Lunar Surface Operations

Volume III:
Robotic Arm for Lunar Surface Vehicle

Dr. William Shields
Dr. Salah Feteih
Dr. Patrick Hollis
and
Mechanical Engineering Students
FAMU/FSU College of Engineering
Tallahassee, Florida

NASA/USRA University Advanced Design Program
Annual Report
July, 1993
List of Student Participants

Robotic Arm for Lunar Surface Vehicle

June Clark
Anthony Cox
Robert Hall
Oscar Hill
Jeff Mayers
Mark Nickel
Tyrene Townsley
Table of Contents

Volume III:
Robotic Arm for Lunar Surface Vehicle

1 Introduction
   Purpose of Document
   Premise for Robotic Arm Design
   Project Organization

3 Mission Statement & Requirements
   Vehicle Mission Statement
   Robotic Arm Mission Statement

4 System Design & Integration
   Performance Objectives
   Robotic Arm Tasks
   Environmental Factors

7 Mechanical Structure
   Subsystem Functions & Requirements
   Mechanical Structure Design
   Calculations

63 Modified Wrist
   Introduction
   Operation
   Drawings
   Materials
   Calculations

68 Structure-to-End-Effector Interface
   Introduction
   Operation
   Drawings
   Materials
   Calculations

95 End-Effectors
   Functions, Subsystem Requirements
   End-Effector Design
   Calculations

106 System Controls
   Introduction
   Control System Selection
   Selected Control System

109 References

110 Bibliography

112 Appendix A: Motor Specifications

118 Appendix B: Time Line
1. INTRODUCTION

1.1 Purpose of Document

The purpose of this design report is three-fold:

- To effectively communicate design concepts.
- To empower the customer and future designers to optimize system performance as well as modify its capabilities based on more specific needs and/or future demands.
- To satisfy course requirements as detailed by Dr. William Shields, instructor of EML4558 for the spring semester of 1993.

1.2 Premise for Robotic Arm Design

The premise for the design is based on the intentions of the National Aeronautics and Space Administration (NASA) to establish a lunar based colony by the year 2010. Provisions for such a colony include developing reliable systems to support lunar operations within this time frame. NASA currently supports a program that encourages competitive designing among college students for a number of these systems. One of these systems is the robotic arm for a lunar surface vehicle.

1.3 Project Organization

For the design of the robotic arm the following company based structure is employed among the design group members. At the head of the design structure is the Project Manager. Reporting to the Project Manager are the Chief Systems Engineer, Deputy Project Manager, and Principal Investigator. Also, in line below the Chief Systems Engineer are five subsystems. These subsystems are the Mechanical Structure, Wrist, Structure-to-End Effector Interface, End Effectors, and Controls, Sensors, and Cameras. A schematic diagram of the company based structure is shown in Figure 1.1.

The following is a description of the five principal positions:

- **Project Manager**: Oversees entire design group.
- **Deputy Project Manager**: Works directly with Chief Systems Engineer and Principal Investigator. Also reports directly to Project Manager.
Documentation Managers: Responsible for report preparation and internal communication.

Chief Systems Engineer: In charge of interface control, requirements analysis, integration, and verification (testing, analysis, etc.).

Principal Investigator: Responsible for application of robotic arm to NASA's needs.

A time line of the activities is given in Appendix B.

Figure 1.1 Company Based Structure
2. MISSION STATEMENT AND REQUIREMENTS

2.1 Vehicle Mission Statement

The previous design of an Extended Mission/Lunar Rover from the Aerospace Final Design Report 1991-92 outlined a vehicle for a 28 earth day mission for four crew members. The time frame for this vehicle is for the years 2010 to 2030 for second generation lunar exploration. Within this report, the design team identified the need for a robotic arm that would be mounted on the rover.

The vehicle's mission requirements as identified by the aforementioned team are "to provide transportation, shelter and working quarters for a crew of four on long duration lunar surface mission." For the design of a robotic arm attached to this vehicle, the pertinent requirements given in their report are as follows:

- Mission Distance: 1000 km round trip
- Mission Duration: 28 earth days (1 lunar day)
- Transport various experimental apparatus
- Possess robotic data sample/data collection capability
- Collect/analyze/store data
- Provide shielding from environmental elements
- Internal navigational support
- Possess path-clearing abilities
- Travel over rough terrain (45° head-on, 20° traverse)
- Provide redundant systems
- Easily maintained

2.2 Robotic Arm Mission Statement

The robotic arm's mission requirements are to "incorporate key issues of compactness, versatility, reliability, accuracy, and weight" to assist in handling cargo and equipment, and to remove obstacles from the path of the vehicle. Mission scenarios would include, but not be limited to the following:

- Exploration
- Lunar sampling
- Replace and remove equipment
- Set-up equipment (e.g. microwave repeater stations)
3. SYSTEM DESIGN AND INTEGRATION

3.1 Performance Objectives

Performance objectives for the robotic arm include a reach of 3 m, accuracy of 1 cm, arm mass of 100 kg, and lifting capability of 50 kg. The arm is able to safely complete a task within a reasonable amount of time; the actual time is dependent upon the task to be performed. The positioning of the arm includes a manual backup system such that the arm can be safely stored in case of failure. No maintenance is required for the duration of the 28 earth day mission. Remote viewing and proximity and positioning sensors are incorporated in the design of the arm.

3.2 Robotic Arm Tasks

3.2.1 Cargo Handling

The end effectors must grip various sizes and shapes of cargo. The said cargo must be no more than the weight capacity of the arm and be equipped with a uniform handle. The handling of cargo requires lifting, lowering, and any other mode of translation from vehicle to vehicle, vehicle to surface, or any combination thereof. This task can be performed only if the origination point, path, and termination point are within the operating range of the robotic arm. Upon completion of the cargo handling task, the end effector will disengage the cargo.

3.2.2 Equipment Setup

During operation, the end effectors must push, pull, turn, lift, or lower various types of equipment. Any equipment to be used must conform to the abilities of the robotic arm (i.e. must be less than 50 kg in mass and must be designed in accordance with the 1 cm precision requirement). The robotic arm must possess the following capabilities:

1. Push objects in all directions in 3-D space within the operating range of the robotic arm, such as pressing a button or knob or sliding objects
2. Pull objects in all directions in 3-D space within the operating range of the robotic arm, including extending an object such as an antenna or an unfolding solar array and unlatching locks and safety devices (pulling is distinguished
from pushing in that the end effector must physically grasp the intended object
3. Turn objects in 3-D space within the operating range of the robotic arm, such as turning a dial or knob
4. Lift or lower objects in 3-D space within the operating range of the robotic arm, such as flipping a switch

3.2.3 Path Clearing

The end effector must clear a path on the lunar surface by shoveling, sweeping aside, or gripping the obstacles present in the desired path. In clearing a path, the materials or obstacles encountered will be moved to a location within the range of the robotic arm. The functions of a path clearing mechanism are listed below.
1. Shovel a maximum volume of regolith (equivalent to 50 kg in mass) to a desired location
2. Sweep aside regolith or small lunar rocks in order to clear a path for travel or equipment placement
3. Transport large lunar rocks or other obstacles that inhibit the desired path of the vehicle

3.3 Environmental Factors

The following environmental information is needed to assure that the design is appropriate. The radiation, temperature range, size of dust, pressure on the surface, magnetic field, and the type of meteorite activity on the moon are values to be considered.

- Radiation = 1000 REM total during the 11 year solar cycle
- Gravity = 1.62 m/s²
- Temperature = 400 K to 80 K
- Soil grain size = 2 to 60 μm, with 50% of grains less than 10 μm
- Pressure = 10⁻¹² Torr (1.3 * 10⁻¹⁰ Pa)
- Magnetic field = no general magnetic field on the moon (dipole field is less than −.5 * 10⁻⁵ times earth's)
3.4 System Power Requirements

The power requirements for the system are detailed by device, subsystem, and total system. The purpose of listing such information is to expedite integrating of the Robotic Arm Design into larger systems such as the Lunar Rover.

Power Requirement Distribution

<table>
<thead>
<tr>
<th>Subsystem</th>
<th>Peak Power</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mechanical Structure</td>
<td>6.0kW</td>
</tr>
<tr>
<td>Motor (4)</td>
<td>1.5kW</td>
</tr>
<tr>
<td>Modified Wrist Joint</td>
<td>4.5kW</td>
</tr>
<tr>
<td>Motor (3)</td>
<td>1.5kW</td>
</tr>
<tr>
<td>Interface Subsystem</td>
<td>1.5kW</td>
</tr>
<tr>
<td>Motor (1)</td>
<td>1.5kW</td>
</tr>
<tr>
<td>End Effector</td>
<td>15.0W</td>
</tr>
<tr>
<td>Instrument (1)</td>
<td>12.0W</td>
</tr>
<tr>
<td>Power Tool (1)</td>
<td>15.0W</td>
</tr>
</tbody>
</table>

System Total (Subsystem Peak Powers) 12.015kW
4. MECHANICAL STRUCTURE

As mentioned, the design of the robotic arm was divided into five distinct subsystems. The mechanical structure of the robotic arm comprises one of these five subsystems. The extent of the robotic arm contained on the mechanical structure ranges from the base, which is attached to the lunar vehicle, to the portion of the robotic arm where the structure and the wrist section meet.

4.1 Subsystem Functions and Requirements

In meeting the requirements and functions for the robotic arm as a whole, the mechanical structure is also required to meet additional stipulations assigned by the design team. As stated in the mission statement and requirements section, the robotic arm is to be able to lift a maximum mass of 50 kg while having a maximum mass of 100 kg. Working within these requirements the subsystem requirements for the mechanical structure are given as follows:

1. Given a total mass of 100 kg, the mechanical structure is not to exceed a maximum of 65% of this total, or 65 kg.
2. Materials selected for the mechanical structure are to possess a balance between material properties and light weight.
3. Structure is to be able to safely withstand applied stresses and allow for containment of cables, wires, controls, and sensors within the structure if needed.
4. Environmental effects on the lunar surface should not affect the material chosen for the structure.
5. Structure should exhibit means of providing protection from dust and other debris for joints, gears, and other mechanisms.

4.2 Mechanical Structure Design

In designing the mechanical structure the first order of business was to make a material selection. The materials utilized should represent a balance between mass density and various other material properties. The parameters involved in making a material choice are listed below.
1. Density - Low mass is of prime concern given the limited ability to transfer equipment and material into space and cost incurred in doing so.

2. Coefficient of thermal expansion - Given the large temperature range in which the arm is to operate, it is of great importance that the materials be capable of withstanding such a range with minimal effects.

3. Yield strength in shear, tension, and compression - The materials must exhibit strengths large enough to safely handle the stresses incurred during operation.

4. Radiation effects - The materials must be able to withstand large doses of radiation on a constant basis with no (or minute) effects to the structure of the materials.

5. Machining capability - The materials need to be machined at acceptable costs and tolerances.

Based on the above mentioned characteristics, the materials selection process was carried out and aluminum 2014-T6 was chosen as the material for the mechanical structure. Of the materials presently in use by the National Aeronautics and Space Administration, aluminum 2014-T6 was selected because it possesses high strengths at a relatively low mass density in comparison with the other materials. Table 4.1 represents the comparison between different materials considered.

Once the material was selected, the layout of the arm was the next step in the design process. After studying the various existing designs in the world of robotics, it was decided that either a three arm or two arm structure would be appropriate. In addition, a configuration utilizing more than three arms presents a much more difficult system to analyze, control, and design. Therefore, only two and three arm systems were considered. After further consideration a three arm configuration was chosen. The main reason a three arm configuration was chosen over a two arm configuration is that the three arm configuration provides a much greater operating envelope around the vehicle.
Table 4.1  Table of material properties for mechanical structure

<table>
<thead>
<tr>
<th>Material</th>
<th>Density kg/cu. m</th>
<th>Yield strength in tension MPa</th>
<th>Machining capability</th>
<th>Coefficient of thermal exp.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum 2014-T6</td>
<td>2800</td>
<td>410</td>
<td>good</td>
<td>23.0E-6</td>
</tr>
<tr>
<td>Aluminum 1100-H14</td>
<td>2710</td>
<td>95</td>
<td>good</td>
<td>23.6E-6</td>
</tr>
<tr>
<td>Stainless Steel 302</td>
<td>7920</td>
<td>520</td>
<td>fair-good</td>
<td>17.3E-6</td>
</tr>
<tr>
<td>Titanium</td>
<td>4460</td>
<td>825</td>
<td>fair</td>
<td>9.5E-6</td>
</tr>
<tr>
<td>Carbon Graphite</td>
<td>1666</td>
<td>833 - 1528</td>
<td>poor-fair</td>
<td></td>
</tr>
</tbody>
</table>

4.2.1 General Layout of Mechanical Structure

With the decision made to use a three arm configuration, the general layout of the mechanical structure was the next step to be completed in the design process. The base of the arm was decided to be mounted on the lunar vehicle on the lower portion of the vehicle. The specified height from the lunar surface to where the base is to be mounted on the vehicle is 1.586 m. The three portions of the mechanical structure shown in Figure 4.1 are called arm one, arm two, and arm three, with arm one being closest to the vehicle. At the base, arm one of the structure is to mount on the base via a revolute joint. Similarly, arm one and arm two, and arm two and arm three are to be connected together by use of revolute joints. In addition to these three revolute joints, a translational joint is to be incorporated into the design of arm one.

The design of the three arms was decided, after looking at alternative configurations, to be of a hollow cylindrical cross section. The basis for this decision was that a hollow cylindrical cross section provides even stress distribution throughout the arm walls and provides space inside the arms for storage of cables, wires, controls, and sensors throughout the length of the structure.
4.2.2 General Layout of Joints

With the layout of the mechanical structure known, it was possible to design the layouts of each joint. In designing the joint layouts many alternatives were considered. Taking into account the harsh lunar environment, many of the alternative solution principles were deemed unacceptable. For instance, because of the great temperature range present in the lunar environment the use of hydraulics was not feasible.

For the three revolute joints, the base joint, and the two joints connecting the three arms, the operating principle is to use gear trains at specified ratios and a motor mounted on top of the preceding arm driving the gears. For the joint at the base, the motor will be mounted to the base itself. For the translational joint, translational motion of the motor is changed into linear motion for extension of the arm using a power screw which is mounted on two sets of rolling bearings. One set of the bearings is mounted to the inside of the two connecting sections of arm one. Figure 4.2, Figure 4.3, and Figure 4.4 illustrate the layouts of the joint connecting arm one and arm two, the joint connecting arm two and arm three, and the translational joint of arm one respectively.

For all components of the joints such as the power screw, axles, gears, or any other component encountering large forces, the material chosen was stainless steel (302). In addition, power for the motors is to be supplied via a power supply located in the lunar vehicle itself and the necessary cables and wires are contained within the arms of the robotic structure. Holes are drilled in the arms at the appropriate locations to allow for the cables and wires to be connected to the motors.

Figure 4.4 Translational Joint
4.2.3 Additional Design Concerns

In addition to the actual design of the joints and arms, consideration must be given to protection of the joints, motors and other sensitive parts. For the gear trains at each joint there is to be a cover composed of thin sheet metal. This sheet metal is to be constructed of stainless steel (302) and can either be bolted or welded to the arms. Furthermore, a protective boot is to be placed at each joint. The boot will cover the connection between the two arms, the gear train (already covered by the sheet metal), and the motor at each joint. The reason for covering the gear trains with the sheet metal coverings is to insure the boot will not be worn down by the rotation of the gears. Table 4.2 illustrates the factors involved in comparing the many different rubber materials available for use in the protective boot.

Table 4.2 Table of material properties for protective boot.

<table>
<thead>
<tr>
<th>key:</th>
<th>E=excellent</th>
<th>G=good</th>
<th>F=fair</th>
<th>P=poor</th>
</tr>
</thead>
<tbody>
<tr>
<td>material</td>
<td>Natural rubber</td>
<td>GR-S</td>
<td>Neoprene</td>
<td>Nitrile rubbers</td>
</tr>
<tr>
<td>resistance to heat</td>
<td>G</td>
<td>F</td>
<td>G</td>
<td>E</td>
</tr>
<tr>
<td>resistance to cold</td>
<td>E</td>
<td>G</td>
<td>G</td>
<td>G</td>
</tr>
<tr>
<td>aging properties</td>
<td>E</td>
<td>E</td>
<td>G</td>
<td>G</td>
</tr>
<tr>
<td>resistance to sunlight</td>
<td>F</td>
<td>F</td>
<td>E</td>
<td>G</td>
</tr>
<tr>
<td>resilience</td>
<td>E</td>
<td>G</td>
<td>G</td>
<td>F</td>
</tr>
</tbody>
</table>

After comparison of the materials given in Table 4.2, it was decided that a butyl based rubber should be used for the protective boot on the robotic arm. Butyl based rubbers are synthetic elastomers and are made from petroleum raw materials. The important properties that butyl based rubbers exhibit are their excellent resistance to both cold and hot temperatures as well as their resistance to sunlight.
Furthermore, the question of reliability needed to be answered. As far as the mechanical structure was concerned, this identified a need to offer a manual backup to the motors at each joint in case of failure. This is accomplished by leaving small openings in the rubber boot and sheet metal casings at each joint. In case of failure the astronauts could then manually turn the motor by means of a wrench type device (similar to an allen wrench but larger) that would be inserted through the holes in the boot and casing and into a groove in the end of the axle protruding from the motor. Concerning possible contamination of the motor and joint area, the holes in the steel casings and rubber boots can have a lift-up type cover that the astronaut simply removes before insertion of the tool.
4.3 Calculations

The analysis for the mechanical structure of the robotic arm is shown below. For the analysis the worst case scenario was taken at all steps to ensure proper performance.

The requirements for the robotic arm stipulated a maximum mass of 100 kg.

For the mechanical structure of the robotic arm aluminum 2014-T6 (4.4% Cu) was chosen. The properties of this material are listed below.

Density: \( \rho = 2800 \text{ kg/m}^3 \)

Coefficient of thermal expansion: \( \alpha_T = 23.0 \times 10^{-6} \text{ K}^{-1} \)

Modulus of elasticity: \( E = 72 \times 10^9 \text{ Pa} \)

Modulus of rigidity: \( R = 27 \times 10^9 \text{ Pa} \)

Tension yield strength: \( S_{yt} = 410 \times 10^6 \text{ Pa} \)

Shear yield strength: \( S_{ys} = 220 \times 10^6 \text{ Pa} \)

Tension ultimate strength: \( S_{ut} = 480 \times 10^6 \text{ Pa} \)

Shear ultimate strength: \( S_{us} = 290 \times 10^6 \text{ Pa} \)

Also, there are properties of the moon that will affect the analysis.

Moon's acceleration due to gravity: \( g = 1.634 \text{ m/sec}^2 \)

Maximum average density of moon surface material: \( \rho_{\text{moon}} = 1.9 \text{ gram/cm}^3 \)

The assigned lengths of each member of the robotic arm.

Arm #1: arm #1 is composed of three sections; a, b, and c. Section a is a hollow cylindrical section beginning at the base. Section a then connects to section b which is the translational joint, composed of a power screw, for the robotic arm. On the other side of section b is section c, which is also a hollow cylinder. Section c ends at Joint A.

Entire length of arm #1: \( L_1 = 2 \text{ m} \)

Length of section a: \( L_a = 0.5 \text{ m} \)

Length of section b: \( L_b = 0.667 \text{ m} \)

Length of section c: \( L_c = 0.833 \text{ m} \)

Arm #2:
Entire length of arm #2: \( L_2 = 1.414 \text{ m} \)

Arm #3:
Entire length of arm #3: \( L_3 = 0.386 \text{ m} \)

Wrist: The wrist is the name given to the section at the end of arm number three where the three joints are located for movement of the interface and tool section. The wrist is divided into three equal parts, one for each of the joints.

Entire length of wrist: \( L_w = 0.2 \text{ m} \)

Interface:

Diameter of interface: \( d_4 = 15 \text{ cm} \)

Combined length of wrist and interface: \( L_4 = L_w + L_{\text{int}} \)

Tool: The length of the tool used is for the maximum size tool as stated by the Tool Subsystem.

Entire length of tool: \( L_5 = 0.5 \text{ m} \)
Assigned masses and weights for each member of the robotic arm.

For arms one, two, and three the masses are to be computed by static analysis.

Mass and weight of tool:

\[ \text{mass}_{\text{tool}} = 10 \text{ kg} \quad \text{mass}_{\text{tool}} \cdot g \cdot \text{w}_{\text{tool}} = 16344 \text{ N} \]

Mass and weight of maximum load:

\[ \text{mass}_{\text{load}} = 50 \text{ kg} \quad \text{mass}_{\text{load}} \cdot g \cdot \text{w}_{\text{load}} = 81722 \text{ N} \]

Combined mass and weight of tool and maximum load (for simpler calculations):

\[ \text{mass}_{5} = \text{mass}_{\text{tool}} + \text{mass}_{\text{load}} \quad \text{mass}_{5} = 60 \text{ kg} \]

\[ \text{w}_{5} = \text{w}_{\text{load}} - \text{w}_{\text{tool}} \quad \text{w}_{5} = 98066 \text{ N} \]

Mass and weight of interface:

\[ \text{mass}_{\text{int}} = 10 \text{ kg} \quad \text{mass}_{\text{int}} \cdot g \cdot \text{w}_{\text{int}} = 16344 \text{ N} \]

Mass and weight of wrist:

\[ \text{mass}_{\text{w}} = 10 \text{ kg} \quad \text{mass}_{\text{w}} \cdot g \cdot \text{w}_{\text{w}} = 16344 \text{ N} \]

Combined mass and weight of wrist and interface:

\[ \text{mass}_{4} = \text{mass}_{\text{int}} - \text{mass}_{\text{w}} \quad \text{mass}_{4} = 20 \text{ kg} \]

\[ \text{w}_{4} = \text{w}_{\text{w}} - \text{w}_{\text{int}} \quad \text{w}_{4} = 32689 \text{ N} \]

In addition to the masses and weights of the robotic arm members, a miscellaneous weight is added at each joint to account for the added weight of the motor, gears, casing, axle, etc. at each joint. This weight is given as a value larger than that actually expected in keeping with the worst case scenario.

\[ \text{mass}_{\text{misc}} = 5 \text{ kg} \quad \text{mass}_{\text{misc}} \cdot g \cdot \text{w}_{\text{misc}} = 8172 \text{ N} \]

Distances from base of robotic arm to centers of gravity of all members.

Since each member of the robotic arm is symmetric in the yz and xy planes, the only direction that is going to affect the center of gravity of each member is the distance in the x-direction from the base of the arm to the center of gravity of each member.

Center of gravity for each section of arm #1.

Section a:

\[ G_a = \frac{L_a}{2} \quad G_a = 0.25 \text{ m} \]

Section b:

\[ G_b = \frac{L_a + L_b}{2} \quad G_b = 0.834 \text{ m} \]

Section c:

\[ G_c = \frac{L_a - L_b + L_c}{2} \quad G_c = 1.584 \text{ m} \]

Center of gravity for arm #2:

\[ G_2 = \frac{L_1 - \frac{L_2}{2}}{2} \quad G_2 = 2.707 \text{ m} \]

Center of gravity for arm #3:

\[ G_3 = \frac{L_1 - L_2 - \frac{L_3}{2}}{2} \quad G_3 = 3.607 \text{ m} \]

Center of gravity for wrist (assuming constant cross section):

\[ G_w = \frac{L_1 + L_2 + L_3 + \frac{L_w}{2}}{2} \quad G_w = 3.9 \text{ m} \]

Center of gravity for interface (assuming constant cross section):

\[ G_{\text{int}} = \frac{L_1 + L_2 + L_3 + \frac{L_{\text{int}}}{2}}{2} \quad G_{\text{int}} = 4.1 \text{ m} \]
Center of gravity for wrist and interface (as a whole to simplify calculations):

\[ G_4 = L_1 - L_2 + L_3 + \frac{L_4}{2} \]

Center of gravity for tool and maximum load:

\[ G_5 = L_1 - L_2 - L_3 - L_4 - \frac{L_5}{2} \]

**Static Analysis**

The static analysis is intended to give the inner and outer diameters of each of the three hollow cylinders that make up the mechanical structure of the arm. From these results, the weights and moments of inertia of each member will be found and the dynamic analysis will then be performed.

By stating a value for either the inner or outer diameter, the remaining diameter can be determined by using an iterative loop until the desired factor of safety is reached. The factor of safety is a function of the maximum stress at a cross section of the arm, and the maximum stress is a function of the inner and outer diameters of the cross section. Therefore, by stating one diameter the remaining diameter can be iteratively guessed until the desired factor of safety is reached.

**Static analysis for arm #3.**

To determine the inner diameter and outer diameter of arm #3

Free body diagram of tool, interface, and wrist sections up to Joint C.

At Joint C:

\[ R_{yc} = w_5 + w_{int} + \frac{1}{3} \cdot w_w + 2 \cdot w_{misc} \]

\[ M_c = \left( \frac{1}{3} \cdot L_w + \frac{L_{int}}{2} \right) \cdot w_5 - \left( \frac{1}{3} \cdot L_w + \frac{L_{int}}{2} \right) \cdot w_{int} - w_{misc} + \left( \frac{1}{3} \cdot L_w + \frac{1}{6} \cdot w_w + w_{misc} \right) \]

Free body diagram of arm #3 and wrist section up to Joint C.
At Joint C:
\[ R_{yc} = w_5 - w_{int} + \frac{1}{3} w_w - 2 w_{misc} \]
\[ R_{yc} = 136.2 \cdot N \]
\[ M_c = \frac{1}{3} L w - L_{int} + \frac{1}{2} L_5 \cdot w_5 - \frac{1}{3} L w - \frac{L_{int}}{2} \cdot w_{int} - \frac{1}{3} L w - \frac{1}{6} w_w - w_{misc} \]
\[ M_c = 55.48 \cdot N \cdot m \]

At Joint B:
\[ R_{yb} = R_{yc} - \frac{2}{3} w_w + 2 w_{misc} - w_3 \]
\[ M_b = M_c + L_3 + \frac{2}{3} L w + \frac{2}{3} w_w \cdot 2 w_{misc} - \frac{L_5}{2} w_3 \]
\[ T = \frac{1}{2} L w \cdot w_w - L w \cdot w_{int} - L w \cdot L_{int} - \frac{1}{2} L_5 w_5 \]

Cross section B-B of arm #3.

