

NASW-4435

IN-37-CR

30476

P-122

**Lunar Surface Operations**  
**Volume III:**  
**Robotic Arm for Lunar Surface Vehicle**

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**NASA/USRA University Advanced Design Program**  
**Annual Report**  
**July, 1993**

(NASA-CR-195553-Vol-3) LUNAR  
SURFACE OPERATIONS. VOLUME 3:  
ROBOTIC ARM FOR LUNAR SURFACE  
VEHICLE Annual Report (Florida  
State Univ.) 122 p

N95-16031

Unclas

G3/37 0030476

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# 1. INTRODUCTION

## 1.1 Purpose of Document

The purpose of this design report is three-fold:

- To effectively communicate design concepts.
- To empower the customer and future designers to optimize system performance as well as modify its capabilities based on more specific needs and/or future demands.
- To satisfy course requirements as detailed by Dr. William Shields, instructor of EML4558 for the spring semester of 1993.

## 1.2 Premise for Robotic Arm Design

The premise for the design is based on the intentions of the National Aeronautics and Space Administration (NASA) to establish a lunar based colony by the year 2010. Provisions for such a colony include developing reliable systems to support lunar operations within this time frame. NASA currently supports a program that encourages competitive designing among college students for a number of these systems. One of these systems is the robotic arm for a lunar surface vehicle.

## 1.3 Project Organization

For the design of the robotic arm the following company based structure is employed among the design group members. At the head of the design structure is the Project Manager. Reporting to the Project Manager are the Chief Systems Engineer, Deputy Project Manager, and Principal Investigator. Also, in line below the Chief Systems Engineer are five subsystems. These subsystems are the Mechanical Structure, Wrist, Structure-to-End Effector Interface, End Effectors, and Controls, Sensors, and Cameras. A schematic diagram of the company based structure is shown in Figure 1.1.

The following is a description of the five principal positions:

Project Manager: Oversees entire design group.

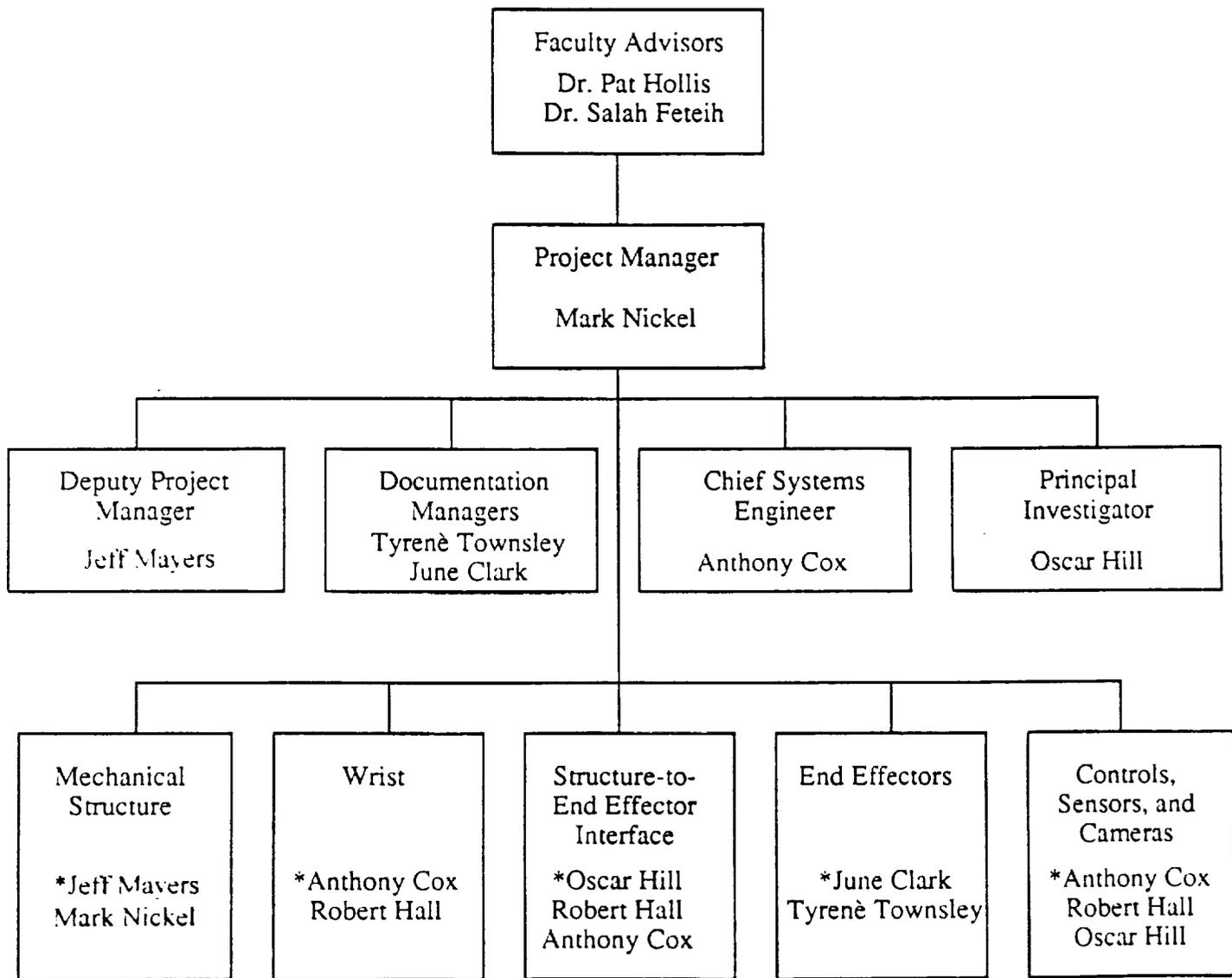
Deputy Project Manager: Works directly with Chief Systems Engineer and Principal Investigator. Also reports directly to Project Manager.

Documentation Managers: Responsible for report preparation and internal communication.

Chief Systems Engineer: In charge of interface control, requirements analysis, integration, and verification (testing, analysis, etc.).

Principal Investigator: Responsible for application of robotic arm to NASA's needs.

A time line of the activities is given in Appendix B.



\* Denotes Sybssystem Manager

Figure 1.1 Company Based Structure

## 2. MISSION STATEMENT AND REQUIREMENTS

### 2.1 Vehicle Mission Statement

The previous design of an Extended Mission/Lunar Rover from the Aerospace Final Design Report 1991-92 outlined a vehicle for a 28 earth day mission for four crew members. The time frame for this vehicle is for the years 2010 to 2030 for second generation lunar exploration. Within this report, the design team identified the need for a robotic arm that would be mounted on the rover.

The vehicle's mission requirements as identified by the aforementioned team are "to provide transportation, shelter and working quarters for a crew of four on long duration lunar surface mission."<sup>1</sup> For the design of a robotic arm attached to this vehicle, the pertinent requirements given in their report are as follows:

- Mission Distance: 1000 km round trip
- Mission Duration: 28 earth days (1 lunar day)
- Transport various experimental apparatus
- Possess robotic data sample/data collection capability
- Collect/analyze/store data
- Provide shielding from environmental elements
- Internal navigational support
- Possess path-clearing abilities
- Travel over rough terrain (45° head-on, 20° traverse)
- Provide redundant systems
- Easily maintained

### 2.2 Robotic Arm Mission Statement

The robotic arm's mission requirements are to "incorporate key issues of compactness, versatility, reliability, accuracy, and weight" to assist in handling cargo and equipment, and to remove obstacles from the path of the vehicle. Mission scenarios would include, but not be limited to the following:

- Exploration
- Lunar sampling
- Replace and remove equipment
- Set-up equipment (e.g. microwave repeater stations)

### 3. SYSTEM DESIGN AND INTEGRATION

#### 3.1 Performance Objectives

Performance objectives for the robotic arm include a reach of 3 m, accuracy of 1 cm, arm mass of 100 kg, and lifting capability of 50 kg. The arm is able to safely complete a task within a reasonable amount of time; the actual time is dependent upon the task to be performed. The positioning of the arm includes a manual backup system such that the arm can be safely stored in case of failure. No maintenance is required for the duration of the 28 earth day mission. Remote viewing and proximity and positioning sensors are incorporated in the design of the arm.

#### 3.2 Robotic Arm Tasks

##### 3.2.1 Cargo Handling

The end effectors must grip various sizes and shapes of cargo. The said cargo must be no more than the weight capacity of the arm and be equipped with a uniform handle. The handling of cargo requires lifting, lowering, and any other mode of translation from vehicle to vehicle, vehicle to surface, or any combination thereof. This task can be performed only if the origination point, path, and termination point are within the operating range of the robotic arm. Upon completion of the cargo handling task, the end effector will disengage the cargo.

