies. Higher frequencies will penetrate less far beneath the surface, while lower frequencies will penetrate further.

Matching the boundary condition with typical HSCT numbers:

- **Speed**: Mach number, \( M = 2.1 \)
  
  \[ \Rightarrow \text{angle of incidence} = \sin^{-1}(1/M) = 28.44^\circ \]

  \[ \Rightarrow \text{exponentially decaying pressure in water, } e^{-A t f z} \]

- **Duration**, \( T = 0.3 \) s
  
  \[ \Rightarrow \text{"fundamental" frequency, } f_0 = 3.33 \text{ Hz:} \]

  Much faster decay for higher \( f \).
By performing a superposition of component frequencies of a real sonic boom, one can further predict what the received noise would look like at various depths under the ocean surface. For example, using the 0.3 second duration boom incident at Mach 2.1, the Fourier analysis results are shown below in the figure. It is assumed that there is a perfectly N shaped sonic boom incident on the ocean surface. One can see that at 2.5 meters depth, the tips of the N are slightly rounded, but the waveform is mostly preserved. At 25 meters depth the waveform’s peak amplitude is decreased to about 45% of its amplitude at the surface, and the waveform is rounded. At a depth of 120 meters or more, the waveform is greatly smoothed but still has an amplitude of as much as 15% of its peak at the surface.
In 1968 the first theory to account for this penetration of sonic boom noise was developed by Sawyers. His theory is in terms of several nondimensionalized variables to represent time, distance, and depth. The theory assumes that the ocean is flat and is so deep that one need not account for bottom reflections. The assumption is also made that N wave shaped booms are incident on the ocean surface.

Sawyers’ Theory

- Boom penetration theory due to Sawyers (1968):

\[
\pi \frac{p}{p_{\text{surface}}} = (2\tau + 2\xi - 1) \tan^{-1}\left(\frac{\tau + \xi - 1}{\zeta}\right) - (2\tau + 2\xi - 1) \tan^{-1}\left(\frac{\tau + \xi}{\zeta}\right) + \zeta \log\left[\frac{\zeta^2 + (\tau + \xi)^2}{\zeta^2 + (\tau + \xi - 1)^2}\right]
\]

where \(\tau\), \(\xi\), and \(\zeta\) are nondimensionalized \(t\), \(x\), and \(z\).

- Theory assumes ocean is perfectly flat, ocean is very deep, and booms are perfectly N shaped.
In 1970 Cook also closely examined the theoretical aspects of sonic boom penetration into the sea. Cook suggested several minor improvements to Sawyers' theory, but Cook noticed that the small differences between the theories were unlikely to be seen in field experiments.

In the early 1970s two separate laboratory experiments validated Sawyers' theory. Waters and Glass exploded small charges in air, modeling sonic booms, above a pond of water and measured the response of hydrophones below the water surface. Their measured waveforms closely matched those predicted by Sawyers' theory, even though their incident waves were small explosions instead of sonic booms.

Further Intrieri and Malcolm investigated the waveforms created by small supersonic projectiles. They sent such projectiles through the air over an aquarium and measured the acoustic pressure waveforms in the water. Intrieri and Malcolm's results were also in good agreement with Sawyers' theory.

- Sawyers' theory was later examined by Cook (1970), who suggested minor improvements.
TODAY'S TALK: EFFECT OF AIRCRAFT SPEED

What hasn’t been completely understood from Sawyers’ theory alone is the effect of aircraft speed on the penetration of sonic boom noise into the ocean. As one examines Sawyers’ nondimensionalized variables one can see that V, the aircraft speed, and T, the duration of the sonic boom, appear several times. Clearly to apply the Sawyers’ theory correctly, one must have accurate speed and duration information.

Effect of aircraft speed?
(The major topic of this talk.)