![Cross section B-B of arm #3](image)

d_o

d_i

Arm #3 has the cross section of a hollow cylinder. By giving the desired outer diameter, the inner diameter can be determined at a desired factor of safety. The iterative loop used for this calculation is given below.

The desired outer diameter of arm #3 is: \[ d_3_o = 100 \cdot mm \]

BEGIN iterative loop

Guess for the inner diameter of arm #3: \[ d_3_i = 95 \cdot mm \]

Area at predetermined cross section of arm:
\[ A_3 = \frac{\pi}{4} \left( d_3_o^2 - d_3_i^2 \right) \]
\[ A_3 = 7.658 \cdot 10^{-4} \cdot m^2 \]

Centroidal moment of inertia for hollow cross section at specified inner and outer diameters:
\[ I_{z3} = \frac{\pi}{64} \left( d_3_o^4 - d_3_i^4 \right) \]
\[ I_{z3} = 9.105 \cdot 10^{-7} \cdot m^4 \]

Polar moment of inertia:
\[ I_{y3} = I_{z3} \quad J_3 = I_{y3} - I_{z3} \]
\[ J_3 = 1.821 \cdot 10^{-6} \cdot m^4 \]
Distance from Neutral Axis to point of maximum tension (top surface) and compression (bottom surface) are the same: \[ \frac{d_3}{c_3} = \frac{d_3_o}{c_3} \]

Maximum magnitude of tensile and compressive stress (when arm is fully extended):

\[ \sigma_3 = \frac{M_b \cdot c_3}{I_z_3} \]
\[ \sigma_3 = 3.738 \cdot 10^6 \cdot \text{Pa} \]

Maximum shearing stress:

\[ Q = \frac{A_3 \cdot 2 \cdot d_3_o^2 - d_3_i^2}{2 \cdot 3} \cdot \frac{d_3_o - d_3_i}{\pi} \cdot \tau \cdot \frac{d_3_o - d_3_i}{\pi} \cdot \frac{I_z_3}{\tau} \]
\[ \tau_3 = \frac{R_y_b \cdot Q}{I_z_3 \cdot \tau_3} \]
\[ \tau_3 = 4.302 \cdot 10^5 \cdot \text{Pa} \]

Maximum torsional stress (when joint W1 is rotated so that rest of arm past Joint C is positioned at a 90 degree angle with respect to axis of the robotic arm resulting in a torsional stress):

\[ \tau_{\text{twist}} = \frac{T}{J_3} \]
\[ \tau_{\text{twist}} = 1.975 \cdot 10^6 \cdot \text{Pa} \]

The maximum total stress in preselected cross section of arm #3 is:

\[ \tau_3^{\text{max}} = \sqrt{\sigma_3^2 + \tau_3^{\text{twist}}\cdot \tau_3^{\text{twist}} + \tau_3^{\text{twist}}^2} \]
\[ \tau_3^{\text{max}} = 2.753 \cdot 10^6 \cdot \text{Pa} \]

The resulting factor of safety is:

\[ n = \frac{0.4 \cdot \text{S}_{\text{ys}}}{\tau_3^{\text{max}}} \]
\[ n = 32 \]

**END of Iterative loop**

When the resulting F.S. equals the desired F.S. then the guess for the needed diameter is appropriate. Therefore the inner diameter for arm #3 is:

\[ d_3_i = 95 \cdot \text{mm} \]

The inner and outer diameters of arm #3 are now known, therefore the volume, mass, weight, and wall thickness of arm #3 can be determined.

**Volume of arm #3**

\[ V_3 = A_3 \cdot L_3 \]
\[ V_3 = 295.6 \cdot \text{cm}^3 \]

**Mass of arm #3**

\[ \text{mass}_3 = V_3 \cdot \rho \]
\[ \text{mass}_3 = 0.828 \cdot \text{kg} \]

**Weight of arm #3**

\[ w_3 = \text{mass}_3 \cdot g \]
\[ w_3 = 1.353 \cdot \text{N} \]

**Wall thickness of arm #3**

\[ t_3 = \frac{d_3_o - d_3_i}{2} \]
\[ t_3 = 2.5 \cdot \text{mm} \]

**Static analysis for arm #2.** To determine the inner diameter and outer diameter of arm #2

**Free body diagram of arm #2.**
At Joint B:

\[ R_{yb} = R_{yc} \cdot \frac{2}{3} w_w - 2 w_{misc} - w_3 \]

\[ R_{yb} = 164.8 \text{ N} \]

\[ M_b = M_c - \frac{1}{2} \frac{2}{3} \frac{L}{L_3} w_w + \frac{2}{3} w_w + 2 w_{misc} = \frac{L_3}{2} w_3 \]

\[ M_b = 68.07 \text{ N m} \]

\[ T = \frac{1}{2} L w w_w - L w - L_{int} w_{int} - \frac{1}{2} L_5 \cdot w_5 \]

At Joint A:

\[ R_{ya} = R_{yb} \cdot w_{misc} - w_2 \]

\[ M_a = \frac{L^2}{2} w_2 - \frac{L^2}{2} R_{yb} w_{misc} - M_b \]

\[ T = \frac{1}{2} L w w_w - L w - L_{int} w_{int} - L w - L_{int} - \frac{1}{2} L_5 \cdot w_5 \]

Cross section A-A of arm #2.

Arm #2 has the cross section of a hollow cylinder. By giving the desired outer diameter, the inner diameter can be determined at a desired factor of safety. The iterative loop used for this calculation is given below.

The desired inner diameter of arm #2 is:

\[ d_2 = 105 \text{ mm} \]

**BEGIN iterative loop**

Guess for the outer diameter of arm #2:

\[ d_2 = 113 \text{ mm} \]

Area at predetermined cross section of arm:

\[ A_2 = \frac{3}{4} \pi (d_2^2 - d_1^2) \quad A_2 = 0.001 \text{ m}^2 \]

Centroidal moment of inertia for hollow cross section at specified inner and outer diameters:

\[ I_{z_2} = \frac{3}{64} \pi [d_2^4 - d_1^4] \quad I_{z_2} = 2.037 \times 10^{-6} \text{ m}^4 \]

Polar moment of inertia:

\[ I_{y_2} = I_{z_2} \quad J_2 = I_{y_2} + I_{z_2} \quad J_2 = 4.074 \times 10^{-6} \text{ m}^4 \]

Distance from Neutral Axis to point of maximum tension (top surface) and compression (bottom surface) are the same:

\[ c_2 = \frac{d_2}{2} \]
Maximum magnitude of tensile and compressive stress:

$$\sigma_2 = \frac{M_2 \cdot c_2}{I\ell_2}$$

$$\sigma_2 = 8.846 \times 10^6 \text{ Pa}$$

Maximum shearing stress:

$$\tau_2 = \frac{R_{ya} \cdot Q_2}{I_\ell_2 \cdot t_2}$$

$$\tau_2 = 2.653 \times 10^5 \text{ Pa}$$

Maximum torsional stress (when Joint W1 is rotated so that rest of arm past Joint C is positioned at a 90 degree angle with respect to axis of the robotic arm resulting in a torsional stress):

$$\tau_{\text{twist}_2} = \frac{T \cdot c_2}{J_2}$$

$$\tau_{\text{twist}_2} = 9.974 \times 10^5 \text{ Pa}$$

The maximum total stress in preselected cross section of arm #2 is:

$$\tau_{\text{max}}^2 = \sigma_2^2 + \tau_2^2 + \left(\tau_{\text{twist}_2}\right)^2$$

$$\tau_{\text{max}}^2 = 4.542 \times 10^6 \text{ Pa}$$

The resulting factor of safety is:

$$n = \frac{0.4 S_{ys}}{\tau_{\text{max}}}$$

$$n = 19.4$$

**END of iterative loop**

When the resulting F.S. equals the desired F.S. then the guess for the needed diameter is appropriate. Therefore the outer diameter for arm #2 is:

$$d_{2o} = 113 \text{ mm}$$

The inner and outer diameters of arm #2 are now known, therefore the volume, mass, weight, and wall thickness of arm #2 can be determined.

**Volume of arm #2**

$$V_2 = A_2 \cdot L_2$$

$$V_2 = 1.937 \times 10^3 \text{ cm}^3$$

**Mass of arm #2**

$$\text{mass}_2 = V_2 \cdot \rho$$

$$\text{mass}_2 = 5.423 \text{ kg}$$

**Weight of arm #2**

$$w_2 = \text{mass}_2 \cdot g$$

$$w_2 = 8.864 \text{ N}$$

**Wall thickness of arm #2**

$$t_2 = \frac{d_{2o} - d_{2i}}{2}$$

$$t_2 = 4 \text{ mm}$$

**Static analysis of arm #1.**

Arm #1 is composed of three parts; sections a, b, and c. Section c is of the form of a hollow cylinder as is arm #2 and arm #3 and connects with arm #2 at Joint A on one end and at section b of arm #1 at the other end. Section a serves as a structural member as do arms #1 and #2. On the other hand, section b of arm #1 is composed of one main power screw and one guide/support bar and serves as a translational joint for greater operating range of the robotic arm and must be able to withstand the forces and loads applied to it. Section b connects on one end to section c and on the other end to section a. Section a is of the form of a hollow cylinder as is section c. Like section c, section a serves as a structural member and is connected at one end to section b and at the other end to the support base of the robotic arm via the Base Joint. The inner diameters of section a and c are the same in order to allow the translational joint supports to be attached to the inside of the cylinder walls. However, the outside diameters of sections a and c will be different in order to safely withstand the different forces applied to each section.
Analysis of section c.
Free body diagram of section c of arm #1

At Joint A:

\[ R_{ya} = R_{yb} + w_{misc} + w_2 \]
\[ R_{ya} = 181.8 \text{ N} \]

\[ M_a = \frac{L_2}{2} w_2 - L_2 \cdot R_{yb} - w_{misc} + M_b \]
\[ M_a = 318.9 \text{ N} \cdot \text{m} \]

\[ T = \frac{1}{2} L_w \cdot w_w - L_w \cdot \text{L_int} \cdot w_{int} + \left( L_w - \text{L_int} \right) \cdot \frac{1}{2} \cdot L_5 \cdot w_5 \]

at cross section M-M of section c of arm #1:

\[ R_{section c} = w_c + w_{misc} - R_{ya} \]
\[ M_{section c} = \frac{L_c}{2} \cdot w_c - L_c \cdot \left( w_{misc} + R_{ya} \right) - M_a \]
\[ T = \frac{1}{2} L_w \cdot w_w - \left( L_w + \text{L_int} \right) \cdot w_{int} + \left( L_w - \text{L_int} \right) \cdot \frac{1}{2} \cdot L_5 \cdot w_5 \]

Cross section M-M of section c of arm #1

The desired inner diameter of section c of arm #1 is: \[ d_{c1} = 118 \text{ mm} \]

BEGIN iterative loop

Guess for the outer diameter of section c of arm #1: \[ d_{c0} = 132 \text{ mm} \]

Area at predetermined cross section of arm:

\[ A_c = \frac{\pi}{4} \cdot (d_{c0}^2 - d_{c1}^2) \]
\[ A_c = 0.003 \text{ m}^2 \]
Centroidal moment of inertia for hollow cross section at specified inner and outer diameters:

\[ I_z = \frac{M_{\text{section}}}{64} \left( d_c^4 - d_i^4 \right), \quad I_z = 5.386 \cdot 10^{-6} \cdot \text{m}^4 \]

Polar moment of inertia:

\[ I_y = I_z, \quad J_c = I_y + I_z, \quad J_c = 1.077 \cdot 10^{-5} \cdot \text{m}^4 \]

Distance from Neutral Axis to point of maximum tension (top surface) and compression (bottom surface) are the same:

\[ c_c = \frac{d_c}{2} \]

Maximum magnitude of tensile and compressive stress:

\[ \sigma_c = \frac{M_{\text{section}}}{I_z} \cdot c_c, \quad \sigma_c = 5.901 \cdot 10^6 \cdot \text{Pa} \]

Maximum shearing stress:

\[ Q_c = A_c \cdot 2 \left( \frac{d_c^2 + d_i \cdot d_c - d_i^2}{2^3} \right), \quad \tau_c = \frac{d_c - d_i}{\pi} \cdot \pi \cdot \pi \]

\[ \tau_c = \frac{R_{\text{section}} \cdot Q_c}{I_z \cdot \pi} \cdot \tau_c = 1.456 \cdot 10^5 \cdot \text{Pa} \]

Maximum torsional stress (when Joint W1 is rotated so that rest of arm past Joint C is positioned at a 90 degree angle with respect to axis of the robotic arm resulting in a torsional stress):

\[ \tau_{\text{twist}} = \frac{T_{\text{twist}}}{J_c} \cdot \tau_{\text{twist}} = 4.406 \cdot 10^5 \cdot \text{length}^{-3} \cdot \text{N m} \]

The maximum total stress in preselected cross section of arm #1 is:

\[ \tau_{\text{c max}} = \sqrt{\left( \frac{\sigma_c}{2} \right)^2 + \left( \tau_c \right)^2 - \tau_{\text{twist}}^2}, \quad \tau_{\text{c max}} = 2.987 \cdot 10^6 \cdot \text{Pa} \]

The resulting factor of safety is:

\[ n = \frac{0.4 \cdot S_{\text{ys}}}{\tau_{\text{c max}}}, \quad n = 29.5 \]

**END of iterative loop**

When the resulting F.S. equals the desired F.S. then the guess for the needed diameter is appropriate. Therefore the outer diameter for section c of arm #1 is:

\[ d_c = 132 \cdot \text{mm} \]

The inner and outer diameter for section c of arm #1 is now known, therefore the volume, mass, weight, and wall thickness of section c of arm #1 can be determined.

**Volume of section c of arm #1**

\[ V_c = A_c \cdot L_c \quad V_c = 2.29 \cdot 10^3 \cdot \text{cm}^3 \]

**Mass of section c of arm #1**

\[ \text{mass}_c = V_c \cdot \rho \quad \text{mass}_c = 6.412 \cdot \text{kg} \]

**Weight of section c of arm #1**

\[ w_c = \text{mass}_c \cdot g \quad w_c = 10.479 \cdot \text{N} \]

**Wall thickness of section c of arm #1**

\[ t_c = \frac{d_c - d_i}{2} \quad t_c = 7 \cdot \text{mm} \]
**Analysis of section b.**

Section b was analyzed in another document. The results gathered in this document were delivered to the group member analyzing section b. In turn, the results from section b were delivered back to the group member performing this analysis. The analysis of section a of arm #1 was then performed and the dynamic analysis was then performed.

For the analysis of section b the properties of the materials used must be defined.

Material for power screw: cold rolled stainless steel (302)

Density of steel: \( \rho_{\text{steel}} = 7920 \frac{\text{kg}}{\text{m}^3} \)

Properties of section b:

- Diameters of screw: \( d_{\text{si}} = 30 \text{ mm} \) \( d_{\text{so}} = 80 \text{ mm} \)

Volume of power screw:

\[
V_{\text{screw}} = \frac{\pi}{4} d_{\text{so}}^2 - d_{\text{si}}^2
\]

\[
\text{mass}_{\text{screw}} = V_{\text{screw}} \rho_{\text{steel}} \quad \text{mass}_{\text{screw}} = 22.819 \cdot \text{kg}
\]

\[
\text{w}_{\text{screw}} = \text{mass}_{\text{screw}} g \quad \text{w}_{\text{screw}} = 37.297 \cdot \text{N}
\]

Volume of section b:

\[
V_b = V_{\text{screw}}
\]

\[
\text{mass}_b = \text{mass}_{\text{screw}} \quad \text{mass}_b = 22.819 \cdot \text{kg}
\]

\[
\text{w}_b = \text{w}_{\text{screw}} \quad \text{w}_b = 37.297 \cdot \text{N}
\]

**Analysis of section a.**

With the data for section b of arm #1 given by section b analyzers, the analysis of section a was carried out.

**Free body diagram of arm #1. All three sections shown.**

\[
M_{\text{base}}
\]

\[
R_{\text{base}} \quad w_a \quad w_b \quad w_c \quad R_y A
\]

At Joint A

\[
R_{ya} = R_{yb} - w_{\text{misc}} - w_2
\]

\[
M_a = \frac{L_2}{2} w_2 + L_2 R_{yb} - w_{\text{misc}} - M_b
\]

\[
T = \frac{1}{2} L_w w_w - L_w L_{\text{int}} - \frac{1}{2} L_w \frac{1}{2} L_5 w_5
\]
At base joint

\[ R_{\text{base}} = R_y + w_a + w_b + w_c \]

\[ M_{\text{base}} = L_1 R_y + \frac{1}{2} L_c w_c - \frac{1}{2} L_b w_b - \frac{1}{2} L_a w_a \]

\[ T = \frac{3}{2} L w \cdot w_w - \frac{3}{2} L w \cdot w_{\text{int}} - \frac{3}{2} L w \cdot w_{\text{int}} + \frac{1}{2} L_w L_s w_s \]

The desired inner diameter of section a of arm #1 is: \( d_{a_i} = d_c, \quad d_{a_i} = 118 \text{ mm} \)

**BEGIN iterative loop**

Guess for the outer diameter of section a of arm #1: \( d_{a_o} = 135 \text{ mm} \)

Area at predetermined cross section of arm:

\[ A_a = \frac{\pi}{4} \left( d_{a_o}^2 - d_{a_i}^2 \right) \quad A_a = 0.003 \text{ m}^2 \]

Centroidal moment of inertia for hollow cross section at specified inner and outer diameters:

\[ I_z = \frac{\pi}{64} \left( d_{a_o}^4 - d_{a_i}^4 \right) \quad I_z = 6.787 \times 10^{-6} \text{ m}^4 \]

Distance from Neutral Axis to point of maximum tension (top surface) and compression (bottom surface) are the same:

\[ c_a = \frac{d_{a_o}}{2} \]

Polar moment of inertia:

\[ J_a = I_z = J_a = \frac{1}{2} \left( d_{a_o}^2 - d_{a_i}^2 \right) \frac{d_{a_o} + d_{a_i}}{2} \]

Maximum magnitude of tensile and compressive stress:

\[ \sigma_a = \frac{M_{\text{base}} c_a}{I_z} \quad \sigma_a = 4.11 \times 10^6 \text{ Pa} \]

Maximum shearing stress:

\[ Q_a = \frac{A_a}{2} \left[ \left( \frac{d_{a_o}^2}{3} \right) \left( \frac{d_{a_o} + d_{a_i}}{2} \right) \right] \tau_a - d_{a_o} d_{a_i} \]

\[ \tau_a = \frac{R_{\text{base}} Q_a}{I_z \tau_a} \quad \tau_a = 1.401 \times 10^5 \text{ Pa} \]

Maximum torsional stress (when Joint W1 is rotated so that rest of arm past Joint C is positioned at a 90 degree angle with respect to axis of the robotic arm resulting in a torsional stress):

\[ \tau_{\text{twista}} = \frac{T \cdot c_c}{J_c} \quad \tau_{\text{twista}} = 4.406 \times 10^5 \text{ Pa} \]

The maximum total stress in preselected cross section of arm #1 is:

\[ \tau_{\text{max}}^a = \sqrt{\frac{\sigma_a^2}{2} + (\tau_a)^2 + \tau_{\text{twista}}^2} \tau_{\text{max}}^a = 2.106 \times 10^6 \text{ Pa} \]

The resulting factor of safety is:

\[ n = \frac{0.4 S_{ys}}{\tau_{\text{max}}} \quad n = 42 \]

**END of iterative loop**
When the resulting F.S. equals the desired F.S. then the guess for the needed diameter is appropriate. Therefore the outer diameter for section a of arm #1 is \( d_{a_o} = 135 \cdot \text{mm} \).

The inner and outer diameter for section a of arm #1 is now known, therefore the volume, mass, weight, and wall thickness of section a of arm #1 can be determined.

Volume of section a of arm #1

\[
V_a = A_a \cdot L_a \quad V_a = 1.689 \cdot 10^3 \cdot \text{cm}^3
\]

Mass of section a of arm #1

\[
\text{mass}_a = \frac{V_a \cdot \rho}{\text{mass}_a} \quad \text{mass}_a = 4.729 \cdot \text{kg}
\]

Weight of section a of arm #1

\[
w_a = \text{mass}_a \cdot g \quad w_a = 7.73 \cdot \text{N}
\]

Wall thickness of section a of arm #1

\[
t_a = \frac{d_{a_o} - d_{a_i}}{2} \quad t_a = 8.5 \cdot \text{mm}
\]

**Arm #1 as a whole.**

Mass of arm #1

\[
\text{mass}_1 = \text{mass}_a + \text{mass}_b + \text{mass}_c
\]

\[
\text{mass}_1 = 33.96 \cdot \text{kg}
\]

Weight of arm #1

\[
w_1 = w_a - w_b - w_c
\]

\[
w_1 = 55.506 \cdot \text{N}
\]

With the static analysis of each arm complete, the mass and weight of the entire structure can be determined.

**Mass of entire structure (including maximum load).**

\[
\text{mass}_{\text{total}} = \sum_{i=1}^{5} \text{mass}_i
\]

\[
\text{mass}_{\text{total}} = 120.2 \cdot \text{kg}
\]

**Weight of entire structure (minus maximum load).**

\[
w_{\text{total}} = \text{mass}_{\text{total}} \cdot g
\]

\[
w_{\text{total}} = 196.477 \cdot \text{N}
\]

**Center of Gravity Calculations.**

**Determination of center of gravity for arm #1 as a whole.**

<table>
<thead>
<tr>
<th>Component</th>
<th>Volume ( V )</th>
<th>Length ( L )</th>
<th>( L \times V )</th>
</tr>
</thead>
<tbody>
<tr>
<td>sect. a</td>
<td>( V_a = 0.002 \cdot \text{m}^3 )</td>
<td>( G_a = 0.25 \cdot \text{m} )</td>
<td>( V_a \cdot G_a = 4.222 \cdot 10^{-4} \cdot \text{m}^4 )</td>
</tr>
<tr>
<td>sect. b</td>
<td>( V_b = 0.003 \cdot \text{m}^3 )</td>
<td>( G_b = 0.834 \cdot \text{m} )</td>
<td>( V_b \cdot G_b = 0.002 \cdot \text{m}^4 )</td>
</tr>
<tr>
<td>sect. c</td>
<td>( V_c = 0.002 \cdot \text{m}^3 )</td>
<td>( G_c = 1.584 \cdot \text{m} )</td>
<td>( V_c \cdot G_c = 0.004 \cdot \text{m}^4 )</td>
</tr>
</tbody>
</table>

\[
V_1 = V_a + V_b + V_c
\]

\[
\text{sumVG} = V_a \cdot G_a + V_b \cdot G_b + V_c \cdot G_c
\]

\[
V_1 = 0.007 \cdot \text{m}^3
\]

\[
\text{sumVG} = 0.006 \cdot \text{m}^4
\]

**Location of center of gravity for arm #1 as a whole.**

\[
G_1 = \frac{\text{sumVG}}{V_1}
\]

\[
G_1 = 0.94 \cdot \text{m}
\]
**Determination of the center of gravity from the base for the mechanical structure as a whole.** For use in calculating the torque at the base of the structure.

In order to perform a dynamic analysis on the robotic arm it is necessary to calculate the center of gravity for each member. To do this the geometry of each member needs to be known. For the maximum load, tool and wrist/interface sections, a simple geometric shape was used to simplify the calculations. All members lie in the same horizontal plane (their geometric axis is coincident with the x-axis) and therefore the calculation of each center of gravity becomes a function of only one length (x direction).

For the maximum load a volume of a square was used as the geometric model, while for the tool a rectangle was the geometric shape used to calculate its volume. A rectangle was used for the tool because the heaviest tool designed was that of a thin rectangular shovel. For the wrist and interface sections, a volume of a solid cylinder was used. Although neither the wrist or the interface is solid, the solid cylinder was used to model these members in an effort to keep with designing for a worst case scenario.

In calculating the volume of the maximum load the density of moon regolith was used. In our research we discovered that the density of moon regolith ranged from 1.4 to 1.9 grams per cubic centimeters. In keeping with the worst case design scenario the maximum density of 1.9 grams per cubic centimeter was used.

<table>
<thead>
<tr>
<th>component</th>
<th>volume</th>
<th>length</th>
<th>length x volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>arm #1</td>
<td>$V_1 = 0.007 \cdot m^3$</td>
<td>$G_1 = 0.94 \cdot m$</td>
<td>$V_1 \cdot G_1 = 0.006 \cdot m^4$</td>
</tr>
<tr>
<td>arm #2</td>
<td>$V_2 = 0.002 \cdot m^3$</td>
<td>$G_2 = 2.707 \cdot m$</td>
<td>$V_2 \cdot G_2 = 0.005 \cdot m^4$</td>
</tr>
<tr>
<td>arm #3</td>
<td>$V_3 = 2.956 \cdot 10^{-4} \cdot m^3$</td>
<td>$G_3 = 3.607 \cdot m$</td>
<td>$V_3 \cdot G_3 = 0.001 \cdot m^4$</td>
</tr>
<tr>
<td>wrist/int.</td>
<td>$\frac{\pi}{4} L_{w} (d_{3})^{2} + \frac{\pi}{4} L_{i n t} (d_{4})^{2}$</td>
<td>$V_4 = 0.005 \cdot m^3$</td>
<td>$V_4 \cdot G_4 = 0.02 \cdot m^4$</td>
</tr>
<tr>
<td>tool/load</td>
<td>$\frac{\text{mass tool}}{\rho} + \frac{\text{mass load}}{\rho_{\text{moon}}}$</td>
<td>$V_5 = 0.03 \cdot m^3$</td>
<td>$V_5 \cdot G_5 = 0.133 \cdot m^4$</td>
</tr>
</tbody>
</table>

\[
\sum_{i=1}^{5} V_i = 0.044 \cdot m^3 \\
\sum_{i} V_i G_i = 0.166 \cdot m^4
\]

Location of center of gravity of entire mechanical structure from the base.

\[
G_{\text{base}} = \frac{\sum_{i} V_i G_i}{\sum_{i} V_i} \\
G_{\text{base}} = 3.769 \cdot m
\]
Determination of center of gravity for mechanical structure excluding arm #1. For use in calculating torque at Joint A.