##### 3.2.2 Equipment Setup

During operation, the end effectors must push, pull, turn, lift, or lower various types of equipment. Any equipment to be used must conform to the abilities of the robotic arm (i.e. must be less than 50 kg in mass and must be designed in accordance with the 1 cm precision requirement). The robotic arm must possess the following capabilities:

1. Push objects in all directions in 3-D space within the operating range of the robotic arm, such as pressing a button or knob or sliding objects
2. Pull objects in all directions in 3-D space within the operating range of the robotic arm, including extending an object such as an antenna or an unfolding solar array and unlatching locks and safety devices (pulling is distinguished

from pushing in that the end effector must physically grasp the intended object)

3. Turn objects in 3-D space within the operating range of the robotic arm, such as turning a dial or knob
4. Lift or lower objects in 3-D space within the operating range of the robotic arm, such as flipping a switch

### 3.2.3 Path Clearing

The end effector must clear a path on the lunar surface by shoveling, sweeping aside, or gripping the obstacles present in the desired path. In clearing a path, the materials or obstacles encountered will be moved to a location within the range of the robotic arm. The functions of a path clearing mechanism are listed below.

1. Shovel a maximum volume of regolith (equivalent to 50 kg in mass) to a desired location
2. Sweep aside regolith or small lunar rocks in order to clear a path for travel or equipment placement
3. Transport large lunar rocks or other obstacles that inhibit the desired path of the vehicle

### 3.3 Environmental Factors

The following environmental information is needed to assure that the design is appropriate. The radiation, temperature range, size of dust, pressure on the surface, magnetic field, and the type of meteorite activity on the moon are values to be considered.

Radiation = 1000 REM total during the 11 year solar cycle

Gravity =  $1.62 \text{ m/s}^2$

Temperature = 400 K to 80 K

Soil grain size = 2 to 60  $\mu\text{m}$ , with 50% of grains less than 10  $\mu\text{m}$

Pressure =  $10^{-12}$  Torr ( $1.3 * 10^{-10}$  Pa)

Magnetic field = no general magnetic field on the moon (dipole field is less than  $\sim 5 * 10^{-5}$  times earth's)

### 3.4 System Power Requirements

The power requirements for the system are detailed by device, subsystem, and total system. The purpose of listing such information is to expedite integrating of the Robotic Arm Design into larger systems such as the Lunar Rover.

#### Power Requirement Distribution

Subsystem	Peak Power
Mechanical Structure	6.0kW
Motor (4)	1.5kW
Modified Wrist Joint	4.5kW
Motor (3)	1.5kW
Interface Subsystem	1.5kW
Motor (1)	1.5kW
End Effector	15.0W
Instrument (1)	12.0W
Power Tool (1)	15.0W
System Total ( Subsystem Peak Powers)	12.015kW

## 4. MECHANICAL STRUCTURE

As mentioned, the design of the robotic arm was divided into five distinct subsystems. The mechanical structure of the robotic arm comprises one of these five subsystems. The extent of the robotic arm contained on the mechanical structure ranges from the base, which is attached to the lunar vehicle, to the portion of the robotic arm where the structure and the wrist section meet.

### 4.1 Subsystem Functions and Requirements

In meeting the requirements and functions for the robotic arm as a whole, the mechanical structure is also required to meet additional stipulations assigned by the design team. As stated in the mission statement and requirements section, the robotic arm is to be able to lift a maximum mass of 50 kg while having a maximum mass of 100 kg. Working within these requirements the subsystem requirements for the mechanical structure are given as follows:

1. Given a total mass of 100 kg, the mechanical structure is not to exceed a maximum of 65 % of this total, or 65 kg.
2. Materials selected for the mechanical structure are to possess a balance between material properties and light weight.
3. Structure is to be able to safely withstand applied stresses and allow for containment of cables, wires, controls, and sensors within the structure if needed.
4. Environmental effects on the lunar surface should not affect the material chosen for the structure.
5. Structure should exhibit means of providing protection from dust and other debris for joints, gears, and other mechanisms.

### 4.2 Mechanical Structure Design

In designing the mechanical structure the first order of business was to make a material selection. The materials utilized should represent a balance between mass density and various other material properties. The parameters involved in making a material choice are listed below.

1. Density - Low mass is of prime concern given the limited ability to transfer equipment and material into space and cost incurred in doing so.
2. Coefficient of thermal expansion - Given the large temperature range in which the arm is to operate, it is of great importance that the materials be capable of withstanding such a range with minimal effects.
3. Yield strength in shear, tension, and compression - The materials must exhibit strengths large enough to safely handle the stresses incurred during operation.
4. Radiation effects - The materials must be able to withstand large doses of radiation on a constant basis with no (or minute) effects to the structure of the materials.
5. Machining capability - The materials need to be machined at acceptable costs and tolerances.

Based on the above mentioned characteristics, the materials selection process was carried out and aluminum 2014-T6 was chosen as the material for the mechanical structure. Of the materials presently in use by the National Aeronautics and Space Administration, aluminum 2014-T6 was selected because it possesses high strengths at a relatively low mass density in comparison with the other materials. Table 4.1 represents the comparison between different materials considered.

Once the material was selected, the layout of the arm was the next step in the design process. After studying the various existing designs in the world of robotics, it was decided that either a three arm or two arm structure would be appropriate. In addition, a configuration utilizing more than three arms presents a much more difficult system to analyze, control, and design. Therefore, only two and three arm systems were considered. After further consideration a three arm configuration was chosen. The main reason a three arm configuration was chosen over a two arm configuration is that the three arm configuration provides a much greater operating envelope around the vehicle.

**Table 4.1 Table of material properties for mechanical structure**

Material	Density kg/cu. m	Yield strength in tension MPa	Machining capability	Coefficient of thermal exp.
Aluminum 2014-T6	2800	410	good	23.0E-6
Aluminum 1100-H14	2710	95	good	23.6E-6
Stainless Steel 302	7920	520	fair-good	17.3E-6
Titanium	4460	825	fair	9.5E-6
Carbon Graphite	1666	833 - 1528	poor-fair	

#### 4.2.1 General Layout of Mechanical Structure

With the decision made to use a three arm configuration, the general layout of the mechanical structure was the next step to be completed in the design process. The base of the arm was decided to be mounted on the lunar vehicle on the lower portion of the vehicle. The specified height from the lunar surface to where the base is to be mounted on the vehicle is 1.586 m. The three portions of the mechanical structure shown in Figure 4.1 are called arm one, arm two, and arm three, with arm one being closest to the vehicle. At the base, arm one of the structure is to mount on the base via a revolute joint. Similarly, arm one and arm two, and arm two and arm three are to be connected together by use of revolute joints. In addition to these three revolute joints, a translational joint is to be incorporated into the design of arm one.

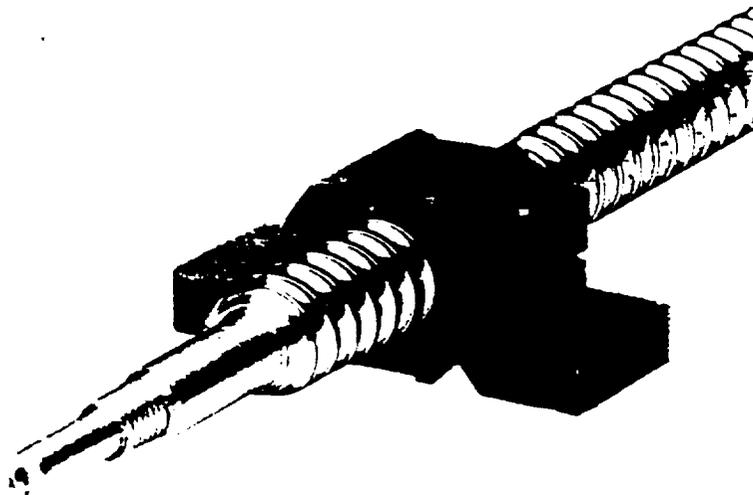
The design of the three arms was decided, after looking at alternative configurations, to be of a hollow cylindrical cross section. The basis for this decision was that a hollow cylindrical cross section provides even stress distribution throughout the arm walls and provides space inside the arms for storage of cables, wires, controls, and sensors throughout the length of the structure.

## 4.2.2 General Layout of Joints

With the layout of the mechanical structure known, it was possible to design the layouts of each joint. In designing the joint layouts many alternatives were considered. Taking into account the harsh lunar environment, many of the alternative solution principles were deemed unacceptable. For instance, because of the great temperature range present in the lunar environment the use of hydraulics was not feasible.

For the three revolute joints, the base joint, and the two joints connecting the three arms, the operating principle is to use gear trains at specified ratios and a motor mounted on top of the preceding arm driving the gears. For the joint at the base, the motor will be mounted to the base itself. For the translational joint, translational motion of the motor is changed into linear motion for extension of the arm using a power screw which is mounted on two sets of rolling bearings. One set of the bearings is mounted to the inside of the two connecting sections of arm one. Figure 4.2, Figure 4.3, and Figure 4.4 illustrate the layouts of the joint connecting arm one and arm two, the joint connecting arm two and arm three, and the translational joint of arm one respectively.