- Examine Sawyers’ nondimensionalized variables:

  \[
  \zeta = z/(mT) \quad \tau = t/T \quad \xi = x/(TV)
  \]

  where

  \[
  m = V \left(1 - V^2/c_{water}^2\right)^{-1/2},
  \]

  \[V - \text{aircraft speed, and } T - \text{boom duration.}\]

- Call \(\hat{p} = p/p_{\text{surface}}\).
It turns out that the duration of the boom, $T$, is a function of the aircraft speed, $V$. This relationship is also more complicated than one might expect. Linear theory would predict that $T = L/V$ where $L$ is the length of the aircraft. Hence for a fixed velocity, longer aircraft (spatially) have longer sonic booms (temporally).

However, such linear theory neglects nonlinear acoustics effects. Because the sound from a sonic boom is reasonably loud, particularly near the aircraft, the sound waveform of a sonic boom lengthens as it propagates toward the ground. Thus one must account for this finite amplitude acoustical nonlinearity when determining the correct relationship between $T$ and $V$.

Must be careful here: $T$ is a function of $V$!

**Linear theory** would say:

$$T = \frac{L}{V} \text{ where } L \text{ is the aircraft’s length.}$$

Must include **nonlinear effects**: 

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Several authors have shown that the duration of a sonic boom due to a projectile has the form shown below, as a function of Mach number, which is equal to $V$ divided by the speed of sound. This $T$ versus $V$ relationship assumes that the aircraft is flying at a fixed altitude in steady flight. The theory has been compared to experimental data with good success. In spite of this, the theory assumes that there are no lift effects as one has for an airplane. More complicated theories are available.

Finite amplitude nonlinear acoustics theory predicts:

$$T = \kappa \frac{M}{(M^2 - 1)^{3/8}}$$

where $\kappa$ is a constant; Pierce (1989), Maglieri and Plotkin (1991).

Assumptions:
- level flight at fixed altitude
- lift effects ignored

This theory is pretty good in describing $T = f(V)$. 
Given that we now have a model relationship between $T$ and $V$, and know how these factors influence the penetration of sonic boom noises into the ocean, it is time to determine what will be the maximum acoustic pressures which occur under the water’s surface. One can determine the maximum pressure by taking the time derivative of the acoustic pressure from Sawyers’ theory using the scaled time variable, setting this result to zero, and determining which value of scaled time satisfies the resulting equation. This procedure is necessary because the value of scaled time at which the maximum pressure occurs varies with depth.

What will be the peak pressure of the boom?

Find from:

$$\frac{\partial \hat{p}}{\partial \tau} = 0$$

...to determine the value of scaled time $\tau$ at which $\hat{p}$ is a maximum.

Call this scaled time $\tau_{\text{MAX}}(\zeta)$.

The maximum value of $\hat{p}$ is then

$$\hat{p}_{\text{MAX}} = \hat{p}(\xi = 0, \zeta, \tau_{\text{MAX}}(\zeta))$$
Shown in the figure below is the maximum pressure plotted as a function of Mach number for three different depths. The depths are 1 m, 10 m, and 100 m below the ocean surface. These curves are based on some SR-71 aircraft sonic boom experimental data in the possession of the author. The curves are based on an SR-71 traveling at a Mach number of 2.6 producing a nearly perfectly shaped N wave having a duration of 200 milliseconds.

As one can see the maximum scaled pressures generally increase with increased Mach numbers. The effect is more noticeable at deeper depths. Along the surface of the ocean the scaled maximum acoustic pressure would be 1 for all Mach numbers.