<table>
<thead>
<tr>
<th>component</th>
<th>volume</th>
<th>length</th>
<th>length x volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>arm #2</td>
<td>( V_2 = 0.002 \cdot m^3 )</td>
<td>( L_2 = 0.707 \cdot m )</td>
<td>( V_2 \cdot \frac{L_2}{2} = 0.001 \cdot m^4 )</td>
</tr>
<tr>
<td>arm #3</td>
<td>( V_3 = 2.956 \cdot 10^{-4} \cdot m^3 )</td>
<td>( L_2 = \frac{L_3}{2} = 1.607 \cdot m )</td>
<td>( V_3 \cdot \frac{L_3}{2} = 4.75 \cdot 10^{-4} \cdot m^4 )</td>
</tr>
<tr>
<td>wrist/int.</td>
<td>( V_4 = \frac{\pi}{4} \cdot L \cdot w \cdot d_0 + \frac{\pi}{4} \cdot L \cdot \text{int} \cdot d_4 )</td>
<td>( L_2 = \frac{L_4}{2} = 0.002 \cdot m )</td>
<td>( V_4 \cdot \frac{L_4}{2} = 0.001 \cdot m^4 )</td>
</tr>
<tr>
<td>tool/load</td>
<td>( V_5 = \frac{\text{mass tool}}{\rho} - \frac{\text{mass load}}{\rho_{moon}} )</td>
<td>( G_4 - L_1 = 2 \cdot m )</td>
<td>( V_5 \cdot G_4 = L_1 = 0.01 \cdot m^4 )</td>
</tr>
</tbody>
</table>

\[
\sum_{i=2}^{5} V_i = 0.044 \cdot m^3 \quad \sum_{i=2}^{5} V_i \cdot G_i = L_1 = 0.085 \cdot m^4
\]

Location of center of gravity of maximum load, tool, interface, wrist, arm #3, and arm #2 from Joint A.

\[
G_{\text{Joint A}} = \frac{\sum_{i=2}^{5} V_i \cdot (G_i - L_1)}{\sum_{i=2}^{5} V_i} = 2.291 \cdot m
\]

Determination of center of gravity for mechanical structure excluding arms #1 and #2. For use in calculating torque at Joint B.

<table>
<thead>
<tr>
<th>component</th>
<th>volume</th>
<th>length</th>
<th>length x volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>arm #3</td>
<td>( V_3 = 2.96 \cdot 10^{-4} \cdot m^3 )</td>
<td>( L_3 = 0.193 \cdot m )</td>
<td>( V_3 \cdot \frac{L_3}{2} = 5.7 \cdot 10^{-5} \cdot m^4 )</td>
</tr>
<tr>
<td>wrist/int.</td>
<td>( V_4 = \frac{\pi}{4} \cdot L \cdot w \cdot d_0 + \frac{\pi}{4} \cdot L \cdot \text{int} \cdot d_4 )</td>
<td>( L_2 = \frac{L_4}{2} = 0.586 \cdot m )</td>
<td>( V_4 \cdot \frac{L_4}{2} = 0.003 \cdot m^4 )</td>
</tr>
<tr>
<td>tool/load</td>
<td>( V_5 = \frac{\text{mass tool}}{\rho} - \frac{\text{mass load}}{\rho_{moon}} )</td>
<td>( L_3 - L_4 = \frac{L_5}{2} = 1.036 \cdot m )</td>
<td>( V_5 \cdot L_3 = L_4 = 0.031 \cdot m^4 )</td>
</tr>
</tbody>
</table>

\[
\sum_{i=3}^{5} V_i = 0.037 \cdot m^3 \quad V_3 \cdot L_3 + V_4 \cdot L_3 = \frac{L_4}{2} \cdot V_5 \cdot L_3 = L_4 = \frac{L_5}{2} \cdot 0.034 \cdot m^4
\]

26
Location of center of gravity of maximum load, tool, interface, wrist, and arm #3 from Joint B.

\[
G_{\text{jointB}} = \frac{V_1 \cdot L_1 - V_4 \cdot L_4 + \frac{L_4}{2} + V_5 \cdot L_4 + \frac{L_5}{2}}{\sum V_1} \cdot \frac{G_{\text{jointB}}}{m} = 0.965 \cdot m
\]

Determination of center of gravity for mechanical structure excluding arms #1, #2, and #3.

For use in calculating torque at Joint W1.

<table>
<thead>
<tr>
<th>component</th>
<th>volume ( V )</th>
<th>length ( L )</th>
<th>length x volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>wrist</td>
<td>( \frac{\pi}{4} \cdot L_w \cdot d_w^2 ) ( \frac{2}{3} )</td>
<td>( L_w )</td>
<td>( \frac{L_w}{3} )</td>
</tr>
<tr>
<td></td>
<td>( V_w = 0.002 \cdot m^3 )</td>
<td>( L_w = 0.1 \cdot m )</td>
<td>( V_w \cdot \frac{L_w}{3} = 3.142 \cdot 10^{-4} \cdot m^4 )</td>
</tr>
<tr>
<td>interface</td>
<td>( \frac{\pi}{4} \cdot L_{\text{int}} \cdot d_{\text{int}}^2 )</td>
<td>( L_{\text{int}} )</td>
<td>( \frac{L_{\text{int}}}{2} )</td>
</tr>
<tr>
<td></td>
<td>( V_{\text{int}} = 0.004 \cdot m^3 )</td>
<td>( L_{\text{int}} = 0.3 \cdot m )</td>
<td>( V_{\text{int}} \cdot \frac{L_{\text{int}}}{2} = 0.001 \cdot m^4 )</td>
</tr>
<tr>
<td>tool/load</td>
<td>( V_5 = \frac{\text{mass tool}}{\rho} \cdot \frac{\text{mass load}}{\rho_{\text{moon}}} )</td>
<td>( L_4 )</td>
<td>( \frac{L_4}{2} )</td>
</tr>
<tr>
<td></td>
<td>( V_5 = 0.03 \cdot m^3 )</td>
<td>( L_4 )</td>
<td>( \frac{L_4}{2} )</td>
</tr>
</tbody>
</table>

sum\( VW1 = V_w + V_{\text{int}} + V_5 \)

sum\( VGW1 = V_w \cdot \frac{L_w}{3} + V_{\text{int}} \cdot \frac{L_{\text{int}}}{2} + V_5 \cdot \frac{L_4}{2} \)

sum\( VW1 = 0.035 \cdot m^3 \)

sum\( VGW1 = 0.021 \cdot m^4 \)

Location of center of gravity of maximum load, tool, interface, wrist from Joint W1.

\[
G_{\text{jointW1}} = \frac{\text{sum\( VGW1 \)}}{\text{sum\( VW1 \)}} \cdot G_{\text{jointW1}} = 0.594 \cdot m
\]

Determination of center of gravity for mechanical structure excluding arms #1, #2, #3, and joint W1. For use in calculating torque at Joint C.

<table>
<thead>
<tr>
<th>component</th>
<th>volume ( V )</th>
<th>length ( L )</th>
<th>length x volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>wrist</td>
<td>( \frac{\pi}{4} \cdot L_w \cdot d_w^2 ) ( \frac{2}{3} )</td>
<td>( L_w )</td>
<td>( \frac{L_w}{3} )</td>
</tr>
<tr>
<td></td>
<td>( V_w = 0.001 \cdot m^3 )</td>
<td>( L_w = 0.067 \cdot m )</td>
<td>( V_w \cdot \frac{L_w}{3} = 6.981 \cdot 10^{-5} \cdot m^4 )</td>
</tr>
<tr>
<td>interface</td>
<td>( \frac{\pi}{4} \cdot L_{\text{int}} \cdot d_{\text{int}}^2 )</td>
<td>( L_{\text{int}} )</td>
<td>( \frac{L_{\text{int}}}{2} )</td>
</tr>
<tr>
<td></td>
<td>( V_{\text{int}} = 0.004 \cdot m^3 )</td>
<td>( 2 \cdot L_w + \frac{L_{\text{int}}}{2} )</td>
<td>( V_{\text{int}} \cdot \frac{2 \cdot L_w + L_{\text{int}}}{2} = 8.247 \cdot 10^{-4} \cdot m )</td>
</tr>
<tr>
<td>tool/load</td>
<td>( V_5 = \frac{\text{mass tool}}{\rho} \cdot \frac{\text{mass load}}{\rho_{\text{moon}}} )</td>
<td>( L_4 )</td>
<td>( \frac{L_4}{2} )</td>
</tr>
<tr>
<td></td>
<td>( V_5 = 0.03 \cdot m^3 )</td>
<td>( 2 \cdot L_w + \frac{L_{\text{int}}}{2} )( \frac{L_5}{2} )</td>
<td>( V_5 \cdot \frac{2 \cdot L_w + L_{\text{int}}}{2} \frac{L_5}{2} = 0.017 \cdot m^4 )</td>
</tr>
</tbody>
</table>

\[
V_5 = \frac{2 \cdot L_w}{3} - L_{\text{int}} + \frac{L_5}{2} = 0.583 \cdot m
\]
\[
\text{sumVC} = V_w \frac{2}{3} - V_{\text{int}} - V_5
\]

\[
\text{sumVC} = 0.034 \cdot \text{m}^3
\]

\[
\text{sumVGC} = V_w \left( \frac{L_w}{3} + V_{\text{int}} \frac{2 L_w}{3} - \frac{L_{\text{int}}}{2} - V_5 \frac{2 L_w}{3} - L_{\text{int}} \frac{L_5}{2} \right)
\]

\[
\text{sumVGC} = 0.018 \cdot \text{m}^4
\]

Location of center of gravity of maximum load, tool, interface, 2/3 of wrist from Joint C.

\[
G_{\text{jointC}} = \frac{\text{sumVGC}}{\text{sumVC}}
\]

\[
G_{\text{jointC}} = 0.537 \cdot \text{m}
\]

**Determination of center of gravity for mechanical structure excluding arms #1, #2, #3, joint W1, and Joint C.** For use in calculating torque at Joint W2.

<table>
<thead>
<tr>
<th>component</th>
<th>volume</th>
<th>length</th>
<th>length x volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>wrist</td>
<td>( V_w = \frac{\pi}{4} L_w d^3 )</td>
<td>( \frac{2}{3} )</td>
<td>( \frac{2}{3} L_w d^3 )</td>
</tr>
<tr>
<td></td>
<td>( V_w = 5.236 \cdot 10^{-4} \cdot \text{m}^3 )</td>
<td>( \frac{L_w}{6} = 0.033 \cdot \text{m} )</td>
<td>( \frac{L_w}{6} \cdot 1.745 \cdot 10^{-3} \cdot \text{m}^4 )</td>
</tr>
</tbody>
</table>

\[
V_{\text{int}} = \frac{\pi}{4} L_{\text{int}} d^2
\]

\[
V_{\text{int}} = 0.004 \cdot \text{m}^3
\]

\[
V_{\text{int}} = \frac{1}{3} L_w + \frac{L_{\text{int}}}{2}
\]

\[
V_{\text{int}} = \frac{1}{3} L_w + \frac{L_{\text{int}}}{2}
\]

\[
V_5 = \frac{\text{mass tool}}{\rho} - \frac{\text{mass load}}{\rho_{\text{moon}}}
\]

\[
V_5 = 0.03 \cdot \text{m}^3
\]

\[
\frac{1}{3} L_w + L_{\text{int}} + \frac{L_5}{2} = 0.517 \cdot \text{m}
\]

\[
\text{sumVW2} = V_w \frac{2}{3} - V_{\text{int}} - V_5
\]

\[
\text{sumVW2} = 0.034 \cdot \text{m}^3
\]

\[
\text{sumVGW2} = V_w \left( \frac{L_w}{6} \right) + V_{\text{int}} \left( \frac{1}{3} L_w + \frac{L_{\text{int}}}{2} \right) + V_5 \left( \frac{1}{3} L_w - L_{\text{int}} - \frac{L_5}{2} \right)
\]

\[
\text{sumVGW2} = 0.016 \cdot \text{m}^4
\]

Location of center of gravity of maximum load, tool, interface, 1/3 of wrist from Joint W2.

\[
G_{\text{jointW2}} = \frac{\text{sumVGW2}}{\text{sumVW2}}
\]

\[
G_{\text{jointW2}} = 0.475 \cdot \text{m}
\]

**Summation of analysis for center of gravity calculations.**

\[
G_{\text{base}} = 3.769 \cdot \text{m}
\]

\[
G_{\text{jointW1}} = 0.594 \cdot \text{m}
\]

\[
G_{\text{jointA}} = 2.291 \cdot \text{m}
\]

\[
G_{\text{jointC}} = 0.537 \cdot \text{m}
\]

\[
G_{\text{jointB}} = 0.965 \cdot \text{m}
\]

\[
G_{\text{jointW2}} = 0.475 \cdot \text{m}
\]

28
Mass Moment of Inertia Calculations.
Determination of the mass moments of inertia at base with respect to the y-axis for each member:

for section a of arm #1

\[ I_a = \frac{1}{12} \text{mass}_a \left( 3 \cdot (d_a^2 - d_{a_1}^2 - L_a^2) \right) + \text{mass}_a G_a^2 \]

\[ I_a = 0.4 \cdot \text{kg.m}^2 \]

for section b of arm #1

\[ I_b = \frac{1}{12} \text{mass}_\text{screw} \left[ 3 \cdot (d_{so}^2 - d_{si}^2 - L_b^2) \right] - \text{mass}_b G_b^2 \]

\[ I_b = 16.73 \cdot \text{kg.m}^2 \]

for section c of arm #1

\[ I_c = \frac{1}{12} \text{mass}_c \left( 3 \cdot (d_c^2 - d_{ci}^2 - L_c^2) \right) - \text{mass}_c G_c^2 \]

\[ I_c = 16.45 \cdot \text{kg.m}^2 \]

for arm #1 as a whole

\[ I_1 = I_a - I_b - I_c \]

\[ I_1 = 33.583 \cdot \text{kg.m}^2 \]

for arm #2

\[ I_2 = \frac{1}{12} \text{mass}_2 \left[ 3 \cdot (d_2^2 - d_{2i}^2) - L_2^2 \right] + \text{mass}_2 G_2^2 \]

\[ I_2 = 40.65 \cdot \text{kg.m}^2 \]

for arm #3

\[ I_3 = \frac{1}{12} \text{mass}_3 \left[ 3 \cdot (d_3^2 - d_{3i}^2) - (L_3)^2 \right] + \text{mass}_3 G_3^2 \]

\[ I_3 = 10.78 \cdot \text{kg.m}^2 \]

for wrist

\[ I_w = \frac{1}{12} \text{mass}_w \left[ 3 \cdot (d_w^2) + (L_w)^2 \right] + \text{mass}_w G_w^2 \]

\[ I_w = 152.2 \cdot \text{kg.m}^2 \]

at Joint C in wrist

\[ I_C = \frac{1}{12} \frac{2 \cdot \text{mass}_w}{3} \left[ 3 \cdot (d_0^2) + \left( \frac{2}{3} L_w \right)^2 \right] - \frac{2 \cdot \text{mass}_w}{3} \left( \frac{2}{3} G_w \right)^2 \]

\[ I_C = 45.1 \cdot \text{kg.m}^2 \]

at Joint W2 in wrist

\[ I_{W2} = \frac{1}{12} \frac{1 \cdot \text{mass}_w}{3} \left[ 3 \cdot (d_3^2) - \left( \frac{1}{3} L_w \right)^2 \right] + \frac{1 \cdot \text{mass}_w}{3} \left( \frac{1}{3} G_w \right)^2 \]

\[ I_{W2} = 5.6 \cdot \text{kg.m}^2 \]
for interface

\[ I_{\text{int}} = \frac{1}{12} \cdot \text{mass}_{\text{int}} \cdot d^2_{\text{int}} - L_{\text{int}}^2 \cdot \text{mass}_{\text{int}} \cdot G_{\text{int}}^2 \]

\[ I_{\text{int}} = 168.2 \cdot \text{kg} \cdot \text{m}^2 \]

for wrist and interface combined

\[ I_4 = I_w + I_{\text{int}} \]

\[ I_4 = 320.3 \cdot \text{kg} \cdot \text{m}^2 \]

for tool and load

\[ I_{\text{tool}} = \frac{1}{12} \cdot \text{mass}_{\text{tool}} \cdot (r_{\text{tool}} - 0.5 \cdot \text{m})^2 - (r_{\text{tool}} - 0.5 \cdot \text{m})^2 - L_{\text{tool}}^2 \cdot \text{mass}_{\text{tool}} \cdot G_{\text{tool}}^2 \]

\[ I_{\text{tool}} = 198.7 \cdot \text{kg} \cdot \text{m}^2 \]

\[ I_{\text{load}} = \frac{1}{12} \cdot \text{mass}_{\text{load}} \cdot (r_{\text{load}} - 0.5 \cdot \text{m})^2 - (r_{\text{load}} - 0.5 \cdot \text{m})^2 - L_{\text{load}}^2 \cdot \text{mass}_{\text{load}} \cdot G_{\text{load}}^2 \]

\[ I_{\text{load}} = 993.3 \cdot \text{kg} \cdot \text{m}^2 \]

\[ I_5 = I_{\text{load}} - I_{\text{tool}} \]

\[ I_5 = 1192 \cdot \text{kg} \cdot \text{m}^2 \]

Mass moment of inertia for entire structure for use in calculating torque required at base of robotic arm

\[ i = 1..5 \]

\[ I_{\text{base}} = \sum_{i} I_i \]

\[ I_{\text{base}} = 1.597 \cdot 10^3 \cdot \text{kg} \cdot \text{m}^2 \]

Mass moment of inertia for entire structure excluding arm #1 for use in calculating torque required at Joint A

\[ j = 2..5 \]

\[ I_{\text{jointA}} = \sum_{j} I_j \]

\[ I_{\text{jointA}} = 1.564 \cdot 10^3 \cdot \text{kg} \cdot \text{m}^2 \]

Mass moment of inertia for entire structure excluding arm #1 and arm #2 for use in calculating torque required at Joint A

\[ I_{\text{jointB}} = I_3 - I_4 - I_5 \]

\[ I_{\text{jointB}} = 1.523 \cdot 10^3 \cdot \text{kg} \cdot \text{m}^2 \]

Mass moment of inertia for maximum load, tool, interface, and wrist for use in calculating torque at Joint W1 in wrist

\[ i = 4..5 \]

\[ I_{\text{jointW1}} = I_4 + I_5 \]

\[ I_{\text{jointW1}} = 1.512 \cdot 10^3 \cdot \text{kg} \cdot \text{m}^2 \]

Mass moment of inertia for maximum load, tool, interface, and 2/3 of wrist for use in calculating torque at Joint C in wrist

\[ I_{\text{jointC}} = I_3 - I_{\text{int}} - I_5 \]

\[ I_{\text{jointC}} = 1.405 \cdot 10^3 \cdot \text{kg} \cdot \text{m}^2 \]

Mass moment of inertia for maximum load, tool, interface, and 1/3 of wrist for use in calculating torque at Joint W2 in wrist

\[ I_{\text{jointW2}} = I_2 - I_{\text{int}} - I_5 \]

\[ I_{\text{jointW2}} = 1.366 \cdot 10^3 \cdot \text{kg} \cdot \text{m}^2 \]
Summation of mass moment of inertia calculations.

\[ I_1 = 33.6 \text{ kg} \cdot \text{m}^2 \quad I_a = 0.399 \text{ kg} \cdot \text{m}^2 \quad I_C = 45.09 \text{ kg} \cdot \text{m}^2 \quad I_{\text{jointW1}} = 1512 \text{ kg} \cdot \text{m}^2 \]
\[ I_2 = 40.6 \text{ kg} \cdot \text{m}^2 \quad I_b = 16.731 \text{ kg} \cdot \text{m}^2 \quad I_{w_2} = 5.64 \text{ kg} \cdot \text{m}^2 \quad I_{\text{jointC}} = 1405 \text{ kg} \cdot \text{m}^2 \]
\[ I_3 = 10.8 \text{ kg} \cdot \text{m}^2 \quad I_c = 16.453 \text{ kg} \cdot \text{m}^2 \quad I_{\text{base}} = 1597 \text{ kg} \cdot \text{m}^2 \quad I_{\text{jointW2}} = 1366 \text{ kg} \cdot \text{m}^2 \]
\[ I_4 = 320.3 \text{ kg} \cdot \text{m}^2 \quad I_w = 152.2 \text{ kg} \cdot \text{m}^2 \quad I_{\text{jointA}} = 1564 \text{ kg} \cdot \text{m}^2 \]
\[ I_5 = 1192 \text{ kg} \cdot \text{m}^2 \quad I_{\text{int}} = 168.2 \text{ kg} \cdot \text{m}^2 \quad I_{\text{jointB}} = 1523 \text{ kg} \cdot \text{m}^2 \]

**Dynamic Analysis**

As a result of the static analysis the geometric structure of the robotic arm is now known. From this, the masses of the members were calculated and then the center of gravity of each member was determined. The next step is to calculate the torque at each joint so that the proper motor selection can be carried out. For the calculation of the torques the accelerations of the members of the robotic arm must be determined. The accelerations are a function of the angles of the operating range of each joint and therefore the orientation of the robotic arm at which the maximum accelerations exist must be found. In addition, the mass moments of inertia for each member must be calculated.

**Range of motion of each joint**  
As specified by members of Mechanical Subsystem

- **Joint at base** \( \theta = 0 \ldots 90 \) in positive and negative directions, for a full range of 180 degrees
- **Joint A** \( \theta_a = 0 \ldots 135 \) in positive and negative directions, for a full range of 270 degrees
- **Joint B** \( \theta_b = 0 \ldots 135 \) in positive and negative directions, for a full range of 270 degrees
- **Joint W1** \( \theta_{w1} = 0 \ldots 180 \) in positive and negative directions, for a full range of 360 degrees
- **Joint C** \( \theta_c = 0 \ldots 135 \) in positive and negative directions, for a full range of 270 degrees
- **Joint W2** \( \theta_{w2} = 0 \ldots 180 \) in positive and negative directions, for a full range of 360 degrees

**Angular accelerations and angular velocities at each joint as specified by members of Mechanical Subsystem**

- **Joint at base**  
  \( \omega_1 = 0.025 \text{ sec}^{-1} \quad \alpha_1 = 0.005 \text{ sec}^{-2} \)
- **Joint A**  
  \( \omega_2 = 0.025 \text{ sec}^{-1} \quad \alpha_2 = 0.005 \text{ sec}^{-2} \)
- **Joint B**  
  \( \omega_3 = 0.025 \text{ sec}^{-1} \quad \alpha_3 = 0.005 \text{ sec}^{-2} \)
- **Joint W1**  
  \( \omega_5 = 0.025 \text{ sec}^{-1} \quad \alpha_5 = 0.005 \text{ sec}^{-2} \)
- **Joint C**  
  \( \omega_4 = 0.025 \text{ sec}^{-1} \quad \alpha_4 = 0.005 \text{ sec}^{-2} \)
- **Joint W2**  
  \( \omega_6 = 0.025 \text{ sec}^{-1} \quad \alpha_6 = 0.005 \text{ sec}^{-2} \)

**Center of gravity of arm #1: velocity and acceleration**

\[
\begin{align*}
 v_{x_1} &= 0.0 \text{ m sec}^{-1} \\
 v_{y_1} &= 0.0 \text{ m sec}^{-1} \\
 v_{z_1} &= -\omega_1 G_1 \\
 a_{x_1} &= -\frac{1}{2} \omega_1^2 G_1 \\
 a_{y_1} &= 0 \text{ m sec}^{-2} \\
 a_{z_1} &= \alpha_1 G_1 \\
 a_{z_1} &= 0.0047 \text{ m sec}^{-2}
\end{align*}
\]
Joint A (end of arm #1): velocity and acceleration

\[ v_{x,a} = 0 \cdot \frac{m}{\text{sec}} \quad a_{x,a} = -\omega_1^2 L_1 \quad a_{x,a} = -0.001 \cdot \frac{m}{\text{sec}^2} \]

\[ v_{y,a} = 0 \cdot \frac{m}{\text{sec}} \quad a_{y,a} = 0 \cdot \frac{m}{\text{sec}^2} \]

\[ v_{z,a} = -\omega_1 L_1 \quad v_{z,a} = -0.05 \cdot \frac{m}{\text{sec}} \quad a_{z,a} = \alpha_1 L_1 \quad a_{z,a} = 0.01 \cdot \frac{m}{\text{sec}^2} \]

Joint B (end of arm #2): velocity and acceleration

Velocities

in the x direction \[ v_{x,b_0a} = -\omega_1 L_2 \sin \theta a \cdot \frac{\pi}{180} \]

in the y direction \[ v_{y,b_0a} = \omega_1 L_2 \cos \theta a \cdot \frac{\pi}{180} \]

in the z direction \[ v_{z,b_0a} = \omega_1 L_1 \cdot L_2 \cos \theta a \cdot \frac{\pi}{180} \]

Graph 4.3.0 Graph of directional velocities versus angle displacement for Joint B.
The angle displacement is the angle between the axis of arm #2 and the axis of arm #1.

Magnitude of velocity for arm #2

\[ v_{b_0a} = \sqrt{v_{x,b_0a}^2 + v_{y,b_0a}^2 + v_{z,b_0a}^2} \]
Graph 4.3.1  Graph of velocity magnitude versus angle of displacement for Joint B

Graph 4.3.1 shows that the maximum velocity occurs when arm #2 is at 0 degrees displacement with respect to arm #1.