For all components of the joints such as the power screw, axles, gears, or any other component encountering large forces, the material chosen was stainless steel (302). In addition, power for the motors is to be supplied via a power supply located in the lunar vehicle itself and the necessary cables and wires are contained within the arms of the robotic structure. Holes are drilled in the arms at the appropriate locations to allow for the cables and wires to be connected to the motors.



**Figure 4.4 Translational Joint**

### 4.2.3 Additional Design Concerns

In addition to the actual design of the joints and arms, consideration must be given to protection of the joints, motors and other sensitive parts. For the gear trains at each joint there is to be a cover composed of thin sheet metal. This sheet metal is to be constructed of stainless steel (302) and can either be bolted or welded to the arms. Furthermore, a protective boot is to be placed at each joint. The boot will cover the connection between the two arms, the gear train (already covered by the sheet metal), and the motor at each joint. The reason for covering the gear trains with the sheet metal coverings is to insure the boot will not be worn down by the rotation of the gears. Table 4.2 illustrates the factors involved in comparing the many different rubber materials available for use in the protective boot.

**Table 4.2 Table of material properties for protective boot.**

key:	E=excellent	G=good	F=fair	P=poor		
material	Natural rubber	GR-S	Neoprene	Nitrile rubbers	Butyl	Thiokol
resistance to heat	G	F	G	E	G	P
resistance to cold	E	G	G	G	E	P
aging properties	E	E	G	G	E	G
resistance to sunlight	F	F	E	G	E	E
resilience	E	G	G	F	F	P

After comparison of the materials given in Table 4.2, it was decided that a butyl based rubber should be used for the protective boot on the robotic arm. Butyl based rubbers are synthetic elastomers and are made from petroleum raw materials. The important properties that butyl based rubbers exhibit are their excellent resistance to both cold and hot temperatures as well as their resistance to sunlight.

Furthermore, the question of reliability needed to be answered. As far as the mechanical structure was concerned, this identified a need to offer a manual backup to the motors at each joint in case of failure. This is accomplished by leaving small openings in the rubber boot and sheet metal casings at each joint. In case of failure the astronauts could then manually turn the motor by means of a wrench type device (similar to an allen wrench but larger) that would be inserted through the holes in the boot and casing and into a groove in the end of the axle protruding from the motor. Concerning possible contamination of the motor and joint area, the holes in the steel casings and rubber boots can have a lift-up type cover that the astronaut simply removes before insertion of the tool.

### 4.3 Calculations

The analysis for the mechanical structure of the robotic arm is shown below. For the analysis the worst case scenario was taken at all steps to ensure proper performance.

The requirements for the robotic arm stipulated a maximum mass of 100 kg.

For the mechanical structure of the arm aluminum 2014-T6 (4.4% Cu) was chosen. The properties of this material are listed below.

Density:  $\rho = 2800 \cdot \frac{\text{kg}}{\text{m}^3}$  Coefficient of thermal expansion:  $\alpha_T = 23.0 \cdot 10^{-6} \cdot \frac{1}{\text{K}}$

Modulus of elasticity:  $E = 72 \cdot 10^9 \cdot \text{Pa}$  Modulus of rigidity:  $R = 27 \cdot 10^9 \cdot \text{Pa}$

Tension yield strength:  $S_{yt} = 410 \cdot 10^6 \cdot \text{Pa}$  Shear yield strength:  $S_{ys} = 220 \cdot 10^6 \cdot \text{Pa}$

Tension ultimate strength:  $S_{ut} = 480 \cdot 10^6 \cdot \text{Pa}$  Shear ultimate strength:  $S_{us} = 290 \cdot 10^6 \cdot \text{Pa}$

Also, there are properties of the moon that will affect the analysis.

Moon's acceleration due to gravity:  $g = \frac{g}{6}$   $g = 1.634 \cdot \frac{\text{m}}{\text{sec}^2}$

Maximum average density of moon surface material:  $\rho_{\text{moon}} = 1.9 \cdot \frac{\text{gram}}{\text{cm}^3}$

The assigned lengths of each member of the robotic arm.

Arm #1: arm #1 is composed of three sections; a, b, and c. Section a is a hollow cylindrical section beginning at the base. Section a then connects to section b which is the translational joint, composed of a power screw, for the robotic arm. On the other side of section b is section c, which is also a hollow cylinder. Section c ends at Joint A.

Entire length of arm #1:  $L_1 = 2 \cdot \text{m}$   
Length of section a:  $L_a = 0.5 \cdot \text{m}$   
Length of section b:  $L_b = 0.667 \cdot \text{m}$   
Length of section c:  $L_c = 0.833 \cdot \text{m}$

Arm #2:  
Entire length of arm #2:  $L_2 = 1.414 \cdot \text{m}$

Arm #3:  
Entire length of arm #3:  $L_3 = 0.386 \cdot \text{m}$

Wrist: The wrist is the name given to the section at the end of arm number three where the three joints are located for movement of the interface and tool section. The wrist is divided into three equal parts, one for each of the joints.

Entire length of wrist:  $L_w = 0.2 \cdot \text{m}$   
Interface:  $L_{\text{int}} = 0.2 \cdot \text{m}$   
Diameter of interface:  $d_4 = 15 \cdot \text{cm}$   
Combined length of wrist and interface:  $L_4 = L_w + L_{\text{int}}$

Tool: The length of the tool used is for the maximum size tool as stated by the Tool Subsystem.  
Entire length of tool:  $L_5 = 0.5 \cdot \text{m}$

### Assigned masses and weights for each member of the robotic arm.

For arms one, two, and three the masses are to be computed by static analysis.

Mass and weight of tool:

$$\text{mass}_{\text{tool}} = 10 \cdot \text{kg} \quad w_{\text{tool}} = \text{mass}_{\text{tool}} \cdot g \quad w_{\text{tool}} = 16.344 \cdot \text{N}$$

Mass and weight of maximum load:

$$\text{mass}_{\text{load}} = 50 \cdot \text{kg} \quad w_{\text{load}} = \text{mass}_{\text{load}} \cdot g \quad w_{\text{load}} = 81.722 \cdot \text{N}$$

Combined mass and weight of tool and maximum load (for simpler calculations).

$$\text{mass}_5 = \text{mass}_{\text{tool}} + \text{mass}_{\text{load}} \quad \text{mass}_5 = 60 \cdot \text{kg}$$

$$w_5 = w_{\text{load}} + w_{\text{tool}} \quad w_5 = 98.066 \cdot \text{N}$$

Mass and weight of interface:

$$\text{mass}_{\text{int}} = 10 \cdot \text{kg} \quad w_{\text{int}} = \text{mass}_{\text{int}} \cdot g \quad w_{\text{int}} = 16.344 \cdot \text{N}$$

Mass and weight of wrist:

$$\text{mass}_w = 10 \cdot \text{kg} \quad w_w = \text{mass}_w \cdot g \quad w_w = 16.344 \cdot \text{N}$$

Combined mass and weight of wrist and interface:

$$\text{mass}_4 = \text{mass}_{\text{int}} + \text{mass}_w \quad \text{mass}_4 = 20 \cdot \text{kg}$$

$$w_4 = w_w + w_{\text{int}} \quad w_4 = 32.689 \cdot \text{N}$$

In addition to the masses and weights of the robotic arm members, a miscellaneous weight is added at each joint to account for the added weight of the motor, gears, casing, axle, etc. at each joint. This weight is given as a value larger than that actually expected in keeping with the worst case scenario.

$$\text{mass}_{\text{misc}} = 5 \cdot \text{kg} \quad w_{\text{misc}} = \text{mass}_{\text{misc}} \cdot g \quad w_{\text{misc}} = 8.172 \cdot \text{N}$$

### Distances from base of robotic arm to centers of gravity of all members.

Since each member of the robotic arm is symmetric in the yz and xy planes, the only direction that is going to affect the center of gravity of each member is the distance in the x-direction from the base of the arm to the center of gravity of each member.