Maximum pressure as a function of Mach number, for different depths:

\[
\hat{p}_{\text{MAX}} \quad z = 1 \\
\hat{p}_{\text{MAX}} \quad z = 10 \\
\hat{p}_{\text{MAX}} \quad z = 100
\]

\(M\)  
1 1.5 2 2.5 3 3.5 4

(Based on data for SR-71 aircraft: \(T = 0.2 \text{ s} \) at \(M = 2.6\).)
Shown in this figure is the same information, but on a relative decibel scale. The curve for each depth is scaled by the maximum pressure at Mach 2.6. One can see that for depths of 10 m or less, the maximum sound pressure level, SPL, will vary less than 6 dB over a wide range of Mach numbers. For greater depths, 100 m for example, increased Mach numbers may increase the SPL by 15 dB or more.

\[ \frac{p_{\text{MAX}}(M)}{p_{\text{MAX}}(2.6)} \]

(Based on data for SR-71 aircraft: \( T = 0.2 \) s at \( M = 2.6 \).)
Instead of peak levels, one may also be interested in the time domain representations of the boom noise. Shown below are waveforms of scaled acoustic pressure versus scaled acoustic time. At the surface of the ocean the scaled pressure would have a value of unity and the duration of the boom in scaled time units would also be unity. The three columns represent an aircraft having speeds of Mach 1.4, 2.4, and 3.4. In each column the waveform is shown at depths of 1 m, 10 m, and 100 m.

One can clearly see that the waveforms at a depth of 1 m are nearly N waves. Further at 10 m the waveforms are similarly shaped and have amplitudes well in excess of 50% of the corresponding amplitude at the surface. The waveforms at 100 m are significantly smoothed out, except for high Mach numbers where a sizable waveform still exists.
CONCLUSIONS

The major conclusion from this study is that faster flying supersonic aircraft produce sonic booms which penetrate more deeply into the ocean. The acoustic pressures experienced under the ocean’s surface increase with increased Mach number.

The key to making these predictions using Sawyers’ theory is an accurate functional relationship between the boom duration, T, and the aircraft velocity, V. Although the functional relationship used here has wide agreement with experimental data, it certainly could be improved upon to account for other factors such as aircraft lift and geometry.

This research is just the beginning, however. Many other factors must be taken into account to ascertain the sonic boom noise impact/non-impact on marine mammals. First of all, Sawyers’ theory assumes that perfectly N shaped waves are incident. A similar theory should be developed for more realistically shaped sonic boom waveforms.

Conclusions

- Faster flying aircraft $\implies$ increased penetration of boom noise.

- Using the correct $T = f(V)$ relationship is the key.

- More work needs to be done; this is only the beginning:

  Issues: Non N shaped waves. Include lift effects.
AN IMPORTANT UNADDRESSSED ISSUE

In addition, another important issue should be addressed. The real ocean surface is rarely, if ever, flat. One should develop a boom penetration theory which takes into account the curvature of the ocean’s surface.

This curvature causes the rippling surface of the ocean to act as a series of lenses, focusing and defocusing the sound under the surface. The focus spots can be considered acoustically hot, where the peak sound pressure levels could be substantially higher than in other regions below the surface. There has been no published research regarding this focusing and defocusing effect.

Another issue:

How often is the ocean perfectly flat?

Instead have focusing and defocusing for real sea surface:

incident boom, propagating

COLD HOT COLD HOT

penetrating boom, evanescent
It would be standard to represent the sea surface as a superposition of Fourier modes as a first pass model. Then one could investigate the focusing effect by a number of analytical and numerical techniques.

For weak waves on the sea, called wind waves, analytical solutions would be tractable via perturbation expansion techniques available in modern symbolic algebra packages.

For wind waves of higher amplitude one could look into either analytical or analytical/numerical solutions to the Kirchhoff-Helmholtz integral equations. Such integral equations have been used to examine the propagation of constant frequency sound from air into water with a rippled surface. The extension of this methodology to sonic booms seems straightforward.

Further one can also perform finite difference calculations with conformal grids to model the penetration of sound in the air into the ocean. Such calculations would be invaluable for validating the perturbation expansion and Kirchhoff approaches. Hence, there are a number of methods available today for predicting the focusing effect.

Describe sea surface by Fourier superposition of many modes.

How to handle:

- Weak wind waves: Analytical perturbation solution via Mathematica.
  (Others have looked at this but not for sonic booms.)
- For comparison: Finite difference time domain calculations.