Acceleration

in the x direction
\[ a_{xb} = \left( \omega_1 \right)^2 L_1 + \left( \omega_2 \right)^2 L_2 \cos \theta a - \alpha_2 L_2 \sin \theta a \]

in the y direction
\[ a_{yb} = \alpha_2 L_2 \cos \left( \theta a \right) \]

in the z direction
\[ a_{zb} = \alpha_1 \left( L_1 + L_2 \cos \left( \theta a \right) \right) - 2 \cdot \omega_1 \cdot \omega_2 \cdot L_2 \sin \left( \theta a \right) \]

Graph 4.3.2  Graph of directional accelerations versus angle displacement for Joint B.

The angle displacement is the angle between the axis of arm #2 and the axis of arm #1.
Magnitude of acceleration for arm #2

\[ a_{b_a} = \sqrt{a_{x b_a}^2 + a_{y b_a}^2 + a_{z b_a}^2} \]

**Graph 4.3.3**  Graph of acceleration magnitude versus angle of displacement for Joint B

Graph 4.3.3 shows that the maximum acceleration of arm #2 occurs when arm #2 is at 0 degrees displacement with respect to arm #1.

**Joint C** (located in wrist section): velocity and acceleration

**Velocity**

- in the x direction  \[ v_{x C_{b b}} = (\alpha_2 + \alpha_3) \left( \frac{2}{3} L_w \right) \sin(\theta b \frac{\pi}{180}) \]

- in the y direction  \[ v_{y C_{b b}} = \omega_2 (L_2 + \omega_3 \frac{2}{3} L_w) \cos(\theta b \frac{\pi}{180}) + \omega_2 \left( \frac{2}{3} L_w \right) \cos(\theta b \frac{\pi}{180}) \]

- in the z direction  \[ v_{z C_{b b}} = \omega_1 \left( L_1 + L_2 \right) - \omega_1 \left( \frac{2}{3} L_w \right) \cos(\theta b \frac{\pi}{180}) \]

**Graph 4.3.4**  Graph of directional velocities versus angle displacement for Joint C.

The angle displacement is the angle between the axis of arm #3 and the axis of arm #2.
Magnitude of velocity for arm #3

\[ v_{C_{0b}} = \sqrt{v_{x_{C_{0b}}}^2 + v_{y_{C_{0b}}}^2 + v_{z_{C_{0b}}}^2} \]

Graph 4.3.5  Graph of velocity magnitude versus angle of displacement for Joint C

![Graph of velocity magnitude versus angle of displacement for Joint C](image)

Graph 4.3.5 shows that the maximum velocity occurs when arm #3 is at 0 degrees displacement with respect to arm #2.

**Acceleration**

in the x direction

\[ X_{1_{0b}} = -\omega_1^2 \left[ L_1 + L_2 + \left( L_3 + \frac{2}{3}L_w \right) \cos \left( \theta b \cdot \frac{\pi}{180} \right) \right] \]

\[ X_{2_{0b}} = -\omega_2^2 \left[ L_2 + \left( L_3 + \frac{2}{3}L_w \right) \cos \left( \theta b \cdot \frac{\pi}{180} \right) \right] - \alpha_2 \left( L_3 + \frac{2}{3}L_w \right) \sin \left( \theta b \cdot \frac{\pi}{180} \right) \]

\[ X_{3_{0b}} = -\alpha_3 \left( L_3 + \frac{2}{3}L_w \right) \sin \left( \theta b \cdot \frac{\pi}{180} \right) - \omega_3^2 \left( L_3 + \frac{2}{3}L_w \right) \cos \left( \theta b \cdot \frac{\pi}{180} \right) \]

\[ a_{x_{C_{0b}}} = X_{1_{0b}} - X_{2_{0b}} - X_{3_{0b}} \]

in the y direction

\[ Y_{1_{0b}} = \alpha_2 \left( L_2 + \left( L_3 + \frac{2}{3}L_w \right) \cos \left( \theta b \cdot \frac{\pi}{180} \right) \right) - \omega_2^2 \left( L_3 + \frac{2}{3}L_w \right) \sin \left( \theta b \cdot \frac{\pi}{180} \right) \]

\[ Y_{2_{0b}} = \alpha_3 \left( L_3 + \frac{2}{3}L_w \right) \cos \left( \theta b \cdot \frac{\pi}{180} \right) - \omega_3^2 \left( L_3 + \frac{2}{3}L_w \right) \sin \left( \theta b \cdot \frac{\pi}{180} \right) \]

\[ a_{y_{C_{0b}}} = Y_{1_{0b}} - Y_{2_{0b}} \]

in the z direction

\[ a_{z_{C_{0b}}} = \alpha_1 \left( L_1 + L_2 + \left( L_3 - \frac{2}{3}L_w \right) \cos \left( \theta b \cdot \frac{\pi}{180} \right) \right) - 2 \omega_1 \omega_3 \left( L_3 + \frac{2}{3}L_w \right) \sin \left( \theta b \cdot \frac{\pi}{180} \right) \]

\[ a_{z_{C_{0b}}} = \alpha_1 \left( L_1 - L_2 + \left( L_3 - \frac{2}{3}L_w \right) \cos \left( \theta b \cdot \frac{\pi}{180} \right) \right) - 2 \omega_1 \omega_3 \left( L_3 + \frac{2}{3}L_w \right) \sin \left( \theta b \cdot \frac{\pi}{180} \right) \]
Graph 4.3.6  Graph of directional accelerations versus angle displacement for Joint C.
The angle displacement is the angle between the axis of arm #3 and the axis of arm #2.

Graph 4.3.7  Graph of acceleration magnitude versus angle of displacement for Joint C

Graph 4.3.7 shows that the maximum acceleration of arm #3 occurs when arm #3 is at
0 degrees displacement with respect to arm #2.

Center of gravity of tool and load: velocity and acceleration

Velocity
in the x direction

\[ v_x^{\theta_c} = \omega_2 - \omega_3 \left( L_3 - \frac{1}{3} L_w - L_{int} \right) \sin \theta_c \frac{\pi}{180} - \omega_2 - \omega_3 \left( \frac{2}{3} L_w + G_5 \right) \sin \theta_c \frac{\pi}{180} \]

in the y direction

\[ v_y^{\theta_c} = \omega_2 L_2 + \omega_3 \left( L_3 - L_4 - G_5 \right) \cos \theta_c \frac{\pi}{180} - \omega_2 L_3 - L_4 - G_5 \cos \theta_c \frac{\pi}{180} \]
in the z direction

\[ V_{z_{0h}} = \omega_1 \cdot L_1 - L_2 \cdot \omega_1 \cdot L_3 - L_4 - G_5 \cdot \cos \theta \cdot \frac{\pi}{180} \]

**Graph 4.3.8**  
Graph of directional velocities versus angle displacement for tool.  
The angle displacement is the angle between the axis of the interface and the axis of arm #3.

**Graph 4.3.9**  
Graph of velocity magnitude versus angle displacement for tool and load.

**Magnitude of velocity at center of gravity of tool and load**

\[ v_{0h}^5 = \sqrt{v_{x_{0h}}^2 + \left( v_{y_{0h}}^2 \right)^2 + \left( v_{z_{0h}}^2 \right)^2} \]

**Graph 4.3.9**  
Graph of velocity magnitude versus angle displacement for tool and load.

Graph 4.3.9 shows that the maximum velocity occurs when tool and load are at 0 degrees displacement with respect to arm #2.
Acceleration

in the x direction

\[ X_{1_{0c}} = -\omega_1^2 L_1 - L_2 - L_3 - L_4 - G_5 \cos \theta_c \frac{\pi}{180} \]
\[ X_{2_{0c}} = -\omega_2^2 L_2 - L_3 - L_4 - G_5 \cos \theta_c \frac{\pi}{180} - \omega_2^2 L_3 - L_4 - G_5 \sin \theta_c \frac{\pi}{180} \]
\[ X_{3_{0c}} = \alpha_3^2 L_3 - L_4 - G_5 \sin \theta_c \frac{\pi}{180} - \omega_3^2 L_3 - L_4 - G_5 \cos \theta_c \frac{\pi}{180} \]

\[ a_{x5_{0c}} = X_{1_{0c}} + X_{2_{0c}} + X_{3_{0c}} \]

in the y direction

\[ Y_{1_{0c}} = \alpha_2^2 L_2 + L_3 - L_4 - G_5 \cos \theta_c \frac{\pi}{180} - \omega_2^2 L_3 - L_4 - G_5 \sin \theta_c \frac{\pi}{180} \]
\[ Y_{2_{0c}} = \alpha_3^2 L_3 - L_4 - G_5 \cos \theta_c \frac{\pi}{180} - \omega_3^2 L_3 - L_4 - G_5 \sin \theta_c \frac{\pi}{180} \]

\[ a_{y5_{0c}} = Y_{1_{0c}} + Y_{2_{0c}} \]

in the z direction

\[ a_{z5_{0c}} = \alpha_1^2 L_1 + L_2 + L_3 - L_4 - G_5 \cos \theta_c \frac{\pi}{180} - \omega_1^2 L_1 - L_2 - L_3 - L_4 - G_5 \sin \theta_c \frac{\pi}{180} \]

Graph 4.3.10 Graph of directional accelerations versus angle displacement for tool and load. The angle displacement is the angle between the axis of the tool and the axis of arm #3.

Magnitude of acceleration for center of gravity for tool and load

\[ a^5_{0c} = \sqrt{a_{x5_{0c}}^2 + a_{y5_{0c}}^2 + a_{z5_{0c}}^2} \]
Graph 4.3.11 shows that the maximum acceleration of tool and load occurs when axis of tool is at 0 degrees displacement with respect to arm #3.

So far the dynamic analysis has proven that the maximum acceleration at each member of the arm occurs when each member of the arm is in the same horizontal plane as the base. Or in other words, the displacement angle at each joint is 0 degrees. This is the expected result. The following calculations are intended to calculate the torques required at each joint so that the appropriate motors can be chosen.

**Torque Calculations.**

**For motor at the base of the robotic arm.**

Tangential acceleration (z direction) at center of gravity of entire structure,

\[ a_{\text{base}} = a_1 - G_{\text{base}} \]

\[ a_{\text{base}} = 0.0188 \text{ m/s}^2 \]

Torque required for motor at base of robotic arm,

\[ T_{\text{base}} = I_{\text{base}} a_{\text{base}} - G_{\text{base}} \text{ mass}_{\text{total}} \]

\[ T_{\text{base}} = 20.1 \text{ N.m} \]

**For motor at Joint A.**

Tangential acceleration (y direction) at center of gravity of structure excluding arm #1,

\[ a_{\text{jointA}} = a_1 - G_{\text{jointA}} \]

\[ a_{\text{jointA}} = 0.0115 \text{ m/s}^2 \]

Mass of entire structure minus arm #1,

\[ \text{mass}_{\text{jointA}} = \text{mass}_{\text{total}} - \text{mass}_1 \]

\[ \text{mass}_{\text{jointA}} = 136.25 \text{ kg} \]

Torque required for motor at joint A,

\[ T_{\text{jointA}} = I_{\text{jointA}} a_{\text{jointA}} - G_{\text{jointA}} \text{ mass}_{\text{jointA}} a_{\text{jointA}} - G_{\text{jointA}} \text{ mass}_{\text{jointA}} g \]

\[ T_{\text{jointA}} = 521.6 \text{ N.m} \]
For motor at Joint B.

Tangential acceleration (y direction) at center of gravity of structure excluding arms #1 and #2.

\[ a_{\text{jointB}} = a_1 G_{\text{jointB}} \]

Mass of entire structure minus arms #1 and #2.

\[ \text{mass}_{\text{jointB}} = \text{mass}_{\text{jointA}} - \text{mass}_2 \]

\[ \text{mass}_{\text{jointB}} = 130.83 \text{ kg} \]

Torque required for motor at Joint B.

\[ T_{\text{jointB}} = I_{\text{jointB}} a_1 G_{\text{jointB}} \text{mass}_{\text{jointB}} a_{\text{jointB}} + G_{\text{jointB}} \text{mass}_{\text{jointB}} g \]

\[ T_{\text{jointB}} = 214.7 \text{ N m} \]

For motor at Joint W1.

Tangential acceleration (z direction) at center of gravity of structure excluding arms #1, #2, and #3.

\[ a_{\text{jointW1}} = a_1 G_{\text{jointW1}} \]

Mass of entire structure minus arms #1, #2, and #3.

\[ \text{mass}_{\text{jointW1}} = \text{mass}_{\text{jointB}} - \text{mass}_3 \]

\[ \text{mass}_{\text{jointW1}} = 130 \text{ kg} \]

Torque required for motor at Joint W1. Weight does play a part because Joint W is rotated 90 degrees.

\[ T_{\text{jointW1}} = I_{\text{jointW1}} a_1 G_{\text{jointW1}} \text{mass}_{\text{jointW1}} a_{\text{jointW1}} + G_{\text{jointW1}} \text{mass}_{\text{jointW1}} g \]

\[ T_{\text{jointW1}} = 134.1 \text{ N m} \]

For motor at Joint C.

Tangential acceleration (y direction) at center of gravity of structure including tool, interface, and wrist.

\[ a_{\text{jointC}} = a_1 G_{\text{jointC}} \]

\[ a_{\text{jointC}} = 2.69 \times 10^{-3} \frac{m}{\text{sec}^2} \]


\[ \text{mass}_{\text{jointC}} = \text{mass}_{\text{tool}} + \text{mass}_{\text{int}} + \frac{2}{3} \text{mass}_W \]

\[ \text{mass}_{\text{jointC}} = 26.7 \text{ kg} \]

Torque required for motor at Joint C.

\[ T_{\text{jointC}} = I_{\text{jointC}} a_1 G_{\text{jointC}} \text{mass}_{\text{jointC}} a_{\text{jointC}} G_{\text{jointC}} \text{mass}_{\text{jointC}} g \]

\[ T_{\text{jointC}} = 30.5 \text{ N m} \]

For motor at Joint W2.

Tangential acceleration (z direction) at center of gravity of structure including tool, load, interface, and joint W2.

\[ a_{\text{jointW2}} = a_1 G_{\text{jointW2}} \]

\[ a_{\text{jointW2}} = 2.38 \times 10^{-3} \frac{m}{\text{sec}^2} \]
Mass of tool, load, interface, and Joint W2:

\[
\text{mass}_{\text{jointW2}} = \text{mass}_{\text{jointC}} + \frac{1}{3} \text{mass}_w
\]

\[
\text{mass}_{\text{jointW2}} = 30 \cdot \text{kg}
\]

Torque required for motor at Joint W2. Weight does not play a part because axis of rotation is the same as centerline axis for members.

\[
T_{\text{jointW2}} = 1_{\text{jointW2}} a_{1} - G_{\text{jointW2}} \text{mass}_{\text{jointW2}} a_{\text{jointW2}}
\]

\[
T_{\text{jointW2}} = 6.9 \cdot \text{N} \cdot \text{m}
\]
Units, dimensions and other constants

\[ \text{MPa} = 1 \times 10^6 \text{ Pa} \quad \text{GPa} = 1 \times 10^9 \text{ Pa} \quad \rho = \frac{\text{kg}}{\text{m}^3} \quad \alpha = 1 \times 10^{-6} \quad g_{\text{moon}} = 1.634 \frac{\text{m}}{\text{sec}^2} \]

Material of the structure is aluminum 2014-T6

\[ \rho_{\text{al}} = 2800 \rho \quad \text{Density:} \]

\[ \text{UST}_{\text{al}} = 480 \text{ MPa} \quad \text{Ultimate Strength (Tension):} \]

\[ \text{USS}_{\text{al}} = 290 \text{ MPa} \quad \text{Ultimate Strength (Shear)} \]

\[ \text{YST}_{\text{al}} = 410 \text{ MPa} \quad \text{Yield Strength (Tension)} \]

\[ \text{YSS}_{\text{al}} = 220 \text{ MPa} \quad \text{Yield Strength (Shear)} \]

\[ E_{\text{al}} = 72 \text{ GPa} \quad \text{Modulus of Elasticity:} \]

\[ G_{\text{al}} = 27 \text{ GPa} \quad \text{Modulus of Rigidity:} \]

\[ \alpha_{\text{al}} = 23 \alpha \quad \text{Coefficient of linear thermal expansion:} \]

\[ \text{Duc}_{\text{al}} = 13 \quad \text{Ductility, percent elongation:} \]

Material of the ball and screw is Cold Rolled Stainless Steel (302)

\[ \rho_{\text{st}} = 7920 \rho \quad \text{Density:} \]

\[ \text{UST}_{\text{st}} = 860 \text{ MPa} \quad \text{Ultimate Strength (Tension):} \]

\[ \text{USS}_{\text{st}} = 430 \text{ MPa} \quad \text{Ultimate Strength (Shear, assumed)} \]

\[ \text{YST}_{\text{st}} = 520 \text{ MPa} \quad \text{Yield Strength (Tension)} \]

\[ \text{YSS}_{\text{st}} = 260 \text{ MPa} \quad \text{Yield Strength (Shear, assumed)} \]

\[ E_{\text{st}} = 190 \text{ GPa} \quad \text{Modulus of Elasticity:} \]

\[ G_{\text{st}} = 73 \text{ GPa} \quad \text{Modulus of Rigidity:} \]

\[ \alpha_{\text{st}} = 17.3 \alpha \quad \text{Coefficient of linear thermal expansion:} \]

\[ \text{Duc}_{\text{st}} = 12 \quad \text{Ductility, percent elongation:} \]
The analysis for the mechanical structure of the robotic arm section #1 (Translational Mechanism) is shown below. For the analysis, the worst case scenario was taken to ensure proper performance.

The analysis of this portion of the arm consists of 6 sections. They are:

1. Material analysis (due to the stresses and strains on the structure, bending and fatigue)
2. Dynamic analysis of the structure and the translating mechanism (velocities, accelerations, Forces and torques)
3. Motor selection (based on torques derived in dynamic analysis)
4. Machine analysis of the translating mechanism (wear and material selection of the mechanism and its components)
5. Thermal considerations (different materials exhibit different coefficients of linear thermal expansion)
6. Radiation considerations (different materials exhibit different reactions to prolonged radiation)

**Material Analysis**

The following is the Material Analysis of the first link of the robotic arm (the translational link). Just as all subsequent components of the analysis, this analysis is performed on each of three sections of the first link. The following analysis is performed on section b of the link arm #1 (the middle section).

**The assigned lengths of of each member of the robotic arm.**

Arm #1: arm #1 is composed of three sections: a, b, and c. Sections a, b, and c are each hollow cylinders. The following analysis is for section b of the arm #1 link (the screw part of the ball and screw translational mechanism)

Entire length of arm #1: \( L_1 = 2 \text{ m} \)
Length of section a: \( L_a = 0.5 \text{ m} \)
Length of section b: \( L_b = 0.667 \text{ m} \)
Length of section c: \( L_c = 0.833 \text{ m} \)

Entire length of arm #2: \( L_2 = 1.414\text{ m} \)

Entire length of tool: \( L_5 = 0.5 \text{ m} \)

**Assigned masses and weights for each member of the robotic arm**

For arms one, two and three, the masses are to be computed by static analysis:

Mass and weight of tool:

\[
\text{mass}_{\text{tool}} = 10\text{ kg} \quad w_{\text{tool}} = \text{mass}_{\text{tool}} g_{\text{moon}} \quad w_{\text{tool}} = 16.34\text{ N}
\]

Mass and weight of maximum load:

\[
\text{mass}_{\text{load}} = 50\text{ kg} \quad w_{\text{load}} = \text{mass}_{\text{load}} g_{\text{moon}} \quad w_{\text{load}} = 81.7\text{ N}
\]

Combined mass and weight of tool and maximum load (for simplified calculations):

\[
\text{mass}_{5} = \text{mass}_{\text{tool}} - \text{mass}_{\text{load}} \quad \text{mass}_{5} = 60\text{ kg}
\]

\[
w_{5} = w_{\text{tool}} + w_{\text{load}} \quad w_{5} = 98.04\text{ N}
\]
Mass and weight of interface:
\[ \text{mass}_{\text{int}} = 10 \text{ kg} \quad \text{w}_{\text{int}} = \text{mass}_{\text{int}} \times \text{g}_{\text{moon}} \quad \text{w}_{\text{int}} = 16.34 \text{ N} \]

Mass and weight of wrist:
\[ \text{mass}_w = 10 \text{ kg} \quad \text{w}_w = \text{mass}_w \times \text{g}_{\text{moon}} \quad \text{w}_w = 16.34 \text{ N} \]

Combined mass and weight of wrist and interface:
\[ \text{mass}_4 = \text{mass}_{\text{int}} - \text{mass}_w \quad \text{mass}_4 = 20 \text{ kg} \]
\[ \text{w}_4 = \text{w}_w + \text{w}_{\text{int}} \quad \text{w}_4 = 32.68 \text{ N} \]

In addition to the masses and weights of the robotic arm members, a miscellaneous weight is added at each joint to account for the added weight of the motor, gears, casing, axle, etc. at each joint. This weight is given as a value larger than that actually expected in keeping with the worst case scenario.
\[ \text{mass}_{\text{misc}} = 5 \text{ kg} \quad \text{w}_{\text{misc}} = \text{mass}_{\text{misc}} \times \text{g}_{\text{moon}} \quad \text{w}_{\text{misc}} = 8.17 \text{ N} \]

Distances from base of robotic arm to centers of gravity of all members

Since each member of the robotic arm is symmetric in the yz and xy planes, the only direction that is going to affect the center of gravity of each member is the distance in x-direction from the base of the arm to the center of gravity of each member.

Center of gravity for each section of arm #1

Section a:
\[ G_a = \frac{L_a}{2} \quad G_a = 0.25 \text{ m} \]

Section b:
\[ G_b = L_a - \frac{L_b}{2} \quad G_b = 0.834 \text{ m} \]

Section c:
\[ G_c = L_a + L_b - \frac{L_c}{2} \quad G_c = 1.584 \text{ m} \]

Center of gravity for arm #2:
\[ G_2 = L_1 + \frac{L_2}{2} \quad G_2 = 2.707 \text{ m} \]

Center of gravity for arm #3:
\[ G_3 = L_1 + L_2 - \frac{L_3}{2} \quad G_3 = 3.607 \text{ m} \]

Center of gravity for wrist (assuming constant cross section):
\[ G_w = L_1 - L_2 - L_3 + \frac{L_w}{2} \quad G_w = 3.9 \text{ m} \]

Center of gravity for interface (assuming constant cross section):
\[ G_{\text{int}} = L_1 - L_2 - L_3 - L_w - \frac{L_{\text{int}}}{2} \quad G_{\text{int}} = 4.1 \text{ m} \]

Center of gravity for wrist and interface (as a whole to simplify calculations)
\[ G_4 = L_1 - L_2 - L_3 - \frac{L_4}{2} \quad G_4 = 4 \text{ m} \]
Center of gravity for tool with maximum load:

\[ G_5 = L_1 \cdot L_2 \cdot L_3 \cdot L_4 \cdot \frac{L_5}{2} \quad G_5 = 4.45 \cdot m \]

Condensed static analysis numbers for the other sections of the arm. Note, these numbers were computed in another Mathcad document so the documentation is not included here.

At Joint C:

\[ R_{yc} = w_5 \cdot w_{int} \cdot \frac{1}{3} w_w + 2 \cdot w_{misc} \quad R_{yc} = 136.167 \cdot N \]

\[ M_c = \frac{1}{3} L_w + \frac{1}{2} L_5 \cdot w_5 + \frac{1}{3} L_w \cdot \frac{L_{int}}{2} \cdot w_{int} \cdot w_{misc} + \frac{1}{3} L_w \cdot \frac{1}{6} w_w \cdot w_{misc} \]

\[ M_c = 55.465 \cdot N \cdot m \]

At Joint B:

\[ R_{yb} = R_{yc} - \frac{2}{3} w_w + 2 \cdot w_{misc} - w_3 \quad R_{yb} = 163.4 \cdot N \]

\[ M_b = M_c + \left[ \frac{1}{2} L_3 \cdot \frac{1}{3} L_w \cdot \frac{2}{3} w_w + 2 \cdot w_{misc} \right] - \frac{L_3}{2} \cdot w_3 \quad M_b = 67.793 \cdot N \cdot m \]

For Torque Load

\[ T = \frac{1}{2} L_w \cdot w_w \cdot \left( L_w \cdot L_{int} \cdot w_{int} - L_w + L_{int} \right) + \frac{1}{2} L_5 \cdot w_5 \quad T = 71.896 \cdot N \cdot m \]

BEGIN iterative loop

The desired outer diameter of arm #3 is:

\[ d_3_o = 100 \text{ mm} \]

Guess for the inner diameter of arm #3:

\[ d_3_i = 95 \text{ mm} \]

Area at predetermined cross section of arm:

\[ A_3 = \frac{\pi}{4} \cdot d_3_o^2 - d_3_i^2 \quad A_3 = 7.658 \cdot 10^{-4} \cdot m^2 \]

Centroidal moment of inertia for hollow cross section at specified inner and outer diameters:

\[ I_z_3 = \frac{\pi}{64} \cdot d_3_o^4 - d_3_i^4 \quad I_z_3 = 9.105 \cdot 10^{-7} \cdot m^4 \]

Polar moment of inertia:

\[ I_y_3 = I_z_3 \quad J_3 = I_y_3 + I_z_3 \quad J_3 = 1.821 \cdot 10^{-6} \cdot m^4 \]

Distance from neutral axis to point of maximum tension (top surface) and compression (bottom surface) are the same:

\[ c_3 = \frac{d_3_o}{2} \quad c_3 = 0.05 \cdot m \]
Maximum magnitude of tensile and compressive stress (when arm is fully extended):

\[ \sigma_3 = \frac{M_{bc} \cdot 3}{I_3} \quad \sigma_3 = 3.723 \times 10^6 \text{ Pa} \]

Maximum shearing stress:

\[ Q = \frac{A_3 \cdot 2 \cdot \frac{d_3^3 - d_3^2}{2} \cdot \frac{d_3^2 - d_3}{2} \cdot \pi}{d_3 \cdot \frac{d_3^2}{2}} \quad \tau_3 = \frac{R_{yb} \cdot Q}{I_3 \cdot \tau_3} \quad \tau_3 = 4.266 \times 10^5 \text{ Pa} \]

Maximum torsional stress (when joint W1 is rotated so that the rest of the arm past Joint C is positioned at a 90 degree angle with respect to the axis of the robotic arm resulting in a torsional stress):

\[ \tau_{\text{twist.3}} = \frac{T \cdot c_3}{J_3} \quad \tau_{\text{twist.3}} = 1.974 \times 10^6 \text{ Pa} \]

The maximum total stress in preselected cross section of arm #3 is:

\[ \tau_{3,\text{max}} = \frac{\sigma_3^2}{2} + \frac{\tau_3^2}{2} + \tau_{\text{twist.3}}^2 \quad \tau_{3,\text{max}} = 2.746 \times 10^6 \text{ Pa} \]

The resulting factor of safety is:

\[ n = \frac{0.4 \cdot YSS \text{ al}}{\tau_{3,\text{max}}} \quad n = 32.041 \]

END iterative loop

When the resulting F.S. equals the desired F.S., then the guess for the needed diameter is appropriate. Therefore the inner diameter for arm #3 is:

\[ d_3^i = 95 \text{ mm} \]

The inner and outer diameters of arm #3 are now known, therefore the volume, mass, weight, and wall thickness of arm #3 can now be determined.