Center of gravity for each section of arm #1.

$$\text{Section a:} \quad G_a = \frac{L_a}{2} \quad G_a = 0.25 \cdot \text{m}$$

$$\text{Section b:} \quad G_b = L_a + \frac{L_b}{2} \quad G_b = 0.834 \cdot \text{m}$$

$$\text{Section c:} \quad G_c = L_a + L_b + \frac{L_c}{2} \quad G_c = 1.584 \cdot \text{m}$$

Center of gravity for arm #2:

$$G_2 = L_1 + \frac{L_2}{2} \quad G_2 = 2.707 \cdot \text{m}$$

Center of gravity for arm #3:

$$G_3 = L_1 + L_2 + \frac{L_3}{2} \quad G_3 = 3.607 \cdot \text{m}$$

Center of gravity for wrist (assuming constant cross section):

$$G_w = L_1 + L_2 + L_3 + \frac{L_w}{2} \quad G_w = 3.9 \cdot \text{m}$$

Center of gravity for interface (assuming constant cross section):

$$G_{\text{int}} = L_1 + L_2 + L_3 + L_w + \frac{L_{\text{int}}}{2} \quad G_{\text{int}} = 4.1 \cdot \text{m}$$

Center of gravity for wrist and interface (as a whole to simplify calculations):

$$G_4 = L_1 + L_2 + L_3 + \frac{L_4}{2} \qquad G_4 = 4 \cdot m$$

Center of gravity for tool and maximum load:

$$G_5 = L_1 + L_2 + L_3 + L_4 + \frac{L_5}{2} \qquad G_5 = 4.45 \cdot m$$

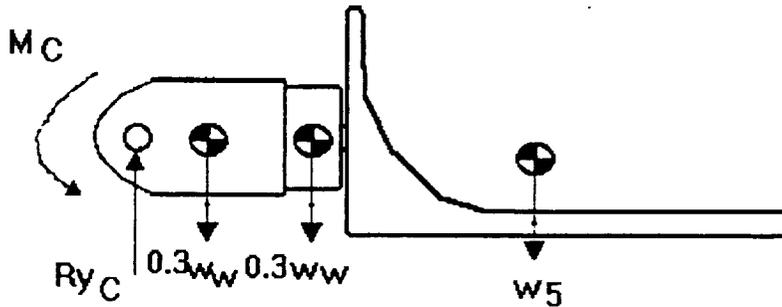
### Static Analysis

The static analysis is intended to give the inner and outer diameters of each of the three hollow cylinders that make up the mechanical structure of the arm. From these results, the weights and moments of inertia of each member will be found and the dynamic analysis will then be performed.

By stating a value for either the inner or outer diameter, the remaining diameter can be determined by using an iterative loop until the desired factor of safety is reached. The factor of safety is a function of the maximum stress at a cross section of the arm, and the maximum stress is a function of the inner and outer diameters of the cross section. Therefore, by stating one diameter the remaining diameter can be iteratively guessed until the desired factor of safety is reached.

**Static analysis for arm #3.** To determine the inner diameter and outer diameter of arm #3

Free body diagram of tool, interface, and wrist sections up to Joint C.

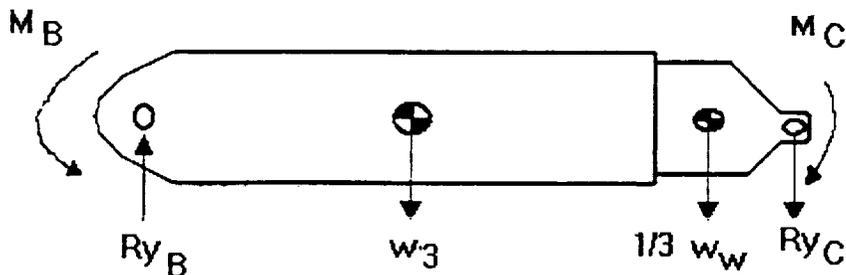


At Joint C:

$$R_{yc} = w_5 + w_{int} + \frac{1}{3} \cdot w_w + 2 \cdot w_{misc}$$

$$M_c = \left( \frac{1}{3} \cdot L_w + L_{int} + \frac{1}{2} \cdot L_s \right) \cdot w_5 + \left( \frac{1}{3} \cdot L_w + \frac{L_{int}}{2} \right) \cdot (w_{int} + w_{misc}) + \left( \frac{1}{3} \cdot L_w \right) \cdot \left( \frac{1}{6} \cdot w_w + w_{misc} \right)$$

Free body diagram of arm #3 and wrist section up to Joint C.



At Joint C:

$$R_{yc} = w_5 + w_{int} + \frac{1}{3} \cdot w_w - 2 \cdot w_{misc}$$

$$R_{yc} = 136.2 \cdot N$$

$$M_c = \left[ \frac{1}{3} \cdot L_w + L_{int} + \frac{1}{2} \cdot L_5 \right] \cdot w_5 + \left[ \frac{1}{3} \cdot L_w + \frac{L_{int}}{2} \right] \cdot w_{int} - w_{misc} + \left[ \frac{1}{3} \cdot L_w \right] \cdot \left[ \frac{1}{6} \cdot w_w + w_{misc} \right]$$

$$M_c = 55.48 \cdot N \cdot m$$

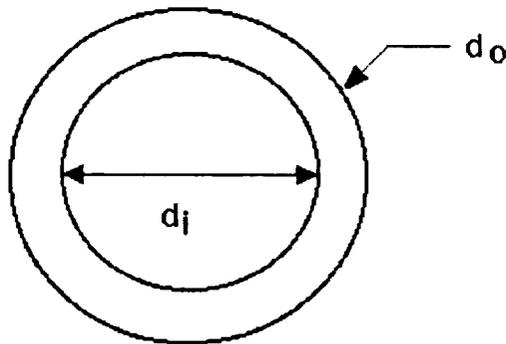
At Joint B:

$$R_{yb} = R_{yc} - \frac{2}{3} \cdot w_w + 2 \cdot w_{misc} + w_3$$

$$M_b = M_c + \left[ L_3 + \frac{1}{2} \cdot \left[ \frac{2}{3} \cdot L_w \right] \right] \cdot \left( \frac{2}{3} \cdot w_w + 2 \cdot w_{misc} \right) + \frac{L_3}{2} \cdot w_3$$

$$T = \frac{1}{2} \cdot L_w \cdot w_w + L_w - L_{int} \cdot w_{int} - L_w - L_{int} - \frac{1}{2} \cdot L_5 \cdot w_5$$

Cross section B-B of arm #3.



Arm #3 has the cross section of a hollow cylinder. By giving the desired outer diameter, the inner diameter can be determined at a desired factor of safety. The iterative loop used for this calculation is given below.

The desired outer diameter of arm #3 is:  $d_{3_o} = 100 \cdot \text{mm}$

**BEGIN iterative loop**

Guess for the inner diameter of arm #3:  $d_{3_i} = 95 \cdot \text{mm}$

Area at predetermined cross section of arm:

$$A_3 = \frac{\pi}{4} \cdot (d_{3_o}^2 - d_{3_i}^2) \quad A_3 = 7.658 \cdot 10^{-4} \cdot \text{m}^2$$

Centroidal moment of inertia for hollow cross section at specified inner and outer diameters:

$$I_{z_3} = \frac{\pi}{64} \cdot (d_{3_o}^4 - d_{3_i}^4) \quad I_{z_3} = 9.105 \cdot 10^{-7} \cdot \text{m}^4$$

Polar moment of inertia:

$$I_{y_3} = I_{z_3} \quad J_3 = I_{y_3} + I_{z_3} \quad J_3 = 1.821 \cdot 10^{-6} \cdot \text{m}^4$$

Distance from Neutral Axis to point of maximum tension (top surface) and compression (bottom surface) are the same:

$$c_3 = \frac{d3_o}{2}$$

Maximum magnitude of tensile and compressive stress (when arm is fully extended):

$$\sigma_3 = \frac{M_b \cdot c_3}{Iz_3}$$

$$\sigma_3 = 3.738 \cdot 10^6 \cdot \text{Pa}$$

Maximum shearing stress:

$$Q = \frac{A_3}{2} \frac{d3_o^2 - d3_i^2}{d3_o + d3_i} \cdot \frac{d3_o - d3_i}{2} \quad \tau_3 = d3_o - d3_i$$

$$\tau_3 = \frac{R_{yb} \cdot Q}{Iz_3 \cdot \tau_3}$$

$$\tau_3 = 4.302 \cdot 10^5 \cdot \text{Pa}$$

Maximum torsional stress (when joint W1 is rotated so that rest of arm past Joint C is positioned at a 90 degree angle with respect to axis of the robotic arm resulting in a torsional stress):

$$\tau_{\text{twist}_3} = \frac{T \cdot c_3}{J_3}$$

$$\tau_{\text{twist}_3} = 1.975 \cdot 10^6 \cdot \text{Pa}$$

The maximum total stress in preselected cross section of arm #3 is:

$$\tau_{3 \text{ max}} = \sqrt{\left(\frac{\sigma_3}{2}\right)^2 + (\tau_3)^2 + (\tau_{\text{twist}_3})^2} \quad \tau_{3 \text{ max}} = 2.753 \cdot 10^6 \cdot \text{Pa}$$

The resulting factor of safety is:

$$n = \frac{0.4 \cdot S_{ys}}{\tau_{3 \text{ max}}}$$

$$n = 32$$

### END of iterative loop

When the resulting F.S. equals the desired F.S. then the guess for the needed diameter is appropriate. Therefore the inner diameter for arm #3 is:

$$d3_i = 95 \cdot \text{mm}$$

The inner and outer diameters of arm #3 are now known, therefore the volume, mass, weight, and wall thickness of arm #3 can be determined.

Volume of arm #3

$$V_3 = A_3 \cdot L_3$$

$$V_3 = 295.6 \cdot \text{cm}^3$$

Mass of arm #3

$$\text{mass}_3 = V_3 \cdot \rho$$

$$\text{mass}_3 = 0.828 \cdot \text{kg}$$

Weight of arm #3

$$w_3 = \text{mass}_3 \cdot g$$

$$w_3 = 1.353 \cdot \text{N}$$

Wall thickness of arm #3

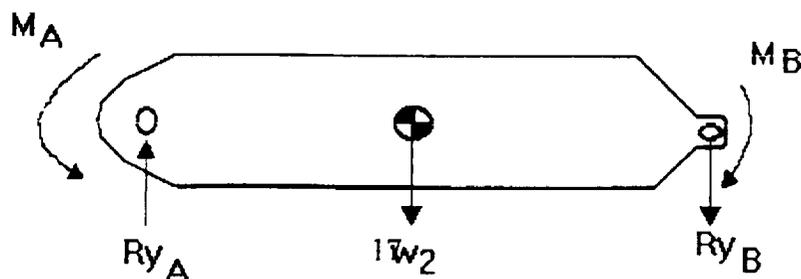
$$t_3 = \frac{d3_o - d3_i}{2}$$

$$t_3 = 2.5 \cdot \text{mm}$$

### Static analysis for arm #2.

To determine the inner diameter and outer diameter of arm #2

### Free body diagram of arm #2.



At Joint B:

$$R_{yb} = R_{yc} + \frac{2}{3} \cdot w_w - 2 \cdot w_{misc} - w_3$$

$$R_{yb} = 164.8 \cdot N$$

$$M_b = M_c - \left[ L_3 + \frac{1}{2} \cdot \frac{2}{3} \cdot L_w \right] \cdot \left[ \frac{2}{3} \cdot w_w + 2 \cdot w_{misc} \right] + \frac{L_3}{2} \cdot w_3$$

$$M_b = 68.07 \cdot N \cdot m$$

$$T = \frac{1}{2} \cdot L_w \cdot w_w + (L_w + L_{int}) \cdot w_{int} + \left( L_w + L_{int} + \frac{1}{2} \cdot L_5 \right) \cdot w_5$$

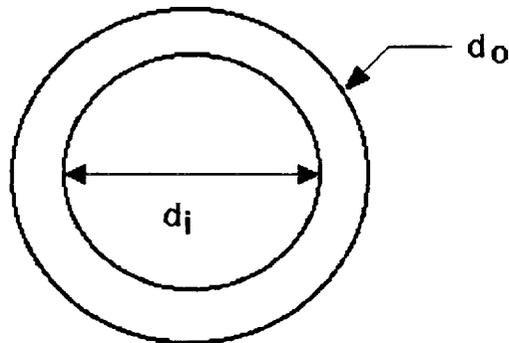
At Joint A:

$$R_{ya} = R_{yb} + w_{misc} + w_2$$

$$M_a = \frac{L_2}{2} \cdot w_2 - L_2 \cdot R_{yb} + w_{misc} \cdot M_b$$

$$T = \frac{1}{2} \cdot L_w \cdot w_w + L_w + L_{int} \cdot w_{int} + \left( L_w + L_{int} + \frac{1}{2} \cdot L_5 \right) \cdot w_5$$

Cross section A-A of arm #2.



Arm #2 has the cross section of a hollow cylinder. By giving the desired outer diameter, the inner diameter can be determined at a desired factor of safety. The iterative loop used for this calculation is given below.

The desired inner diameter of arm #2 is:  $d_{2_i} = 105 \cdot \text{mm}$

**BEGIN iterative loop**

Guess for the outer diameter of arm #2:  $d_{2_o} = 113 \cdot \text{mm}$

Area at predetermined cross section of arm:

$$A_2 = \frac{\pi}{4} \cdot (d_{2_o}^2 - d_{2_i}^2) \quad A_2 = 0.001 \cdot \text{m}^2$$

Centroidal moment of inertia for hollow cross section at specified inner and outer diameters:

$$I_{z_2} = \frac{\pi}{64} \cdot (d_{2_o}^4 - d_{2_i}^4) \quad I_{z_2} = 2.037 \cdot 10^{-6} \cdot \text{m}^4$$

Polar moment of inertia:

$$I_{y_2} = I_{z_2} \quad J_2 = I_{y_2} + I_{z_2} \quad J_2 = 4.074 \cdot 10^{-6} \cdot \text{m}^4$$

Distance from Neutral Axis to point of maximum tension (top surface) and compression (bottom surface) are the same:

$$c_2 = \frac{d_{2_o}}{2}$$

Maximum magnitude of tensile and compressive stress:

$$\sigma_2 = \frac{M_a \cdot c_2}{I_{z_2}} \quad \sigma_2 = 8.846 \cdot 10^6 \cdot \text{Pa}$$

Maximum shearing stress:  $Q_2 = \frac{A_2}{2} \cdot \frac{d_2^2 - d_1^2}{d_2 + d_1} \cdot \pi \cdot t_2$   $t_2 = d_2 - d_1$

$$\tau_2 = \frac{R_{ya} \cdot Q_2}{I_{z_2} \cdot t_2} \quad \tau_2 = 2.653 \cdot 10^5 \cdot \text{Pa}$$

Maximum torsional stress (when Joint W1 is rotated so that rest of arm past Joint C is positioned at a 90 degree angle with respect to axis of the robotic arm resulting in a torsional stress):

$$\tau_{\text{twist}_2} = \frac{T \cdot c_2}{J_2} \quad \tau_{\text{twist}_2} = 9.974 \cdot 10^5 \cdot \text{Pa}$$

The maximum total stress in preselected cross section of arm #2 is:

$$\tau_{2 \text{ max}} = \sqrt{\left(\frac{\sigma_2}{2}\right)^2 + (\tau_2)^2 + (\tau_{\text{twist}_2})^2} \quad \tau_{2 \text{ max}} = 4.542 \cdot 10^6 \cdot \text{Pa}$$

The resulting factor of safety is:

$$n = \frac{0.4 \cdot S_{ys}}{\tau_{2 \text{ max}}} \quad n = 19.4$$

### END of iterative loop

When the resulting F.S. equals the desired F.S. then the guess for the needed diameter is appropriate. Therefore the outer diameter for arm #2 is:

$$d_{2_o} = 113 \cdot \text{mm}$$

The inner and outer diameters of arm #2 are now known, therefore the volume, mass, weight, and wall thickness of arm #2 can be determined.

Volume of arm #2  $V_2 = A_2 \cdot L_2$   $V_2 = 1.937 \cdot 10^3 \cdot \text{cm}^3$

Mass of arm #2  $\text{mass}_2 = V_2 \cdot \rho$   $\text{mass}_2 = 5.423 \cdot \text{kg}$

Weight of arm #2  $w_2 = \text{mass}_2 \cdot g$   $w_2 = 8.864 \cdot \text{N}$

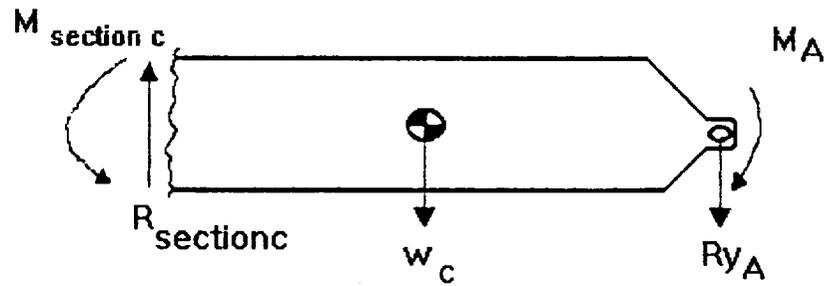
Wall thickness of arm #2  $t_2 = \frac{d_{2_o} - d_{2_i}}{2}$   $t_2 = 4 \cdot \text{mm}$

### Static analysis of arm #1.

Arm #1 is composed of three parts; sections a, b, and c. Section c is of the form of a hollow cylinder as is arm #2 and arm #3 and connects with arm #2 at Joint A on one end and at section b of arm #1 at the other end. Section a serves as a structural member as do arms #1 and #2. On the other hand, section b of arm #1 is composed of one main power screw and one guide/support bar and serves as a translational joint for greater operating range of the robotic arm and must be able to withstand the forces and loads applied to it. Section b connects on one end to section c and on the other end to section a. Section a is of the form of a hollow cylinder as is section c. Like section c, section a serves as a structural member and is connected at one end to section b and at the other end to the support base of the robotic arm via the Base Joint. The inner diameters of section a and c are the same in order to allow the translational joint supports to be attached to the inside of the cylinder walls. However, the outside diameters of sections a and c will be different in order to safely withstand the different forces applied to each section.