Volume of arm #3:

\[ V_3 = A_3 \cdot L_3 \quad V_3 = 295.585 \cdot \text{cm}^3 \]

Mass of arm #3:

\[ \text{mass}_3 = V_3 \cdot \rho_{\text{al}} \quad \text{mass}_3 = 0.828 \cdot \text{kg} \]

Weight of arm #3:

\[ w_3 = \text{mass}_3 \cdot g_{\text{moon}} \quad w_3 = 1.352 \cdot \text{N} \]

Wall thickness of arm #3:

\[ t_3 = \frac{d_3^o - d_3^i}{2} \quad t_3 = 2.5 \text{ mm} \]

At joint B:

\[ R_{yb} = R_{yc} \cdot \frac{2}{3} \cdot w_w + 2 \cdot w_{\text{misc}} - w_3 \quad R_{yb} = 164.752 \cdot \text{N} \]

\[ M_b = M_c \cdot \frac{L_3}{2} \cdot \frac{2}{3} \cdot w_w + 2 \cdot w_{\text{misc}} + \frac{L_3}{2} \cdot w_3 \quad M_b = 68.054 \cdot \text{N m} \]
BEGIN iterative loop

The desired inner diameter of arm #2 is:  
\[ d_{2i} = 105 \text{ mm} \]

Guess for the outer diameter of arm #2 is:  
\[ d_{2o} = 113 \text{ mm} \]

Area at predetermined cross section of arm:  
\[ A_2 = \frac{\pi}{4} (d_{2o}^2 - d_{2i}^2) \quad A_2 = 0.001 \text{ m}^2 \]

Centroidal moment of inertia for hollow cross section at specified inner and outer diameters:  
\[ I_{z2} = \frac{\pi}{64} (d_{2o}^4 - d_{2i}^4) \quad I_{z2} = 2.037 \times 10^{-6} \text{ m}^4 \]

Polar moment of inertia:
\[ I_{y2} = I_{z2} \quad J_2 = I_{y2} + I_{z2} \quad J_2 = 4.074 \times 10^{-6} \text{ m}^4 \]

Distance from neutral axis to point of maximum tension (top surface) and compression (bottom surface) are the same:
\[ c_2 = \frac{d_{2o}}{2} \quad c_2 = 0.057 \text{ m} \]

Maximum magnitude of tensile and compressive stress:
\[ \sigma_2 = \frac{M_a c_2}{I_{z2}} \quad \sigma_2 = 8.67 \times 10^6 \text{ Pa} \]

Maximum shearing stress:
\[ Q_2 = \frac{A_2}{2} \left[ \frac{d_{2o}^2 - d_{2i}^2}{d_{2o} - d_{2i}} \cdot \pi \right] \quad \tau_2 = \frac{R_{ya} Q_2}{I_{z2} \tau_2} \quad \tau_2 = 2.523 \times 10^5 \text{ Pa} \]
Maximum torsional stress (when Joint W1 is rotated so that rest of arm past Joint C is positioned at a 90 degree angle with respect to axis of the robotic arm resulting in a torsional stress):

\[ \tau_{\text{twist,2}} = \frac{T \cdot c_2}{J_2} \]

\[ \tau_{\text{twist,2}} = 9.971 \cdot 10^5 \text{ Pa} \]

The maximum total stress in preselected cross section of arm #2 is:

\[ \tau_{2 \text{ max}} = \sqrt{\frac{\sigma_2^2}{2} + \tau_{\text{twist,2}}^2} \]

\[ \tau_{2 \text{ max}} = 4.455 \cdot 10^6 \text{ Pa} \]

The resulting factor of safety is:

\[ n = \frac{0.4 \cdot YSS_{al}}{\tau_{2 \text{ max}}} \]

\[ n = 19.752 \]

**END iterative loop**

When the resulting F.S. equals the desired F.S, then the guess for the needed diameter is appropriate. Therefore, the outer diameter for arm #2 is:

\[ d_{2o} = 113 \text{ mm} \]

The inner and outer diameters of arm #2 are now known. Therefore, the volume, mass weight, and wall thickness of arm #2 can be determined:

**Volume of arm #2:**

\[ V_2 = A_2 \cdot L_2 \]

\[ V_2 = 1.937 \cdot 10^3 \text{ cm}^3 \]

**Mass of arm #2:**

\[ \text{mass}_2 = V_2 \cdot \rho_{al} \]

\[ \text{mass}_2 = 5.423 \text{ kg} \]

**Weight of arm #2:**

\[ w_2 = \text{mass}_2 \cdot g \_\text{moon} \]

\[ w_2 = 8.861 \text{ N} \]

**Wall thickness of arm #2:**

\[ t_2 = \frac{d_{2o} - d_{2i}}{2} \]

\[ t_2 = 4 \text{ mm} \]

At Joint A:

\[ R_{ya} = R_{yb} + w_{\text{misc}} + w_2 \]

\[ R_{ya} = 172.922 \text{ N} \]

\[ M_a = \frac{L_2}{2} \cdot w_2 - L_2 \cdot R_{yb} + w_{\text{misc}} + M_b \]

\[ M_a = 312.566 \text{ N\cdot m} \]

\[ T = \frac{1}{2} L_5 \cdot w_5 - L_4 \cdot w_4 - L_6 \cdot w_6 + \left( L_4 - L_6 \right) \frac{1}{2} L_5 \cdot w_5 \cdot \frac{1}{2} L_5 \cdot w_5 \]

\[ T = 71.896 \text{ N\cdot m} \]

At cross section M-M of section c of arm #1:

\[ R_{\text{section c}} = w_c + w_{\text{misc}} + R_{ya} \]

\[ R_{\text{section c}} = 190.954 \text{ N} \]

\[ M_{\text{section c}} = \frac{L_c}{2} \cdot w_c - L_c \cdot R_{ya} + M_a \]

\[ M_{\text{section c}} = 477.479 \text{ N\cdot m} \]

\[ T = \frac{1}{2} L_5 \cdot w_5 - L_4 \cdot w_4 - L_6 \cdot w_6 + \left( L_4 - L_6 \right) \frac{1}{2} L_5 \cdot w_5 \cdot \frac{1}{2} L_5 \cdot w_5 \]

\[ T = 71.896 \text{ N\cdot m} \]
BEGIN iterative loop

The desired inner diameter of section c of arm #1 is: \[ dc_i = 118 \text{ mm} \]

Guess for the outer diameter of section c of arm #1: \[ dc_o = 132 \text{ mm} \]

Area at predetermined cross section of arm:
\[ A_c = \frac{\pi}{4} (dc_o^2 - dc_i^2) \quad A_c = 0.003 \text{ m}^2 \]

Centroidal moment of inertia for hollow cross section at specified inner and outer diameters:
\[ I_{zc} = \frac{\pi}{64} (dc_o^4 - dc_i^4) \quad I_{zc} = 5.386 \times 10^{-6} \text{ m}^4 \]

Polar moment of inertia:
\[ J_c = I_{zc} \quad J_c = 1.077 \times 10^{-5} \text{ m}^4 \]

Distance from neutral axis to point of maximum tension (top surface) and compression (bottom surface) are the same:
\[ c_c = \frac{dc_o - dc_i}{2} \quad c_c = 0.066 \text{ m} \]

Maximum magnitude of tensile and compressive stress:
\[ \sigma_c = \frac{M_{section c \ c}}{I_{zc}} \quad \sigma_c = 5.851 \times 10^6 \text{ Pa} \]

Maximum shearing stress:
\[ Q_c = \frac{A_c}{2} \left( \frac{dc_o^2 - dc_i^2}{dc_o - dc_i} \right) \quad \tau_c = \frac{dc_o - dc_i}{dc_o - dc_i + 2} \]

\[ \tau_c = \frac{R_{section c \ c} Q_c}{I_{zc} \cdot \tau_c} \quad \tau_c = 1.386 \times 10^5 \text{ Pa} \]

Maximum torsional stress (when joint W1 is rotated so that the rest of arm past Joint C is positioned at a 90 degree angle with respect to axis of the robotic arm resulting in a torsional stress):
\[ \tau_{twistc} = \frac{T \cdot c_c}{J_c} \quad \tau_{twistc} = 4.405 \times 10^5 \text{ Pa} \]

\[ \tau_{c, max} = \sqrt{\sigma_c^2 + \frac{\tau_c^2 + \tau_{twistc}^2}{2}} \quad \tau_{c, max} = 2.962 \times 10^6 \text{ Pa} \]

The resulting factor of safety is:
\[ n = \frac{YSS_{al}}{\tau_{c, max}} \quad n = 29.711 \]

END iterative loop
When the resulting F.S. equals the desired F.S then the guess for the needed diameter is appropriate. Therefore, the outer diameter for section c of arm #1 is:

\[ d_{c0} = 132 \text{ mm} \]

The inner and outer diameter for section c of arm #1 is now known. Therefore, the volume, mass, weight, and wall thickness of section c of arm #1 can be determined.

\[
\begin{align*}
\text{Volume of section c of arm #1} & = V_c = A_c L_c = 2.29 \times 10^3 \text{ cm}^3 \\
\text{Mass of section c of arm #1} & = \text{mass}_c = V_c \rho_al = 6.412 \text{ kg} \\
\text{Weight of section c of arm #1} & = w_c = \text{mass}_c g \text{moon} = 10.476 \text{ N} \\
\text{Wall thickness of section c of arm #1} & = t_c = \frac{d_{c0} - d_{ci}}{2} = 7 \text{ mm}
\end{align*}
\]

At cross section M-M of section c of arm #1:

\[
\begin{align*}
R_{sectionc} & = w_c - w_{misc} - R_{ya} \\
M_{sectionc} & = \frac{L_c}{2} (w_c - L_c (w_{misc} - R_{ya})) + M_a \\
T & = \frac{1}{2} L_w w_w + \left( \frac{1}{2} L_w + L_{int} \right) w_{int} + \left( \frac{1}{2} L_w + L_{int} + \frac{1}{2} L_5 \right) w_5 \\
T & = 71.896 \text{ N\cdot m}
\end{align*}
\]

At cross section N-N of section b of arm #1:

\[
\begin{align*}
R_{ysb} & = R_{sectionc} + w_b - w_{misc} \\
M_{sb} & = \frac{L_b}{2} (w_b + L_b (w_{misc} + R_{sectionc}) + M_{sectionc} \\
M_{sb} & = 620.896 \text{ N\cdot m}
\end{align*}
\]

Arm #1 section b has the cross section of a hollow cylinder. By giving the desired outer diameter, the inner diameter can be determined adesired factor of safety. The iterative loop used for this calculation is given below.

The desired inner diameter of arm #1 section b is:

\[ d_{si} = 30 \text{ mm} \]

BEGIN Iterative Loop

Guess for the outer diameter of arm #1 section b:

\[ d_{so} = 80 \text{ mm} \]

Area at predetermined cross section of arm:

\[ A_{sb} = \frac{\pi}{4} (d_{so}^2 - d_{si}^2) = 0.004 \text{ m}^2 \]

Centriodal moment of inertia for hollow cross section at specified inner and outer diameters

\[ I_{sb} = \frac{\pi}{64} (d_{so}^4 - d_{si}^4) = 1.971 \times 10^{-6} \text{ m}^4 \]

50
Polar moment of inertia:

\[ I_{sb} = I_{y_{sb}} - I_{z_{sb}} = J_{sb} = 3.942 \times 10^{-6} \cdot m^4 \]

Distance from Neutral Axis to point of maximum tension (top surface) and compression (bottom surface) are the same:

\[ c_{sb} = \frac{d_{so}}{2}, \quad c_{sb} = 0.04 \cdot m \]

Maximum magnitude of tensile and compressive stress (when arm is fully extended):

\[ \sigma_{sb} = \frac{M_{sb} c_{sb}}{I_{z_{sb}}} \quad \sigma_{sb} = 1.26 \times 10^{7} \cdot Pa \]

Maximum shearing stress:

\[ \tau_{sb} = \frac{R_{ysb} Q}{I_{z_{sb}} t_{tsb}} \quad \tau_{sb} = 8.597 \times 10^{4} \cdot Pa \]

Maximum torsional stress (occurs when joint W1 is rotated so that the rest of arm past Joint c is positioned at a 90 degree angle with respect to longitudinal axis of the robotic arm structure)

\[ \tau_{twist, sb} = \frac{T \cdot c_{sb}}{J_{sb}} \quad \tau_{twist, sb} = 7.296 \times 10^{5} \cdot Pa \]

The maximum total stress in preselected cross section of arm #3 is:

\[ \tau_{sb, max} = \sqrt{\left( \frac{\sigma_{sb}}{2} \right)^2 + \left( \tau_{sb} \right)^2 + \left( \tau_{twist, sb} \right)^2} \quad \tau_{sb, max} = 6.343 \times 10^{6} \cdot Pa \]

The resulting factor of safety is:

\[ n = \frac{0.4 \times Y_{SS}}{\tau_{sb, max}} \quad n = 16.395 \]

END of iterative loop

When the resulting F.S. equals the desired F.S. then the guess for the needed diameter is appropriate. Therefore, the outer diameter for arm #1 section b is:

\[ d_{so} = 0.08 \cdot m \]

The inner and outer diameters of arm #1 section b are now known, therefore, the volume, mass, weight, and wall thickness of arm #1 section b can be determined.

Volume of arm #1 section b:

\[ V_{sb} = A_{sb} L_{b} \quad V_{sb} = 2.881 \times 10^{3} \cdot cm^{3} \]

Mass of arm #1 section b:

\[ \text{mass}_{sb} = V_{sb} \cdot \rho_{st} \quad \text{mass}_{sb} = 22.819 \cdot kg \]

Weight of arm #1 section b:

\[ w_{sb} = \text{mass}_{sb} \cdot g_{moon} \quad w_{sb} = 37.287 \cdot N \]

Wall thickness of arm #1 section b:

\[ t_{sb} = \frac{d_{so} - d_{si}}{2} \quad t_{sb} = 0.025 \cdot m \]
Calculations for Joint A.

**GIVEN:**
- Angular velocity of gear: \( \omega = 0.025 \text{ sec}^{-1} \)
- Angular acceleration of gear: \( \alpha = 0.005 \text{ sec}^{-2} \)
- Torque on each gear at Joint A as given by dynamic analysis of robotic arm: \( T = 521.6 \text{ N m} \)
- Shear force exerted on each axle at Joint A as given by static analysis of robotic arm: \( R_{ya} = 181.8 \text{ N} \)
- Bending moment applied on each axle at Joint A as given by static analysis of robotic arm: \( M_a = 318.9 \text{ N m} \)
- Torsion on each axle at Joint A as given by static analysis of robotic arm: \( T_{\text{twist}} = 71.92 \text{ N m} \)
- Wall thickness of arm \#1: \( t_{\text{wall}} = 7 \text{ mm} \)
- Clearance between arms \#1 and \#2: \( t_{\text{armtol}} = 2.5 \text{ mm} \)
- Clearance between arm \#1 and gear: \( t_{\text{gtol}} = 2 \text{ mm} \)

**STATED values for gears:**
- Material: Cold rolled stainless steel (302)
- Pitch diameter of gear at Joint B: \( d_p = 212 \text{ mm} \)
- Pitch radius of gears: \( r_p = \frac{d_p}{2} = 106 \text{ mm} \)
- Force on gear resulting from chosen pitch diameter: \( F = \frac{T}{r_p} = 4.921 \text{ kN} \)
- Thickness of gears: \( t_g = 5 \text{ mm} \)

**STATED values for axle which gears are connected to:**
- Material: Aluminum 2014-T6
- Yield strength in shear: \( S_{ys} = 220 \cdot 10^6 \text{ Pa} \)
- Length of each axle: \( l_a = t_{\text{armtol}} - t_{\text{wall}} - t_{\text{gtol}} - t_g = 16.5 \text{ mm} \)
STATICALYSIS

of axle at Joint A to determine thickness using Maximum Shear Stress Theory.

Assume a diameter for axle: 
\[ d_a = 15.8 \text{ mm} \]

BEGIN iterative loop

Area of cross section at assumed diameter:
\[ A = \frac{\pi}{4} d_a^2 \quad A = 196.067 \times \text{mm}^2 \]

Moment of inertia about y axis for circular cross section:
\[ I_y = \frac{\pi}{4} d_a^4 \quad I_y = 4.895 \times 10^4 \times \text{mm}^4 \]

Moment of inertia about z axis for circular cross section:
\[ I_z = \frac{\pi}{4} d_a^4 \quad I_z = 4.895 \times 10^4 \times \text{mm}^4 \]

Polar moment of inertia for circular cross section:
\[ J_a = I_y - I_z \quad J_a = 9.789 \times 10^4 \times \text{mm}^4 \]

Thickness at Neutral Axis of circular cross section:
\[ t = d_a \quad t = 15.8 \times \text{mm} \]

Shear stress on axle:
\[ \tau = \frac{0.5 \frac{R_y a - F}{A}}{A} \quad \tau = 2.556 \times 10^7 \times \text{Pa} \]

Torsional stress on axle:
\[ \tau_{\text{tension}} = \frac{T - M_a \cdot 0.5 \cdot d_a}{J_a} \]
\[ \tau_{\text{tension}} = 6.783 \times 10^7 \times \text{Pa} \]

Bending moment stress on axle:
\[ \sigma = \frac{T_{\text{twist}} \cdot (0.5 \cdot d_a)}{I_z} \]
\[ \sigma = 1.161 \times 10^7 \times \text{Pa} \]

Maximum shear stress:
\[ \tau_{\text{max}} = \sqrt{\frac{\sigma^2}{2} + \tau_{\text{tension}}^2} \]
\[ \tau_{\text{max}} = 7.272 \times 10^7 \times \text{Pa} \]

Factor of safety resulting from chosen diameter:
\[ n = \frac{0.4 S_{\text{ys}}}{\tau_{\text{max}}} \quad n = 1.2 \]

END iterative loop

For a factor of safety of \( n = 1.2 \) a diameter of \( d_a = 15.8 \times \text{mm} \) was reached for the axle at Joint B.

STATIC ANALYSIS of arm #1 to determine whether wall thickness of hinge is adequate for given diameter of axle.

Nominal bearing area:
\[ A_{\text{bearing}} = t_{\text{wall}} \cdot d_a \quad A_{\text{bearing}} = 110.6 \times \text{mm}^2 \]
Bearing stress in hinge at location of axle:

\[
\sigma_s = \frac{0.5 \cdot R\cdot y_a - F}{A_{bearing}}
\]

\[
\sigma_s = 4.531 \cdot 10^7 \cdot \text{Pa}
\]

Factor of safety:

\[
n_{bearing} = \frac{0.4 \cdot S_{ys}}{\sigma_s}
\]

\[
n_{bearing} = 1.9
\]

With a wall thickness of \( t_{wall} = 7 \cdot \text{mm} \) a factor of safety of \( n_{bearing} = 1.9 \) was reached at the bearing location in the hinge of arm #2.

**STATIC ANALYSIS** of arm #2 to determine if hinge width is adequate.

Since the diameter of the axle is \( d_a = 15.8 \cdot \text{mm} \) then the width in the hinge on arm #1 must be greater than 15.8 mm. Therefore, a width will be chosen and the analysis will be carried out to determine the factor of safety on that width.

Chosen width of hinge on arm #2:

\[
w = 3.0 \cdot d_a \quad w = 47.4 \cdot \text{mm}
\]

Cross sectional area at location of axle on hinge:

\[
A_h = t_{wall} \cdot w - A_{bearing} \quad A_h = 221.2 \cdot \text{mm}^2
\]

Shear stress on hinge at location of axle:

\[
\sigma_h = \frac{0.5 \cdot R\cdot y_a + F}{A_h} \quad \sigma_h = 2.266 \cdot 10^7 \cdot \text{Pa}
\]

Resulting factor of safety:

\[
n_h = \frac{0.4 \cdot S_{ys}}{\sigma_h} \quad n_h = 3.9
\]

With a hinge width of \( w = 47.4 \cdot \text{mm} \) on arm #1 the resulting factor of safety is \( n_h = 3.9 \).

Now that the hinge width and thickness are chosen, the hinge must be analyzed to ensure that the hinge will not fail at the point of maximum bending.

Choose a length for the hinge from the base of arm #1 to center of axle:

\[
l_{hinge} = 0.5 \cdot d_a - 3 \cdot \text{mm} \quad l_{hinge} = 10.9 \cdot \text{mm}
\]

Bending stress on hinge (where hinge and arm #1 meet):

\[
\sigma_{hinge} = \frac{F - 0.5 \cdot R\cdot y_a \cdot l_{hinge} \cdot 0.5 \cdot d_a}{\frac{1}{3} \cdot t_{wall} \cdot w} \quad \sigma_{hinge} = 1.265 \cdot 10^8 \cdot \text{Pa}
\]

Shear stress on hinge (where hinge and arm #1 meet):

\[
\tau_{hinge} = \frac{0.5 \cdot R\cdot y_a}{t_{wall} \cdot w} \quad \tau_{hinge} = 3.196 \cdot 10^5 \cdot \text{Pa}
\]

Maximum stress on hinge at base of arm #1:

\[
\tau_{maxhinge} = \sqrt{\left(\frac{\sigma_{hinge}}{2}\right)^2 + \tau_{hinge}^2} \quad \tau_{maxhinge} = 6.323 \cdot 10^7 \cdot \text{Pa}
\]
Factor of safety: 

$$n_{\text{hinge}} = \frac{0.4 \cdot S_y}{t_{\text{max hinge}}} \quad n_{\text{hinge}} = 14$$

With a hinge length of $$l_{\text{hinge}} = 10.9 \cdot \text{mm}$$ for the hinge on arm #1 the resulting factor of safety is $$n_{\text{hinge}} = 14$$ at the base of the hinge.

**Determination of motor specification:**

Voltage: $$\text{voltage} = 12\cdot\text{volt}$$

Torque on gear at axle: $$T = 521.6 \cdot \text{N m}$$

Pitch diameter of gear attached to motor: $$d_{\text{pinion}} = 20 \cdot \text{mm}$$

Pinion radius: $$r_{\text{pinion}} = \frac{d_{\text{pinion}}}{2} \quad r_{\text{pinion}} = 10 \cdot \text{mm}$$

Gear ratio of pinion to output gear: $$\text{gear ratio} = \frac{d_p}{d_{\text{pinion}}} \quad \text{gear ratio} = 10.6$$

Angular velocity of pinion: $$\omega_{\text{pinion}} = \frac{\omega_p}{r_{\text{pinion}}} \quad \omega_{\text{pinion}} = 0.265 \cdot \text{sec}^{-1}$$

Revolutions per minute of motor: $$\text{rpm} = 60 \cdot \frac{180}{\omega_{\text{pinion}}} \quad \text{rpm} = 911 \cdot \frac{\text{rev}}{\text{min}}$$

Number of stages in gearhead: $$n = 1$$

Torque of the motor to be chosen: $$T_{\text{motor}} = \frac{T}{0.85^n \cdot \text{gear ratio}} \quad T_{\text{motor}} = 57.9 \cdot \text{N m}$$

For Joint A a motor capable of delivering a torque of $$T_{\text{motor}} = 57.9 \cdot \text{N m}$$ at a speed of $$\text{rpm} = 911 \cdot \frac{\text{rev}}{\text{min}}$$ is needed. The ratio of the pinion to the output gear is 1:10.6.

With the data known for the specification of the motor at joint A the process of choosing a motor was begun. All available motors are not space rated so therefore a decision was made to base our selection on modern available motors. It is presumed that by the time frame of this robot that an equivalent space rated motor will be available as instructed by Dr. Hollis.

The Compumotor Digiplan model Z-940 fits to the needed torque and rpm requirements. The Z-940 model is a brushless, three phase motor which provides 63.7 Nm of continuous stall torque and 127.5 Nm of peak torque at speeds up to 1500 rpm. The mass of the Z-940 motor is stated as 51.0 kg and the operating temperature range is from -40 degrees Celsius to 125 degrees Celsius. As stated it is assumed that technological advances in the years leading to the development of the robotic arm will lead to a greater operating temperature range and decrease in mass. In Appendix A is photocopied information on the Z-940 motor.
protective boot, gears, and motors not shown for clarity
Calculations for Joint B.