**Analysis of section c.**

Free body diagram of section c of arm #1



At Joint A:

$$R_{ya} = R_{yb} + w_{misc} + w_2$$

$$R_{ya} = 181.8 \cdot N$$

$$M_a = \frac{L_2}{2} \cdot w_2 + L_2 \cdot (R_{yb} + w_{misc}) + M_b$$

$$M_a = 318.9 \cdot N \cdot m$$

$$T = \frac{1}{2} \cdot L_w \cdot w_w + (L_w + L_{int}) \cdot w_{int} + \left( L_w + L_{int} - \frac{1}{2} \cdot L_s \right) \cdot w_s$$

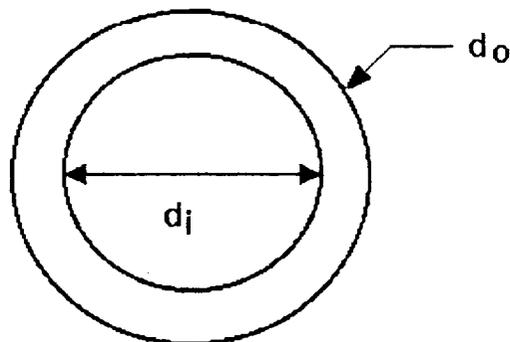
at cross section M-M of section c of arm #1:

$$R_{sectionc} = w_c + w_{misc} + R_{ya}$$

$$M_{sectionc} = \frac{L_c}{2} \cdot w_c + L_c \cdot (w_{misc} + R_{ya}) - M_a$$

$$T = \frac{1}{2} \cdot L_w \cdot w_w + (L_w + L_{int}) \cdot w_{int} + \left( L_w + L_{int} - \frac{1}{2} \cdot L_s \right) \cdot w_s$$

Cross section M-M of section c of arm #1



The desired inner diameter of section c of arm #1 is:  $dc_i = 118 \cdot mm$

**BEGIN iterative loop**

Guess for the outer diameter of section c of arm #1:  $dc_o = 132 \cdot mm$

Area at predetermined cross section of arm:

$$A_c = \frac{\pi}{4} \cdot (dc_o^2 - dc_i^2) \quad A_c = 0.003 \cdot m^2$$

Centroidal moment of inertia for hollow cross section at specified inner and outer diameters:

$$I_{z_c} = \frac{\pi}{64} (dc_o^4 - dc_i^4) \quad I_{z_c} = 5.386 \cdot 10^{-6} \cdot m^4$$

Polar moment of inertia:

$$I_{y_c} = I_{z_c} \quad J_c = I_{y_c} + I_{z_c} \quad J_c = 1.077 \cdot 10^{-5} \cdot m^4$$

Distance from Neutral Axis to point of maximum tension (top surface) and compression (bottom surface) are the same:

$$c_c = \frac{dc_o}{2}$$

Maximum magnitude of tensile and compressive stress:

$$\sigma_c = \frac{M_{section} \cdot c_c}{I_{z_c}} \quad \sigma_c = 5.901 \cdot 10^6 \cdot Pa$$

$$\text{Maximum shearing stress: } Q_c = \frac{A_c}{2} \left[ \frac{dc_o^2 + dc_i \cdot dc_o + dc_i^2}{dc_o + dc_i} \right] t_c = dc_o - dc_i$$

$$\tau_c = \frac{R_{section} \cdot Q_c}{I_{z_c} \cdot t_c} \quad \tau_c = 1.456 \cdot 10^5 \cdot Pa$$

Maximum torsional stress (when Joint W1 is rotated so that rest of arm past Joint C is positioned at a 90 degree angle with respect to axis of the robotic arm resulting in a torsional stress):

$$\tau_{twistc} = \frac{T \cdot c_c}{J_c} \quad \tau_{twistc} = 4.406 \cdot 10^5 \cdot \text{length}^{-3} \cdot N \cdot m$$

The maximum total stress in preselected cross section of arm #1 is:

$$\tau_{c_{max}} = \sqrt{\left(\frac{\sigma_c}{2}\right)^2 + (\tau_c)^2 + \tau_{twistc}^2} \quad \tau_{c_{max}} = 2.987 \cdot 10^6 \cdot Pa$$

The resulting factor of safety is:

$$n = \frac{0.4 \cdot S_{ys}}{\tau_{c_{max}}} \quad n = 29.5$$

### END of iterative loop

When the resulting F.S. equals the desired F.S. then the guess for the needed diameter is appropriate. Therefore the outer diameter for section c of arm #1 is:

$$dc_o = 132 \cdot mm$$

The inner and outer diameter for section c of arm #1 is now known, therefore the volume, mass, weight, and wall thickness of section c of arm #1 can be determined.

$$\text{Volume of section c of arm \#1} \quad V_c = A_c \cdot L_c \quad V_c = 2.29 \cdot 10^3 \cdot cm^3$$

$$\text{Mass of section c of arm \#1} \quad mass_c = V_c \cdot \rho \quad mass_c = 6.412 \cdot kg$$

$$\text{Weight of section c of arm \#1} \quad w_c = mass_c \cdot g \quad w_c = 10.479 \cdot N$$

$$\text{Wall thickness of section c of arm \#1} \quad t_c = \frac{dc_o - dc_i}{2} \quad t_c = 7 \cdot mm$$

**Analysis of section b.**

Section b was analyzed in another document. The results gathered in this document were delivered to the group member analyzing section b. In turn, the results from section b were delivered back to the group member performing this analysis. The analysis of section a of arm #1 was then performed and the dynamic analysis was then performed.

For the analysis of section b the properties of the materials used must be defined.

Material for power screw: cold rolled stainless steel (302)

$$\text{Density of steel: } \rho_{\text{steel}} = 7920 \cdot \frac{\text{kg}}{\text{m}^3}$$

Properties of section b:

Diameters of screw:  $d_{si} = 30 \text{ mm}$      $d_{so} = 80 \text{ mm}$

Volume of power screw:  $V_{\text{screw}} = L_b \cdot \left[ \frac{\pi}{4} \cdot (d_{so}^2 - d_{si}^2) \right]$

$$\text{mass}_{\text{screw}} = V_{\text{screw}} \cdot \rho_{\text{steel}} \quad \text{mass}_{\text{screw}} = 22.819 \cdot \text{kg}$$

$$w_{\text{screw}} = \text{mass}_{\text{screw}} \cdot g \quad w_{\text{screw}} = 37.297 \cdot \text{N}$$

Volume of section b:  $V_b = V_{\text{screw}} \quad V_b = 0.003 \cdot \text{m}^3$

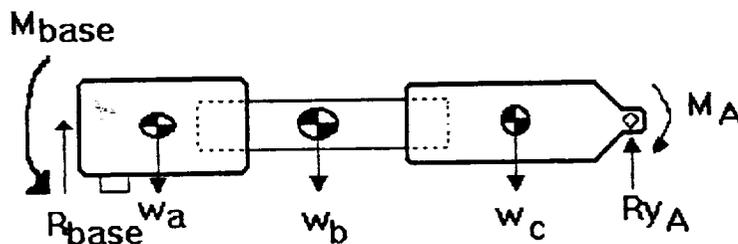
$$\text{mass}_b = \text{mass}_{\text{screw}} \quad \text{mass}_b = 22.819 \cdot \text{kg}$$

$$w_b = w_{\text{screw}} \quad w_b = 37.297 \cdot \text{N}$$

**Analysis of section a.**

With the data for section b of arm #1 given by section b analyzers, the analysis of section a was carried out.

Free body diagram of arm #1. All three sections shown.



At Joint A

$$R_{ya} = R_{yb} - w_{\text{misc}} - w_2$$

$$M_a = \frac{L_2}{2} \cdot w_2 + L_2 \cdot (R_{yb} - w_{\text{misc}}) + M_b$$

$$T = \frac{1}{2} L_w \cdot w_w + L_w + L_{\text{int}} \cdot w_{\text{int}} + \left( L_w + L_{\text{int}} + \frac{1}{2} L_s \right) \cdot w_s$$

At base joint

$$R_{\text{base}} = R_{ya} + w_a + w_b + w_c$$

$$M_{\text{base}} = L_1 \cdot R_{ya} + \left( L_1 - \frac{1}{2} L_c \right) \cdot w_c + \left( L_a + \frac{1}{2} L_b \right) \cdot w_b + \frac{1}{2} L_a \cdot w_a$$

$$T = \frac{1}{2} L_w \cdot w_w + \left( L_w - L_{\text{int}} \right) \cdot w_{\text{int}} + \left( L_w + L_{\text{int}} + \frac{1}{2} L_5 \right) \cdot w_5$$

The desired inner diameter of section a of arm #1 is:  $da_i = dc_j$   $da_i = 118 \text{ mm}$

**BEGIN iterative loop**

Guess for the outer diameter of section a of arm #1:  $da_o = 135 \text{ mm}$

Area at predetermined cross section of arm:

$$A_a = \frac{\pi}{4} \cdot (da_o^2 - da_i^2) \quad A_a = 0.003 \cdot \text{m}^2$$