GIVEN:
Angular velocity of gear:
Angular acceleration of gear:
Torque on each gear at Joint b as given by dynamic analysis of robotic arm:
Shear force exerted on each axle at Joint B as given by static analysis of robotic arm:
Bending moment applied on each axle at Joint B as given by static analysis of robotic arm:
Torsion on each axle at Joint B as given by static analysis of robotic arm:
Wall thickness of arm #2:
Clearance between arms #2 and #3:
Clearance between arm #2 and gear:

STATED values for gears:
material:
Pitch diameter of gear at Joint B:
Pitch radius of gears:
Force on gear resulting from chosen pitch diameter:
Thickness of gears:

STATED values for axle which gears are connected to:
material:
Yield strength in shear:
Length of each axle:

STATIC ANALYSIS of axle at Joint A to determine thickness using Maximum Shear Stress Theory.

Assume a diameter for axle:
BEGIN iterative loop

Area of cross section at assumed diameter:

\[ A = \pi \frac{d_a^2}{4} \]

\[ A = 114.99 \text{ mm}^2 \]

Moment of inertia about y axis for circular cross section:

\[ I_y = \frac{\pi d_a^4}{4} \]

\[ I_y = 1.684 \times 10^4 \text{ mm}^4 \]

Moment of inertia about z axis for circular cross section:

\[ I_z = \frac{\pi d_a^4}{4} \]

\[ I_z = 1.684 \times 10^4 \text{ mm}^4 \]

Polar moment of inertia for circular cross section:

\[ J_a = I_y - I_z \]

\[ J_a = 3.367 \times 10^4 \text{ mm}^4 \]

Thickness at Neutral Axis of circular cross section:

\[ t = \frac{d_a}{2} \]

\[ t = 12.1 \text{ mm} \]

Shear stress on axle:

\[ \tau = \frac{0.5 R_{yb} F}{A} \]

\[ \tau = 1.987 \times 10^7 \text{ Pa} \]

Torsional stress on axle:

\[ \tau_{\text{tension}} = \frac{T_{\text{torsion}} - 0.5 d_a}{0.5 d_a \sqrt{\frac{t^2}{2} + \tau_{\text{tension}}^2}} \]

\[ \tau_{\text{tension}} = 5.081 \times 10^7 \text{ Pa} \]

Bending moment stress on axle:

\[ \sigma = \frac{T_{\text{twist}} \cdot 0.5 d_a}{I_z} \]

\[ \sigma = 2.584 \times 10^7 \text{ Pa} \]

Maximum shear stress:

\[ \tau_{\text{max}} = \sqrt{\frac{\sigma^2}{2} + \tau_{\text{tension}}^2} \]

\[ \tau_{\text{max}} = 5.606 \times 10^7 \text{ Pa} \]

Factor of safety resulting from chosen diameter:

\[ n = \frac{0.4 S_{ys}}{\tau_{\text{max}}} \]

\[ n = 1.6 \]

END iterative loop

For a factor of safety of \( n = 1.6 \) a diameter of \( d_a = 12.1 \text{ mm} \) was reached for the axle at Joint B.

STATIC ANALYSIS of arm #2 to determine whether wall thickness of hinge is adequate for given diameter of axle.

Nominal bearing area:

\[ A_{\text{bearing}} = t_{\text{wall}} d_a \]

\[ A_{\text{bearing}} = 60.5 \text{ mm}^2 \]

Bearing stress in hinge at location of axle:

\[ \sigma_s = \frac{0.5 R_{yb} + F}{A_{\text{bearing}}} \]

\[ \sigma_s = 3.776 \times 10^7 \text{ Pa} \]

Factor of safety:

\[ n_{\text{bearing}} = \frac{0.4 S_{ys}}{\sigma_s} \]

\[ n_{\text{bearing}} = 2.3 \]

With a wall thickness of \( t_{\text{wall}} = 5 \text{ mm} \), a factor of safety of \( n_{\text{bearing}} = 2.3 \) was reached at the bearing location in the hinge of arm #2.
STATIC ANALYSIS of arm #2 to determine if hinge width is adequate.

Since the diameter of the axle is \( d_a = 121 \, \text{mm} \) then the width in the hinge on arm #2 must be greater than 165 mm. Therefore, a width will be chosen and the analysis will be carried out to determine the factor of safety on that width.

Chosen width of hinge on arm #2:

\[ w = 20 \cdot d_a \quad w = 242 \, \text{mm} \]

Cross sectional area at location of axle on hinge:

\[ A_h = \frac{1}{4} \text{wall} \cdot w - A_{\text{bearing}} \quad A_h = 605 \, \text{mm}^2 \]

Shear stress on hinge at location of axle:

\[ \sigma_h = \frac{0.5 \cdot R_{yb} \cdot F}{A_h} \quad \sigma_h = 3.776 \times 10^7 \, \text{Pa} \]

Resulting factor of safety:

\[ n_h = \frac{0.4 \cdot S_{ys}}{\sigma_h} \quad n_h = 2.3 \]

With a hinge width of \( w = 242 \, \text{mm} \) on arm #2 the resulting factor of safety is \( n_h = 2.3 \).

Now that the hinge width and thickness are chosen the hinge must be analyzed to ensure that the hinge will not fail at the point of maximum bending.

Choose a length for the hinge from the base of arm #2 to center of axle:

\[ l_{\text{hinge}} = 0.5 \cdot d_a - 3 \, \text{mm} \quad l_{\text{hinge}} = 905 \, \text{mm} \]

Bending stress on hinge (where hinge and arm #2 meet):

\[ \sigma_{\text{hinge}} = \frac{F - 0.5 \cdot R_{yb} \cdot l_{\text{hinge}}}{0.5 \cdot d_a \cdot \frac{1}{3} \text{wall} \cdot w} \quad \sigma_{\text{hinge}} = 7.179 \times 10^7 \, \text{Pa} \]

Shear stress on hinge (where hinge and arm #2 meet):

\[ \tau_{\text{hinge}} = \frac{0.5 \cdot R_{yb}}{l_{\text{wall}} \cdot w} \quad \tau_{\text{hinge}} = 5.675 \times 10^5 \, \text{Pa} \]

Maximum stress on hinge at base of arm #2:

\[ \tau_{\text{maxhinge}} = \sqrt{\frac{\sigma_{\text{hinge}}^2}{2} + \tau_{\text{hinge}}^2} \quad \tau_{\text{maxhinge}} = 3.59 \times 10^7 \, \text{Pa} \]

Factor of safety:

\[ n_{\text{hinge}} = \frac{0.4 \cdot S_{ys}}{\tau_{\text{maxhinge}}} \quad n_{\text{hinge}} = 2.5 \]

With a hinge length of \( l_{\text{hinge}} = 905 \, \text{mm} \) for the hinge on arm #1 the resulting factor of safety is \( n_{\text{hinge}} = 2.5 \) at the base of the hinge.
Determination of motor specification:

- **Voltage**: voltage = 12 volt
- **Torque on gear at axle**: $T = 214.7 \cdot N \cdot m$
- **Pitch diameter of gear attached to motor**: $d_{\text{pinion}} = 17 \text{ mm}$
- **Pinion radius**: $r_{\text{pinion}} = \frac{d_{\text{pinion}}}{2}$, $r_{\text{pinion}} = 8.5 \cdot \text{ mm}$
- **Gear ratio of pinion to output gear**: gear ratio = \( \frac{d_{p}}{d_{\text{pinion}}} \), gear ratio = 11.5
- **Angular velocity of pinion**: $\omega_{\text{pinion}} = \frac{\omega_{p}}{r_{\text{pinion}}}$, $\omega_{\text{pinion}} = 0.287 \cdot \text{ sec}^{-1}$
- **Revolutions per minute of motor**: $\text{rpm} = 60 \cdot \omega_{\text{pinion}} \frac{180}{\pi}$, $\text{rpm} = 985.8 \cdot \text{ rev} \frac{\text{rev}}{\text{min}}$
- **Number of stages in gearhead**: $n = 1$
- **Torque of the motor to be chosen**: $T_{\text{motor}} = \frac{T}{0.85^{n}} \cdot \text{ gear ratio}$, $T_{\text{motor}} = 22 \cdot N \cdot m$

For Joint A a motor capable of delivering a torque of $T_{\text{motor}} = 22 \cdot N \cdot m$ at a speed of $\text{rpm} = 985.8 \cdot \text{ rev} \frac{\text{rev}}{\text{min}}$ is needed. The ratio of the pinion to the output gear is 1:11.5.

With the data known for the specification of the motor at joint A the process of choosing a motor was begun. All available motors are not space rated so therefore a decision was made to base our selection on modern available motors. It is assumed that by the time frame of this robot that an equivalent space rated motor will be available as instructed by Dr. Hollis.

The Compumotor Digiplan model Z-640 fits to the needed torque and rpm requirements. The Z-640 model is a brushless, three phase motor which provides 29 Nm of continuous stall torque and 58 Nm of peak torque at speeds up to 1600 rpm. The mass of the Z-640 motor is stated as 23.2 kg and the operating temperature range is from -40 degrees Celsius to 125 degrees Celsius. As stated it is assumed that technological advances in the years leading to the development of the robotic arm will lead to a greater operating temperature range and decrease in mass. In Appendix A is photocopied information on the Z-640 motor.
5. MODIFIED WRIST

5.1 Introduction

In a continued effort to satisfy the stated performance objectives, which together comprise the mission of the robotic arm, a modified wrist is required. The primary purpose of this subsystem is to provide crucial degrees of freedom to the system. Located between the primary mechanical structure and the structure-to-end effector interface, the modified wrist enhances the overall dexterity.

5.2 Operation

The way in which the modified wrist joint operates is centered on the fact that it adds three (3) degrees of freedom to the robotic arm. Its operation will, therefore, be explained by detailing the three (3) mechanisms that each contribute one degree of freedom.

In general, each mechanism consists of a motor, one or two gear sets, and its object of manipulation.

Gear set #1 of the modified wrist joint is specifically a worm gear set. The worm gear is welded to its axle which rigidly connects the modified wrist joint to the mechanical structure. The worm is shaft-attached to the motor and oriented to provide a torque through the page (or about the y-axis). The purpose of gear set #1 is to rotate the higher level members of the arm one hundred eighty (180) degrees in both directions.

Gear set #2 is also a worm gear which is rigidly attached to the encasement of the higher level mechanisms. This worm gear rotates about an axle which partially traverses the cross-section of the encasement. The worm and motor are situated similarly to that of the gear set #1, providing a torque along the y-axis also. The purpose of this gear set is to provide a pitch range of one hundred thirty-five (135) degrees in both directions.

The third mechanism for the modified wrist is a gear chain. It consists of a worm gear set (gear set #3) and a bevel gear set (gear set #4). The worm gear rotates freely about an axle which partially traverses the cross-section of the encasement. The worm and motor are again oriented such that the torque is out of or in the page. Additionally, the bevel gear set is used to redirect the output about the x-axis. This output rotates the interface and the installed end effector.

5.3 Drawings
5.4 Materials

5.4.1 Make-up

Space-qualified materials are required. From correspondences with NASA engineers, it has been advised to use an aluminum alloy. Aluminum 2014-T6 has been selected to make-up the encasement for the mechanisms. The axles, gears, worms, and fasteners are composed of stainless steel. This was chosen because of its strength and resilience, which is crucial in areas of gear teeth interfacing.

5.4.2 Motors

Motor specifications are a result of the analyses performed. The requirements were exacted and the specific motor specifications were gathered from company publications.²

<table>
<thead>
<tr>
<th>Type</th>
<th>Digital Brushless Servo Motors</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peak Duty Torque</td>
<td>160 oz. in.</td>
</tr>
<tr>
<td>Continuous Duty Torque</td>
<td>105 oz. in.</td>
</tr>
<tr>
<td>Step Resolution</td>
<td>0.07 degrees</td>
</tr>
<tr>
<td>Weight</td>
<td>1.95 kg</td>
</tr>
</tbody>
</table>

Due to present design conventions, standard motors do not satisfy the torque requirements without violating the weight/size parameters for proper integration of the components. The problem arises with motors being commonly designed to carry a relatively constant torque over an extensive range of velocities, revolutions per seconds (rps). The motors for these systems require similar torques, but at much smaller velocities. Research has shown a correlation in large velocities and large motors. Although a dependence has not been determined, an assumption has been made based on this correlation.

It is assumed that the torque requirements can be satisfied without violating weight/size parameters using present technology. The solution to this problem involves custom designing smaller motors to provide only the needed torques and velocities.

5.4.3 Bearing

Bearings are used to minimize friction at all gear-to-axis interfaces.
5.5 Calculations

The following calculations are to determine the motor sizes at the wrist joints.

Distance to maximum estimated load on wrist:

\[ l_{\text{load}} = 1.5 \text{ m} \]

Acceleration of load due to gravity of moon and arm:

\[ a_A = 1.639 \frac{\text{m}}{\text{sec}^2} \]

Mass of load and wrist estimated at distance above:

\[ m_{\text{load}} = 50 \text{ kg} \]

Coefficient of friction for worm-gear assembly:

\[ \mu_s = 0.58 \]

Pitch of worm-gear and worm:

\[ p = 5 \text{ mm} \]

Efficiency of steel to steel gear assembly:

\[ e_s = 0.9 \]

Mean diameter of gear:

\[ D_{\text{gear}} = 9.5 \text{ cm} \]

Maximum desired rotation of wrist joints:

\[ \omega = 0.05 \frac{\text{rad}}{\text{sec}} \]

Force of load on gear set:

\[ F = m_{\text{load}} a_A \frac{l_{\text{load}}}{D_{\text{gear}}} \]

\[ F = 290.891 \cdot \text{lbf} \]

Polar moment of inertia for load rotating about wrist joints:

\[ J_{\text{load}} = m_{\text{load}} l_{\text{load}}^2 a_A \]

Rotational velocity of worm screw:

\[ \omega_s = \frac{\pi D_{\text{gear}}}{p} \]

\[ \omega_s = 2.985 \frac{\text{rad}}{\text{sec}} \]

\[ \omega_s = 28.5 \cdot \text{rpm} \]

Gear ratio (power ratio) of worm-gear assembly:

\[ \text{GearRatio} = \frac{\omega_s}{\omega} \]

\[ \text{GearRatio} = 59.69 \]

Torque of load due to accelerations:

\[ T_{\text{accel}} = m_{\text{load}} a_A l_{\text{load}} \text{ GearRatio} \]

\[ T_{\text{accel}} = 291.633 \cdot \text{oz} \cdot \text{in} \]

Torque required to overcome friction:

\[ T_{\text{friction}} = \mu_s F p \]

\[ T_{\text{friction}} = 93.971 \cdot \text{oz} \cdot \text{in} \]

Total torque required by motor:

\[ T_{\text{total}} = T_{\text{friction}} - T_{\text{accel}} \]

\[ T_{\text{total}} = 385.603 \cdot \text{oz} \cdot \text{in} \]

\[ T_{\text{total}} = 2.723 \cdot \text{N m} \]
6. STRUCTURE-TO-END EFFECTOR INTERFACE

6.1 Introduction

In a continued effort to satisfy the stated performance objectives, which together comprise the mission of the robotic arm, a structure-to-end effector interface subsystem is required for a number of reasons. The primary purpose of such an interface involves supporting a variety of interchangeable end effectors individually and actuating the innate functions of each end effector.

6.2 Operation

In its most basic sense, the interface consists of a cylindrical shell which encloses the fit-locking mechanism, the end effector actuation mechanism, and the motor. The interface also operates as an electric outlet, channeling power to the end effectors that require it.

6.2.1 Fit-locking Mechanism

This mechanism is responsible for securing the fit between the end effector’s outer mating surface and the inner mating surface of the shell. The fit-lock consists of the actuating link, the followers (2), and the clamps (2) shown in Figure 6.1.

Each clamp is spring loaded and grounded at one revolute joint (hashed). The clamps are spring loaded in such a way, that the flange portion of the clamps are restored to the 'closed' position when no other force is applied at its follower-attached revolute joint. The closed position is achieved when the flange of each clamp protrudes from the shell window to the extent that the end effector (if present) would be adequately secured to perform the most demanding of system operations.

The actuating link is connected to the clamps via the followers. This link slides along the unthreaded portion of the motor shaft and transmits the force needed to restrain the clamps in the 'open' position. This force originates from the rotational motion of the motor shaft, which is then converted into linear motion at the threaded actuating rod. This rod in turn translates toward the motor, sliding the actuating link backward and establishing the open position of the clamps.

The open position is established prior to engagement of the end effector and constitutes disengagement of the same end effector.
6.2.2 End Effector Actuation Mechanism

This mechanism is responsible for the actuation of innate end effector functions. It consists of the actuating rod (of square cross-section) and the motor-driven threaded shaft.

Since end effectors, such as the small and large grippers are actuated by the displacement of their respective piston, the actuation mechanism must deliver the force necessary to induce such a displacement. This task is made possible by the conversion of the shaft's rotational energy into the translational energy of the rod. This translating actuating rod in turn performs the required work on the end effector's piston.

Controlling the precision of this rod motion and its consequential effect on the end effector involves understanding pertinent motor characteristics (i.e. step size, running torque, etc.) and programming supporting controls hardware accordingly.

6.2.3 Motor

The motor provides the mechanical power needed to perform major subsystem functions. By operating in both directions, the motor powers the fit-locking mechanism to its open position and powers end effector actuation as well.

6.2.4 Electric Power

Electric power, if required, is channeled to the end effector through the ports (2) located in the vertical mating surface of the shell. These ports also serve to align the end effector during engagement and absorb torques during operation.

6.3 Drawings
Figure 6.4

Date: April 1993

Scale: All dimensions in centimeters

Title: Clamp

Notes:
1. Clamp is made of Aluminium (2014loys).
2. All unlined lines are 0.1 centimeters.
Notes:
1. All unlabeled fillets are 0.1 centimeters.
2. Follower is made of Aluminum (2014-16).

Figure 6.5  Date: April, 1993  Scale: All dimensions in centimeters  Title: Follower
Jim,

Front View

Side View

Notes:
1. All unlabelled lines are 0.1 centimeters
2. Actuating Rod is made of Stainless Steel

Figure 6.7  Date: April, 1993  Scale: All dimensions in centimeters  Title: Actuating Rod
6.4 Materials

6.4.1 Make-up

Like the entire design, space-qualified materials are required. Aluminum 2014-T6 was selected as the primary material for this subsystem for a number of reasons. In addition to the fundamental demands on weight, strength, and cost, two more characteristics were considered: resistance to thermal expansion and resistance to outgassing.

From correspondences with NASA engineers, it is understood that an aluminum alloy is commonly used in similar applications. Interface designers decided to follow the agency’s example, composing the entire shell of aluminum 2014-T6. This particular alloy was selected because of its high strength and low coefficient of thermal expansion among aluminum alloys.

The links, and other components of the interface mechanisms are also composed of aluminum 2014-T6.

The effect of thermal expansion is critical at surfaces such as the thread-to-thread interfacing of the motor shaft and the actuating rod. To minimize the threat of locking at extreme temperatures, the shaft and rod are made of the same material: stainless steel.

6.4.2 Motor

Motor specifications are highly dependent on the requirements of the interface-based mechanisms in addition to those of the end effectors. As a result of analyses performed on these subsystems, the requirements were exacted and the consequential motor specifications are as follows:

<table>
<thead>
<tr>
<th>Type</th>
<th>Digital Brushless Servo Motors</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peak Duty Torque</td>
<td>160 oz. in.</td>
</tr>
<tr>
<td>Continuous Duty Torque</td>
<td>105 oz. in.</td>
</tr>
<tr>
<td>Step Resolution</td>
<td>0.07 degrees</td>
</tr>
<tr>
<td>Weight</td>
<td>1.95 kg</td>
</tr>
</tbody>
</table>

Due to present design conventions, standard motors do not satisfy the torque requirements without violating the weight/size parameters for proper integration of the
components. The problem arises with motors being commonly designed to carry a relatively constant torque over an extensive range of velocities, revolutions per seconds (rps). The motors for these systems require similar torques, but at much smaller velocities. Research has shown a correlation in large velocities and large motors. Although a dependence has not been determined, an assumption has been made based on this correlation.

It is assumed that the torque requirements can be satisfied without violating weight/size parameters using present technology. The solution to this problem involves custom designing smaller motors to provide only the needed torques and velocities.

6.4.3 Fasteners

Fasteners are used in two instances: to secure the shell halves and to attach the interface to the modified wrist subsystem. For the assembly of the shell, eight (8) stainless steel fillister head cap screws are used. For the interface attachment to the wrist, four (4) stainless steel fillister head cap screws are used. Additional details on these fasteners are included in the Calculations section of this document.
6.5 Calculations

The following calculations are for the interface system of the robotic arm. The proposed weight of the interface system is not to exceed 10 kg.

The calculations below are for each part are documented and describe the concerns and/or weaknesses of each part in order to determine the size of the whole interface and its individual parts.

**Properties of Aluminum 2014-T6 (4.4% Cu):** *Aluminum 2014-T6 will be used until better or necessary material changes are made.*

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gravity on Moon</td>
<td>$g = \frac{1}{6} \cdot g$</td>
</tr>
<tr>
<td></td>
<td>$g = 1.634 \cdot \frac{m}{s^2}$</td>
</tr>
<tr>
<td>Density of Aluminum</td>
<td>$\rho = 2800 \frac{kg}{m^3}$</td>
</tr>
<tr>
<td>Modulus of Elasticity</td>
<td>$E = 72 \cdot 10^9 \cdot Pa$</td>
</tr>
<tr>
<td>Modulus of Rigidity</td>
<td>$G = 27 \cdot 10^9 \cdot Pa$</td>
</tr>
<tr>
<td>Coefficient of Thermal Expansion</td>
<td>$\alpha = 23 \frac{10^6}{K}$</td>
</tr>
<tr>
<td>Ultimate Strength under Tension</td>
<td>$S_{utT} = 480 \cdot 10^6 \cdot Pa$</td>
</tr>
<tr>
<td>Ultimate Strength under Shear</td>
<td>$S_{uts} = 290 \cdot 10^6 \cdot Pa$</td>
</tr>
<tr>
<td>Yield Strength under Tension</td>
<td>$S_{yt} = 410 \cdot 10^6 \cdot Pa$</td>
</tr>
<tr>
<td>Yield Strength under Shear</td>
<td>$S_{ys} = 220 \cdot 10^6 \cdot Pa$</td>
</tr>
<tr>
<td>Factor of Safety for all parts</td>
<td>$\eta = 2.5$</td>
</tr>
</tbody>
</table>
The following calculations are to determine the maximum force acting on each clamp. The figure below shows the proposed orientation of the Robotic Arm which should produce the forces of the worst-case scenario acting on the interface and its components.

An assumption that a 1 meter maximum distance may exist from the end of the tool to the center of gravity of the load. Link #4 can rotate at a maximum angular velocity in the worst-case scenario.

Length of link #4:  
\[ L_4 = 0.558 \text{ m} \]

Length of wrist and interface:  
\[ L_{\text{int}} = 22 \text{ cm} \]

Radius of rotation from joint 4 to load:  
\[ r_4 = L_4 - L_{\text{int}} - 1 \text{ m} \]

Maximum rotational velocity of joint 4:  
\[ \omega_4 = 0.05 \text{ rad/sec} \]

Normal acceleration of load due to rotation of joint 4:  
\[ a_{4,n} = \omega_4^2 r_4 \]

Maximum mass of load:  
\[ m_{\text{load}} = 50 \text{ kg} \]

Total acceleration of load:  
\[ a_A = g + a_{4,n} \]

Maximum force on holding clamps due to load:  
\[ F_{\text{max}} = m_{\text{load}} a_A \]

\[ F_{\text{max}} = 81.944 \text{ N} \]

Force acting on one side of clamp:  
\[ F_{\text{maxA}} = \frac{F_{\text{max}}}{2} \]

\[ F_{\text{maxA}} = 40.972 \text{ N} \]
The following calculations are to determine the cross-sectional area of section A-A.

The size of the clamp face (section A-A) is tested below and found to be under no danger of shearing.

Figure of section A-A:

Maximum expected stress through face: \[ \sigma_{\text{maxS}} = \frac{0.4 S_y S}{\eta} \quad \sigma_{\text{maxS}} = 3.52 \times 10^7 \text{ Pa} \]

Minimum area of clamp surface: \[ \text{area C.F.} = \frac{F_{\text{maxA}}}{\sigma_{\text{maxS}}} \quad \text{area C.F.} = 1.164 \text{ mm}^2 \]

Based on the calculations and results found, the cross-section A-A of the clamp face can be virtually any realistic size in order to satisfy our needs.

The following calculations are to determine the thickness (b) of the clamp at Point B to prevent the part from failing.

Point B shown is tested for failure under the load conditions shown:

Maximum Yield Stress under Tension:

Stress due to bending:
- Distance from pin to Force vector: \( r_M = 2 \text{ cm} \)
- Moment due to force: \( M = F_{\text{maxA}} r_M \)
- Height of small arm of clamp: \( h = 1 \text{ cm} \)
- Distance to Point B from neutral axis: \( c = \frac{h}{2} \)
- Moment of inertia for section containing Point B: \( I = \frac{b h^3}{12} \)
- General Stress Form for bending: \[ \sigma_{\text{bend}} = \frac{M c}{I} = \frac{12 M c}{b h^3} \]

Stress due to tension:
- General Stress Form for tension: \[ \sigma_{\text{tension}} = \frac{F}{A} \]

The thickness (b) of the part at Point B is found to be less than 1.0 cm and is calculated in the following fashion:

Maximum total expected stress at Point B: \[ \sigma_{\text{maxT}} = \sigma_{\text{tension}} + \sigma_{\text{bending}} \]

Substituting general stress equations for bending and tension: \[ \sigma_{\text{maxT}} = \frac{12 M c}{b h^3} + \frac{F_{\text{maxA}}}{b h} \]
Therefore: \[
    b = \frac{12 \cdot M \cdot c - F_{\text{maxA}} h^2}{\sigma_{\text{maxT}} h^3} \quad b = 0.722 \cdot \text{mm}
\]

Based on the calculations and results found for the thickness \(b\) of the part at Point B, virtually any thickness \(b\) can be used. Keeping in mind the weight constraint, a thickness \(b\) of 1.0 cm has been chosen.