Centroidal moment of inertia for hollow cross section at specified inner and outer diameters:

$$I_{z_a} = \frac{\pi}{64} \cdot (da_o^4 - da_i^4) \quad I_{z_a} = 6.787 \cdot 10^{-6} \cdot \text{m}^4$$

Distance from Neutral Axis to point of maximum tension (top surface) and compression (bottom surface) are the same:

$$c_a = \frac{da_o}{2}$$

Polar moment of inertia:

$$I_{y_a} = I_{z_a} \quad J_a = I_{y_a} + I_{z_a} \quad J_a = 1.357 \cdot 10^{-5} \cdot \text{m}^4$$

Maximum magnitude of tensile and compressive stress:

$$\sigma_a = \frac{M_{\text{base}} \cdot c_a}{I_{z_a}} \quad \sigma_a = 4.11 \cdot 10^6 \cdot \text{Pa}$$

$$\text{Maximum shearing stress: } Q_a = \frac{A_a}{2} \cdot \frac{2}{3} \cdot \left[ \frac{da_o^2 + da_i \cdot da_o + da_i^2}{(da_o + da_i) \cdot \pi} \right] \cdot tt_a = da_o - da_i$$

$$\tau_a = \frac{R_{\text{base}} \cdot Q_a}{I_{z_a} \cdot tt_a} \quad \tau_a = 1.401 \cdot 10^5 \cdot \text{Pa}$$

Maximum torsional stress (when Joint W1 is rotated so that rest of arm past Joint C is positioned at a 90 degree angle with respect to axis of the robotic arm resulting in a torsional stress):

$$\tau_{\text{twista}} = \frac{T \cdot c_c}{J_c} \quad \tau_{\text{twista}} = 4.406 \cdot 10^5 \cdot \text{Pa}$$

The maximum total stress in preselected cross section of arm #1 is:

$$\tau_{a \text{ max}} = \sqrt{\left( \frac{\sigma_a}{2} \right)^2 + (\tau_a)^2} + \tau_{\text{twista}} \quad \tau_{a \text{ max}} = 2.106 \cdot 10^6 \cdot \text{Pa}$$

The resulting factor of safety is:

$$n = \frac{0.4 \cdot S_{ys}}{\tau_{a \text{ max}}} \quad n = 42$$

**END of iterative loop**

When the resulting F.S. equals the desired F.S. then the guess for the needed diameter is appropriate. Therefore the outer diameter for section a of arm #1 is:  $da_o = 135 \cdot \text{mm}$

The inner and outer diameter for section a of arm #1 is now known, therefore the volume, mass, weight, and wall thickness of section a of arm #1 can be determined.

Volume of section a of arm #1  $V_a = A_a \cdot L_a$   $V_a = 1.689 \cdot 10^{-3} \cdot \text{cm}^3$

Mass of section a of arm #1  $\text{mass}_a = V_a \cdot \rho$   $\text{mass}_a = 4.729 \cdot \text{kg}$

Weight of section a of arm #1  $w_a = \text{mass}_a \cdot g$   $w_a = 7.73 \cdot \text{N}$

Wall thickness of section a of arm #1  $t_a = \frac{da_o - da_i}{2}$   $t_a = 8.5 \cdot \text{mm}$

**Arm #1 as a whole.**

Mass of arm #1  $\text{mass}_1 = \text{mass}_a + \text{mass}_b + \text{mass}_c$

$\text{mass}_1 = 33.96 \cdot \text{kg}$

Weight of arm #1  $w_1 = w_a + w_b + w_c$

$w_1 = 55.506 \cdot \text{N}$

With the static analysis of each arm complete, the mass and weight of the entire structure can be determined.

**Mass of entire structure (including maximum load). i = 1..5**

$\text{mass}_{\text{total}} = \sum_i \text{mass}_i$   $\text{mass}_{\text{total}} = 120.2 \cdot \text{kg}$

**Weight of entire structure (minus maximum load).**

$w_{\text{total}} = \text{mass}_{\text{total}} \cdot g$   $w_{\text{total}} = 196.477 \cdot \text{N}$

**Center of Gravity Calculations.**

Determination of center of gravity for arm #1 as a whole.

component	volume	length	length x volume
sect. a	$V_a = 0.002 \cdot \text{m}^3$	$G_a = 0.25 \cdot \text{m}$	$V_a \cdot G_a = 4.222 \cdot 10^{-4} \cdot \text{m}^4$
sect. b	$V_b = 0.003 \cdot \text{m}^3$	$G_b = 0.834 \cdot \text{m}$	$V_b \cdot G_b = 0.002 \cdot \text{m}^4$
sect. c	$V_c = 0.002 \cdot \text{m}^3$	$G_c = 1.584 \cdot \text{m}$	$V_c \cdot G_c = 0.004 \cdot \text{m}^4$

$V_1 = V_a + V_b + V_c$

$\text{sumVG} = V_a \cdot G_a + V_b \cdot G_b + V_c \cdot G_c$

$V_1 = 0.007 \cdot \text{m}^3$

$\text{sumVG} = 0.006 \cdot \text{m}^4$

**Location of center of gravity for arm #1 as a whole.**

$G_i = \frac{\text{sumVG}}{V_1}$

$G_i = 0.94 \cdot \text{m}$

Determination of the center of gravity from the base for the mechanical structure as a whole. For use in calculating the torque at the base of the structure.

In order to perform a dynamic analysis on the robotic arm it is necessary to calculate the center of gravity for each member. To do this the geometry of each member needs to be known. For the maximum load, tool and wrist/interface sections, a simple geometric shape was used to simplify the calculations. All members lie in the same horizontal plane (their geometric axis is coincident with the x-axis) and therefore the calculation of each center of gravity becomes a function of only one length (x direction).

For the maximum load a volume of a square was used as the geometric model, while for the tool a rectangle was the geometric shape used to calculate its volume. A rectangle was used for the tool because the heaviest tool designed was that of a thin rectangular shovel. For the wrist and interface sections, a volume of a solid cylinder was used. Although neither the wrist or the interface is solid, the solid cylinder was used to model these members in an effort to keep with designing for a worst case scenario.

In calculating the volume of the maximum load the density of moon regolith was used. In our research we discovered that the density of moon regolith ranged from 1.4 to 1.9 grams per cubic centimeters. In keeping with the worst case design scenario the maximum density of 1.9 grams per cubic centimeter was used.

component	volume	length	length x volume
arm #1	$V_1 = 0.007 \cdot m^3$	$G_1 = 0.94 \cdot m$	$V_1 \cdot G_1 = 0.006 \cdot m^4$
arm #2	$V_2 = 0.002 \cdot m^3$	$G_2 = 2.707 \cdot m$	$V_2 \cdot G_2 = 0.005 \cdot m^4$
arm #3	$V_3 = 2.956 \cdot 10^{-4} \cdot m^3$	$G_3 = 3.607 \cdot m$	$V_3 \cdot G_3 = 0.001 \cdot m^4$
wrist/int.	$V_4 = \frac{\pi}{4} \cdot L_w \cdot (d_3)_o^2 + \frac{\pi}{4} \cdot L_{int} \cdot (d_4)^2$		
	$V_4 = 0.005 \cdot m^3$	$G_4 = 4 \cdot m$	$V_4 \cdot G_4 = 0.02 \cdot m^4$
tool/load	$V_5 = \frac{\text{mass tool}}{\rho} + \frac{\text{mass load}}{\rho_{\text{moon}}}$		
	$V_5 = 0.03 \cdot m^3$	$G_5 = 4.45 \cdot m$	$V_5 \cdot G_5 = 0.133 \cdot m^4$

$$i = 1 \dots 5 \quad \sum_i V_i = 0.044 \cdot m^3 \quad \sum_i V_i \cdot G_i = 0.166 \cdot m^4$$

Location of center of gravity of entire mechanical structure from the base.

$$G_{\text{base}} = \frac{\sum_i V_i \cdot G_i}{\sum_i V_i} \quad G_{\text{base}} = 3.769 \cdot m$$

Determination of center of gravity for mechanical structure excluding arm #1. For use in calculating torque at Joint A.

component	volume	length	length x volume
arm #2	$V_2 = 0.002 \cdot \text{m}^3$	$\frac{L_2}{2} = 0.707 \cdot \text{m}$	$V_2 \cdot \left(\frac{L_2}{2}\right) = 0.001 \cdot \text{m}^4$
arm #3	$V_3 = 2.956 \cdot 10^{-4} \cdot \text{m}^3$	$L_2 + \frac{L_3}{2} = 1.607 \cdot \text{m}$	$V_3 \cdot (G_3 - L_1) = 4.75 \cdot 10^{-4} \cdot \text{m}^4$
wrist/int.	$V_4 = \frac{\pi}{4} \cdot L_w \cdot (d_3 o)^2 + \frac{\pi}{4} \cdot L_{int} \cdot d_4^2$		
	$V_4 = 0.005 \cdot \text{m}^3$	$G_4 - L_1 = 2 \cdot \text{m}$	$V_4 \cdot (G_4 - L_1) = 0.01 \cdot \text{m}^4$
tool/load	$V_5 = \frac{\text{mass tool}}{\rho} - \frac{\text{mass load}}{\rho_{\text{moon}}}$		
	$V_5 = 0.03 \cdot \text{m}^3$	$G_5 - L_1 = 2.45 \cdot \text{m}$	$V_5 \cdot (G_5 - L_1) = 0.073 \cdot \text{m}^4$

$$i = 2 \dots 5 \quad \sum_i V_i = 0.044 \cdot \text{m}^3 \quad \sum_i V_i \cdot (G_i - L_1) = 0.085 \cdot \text{m}^4$$