The following calculations are to determine the diameter of Pin A which the clamp pivots around.

The required cross-sectional area of Pin A was found to be extremely small. Therefore a diameter of 0.5 cm was chosen as the diameter to be used for the design.

A diagram of the position of Pin A on the clamp and the force acting on it is shown below:

Minimum cross-sectional area of Pin A:
\[
    \text{area}_{\text{pinA}} = \frac{F_{\text{maxA}}}{2 \cdot \sigma_{\text{maxS}}} \quad \text{area}_{\text{pinA}} = 0.582 \cdot \text{mm}^2
\]

Minimum diameter of Pin A:
\[
    d_{\text{pinA}} = \frac{4 \cdot \text{area}_{\text{pinA}}}{\pi} \quad d_{\text{pinA}} = 0.861 \cdot \text{mm}
\]

Because such a small force is being applied, the diameter of Pin A was found to be extremely small. Therefore, the diameter of the pin to be used will be 0.5 cm. This diameter was chosen since virtually any realistic size could be used and weight constraints exist.

The following calculations are to determine the thickness \(b\) of the clamp at the location of Pin A.

The cross-section of the clamp at Pin A is shown. The necessary thickness of this part at Pin A was calculated to be extremely small. Therefore, again, a thickness \(b\) of 1.0 cm will be used in the design.

The figure below shows the force being applied and the location where the thickness \(b\) is to be found:

Maximum expected actual stress:
\[
    \sigma_{\text{maxT}} = \frac{0.45 \cdot S_{\text{YT}}}{\eta}
\]
General Stress Form under tension:

\[
\sigma_{\text{maxT}} = \frac{F_{\text{maxA}}}{A \cdot b \cdot h \cdot d_{\text{pinA}}}
\]

Therefore, the thickness \(b\) of clamp can be solved for at location of Pin A as:

\[
b = \frac{F_{\text{maxA}}}{(h \cdot d_{\text{pinA}}) \cdot \sigma_{\text{maxT}}}
\]

\(b = 0.061 \text{ mm}\)

Because such a small force is being applied, the diameter of pin A was found to be extremely small. Therefore, the thickness \(b\) of the clamp at the location of Pin A will be set to a distance of 1.0 cm. This distance was chosen since virtually any realistic size could be used and weight constraints exist.

The following calculations are to determine the minimum spring torque that would hold the tool in place with the maximum angular velocities and the maximum load of the robotic arm being applied.

The clamps are needed to hold the tool within the interface and to keep the tool from spinning within the interface. The clamps will be spring loaded at Pin A. These springs must produce at least the following calculated torque:

The figure below shows where the torque is being applied:

Slope of the sides measured CCW from the vertical:

\(\theta = 5 \text{ deg}\)

Distance of lever arm (horizontal distance from Pin A to location of applied force):

\(L = 3 \text{ cm}\)

Force acting on each clamp:

\(F_{\text{maxA}} = 40.972 \text{ N}\)

Minimum spring force needed to keep the clamp in place:

\(f_{\text{spring}} = F_{\text{maxA}} \cdot \sin(\theta) \quad f_{\text{spring}} = 3.571 \text{ N}\)

Minimum torque needed to keep the clamp in place:

\(\text{Torque}_{\text{spring}} = f_{\text{spring}} \cdot L \quad \text{Torque}_{\text{spring}} = 0.107 \text{ N-m}\)

From the calculations above, the spring must exert a minimum of 0.107 Nm to maintain the clamp in the locked position. The effect of friction was neglected for the calculations above. By neglecting the effect of friction, a larger spring torque would be calculated. Therefore, this would ensure that the spring would hold.
The following calculations are to determine the force in the link of the clamping system.

The Figure below shows the torque being applied and the force being determined:

\[
\text{Torque spring} = \frac{\text{Torque spring}}{1.414\text{ cm}} = 7.576\text{ N}
\]

From the calculations above, the force in the link was found to be 7.576 N. From knowing this force, the thickness \(b\) of the link can be determined.

The following calculations are to determine the thickness \(b\) of the link within the clamping system.

The Figure below helps to understand the calculations being made:

\[
f_{\text{link}} = 7.576\text{ N}
\]

\[
h = 0.25\text{ cm}
\]

\[
\text{area}(b) = bh
\]

\[
\sigma_f(b) = \frac{f_{\text{link}}}{\text{area}(b)}
\]

\[
\eta(b) = \frac{0.45 \cdot S_T}{\sigma_f(b)}
\]

Using a root function to solve for the thickness \(b\) at a factor of safety of 2.5:

\[
b = \sqrt{\eta(b) - 2.5}, b = 0.004\text{ cm}
\]

Therefore, the thickness of the part will be set to \(b=1.0\text{ cm}\) and will encounter no potential problems.
The following calculations are made to test the inside of the clamp curve (Point A) as shown. The situation occurs while disengaging the clamp from the tool.

The following Figure below aids in the understanding of the calculations to follow. Point A shown experiences tearing from bending and tensile stresses:

- Distance from Pin A to Force vector of link: \( r = 1.414 \text{ cm} \)
- Height from top of Pin A to Point A: \( h_1 = 0.25 \text{ cm} \)
- Height from bottom of Pin A to bottom of clamp: \( h_2 = 0.25 \text{ cm} \)
- Angle that the short end of the clamp makes with the horizontal (refer to detailed description of clamp): \( \phi = 27.469 \text{ deg} \)
- Tensile force in the short end of the clamp: \( f_{\text{ten}} = f_{\text{link}} \sin(\phi) \quad f_{\text{ten}} = 3.495 \text{ N} \)
- Shear force in the short end of the clamp: \( f_{\text{shear}} = f_{\text{link}} \cos(\phi) \quad f_{\text{shear}} = 6.722 \text{ N} \)
- Moment due to tangential force in link: \( M = f_{\text{shear}} r \quad M = 0.095 \text{ N m} \)
- Diameter of Pin A: \( d_{\text{pin}} = 0.5 \text{ cm} \)
- Distance of Point A from the neutral axis as a function of the thickness (b): \( c(b) = \frac{h_1 b h_1 + h_2 b h_2}{(h_1 - h_2)^2 b} \)
- Moment of inertia for the cross-sectional area as a function of the thickness (b): \( I(b) = \frac{1}{12} b h_1^3 + \frac{1}{12} b h_2^3 - b h_1 \left( c(b) - \frac{h_1}{2} \right)^2 + b h_2 \left( h_1 - d_{\text{pin}} - \frac{h_2}{2} - c(b) \right)^2 \)
- Stress due to tensile force caused by link as a function of the thickness (b): \( \sigma_{\text{ten}}(b) = \frac{f_{\text{ten}}}{b (h_1 - h_2)} \)
- Stress caused by the bending moment as a function of the thickness (b): \( \sigma_M(b) = \frac{M c(b)}{I(b)} \)
Stress due to shear force as a function of the thickness \( b \):
\[
\tau_{\text{shear}}(b) = \frac{f_{\text{shear}}}{b \cdot h_1 - h_2}
\]

Maximum combined stress at Point A as a function of the thickness \( b \):
\[
\sigma(b) = \sqrt{\sigma_{\text{ten}}(b) - \sigma_M(b)^2 - \tau_{\text{shear}}(b)^2}
\]

Factor of safety as a function of the thickness \( b \):
\[
\eta(b) = \frac{0.40 \cdot S_{NT}}{\sigma(b)}
\]

Using a root function to solve for the thickness \( b \) at a factor of safety of 2.5:
\[
b = \text{root}(\eta(b) - 2.5, b) \quad b = 0.011 \, \text{cm}
\]

Based on the calculations, the thickness \( b \) was found to be 0.011 cm. The design thickness is 1.0 cm. Due to small forces applied, the calculated thickness was extremely small.

The following calculations are to determine the thickness \( b \) of the actuating component.

This Figure shows the appropriate forces being applied on the component and the location of the point where the maximum stress occurs:

- Width of the actuating link: \( w = 0.25 \, \text{cm} \)
- Vertical distance of the actuating link: \( L = 4 \, \text{cm} \)
- Vertical distance from where force is being applied to center of pin: \( r = \frac{L}{2} \)
- Distance from neutral axis to edge of actuating link: \( c = \frac{w}{2} \)
- Moment about Point A caused by force in link as a function of the thickness \( b \): \( M = 2 \, \text{cm} \cdot f_{\text{link}} \cdot \cos(45\,\text{deg}) - 0.5 \, \text{cm} \cdot f_{\text{link}} \cdot \sin(45\,\text{deg}) \)
- Moment of inertia for actuating link as a function of the thickness \( b \): \( I(b) = \frac{b \cdot w^3}{12} \)
- Stress due to tensile force in link as a function of the thickness \( b \):
\[
\sigma_t(b) = \frac{f_{\text{link}} \cdot \cos(45\,\text{deg})}{b \cdot w}
\]
Stress due to shear force in link as a function of the thickness (b):
\[ \tau(b) = \frac{f_{\text{link}} \sin(45 \, \text{deg})}{b \cdot w} \]

Stress due to bending moments caused by the force in the link as a function of the thickness (b):
\[ \sigma_b(b) = \frac{M}{I(b)} \]

Maximum stress in actuating link at Point A as a function of the thickness (b):
\[ \sigma_{\text{actual}}(b) = \sqrt{\sigma_f(b) - \sigma_b(b)^2 + \tau(b)^2} \]

Factor of safety as a function of the thickness (b):
\[ \eta(b) = \frac{0.40 \cdot S_y T}{\sigma_{\text{actual}}(b)} \]

Using a root function to solve for the thickness (b) at a factor of safety of 2.5:
\[ b = \sqrt[2]{\eta(b)} \cdot 2.5, b = 0.121 \cdot \text{cm} \]

Based on the calculations above, it was again found that the required thickness was much less than the specified thickness (2.0 cm) of the actuating link.

The following calculations are to determine the velocity and torque required by the motor.

The following Figure below aids in the understanding of the calculations to follow. The Figure shows the motor, power screw (motor shaft), actuating rod, and appropriate forces being applied:

Force acting on end of actuating rod: \[ P_{\text{rod}} = 200 \cdot \text{N} \]

Major diameter of the power screw: \[ d = 5 \, \text{mm} \]

Pitch of the power screw: \[ p = 3 \, \text{mm} \]

Mean diameter of the power screw: \[ d_m = d - \frac{p}{2} \quad d_m = 3.5 \cdot \text{mm} \]

Number of threads (single threaded): \[ n = 1 \]

Lead angle of power screw: \[ 1 = n \cdot p \]

Coefficient of friction between power screw and actuating rod: \[ \mu = 0.58 \]

Coefficient of friction of motor: \[ \mu_c = 0.58 \]
Torque required by the motor:

\[ T = \frac{P_{rod} d_m \mu \pi}{\mu_1 \pi d_m \mu 1} \cdot \frac{P_{rod} \mu e^{d}}{2} \]

\[ T = 0.645 \text{ Nm} \]

Translation distance of actuating rod:

Dist = 4 cm

Time taken to fully engage actuating rod and subsequently the tool:

Time = 4 sec

Work done by the actuating rod:

\[ W_{rod} = P_{rod} \text{ Dist} \]

Power required by a motor with 100% efficiency:

\[ \text{Power}_{rod} = \frac{W_{rod}}{\text{Time}} \]

\[ \text{Power}_{rod} = \frac{20.944 \text{ rad}}{4 \text{ sec}} \]

\[ \text{Power}_{rod} = 2 \text{ watt} \]

Maximum desired angular velocity of the power screw:

\[ \omega = \frac{2 \pi \text{ rad}}{P} \cdot \frac{\text{Dist}}{\text{Time}} \]

\[ \omega = 20.944 \text{ rad} \]

\[ \text{RPM's} = \frac{\omega}{2 \pi \text{ rad}} \]

\[ \text{RPM's} = 3.333 \text{ sec} \]

The following calculations are to determine the necessary screw size and pre-load required to hold the shell of the interface system together while a maximum load is applied to the tool.

The following figure shows the front half of the shell of the interface system and the estimated maximum loads to cause failure of the securing screws.

Modulus of elasticity of aluminum (2014-T6) shell:

\( E = 7.2 \times 10^{10} \text{ Pa} \)

Screw type used:

Fillister head cap screws:

\( M5 \times 0.8 \)

Maximum diameter of screws:

\( d = 5 \text{ mm} \)
Major diameter of cap screws: 
\[ A_d = \frac{\pi d^2}{4} \]

Tensile stress area of screws: 
\[ A_t = 14.2 \text{ mm}^2 \]

Pitch length of screws: 
\[ p = 0.8 \text{ mm} \]

Thread angle: 
\[ \alpha = 30 \text{ deg} \]

Modulus of elasticity of stainless steel screws: 
\[ E_s = 190 \times 10^9 \text{ Pa} \]

Minimum proof strength of cap screws: 
\[ S_p = 830 \times 10^6 \text{ Pa} \]

Endurance limit for screws with rolled threads: 
\[ S_c = 162 \times 10^6 \text{ Pa} \]

Ultimate strength of stainless steel screws: 
\[ S_{uts} = 1040 \times 10^6 \text{ Pa} \]

Thickness of top component: 
\[ t_1 = 1 \text{ cm} \]

Reach of screw into bottom component: 
\[ t_2 = 1 \text{ cm} \]

Length of threaded portion of screws: 
\[ l_t = t_1 + t_2 = 1 \text{ mm} \]

Length of unthreaded portion of screws: 
\[ l_d = 1 \text{ mm} \]

Effective grip length: 
\[ l = t_1 - \frac{d}{2} \text{ for diameter of screw} < t_2 \]

Maximum width of frustra: 
\[ D_1 = 1.5d + 0.577l \]

Minimum width of frustra: 
\[ D_2 = 1.5d \]

Minimum diameter of frustra: 
\[ D = D_2 \]

Stiffness of shell material being compressed: 
\[ k_m = \frac{0.577 \pi E d}{\ln \left( \frac{1.15(0.5l + D - d)(D - d)}{((1.15(0.5l + D) + d)(D - d))} \right)} \]

Effective stiffness of the cap screws: 
\[ k_b = \frac{A_d A_t E_s}{A_d l_t + A_t l_d} \]

Joint constant: 
\[ C = \frac{k_b}{k_b - k_m} \]

Pre-load for screws: 
\[ f_i = 0.75 A_t S_p \]

Number of screws that load is applied to: 
\[ n_s = 2 \text{ may possibly be distributed over 6 screws.} \]

Load of screws due to tool bending moment: 
\[ P = \frac{1m}{n_s \times 5.1 \text{ cm}} \text{ m load } A \]

Factor of safety for proof load: 
\[ \eta_p = \frac{S_p A_t - f_i}{C P} \quad \eta_p = 23.102 \]
Factor of safety for separation:

\[ \eta_s = \frac{f_i}{P(1 - C)} \]

\[ \eta_s = 13.079 \]

Strengths for Goodman's factor of safety:

\[ S_a = \frac{\frac{f_i}{A_t}}{\frac{1}{S_{uts}} + \frac{S_{uts}}{S_e}} \]

\[ S_m = S_{uts} \left( 1 - \frac{S_a}{S_e} \right) \]

Estimated stress in the shell around screws:

\[ \sigma_a = \frac{C \cdot P}{2 \cdot A_t} \]

Goodman's Factor of Safety for the shell:

\[ \eta_g = \frac{S_a}{\sigma_a} \]

\[ \eta_g = 12.529 \]

The results from the factor of safety above show that the interface shell screws will all be safe from failure due to the proof load, the separation strength, and the Goodman Criterion.

The following calculations are to determine the necessary screw size and pre-load required to hold the shell of the interface system attached to the wrist sub-system while a maximum load is applied to the tool.

The following figure shows the back half of the shell of the interface system and connection plate for the wrist sub-system.

Modulus of elasticity of aluminium (2014-T6) shell:

\[ E = 7.2 \cdot 10^{10} \text{ Pa} \]

Screw type used:

Fillister head cap screws:

M5 X 0.8

Maximum diameter of screws:

\[ d = 5 \text{ mm} \]

Major diameter of cap screws:

\[ A_d = \pi \cdot \frac{d^2}{4} \]
Tensile stress area of screws: \( A_t = 14.2 \, \text{mm}^2 \)

Pitch length of screws: \( p = 0.8 \, \text{mm} \)

Thread angle: \( \alpha = 30\, \text{deg} \)

Modulus of elasticity of stainless steel screws: \( E_s = 190 \times 10^9 \, \text{Pa} \)

Minimum proof strength of cap screws: \( S_p = 830 \times 10^6 \, \text{Pa} \)

Endurance limit for screws with rolled threads: \( S_e = 162 \times 10^6 \, \text{Pa} \)

Ultimate strength of stainless steel screws: \( S_{uts} = 1040 \times 10^6 \, \text{Pa} \)

Thickness of top component: \( t_1 = 1 \, \text{cm} \)

Reach of screw into bottom component: \( t_2 = 1 \, \text{cm} \)

Length of threaded portion of screws: \( l_t = t_1 \cdot t_2 = 1 \, \text{mm} \)

Length of unthreaded portion of screws: \( l_d = 1 \, \text{mm} \)

Effective grip length: \( l = t_1 + \frac{d}{2} \) for diameter of screw < \( t_2 \)

Maximum width of frustra: \( D_1 = 1.5 \cdot d + 0.577 \cdot l \)

Minimum width of frustra: \( D_2 = 1.5 \cdot d \)

Minimum diameter of frustra: \( D = D_2 \)

Stiffness of shell material being compressed: \( k_m = \frac{0.577 \cdot \pi \cdot E \cdot d}{\ln \left( \frac{(1.15 \cdot (0.5) \cdot D - d) \cdot (D - d)}{(1.15 \cdot (0.5) \cdot D - d) \cdot (D - d)} \right)} \)

Effective stiffness of the cap screws: \( k_b = \frac{A_d \cdot A_t \cdot E_s}{A_d \cdot l_t \cdot A_t \cdot l_d} \)

Joint constant: \( C = \frac{k_b}{k_b - k_m} \)

Pre-load for screws: \( f_i = 0.75 \cdot A_t \cdot S_p \)

Number of screws that load is applied to: \( n_s = 1 \)

Load of screw caused by bending from load at end of tool: \( P = \frac{1.2 \cdot m}{n_s \cdot 14 \, \text{cm}} \cdot m_{load} \cdot a \cdot A \)

Factor of safety for proof load: \( \eta_p = \frac{S_p \cdot A_t - f_i}{C \cdot P} \), \( \eta_p = 26.424 \)
Factor of safety for separation: 
\[ \eta_s = \frac{f_i}{P (1 - C)} \]
\[ \eta_s = 14.96 \]

Strengths for Goodman's factor of safety:
\[ S_a = \frac{f_i}{A_t} \]
\[ S_{uts} = S_a \left( S_{uts} - 1 - \frac{S_a}{S_e} \right) \]
\[ S_m = S_{uts} - 1 - \frac{S_a}{S_e} \]

Estimated stress in the shell around screws:
\[ \sigma_a = \frac{C \cdot P}{2 \cdot A_t} \]

Goodman's Factor of Safety for the shell:
\[ \eta_g = \frac{S_{uts}}{\sigma_a} \]
\[ \eta_g = 14.33 \]

The results from the factor of safeties above show that the screws holding the interface shell to the wrist sub-system are all safe from failure due to the proof load, the separation strength, and the Goodman Criterion.

The following calculations are made in order to determine the minimum acceptable thickness of the shell walls.

This figure shows the interface shell, the location of interest for failure in the shell wall, and the worst-case scenario for a load on the tool that will cause bending, torque, and shear throughout the shell walls.

The following section calculates the factors of safety for the worst-case scenario in which the interface is located in a horizontal position relative to the ground.

Outer diameter of interface shell: \[ D_{\text{int}} = 15 \text{ cm} \]

Inner diameter of interface shell: \[ d_{\text{int}} = 14.5 \text{ cm} \]
Moment of inertia of interface shell about z-axis as shown in figure:

\[ I_{z\text{Wall}} = \frac{\pi}{64} \left( D_{\text{int}}^4 - d_{\text{int}}^4 \right) \]

Polar moment of inertia for the interface shell:

\[ J_{\text{Wall}} = 2.1 \cdot I_{z\text{Wall}} \]

Force of load located at distance shown:

\[ V = m_{\text{load}} \cdot a \cdot A \]

Moment caused by load at interface about z-axis:

\[ M = (1.2 \text{ m}) \cdot V \]

Distance from center of shell to outer edge element of interest:

\[ c = \frac{D_{\text{int}}}{2} \]

Torque exerted on shell due to load shown:

\[ T = V \cdot (25 \text{ cm}) \]

Distance from center to centroid of top half of wall:

\[ a_{\text{top}} = \frac{\pi}{2.4} \left( D_{\text{int}}^2 - d_{\text{int}}^2 \right) \]

Shear flow parameter needed for shear at point of interest:

\[ Q = a_{\text{top}} \cdot \frac{2}{3} \frac{D_{\text{int}}^2 - d_{\text{int}}^2}{(D_{\text{int}} - d_{\text{int}})^2} \pi \]

Thickness of shell cross-section at equator:

\[ t = D_{\text{int}} - d_{\text{int}} \]

Maximum shear stress for shell wall:

\[ \tau_{\text{max}} = \sqrt{\left( \frac{M \cdot c}{2.1 \cdot I_{z\text{Wall}}} \right)^2 + \left( \frac{T \cdot c}{J_{\text{Wall}}} \right)^2 + \left( \frac{V \cdot Q}{I_{z\text{Wall}}} \right)^2} \]

Factor of safety for point of interest:

\[ \eta = \frac{0.4 \cdot S_{\text{ys}}}{\tau_{\text{max}}} \]

\[ \eta = 73.115 \]
7. END EFFECTORS

Tools are located at the end of the robotic arm and will perform their functions by immediately touching the equipment, cargo, etc. that needs to be moved or activated. These interchangeable end effectors will be connected to the robotic arm by an interface and will be stored in an area accessible to the arm. The interface will contain an activation rod to supply both mechanical and electrical power to the tool. Several end effector designs will be presented in this section; however the versatility of the interface allows for additional tools to be designed and implemented as the need arises.

7.1 Functions, Sub-system Requirements

Given the system tasks to be performed, the end effectors must perform the following functions:

- Shovel regolith
- Level a 2 m X 2 m area by distributing regolith
- Extend antennae, push buttons, pull levers and/or turn dials on specially adapted equipment
- Move lunar rocks
- Grasp specialized handles on cargo
- Drill into lunar surface for the purpose of sampling

In light of these functions, there are many sub-system requirements that must be met. Each end effector must retain its shape while performing a task. The tools need to contain a connection to the interface and must stay connected until ejected by the interface. Tools with moving parts should have emergency positioning capabilities in case of failure and these emergency positions need to have manual releases. In addition, the mass of the end effector must be such that the combined total mass of the robot arm does not exceed 100 kg. A mass limit of 7 kg is proposed for each end effector.
7.2 End Effector Design

7.2.1 Shovel

The shovel is flat with two different sides, as shown in Figure 7.1. Its purpose is to push aside large moon rocks as well as to shovel piles of regolith. The material selected was aluminum (Al 2014-T6) due to its tensile strength and low density. The surface area of the shovel is such that a relatively thin layer of regolith can be handled. A thickness of 1 cm, which is greater than the thickness required to overcome stress, was chosen to insure the prevention of deformation due to unforeseen collisions. The shovel will not require any mechanical or electrical power to be supplied to it from the interface and will possess only a handle located at the vertical mass center. The shovel is oriented in a manner such that the interface does not come in contact with objects during operation of the shovel.

![Figure 7.1 Shovel](image)

7.2.2 Large Gripper

The large gripper, shown in Figure 7.2, is a three-pronged device used for moving large rocks and cargo. The large gripper is capable of carrying a spherical lunar rock with a mass of 50 kg. Cargo is adapted with handles, shown in Figure 7.3, designed for use with the large gripper. A cage-like enclosure on the end of the prongs is provided to insure that objects are completely encased by the gripper while being transported. The interface provides mechanical power that activates the piston to open the gripper. Springs are placed between each of the prongs to restore the gripper to the closed
position when the piston is not activated. Steel (ASTM-A514) was used due to its strength. The 2 cm by 2 cm prong cross-sectional area was selected to provide maximum strength while remaining under the mass constraint for the end effector.

Figure 7.2 Large Gripper
7.2.3 Small Gripper

The small gripper, shown in Figure 7.4, is a two-pronged device requiring mechanical power from the interface. The gripper is able to activate specially adapted equipment as well as being able to grasp small objects. Springs keep the gripper in a closed position until the piston activates the prongs. Steel (ASTM-A514) was selected as a material for construction due to its strength. The tips of the prongs are fitted with a space-rated rubber-like material to provide a large coefficient of friction.
7.2.4 Drill

A drill, shown in Figure 7.5, is provided as an example of the interface's capability to support power tools. This drill is used for obtaining mine samples from the lunar surface. The design depicted is based on a Black & Decker #7014 electric drill. A comparable space-rated motor possessing the same specifications will be used in actual construction.

![Figure 7.5 Drill](image)

Figure 7.5 Drill
7.3 Calculations

7.3.1 Shovel

mass of maximum allowable load \( m_{\text{max}} = 50 \text{ kg} \)

gravity on the moon \( g_{\text{m}} = 1.62 \frac{m}{\text{sec}^2} \)

thickness of the shovel \( t = 0.01 \text{ m} \)

length of the shovel \( l = 0.5 \text{ m} \)

Calculating the stress:

\[
\sigma = \frac{g_{\text{m}} m_{\text{max}}}{l \cdot t} \quad \sigma = 1.62 \times 10^4 \cdot \text{Pa}
\]

This stress is much lower than the tensile yield strength of 410 MPa.

Finding the center of gravity

<table>
<thead>
<tr>
<th>Portion</th>
<th>Area (m²)</th>
<th>y</th>
<th>yA</th>
</tr>
</thead>
<tbody>
<tr>
<td>rectangle</td>
<td>((0.5 \text{ m}) \cdot (0.35 \text{ m}))</td>
<td>(y_r = \frac{1}{2} (0.35 \text{ m}))</td>
<td>(A_r y_r = 0.031 \cdot \text{m}^3)</td>
</tr>
<tr>
<td>parabola</td>
<td>(\frac{4}{3} (0.15 \text{ m}) \cdot \frac{0.5}{2} \cdot \text{m} \cdot y_p = 0.5 \text{ m} - \frac{3}{5} (0.15 \text{ m}))</td>
<td>(A_p y_p = 0.021 \cdot \text{m}^3)</td>
<td></td>
</tr>
</tbody>
</table>

\[A_s = A_r - A_p\]
\[y_s = y_r + y_p\]
\[yA = A_r y_r - A_p y_p\]
\[A_s = 0.225 \cdot \text{m}^2\]
\[y_s = 0.585 \cdot \text{m}\]
\[yA = 0.051 \cdot \text{m}^3\]

Distance to center of gravity

\[Y = \frac{yA}{A_s} = 0.227 \cdot \text{m}\]

Calculating the weight:

Volume

\[V_s = A_s \cdot t \quad V_s = 0.002 \cdot \text{m}^3\]

Density of 2014-T6 Al \( \rho = 2800 \frac{\text{kg}}{\text{m}^3} \)

Mass of Shovel: \( \text{Mass}_s = V_s \cdot \rho \quad \text{Mass}_s = 6.3 \cdot \text{kg} \)

The actual mass of the shovel is within the given mass for each end effector.
7.3.2 Large Gripper

number of prongs $p = 3$

thickness of prongs $t_p = 0.2 \text{ m}$

density of lunar rocks $\rho_r = 1400 \text{ kg/m}^3$

for a spherical rock:

$$V = \frac{m_{\text{max}}}{\rho_r} \quad V = 0.036 \cdot m^3 \quad r = \frac{3}{4} \cdot V^{\frac{1}{3}} \quad r = 0.204 \cdot m$$

A spherical rock with a mass of 50 kg has a radius of 20.4 cm. This is the maximum size rock the arm would be required to lift.

Calculating the stress on each prong:

$$\sigma = \frac{8m_{\text{max}}}{tp^2 \cdot p} \quad \sigma = 6.75 \cdot 10^4 \cdot \text{Pa}$$

This stress is much lower than the tensile yield strength of 690 MPa.

Volume of each part

length of each bar $l_A = 0.205 \text{ m}$

$$l_B = 0.250 \text{ m}$$

$$l_C = 0.131 \text{ m}$$

$$l_D = 0.050 \text{ m}$$

total lengths $l_t = l_A + l_B + l_C + l_D$

$$V = l_t \cdot t_p^2 \cdot \rho \quad V = 2.472 \cdot 10^{-4} \cdot \text{m}^3$$

Density of Steel ASTM-AS14 $\rho = 7860 \text{ kg/m}^3$

Mass of Large Gripper: $\text{Mass}_{lg} = V \cdot \rho \cdot \rho \quad \text{Mass}_{lg} = 5.83 \cdot \text{kg}$

The actual mass of the large gripper is within the given mass for each end effector.

Calculate the force required to open the gripper. $N = \text{newton}$

forces $F_A = l_A \cdot \rho \cdot t_p^2 \cdot g \quad F_A = 1.044 \cdot N \quad r_A = 0.2881 \text{ m}$

$F_B = l_B \cdot \rho \cdot t_p^2 \cdot g \quad F_B = 1.273 \cdot N \quad r_B = 0.2015 \text{ m}$

$F_C = l_C \cdot \rho \cdot t_p^2 \cdot g \quad F_C = 0.576 \cdot N \quad r_C = 0.05655 \text{ m}$

$F_D = l_D \cdot \rho \cdot t_p^2 \cdot g \quad F_D = 0.255 \cdot N \quad r_F = 0.05 \text{ m}$

sum of the moments about the pin while the prong is in an open position.

$$F = \frac{F_A \cdot r_A + F_B \cdot r_B + F_C \cdot r_C + F_D \cdot r_D}{r_F} \quad F = 11.672 \cdot \text{N} \quad \text{on each prong}$$

Total force needed to open prongs: $p \cdot F = 35.016 \cdot N$

The interface actuating rod supplies a much greater amount of force.
7.3.3 Small Gripper

- number of prongs: \( p = 2 \)
- thickness of prongs: \( t_p = 0.01 \text{ m} \)

**Volume of each part**

- length of each bar:
  - \( l_A = 0.0796 \text{ m} \)
  - \( l_B = 0.0762 \text{ m} \)
  - \( l_C = 0.0584 \text{ m} \)
- total lengths:
  - \( l_t = l_A - l_B - l_C \)

\[
V = l_t \cdot t_p^2 \cdot \rho = 2.142 \times 10^{-5} \text{ m}^3
\]

*Density of Steel ASTM-AS14*

\( \rho = 7860 \text{ kg/m}^3 \)

**Mass of Small Gripper**

\[
\text{Mass}_{sm} = V \cdot \rho \cdot p = 0.337 \text{ kg}
\]

The actual mass of the small gripper is within the given mass for each end effector.

Calculating the stress on each prong:

\[
\sigma = \frac{g_m \cdot \rho}{t_p^2 \cdot p} \quad \sigma = 4.05 \times 10^5 \text{ Pa}
\]

This stress is much lower than the tensile yield strength of 690 MPa.

Calculate the spring needed.

- Coefficient of friction for rubber on concrete: \( \mu = 0.75 \)

**Weight of object**

\( W = m_{\text{max}} \cdot g \quad W = 81 \text{ N} \)

**Frictional force**

\( F_f = \frac{W}{p} \quad F_f = 40.5 \text{ N} \)

**Normal force**

\( N_f = \frac{F_f}{\mu} \quad N_f = 54 \text{ N} \)

Calculate the force required to open the gripper. \( N = \text{newton} \)

\[
F_A = l_A \cdot p \cdot t_p^2 \cdot g \quad F_A = 0.101 \text{ N} \\
F_B = l_B \cdot p \cdot t_p^2 \cdot g \quad F_B = 0.097 \text{ N} \\
F_C = l_C \cdot p \cdot t_p^2 \cdot g \quad F_C = 0.074 \text{ N}
\]

\( r_A = 0.0762 \text{ m} \)

\( r_B = 0.0381 \text{ m} \)

\( r_C = 0.0292 \text{ m} \)

\( r_s = 0.0584 \text{ m} \)

\( r_N = 0.0796 \text{ m} \)

\( r_p = 0.0584 \text{ m} \)
sum of the moments about the pin while the prong is in an open position and holding an object.

\[ F_s = \frac{F_A \cdot r_A - F_B \cdot r_B - F_C \cdot r_C - N \cdot r_N}{r_s} \]

\[ F_s = 73.761 \cdot N \quad \text{spring force on each prong} \]

calculate the spring constant

maximum displacement \( x = 0.04 \cdot m \)

\[ k = \frac{F_s}{x} \quad k = 1.844 \cdot 10^3 \cdot \frac{N}{m} \]

sum of the moments about the pin while the prong is in an open position.

\[ F_p = \frac{F_A \cdot r_A + F_B \cdot r_B - F_C \cdot r_C - F_s \cdot r_s}{r_p} \]

\[ F_p = 73.919 \cdot N \quad \text{piston force on each prong} \]

Total force needed to open prongs: \( p \cdot F_p = 147.839 \cdot N \)

The interface actuating rod supplies a greater amount of force.
Figure 7.6 Large Gripper Dimensions
Figure 7.6 Small Gripper Dimensions

ALL DIMENSIONS IN CENTIMETERS
8. SYSTEM CONTROLS

8.1 Introduction

Proper operation of the mechanical systems requires the integration of supportive hardware. Such hardware must serve a number of functions which help satisfy the performance objectives of the robotic arm. These functions include providing a means of controlling system processes.

8.2 Controls System Selection

The controls system places the functionality of the entire robotic arm in the hands of the user. With the agitation of hand held joysticks at a computer console the input is received, processed and delivered to the output devices by this system.

The selection of this system involved many important considerations. Compumotor and Digiplan, Incorporated offers motion control systems based on the following technologies.

8.2.1 Position Control

"The position of the motor is controlled digitally by the indexer. The incremental nature of a causes the amount of motor movement to be equal to the number of pulses applied. For example, a motor with a resolution of 25,000 steps/rev will move 2 revolutions upon receipt of 50,000 pulses" (E5).

Note: An Indexer is a programmable motion controller used for single or multi-axis motion control with I/O as an auxiliary function (A78).

8.2.2 Velocity

Incremental motion systems characteristically have discrete increments of motion which respond to each pulse generated by the indexer. "A string of pulses of a given frequency will cause the motor to move at a velocity proportional to that frequency. For example, an applied 25 kHz pulse train will cause a motor with a resolution of 25000 steps/rev to rotate at exactly one rev/sec. Several axes of motion can be ratioed synchronously with simple frequency control" (E5).
8.2.3 Acceleration/Deceleration Control

A typical indexer controlled motion profile (velocity vs. time) is trapezoidal. The motor accelerates then continues to run at a constant velocity for a period of time, then decelerates to a stop.

All Compumotor and Digiplan indexers automatically select the optimum move profile based on the commanded acceleration, deceleration, velocity and distance parameters.

8.2.4 Indexer

All Compumotor indexers provide programmable acceleration, velocity, and position control. The right indexer for a given application can be determined by considering the following factors:

8.2.4.1 Interface Type

"The source of motion control commands may be a computer programmable controller, custom logic or an operator" (E5).

8.2.4.2 Packaging

"It is important to consider how the indexer's physical configuration fits with other equipment" (E5).

8.2.4.3 Front Panel Control

Some indexers offer front panel thumbwheels for selecting acceleration, velocity, and position. These controls are useful for testing and operator setup. Dedicated computer or programmable controller interfaces often eliminate the need for front control panels.

8.2.4.4 Open vs. Closed Loop Control

"Critical positioning applications may require constant motor monitoring or positioning capabilities in excess of the microstepping motor/drive's normal capabilities. Some indexers provide the electronic circuitry necessary to utilize a position sensor--typically an optical encoder--to detect stalls and correct positional errors. Servo-based motors/drives--like those used throughout this design--include an integral brushless resolver feedback element, eliminating the need for encoder or tachometer circuitry in the indexer" (E5).
8.3 Selected Control System

The controller selected for the robotic arm is that of a ZXF Series for brushless series systems offered by the Compumotor Digiplan Corporation. Features of the ZXF Series is listed below:

- Digital brushless servo system
- RS-232C command interface; up to 16 devices per port
- Optical isolation for high noise immunity
- Battery backed RAM for storage of up to 99 sequences
- User definable resolutions up to 65,536 steps per revolution
- Built-in power supply
- Diagnostic display
- Extended edition X programming language
- Motion Profiling - allows change of velocity, distance or output base on distance travelled without stopping
- Conditional branching commands: IF/THEN/ELSE, WHILE, REPEAT/UNTIL, GOTO, GOSUB
- Complex evaluations such as input states, boolean logic, and mathematical comparisons for program branching
- BCD thumbwheel interface for entering motion or program parameters
- Program debugging tools - single step, trace and I/O simulation
- Separate acceleration and deceleration commands
- 50 user defined variables
- Math functions - add, subtract, multiply and divide
- Registration inputs - high level interrupt for repeatable registration sensing
- Control of speed base at a ratio of a master axis speed
- Makes preset moves at a velocity ratio of a master axis
- Synchronize speed to a master axis based on registration marks
- Jog in the following mode of assist set up
- Incrementally change the following ratio using the front panel pushbuttons or thumbwheels
- Change following ratios on the fly without affecting distance move accuracy
- Motion profiling capabilities based on the primary encoder position
REFERENCES


3 Ibid.
BIBLIOGRAPHY


Appendix A:
Motor Specifications
### Specifications—Z Drive

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Performance</td>
<td></td>
</tr>
<tr>
<td>Repeatability</td>
<td>±0.088°, unloaded</td>
</tr>
<tr>
<td>Resolver accuracy</td>
<td>±7 arc min (range 200-65,536 steps/rev)</td>
</tr>
<tr>
<td>R/D converter accuracy</td>
<td>±8 arc min</td>
</tr>
<tr>
<td>Speed/torque</td>
<td>Performance curves for each model on page B38 and B39</td>
</tr>
<tr>
<td>Resolution</td>
<td>5,000 steps/rev factory default—65,536 steps/rev (max)</td>
</tr>
<tr>
<td>Input Power</td>
<td></td>
</tr>
<tr>
<td>Voltage nominal</td>
<td>208 to 252 VAC 3-phase</td>
</tr>
<tr>
<td>Frequency</td>
<td>47 to 66 Hz</td>
</tr>
<tr>
<td>Current</td>
<td>15 amps max continuous (RMS)—Z 600 series</td>
</tr>
<tr>
<td></td>
<td>30 amps max continuous (RMS)—Z 900 series</td>
</tr>
<tr>
<td>Command interface</td>
<td></td>
</tr>
<tr>
<td>Step input</td>
<td>Low going pulse, Minimum pulse width is 200 nSec. Maximum pulse rate is 2.5 MHz.</td>
</tr>
<tr>
<td>Direction input</td>
<td>Logic high = CW rotation</td>
</tr>
<tr>
<td></td>
<td>Logic low = CCW rotation</td>
</tr>
<tr>
<td></td>
<td>Can be reversed through software (SSM Command)</td>
</tr>
<tr>
<td>Shutdown</td>
<td>Logic high = Amplifier disable</td>
</tr>
<tr>
<td></td>
<td>Logic low = Normal operation</td>
</tr>
<tr>
<td>Analog input</td>
<td>+/-10V differential signals</td>
</tr>
<tr>
<td>All inputs are optically</td>
<td>Voltage low = 0.4V maximum</td>
</tr>
<tr>
<td>isolated and require TTL</td>
<td>Voltage high = 2.5-5.0V</td>
</tr>
<tr>
<td>level signals to operate</td>
<td></td>
</tr>
<tr>
<td>Outputs</td>
<td>Differential, optically isolated signals</td>
</tr>
<tr>
<td>CHA, CHB, CHZ</td>
<td>Voltage low = 0.5V max. referenced to isolated ground</td>
</tr>
<tr>
<td>RTO</td>
<td>Voltage high = 2.5-5.0V referenced to isolated ground</td>
</tr>
<tr>
<td>Output voltages</td>
<td></td>
</tr>
<tr>
<td>Interface RS-232C</td>
<td>9,600 Baud (configurable)</td>
</tr>
<tr>
<td>Baud</td>
<td></td>
</tr>
<tr>
<td>Data bits</td>
<td>8</td>
</tr>
<tr>
<td>Stop bits</td>
<td>1</td>
</tr>
<tr>
<td>Parity</td>
<td>None</td>
</tr>
<tr>
<td>Environmental</td>
<td></td>
</tr>
<tr>
<td>Operating Driver</td>
<td>32°F to 122°F max (0°C to 50°C) with adequate air flow (10 cfm)</td>
</tr>
<tr>
<td></td>
<td>Maximum heatsink temperature is 160°F (71.1°C)</td>
</tr>
<tr>
<td>Motor</td>
<td>32°F to 104°F max (0°C to 40°C). Max motor case temperature is 25°F (1259).</td>
</tr>
<tr>
<td>Storage</td>
<td>-4°F to 185°F (-40°C to 85°C)</td>
</tr>
<tr>
<td>Humidity</td>
<td>0-95% non-condensing</td>
</tr>
<tr>
<td>Technical data</td>
<td>Complete motor data on each model on pages B41 and B42.</td>
</tr>
</tbody>
</table>

---

![Diagram of communication components](image)
Series
Brushless Servo Systems

Torque/Speed Curves

Z-605
ZX-605
ZXF-605

Z-606
ZX-606
ZXF-606

Z-610
ZX-610
ZXF-610

Z-620
ZX-620
ZXF-620

Z-630
ZX-630
ZXF-630

Z-635
ZX-635
ZXF-635

Z-640
ZX-640
ZXF-640

ORIGINAL PAGE IS OF POOR QUALITY

B38
PROCEEDING PAGE BLANK NOT FILMED
Torque/Speed Curves

Z Series Shunt Regulator
The Z shunt regulator monitors the Z Drive's internal DC bus voltage. If the bus voltage rises above a preset value, the regulator dumps some of the excess power into resistors to reduce the bus voltage and prevent an overvoltage fault. Rapidly decelerating high inertial loads from high velocities can produce these conditions. Shunt regulators are not required for most applications; they simply enhance the system allowing higher deceleration rates than are otherwise possible.

There are two sizes of shunt regulators, a 400-watt (Z-shunt-400W H14'' x W2'' x D11.5'') and an 800-watt (Z-shunt-800W H14'' x W4'' x D11.5'') version. The higher wattage shunt regulator dumps more regenerated energy and generally allows lower move times. Multiple shunt regulators may be connected to one drive or multiple drives to a single shunt regulator. The Z-drive and shunt regulator combination give the highest system performance possible with the Z Series brushless servo systems.

To determine if your application can benefit from the addition of a shunt regulator, contact your local Automation Technology Center or call our application engineering department at 1-800-358-9070 and ask for technical bulletin #169.
Features of Compumotor Servo Motors

The Z motor family consists of brushless, 3-phase, AC motors. The basic construction of a Z motor is shown to the right. The permanent magnets are securely held in place by metal bands to allow high speed performance. The rotors are precision balanced, resulting in both low and high speed smoothness. The windings are located in the outer portion of the motor (stator). This "inside-out" construction allows better heat dissipation than conventional brush type motors. As a result, higher continuous torque and horsepower ratings are achieved for a given motor size.

### Technical Data

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Continuous stall torque oz-in</td>
<td>346</td>
<td>633</td>
<td>867</td>
<td>1,743</td>
<td>2,475</td>
<td>2,319</td>
<td>4,114</td>
<td>2,407</td>
<td>4,263</td>
<td>5,990</td>
<td>9,02</td>
</tr>
<tr>
<td></td>
<td>Nm</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Peak torque oz-in</td>
<td>1,083</td>
<td>1,954</td>
<td>1,733</td>
<td>3,486</td>
<td>4,951</td>
<td>4,638</td>
<td>8,228</td>
<td>5,205</td>
<td>8,525</td>
<td>11,980</td>
<td>18,04</td>
</tr>
<tr>
<td></td>
<td>Nm</td>
<td>66</td>
<td>122</td>
<td>108</td>
<td>218</td>
<td>309</td>
<td>514</td>
<td>325</td>
<td>533</td>
<td>749</td>
<td>1,12</td>
</tr>
<tr>
<td>Rated power hp</td>
<td>2</td>
<td>2.1</td>
<td>4.2</td>
<td>5.6</td>
<td>5.4</td>
<td>5.4</td>
<td>5.9</td>
<td>9.6</td>
<td>10.4</td>
<td>11.0</td>
<td>11.0</td>
</tr>
<tr>
<td></td>
<td>k Watts</td>
<td>1.49</td>
<td>1.57</td>
<td>3.13</td>
<td>4.18</td>
<td>4.03</td>
<td>4.24</td>
<td>4.40</td>
<td>7.2</td>
<td>7.8</td>
<td>8.2</td>
</tr>
<tr>
<td>Rated speed rpm</td>
<td>6,200</td>
<td>3,600</td>
<td>7,000</td>
<td>3,700</td>
<td>2,500</td>
<td>3,000</td>
<td>1,600</td>
<td>5,000</td>
<td>3,150</td>
<td>2,300</td>
<td>1,50</td>
</tr>
<tr>
<td></td>
<td>rps</td>
<td>103</td>
<td>60</td>
<td>117</td>
<td>62</td>
<td>42</td>
<td>50</td>
<td>27</td>
<td>83.3</td>
<td>52.5</td>
<td>38.3</td>
</tr>
<tr>
<td>Rated current A (rms)</td>
<td>5</td>
<td>5.3</td>
<td>14.1</td>
<td>14.1</td>
<td>14.1</td>
<td>14.1</td>
<td>14.1</td>
<td>14.1</td>
<td>27.2</td>
<td>27.7</td>
<td>28.3</td>
</tr>
<tr>
<td>Peak current (3.3 sec max) A (rms)</td>
<td>16.6</td>
<td>17.2</td>
<td>28.2</td>
<td>28.2</td>
<td>28.2</td>
<td>28.2</td>
<td>28.2</td>
<td>28.2</td>
<td>56.6</td>
<td>56.6</td>
<td>56.6</td>
</tr>
<tr>
<td>Max cont AC input power (3 phase 240 VAC) A (rms)</td>
<td>6</td>
<td>6</td>
<td>15</td>
<td>15</td>
<td>15</td>
<td>15</td>
<td>30</td>
<td>30</td>
<td>30</td>
<td>30</td>
<td>30</td>
</tr>
<tr>
<td>Rotor inertia oz-in (mass)</td>
<td>5.45</td>
<td>9.45</td>
<td>13.73</td>
<td>35.87</td>
<td>50.79</td>
<td>56.21</td>
<td>111.21</td>
<td>50.79</td>
<td>111.21</td>
<td>166.21</td>
<td>459.4</td>
</tr>
<tr>
<td></td>
<td>kg m^2 x 10^6</td>
<td>99.6</td>
<td>172.9</td>
<td>251.2</td>
<td>656</td>
<td>929</td>
<td>1,028</td>
<td>2,034</td>
<td>929</td>
<td>2,034</td>
<td>3,040</td>
</tr>
<tr>
<td>Motor weight lbs</td>
<td>10.0</td>
<td>14.0</td>
<td>17.0</td>
<td>29.0</td>
<td>32.0</td>
<td>37</td>
<td>51.0</td>
<td>32.0</td>
<td>57.0</td>
<td>65.0</td>
<td>112.0</td>
</tr>
<tr>
<td></td>
<td>kg</td>
<td>4.5</td>
<td>6.4</td>
<td>7.7</td>
<td>13.2</td>
<td>14.5</td>
<td>16.8</td>
<td>23.2</td>
<td>15.0</td>
<td>26.0</td>
<td>29.0</td>
</tr>
<tr>
<td>Shipping weight lbs</td>
<td>52.0</td>
<td>55.0</td>
<td>58.0</td>
<td>71.0</td>
<td>74.0</td>
<td>79.0</td>
<td>93.0</td>
<td>89.0</td>
<td>114.0</td>
<td>122.0</td>
<td>169.0</td>
</tr>
<tr>
<td></td>
<td>kg</td>
<td>23.6</td>
<td>25.0</td>
<td>26.4</td>
<td>32.3</td>
<td>33.6</td>
<td>35.9</td>
<td>42.3</td>
<td>40.0</td>
<td>52.0</td>
<td>55.0</td>
</tr>
</tbody>
</table>

**ORIGINAL PAGE IS OF POOR QUALITY**
### Dimensions

<table>
<thead>
<tr>
<th>Model</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>Z635</td>
<td>11.78 (299.2)</td>
<td>6.70 (170.2)</td>
<td>1.967 (49.96)</td>
<td>1.45 (37.01)</td>
<td>0.945 (24.03)</td>
</tr>
<tr>
<td>Z640</td>
<td>14.48 (367.8)</td>
<td>9.49 (241.0)</td>
<td>1.967 (49.96)</td>
<td>1.457 (37.01)</td>
<td>0.945 (24.03)</td>
</tr>
<tr>
<td>Z920</td>
<td>14.48 (367.8)</td>
<td>9.49 (241.0)</td>
<td>1.967 (49.96)</td>
<td>1.457 (37.01)</td>
<td>0.945 (24.03)</td>
</tr>
<tr>
<td>Z930</td>
<td>17.18 (436.4)</td>
<td>10.87 (276.2)</td>
<td>3.00 (76.20)</td>
<td>2.25 (57.15)</td>
<td>1.260 (32.00)</td>
</tr>
</tbody>
</table>

#### Keyway Detail

**Z635, Z640, Z920**

- **A**
  - 0.004 (0.10) TIR
  - 0.68 (17.5)
  - 0.25 (6.3)
  - 0.998 (24.9)

**Z930**

- **A**
  - 0.004 (0.10) TIR
  - 0.68 (17.5)
  - 0.25 (6.3)
  - 0.998 (24.9)

---

PRECEDING PAGE BLANK NOT FILMED

B42  
ORIGINAL PAGE IS OF POOR QUALITY
Appendix B:
Time Line
<table>
<thead>
<tr>
<th>Activities</th>
<th>January 1993</th>
<th>February 1993</th>
</tr>
</thead>
<tbody>
<tr>
<td>Preliminary Design Phase</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Project Selected</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Company Based Structure Developed</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Design-Build Officers Assigned</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Subsystem Alternatives Discussed</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Preliminary Research</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Written Proposal Report Submitted</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Oral Progress Report Given</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Conceptual Design Phase</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Research</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Environmental Concerns &amp; Constraints</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Material Selection &amp; Attributes</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lunar Environment</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vehicle Identification</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mission Objective Defined</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Performance Objectives Defined</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Environmental Requirements Defined</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Design Subsystems Defined</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Subsystem Groups and Managers Assigned</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Subsystem Requirements Defined</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Subsystem Functions Defined</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Review of Conceptual Design</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Oral Progress Review Given</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intermediate Design Phase</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Subsystems Analysis &amp; Redesign</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Subsystems Design Requirements Specified</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mass, Volume, &amp; Power Estimates Defined</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Power Availability Defined</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Integration of Subsystems</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intermediate Design Review</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Oral Progress Review Given</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Final Design Phase</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Completion of Subsystems Analysis</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Completion of Integration of Subsystems</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Final Design Complete</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Solid Modeling of System on CMS</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Final Design Review</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Report Integration Phase</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Identification of Report Team</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Report Director Assigned</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Report Format Decided On</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Preliminary Draft Completed</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Preliminary Draft Revised</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Preliminary Draft Corrected</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Final Draft Completed</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Practice Report Presentation</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Final Report Presentation</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>