Location of center of gravity of maximum load, tool, interface, wrist, arm #3, and arm #2 from Joint A.

$$G_{\text{jointA}} = \frac{\sum_i V_i \cdot (G_i - L_1)}{\sum_i V_i} \quad G_{\text{jointA}} = 2.291 \cdot \text{m}$$

Determination of center of gravity for mechanical structure excluding arms #1 and #2. For use in calculating torque at Joint B.

component	volume	length	length x volume
arm #3	$V_3 = 2.96 \cdot 10^{-4} \cdot \text{m}^3$	$\frac{L_3}{2} = 0.193 \cdot \text{m}$	$V_3 \cdot \frac{L_3}{2} = 5.7 \cdot 10^{-5} \cdot \text{m}^4$
wrist/int.	$V_4 = \frac{\pi}{4} \cdot L_w \cdot (d_3 o)^2 + \frac{\pi}{4} \cdot L_{int} \cdot d_4^2$		
	$V_4 = 0.005 \cdot \text{m}^3$	$L_3 + \frac{L_4}{2} = 0.586 \cdot \text{m}$	$V_4 \cdot \left(L_3 + \frac{L_4}{2}\right) = 0.003 \cdot \text{m}^4$
tool/load	$V_5 = \frac{\text{mass tool}}{\rho} - \frac{\text{mass load}}{\rho_{\text{moon}}}$		
	$V_5 = 0.03 \cdot \text{m}^3$	$L_3 + L_4 + \frac{L_5}{2} = 1.036 \cdot \text{m}$	$V_5 \cdot \left(L_3 + L_4 + \frac{L_5}{2}\right) = 0.031 \cdot \text{m}^4$

$$i = 3 \dots 5 \quad \sum_i V_i = 0.037 \cdot \text{m}^3 \quad V_3 \cdot L_3 + V_4 \cdot \left(L_3 + \frac{L_4}{2}\right) + V_5 \cdot \left(L_3 + L_4 + \frac{L_5}{2}\right) = 0.034 \cdot \text{m}^4$$

Location of center of gravity of maximum load, tool, interface, wrist, and arm #3 from Joint B.

$$G_{\text{jointB}} = \frac{V_3 \cdot L_3 + V_4 \cdot L_3 + \frac{L_4}{2} + V_5 \cdot L_3 \cdot L_4 + \frac{L_5}{2}}{\sum_i V_i} G_{\text{jointB}} = 0.965 \cdot m$$

Determination of center of gravity for mechanical structure excluding arms #1, #2, and #3. For use in calculating torque at Joint W1.

component	volume	length	length x volume
wrist	$V_w = \frac{\pi}{4} \cdot L_w \cdot d_3^2$ $V_w = 0.002 \cdot m^3$	$\frac{L_w}{2} = 0.1 \cdot m$	$V_w \cdot L_w = 3.142 \cdot 10^{-4} \cdot m^4$
interface	$V_{\text{int}} = \frac{\pi}{4} \cdot L_{\text{int}} \cdot d_4^2$ $V_{\text{int}} = 0.004 \cdot m^3$	$L_w + \frac{L_{\text{int}}}{2} = 0.3 \cdot m$	$V_{\text{int}} \cdot \left( L_w + \frac{L_{\text{int}}}{2} \right) = 0.001 \cdot m^4$
tool/load	$V_5 = \frac{\text{mass tool}}{\rho} + \frac{\text{mass load}}{\rho_{\text{moon}}}$ $V_5 = 0.03 \cdot m^3$	$L_4 + \frac{L_5}{2} = 0.65 \cdot m$	$V_5 \cdot \left( L_4 + \frac{L_5}{2} \right) = 0.019 \cdot m^4$

$$\begin{aligned} \text{sumVW1} &= V_w + V_{\text{int}} + V_5 & \text{sumVGW1} &= V_w \cdot L_w + V_{\text{int}} \cdot \left( L_w + \frac{L_{\text{int}}}{2} \right) + V_5 \cdot \left( L_4 + \frac{L_5}{2} \right) \\ \text{sumVW1} &= 0.035 \cdot m^3 & \text{sumVGW1} &= 0.021 \cdot m^4 \end{aligned}$$

Location of center of gravity of maximum load, tool, interface, wrist from Joint W1.

$$G_{\text{jointW1}} = \frac{\text{sumVGW1}}{\text{sumVW1}} \quad G_{\text{jointW1}} = 0.594 \cdot m$$

Determination of center of gravity for mechanical structure excluding arms #1, #2, #3, and joint W1. For use in calculating torque at Joint C.

component	volume	length	length x volume
wrist	$V_w = \frac{\pi}{4} \cdot L_w \cdot d_3^2 \cdot \frac{2}{3}$ $V_w = 0.001 \cdot m^3$	$\frac{L_w}{3} = 0.067 \cdot m$	$V_w \cdot \left( \frac{L_w}{3} \right) = 6.981 \cdot 10^{-5} \cdot m^4$
interface	$V_{\text{int}} = \frac{\pi}{4} \cdot L_{\text{int}} \cdot d_4^2$ $V_{\text{int}} = 0.004 \cdot m^3$	$\frac{2 \cdot L_w}{3} + \frac{L_{\text{int}}}{2} = 0.233 \cdot m$	$V_{\text{int}} \cdot \left( \frac{2 \cdot L_w}{3} + \frac{L_{\text{int}}}{2} \right) = 8.247 \cdot 10^{-4} \cdot m^4$
tool/load	$V_5 = \frac{\text{mass tool}}{\rho} + \frac{\text{mass load}}{\rho_{\text{moon}}}$ $V_5 = 0.03 \cdot m^3$	$\frac{2 \cdot L_w}{3} + L_{\text{int}} + \frac{L_5}{2} = 0.583 \cdot m$	$V_5 \cdot \left( \frac{2 \cdot L_w}{3} + L_{\text{int}} + \frac{L_5}{2} \right) = 0.017 \cdot m^4$

$$\text{sumVC} = V_w \frac{2}{3} + V_{\text{int}} + V_s$$

$$\text{sumVC} = 0.034 \cdot \text{m}^3$$

$$\text{sumVGC} = V_w \left( \frac{L_w}{3} \right) + V_{\text{int}} \left( \frac{2 \cdot L_w}{3} + \frac{L_{\text{int}}}{2} \right) + V_s \left( \frac{2 \cdot L_w}{3} + L_{\text{int}} + \frac{L_s}{2} \right)$$

$$\text{sumVGC} = 0.018 \cdot \text{m}^4$$

Location of center of gravity of maximum load, tool, interface, 2/3 of wrist from Joint C.

$$G_{\text{jointC}} = \frac{\text{sumVGC}}{\text{sumVC}} \quad G_{\text{jointC}} = 0.537 \cdot \text{m}$$

Determination of center of gravity for mechanical structure excluding arms #1, #2, #3, joint W1, and Joint C. For use in calculating torque at Joint W2.

component	volume	length	length x volume
wrst	$V_w = \frac{\pi}{4} \cdot L_w \cdot d_3^2 \cdot \frac{1}{3}$		
	$V_w = 5.236 \cdot 10^{-4} \cdot \text{m}^3$	$\frac{L_w}{6} = 0.033 \cdot \text{m}$	$V_w \left( \frac{L_w}{6} \right) = 1.745 \cdot 10^{-5} \cdot \text{m}^4$
interface	$V_{\text{int}} = \frac{\pi}{4} \cdot L_{\text{int}} \cdot d_4^2$		
	$V_{\text{int}} = 0.004 \cdot \text{m}^3$	$\frac{1 \cdot L_w}{3} + \frac{L_{\text{int}}}{2}$	$V_{\text{int}} \left( \frac{1 \cdot L_w}{3} + \frac{L_{\text{int}}}{2} \right) = 5.89 \cdot 10^{-4} \cdot \text{m}^4$
tool/load	$V_s = \frac{\text{mass tool}}{\rho} + \frac{\text{mass load}}{\rho_{\text{moon}}}$		
	$V_s = 0.03 \cdot \text{m}^3$	$\frac{1}{3} \cdot L_w + L_{\text{int}} + \frac{L_s}{2} = 0.517 \cdot \text{m}$	$V_s \left( \frac{1}{3} \cdot L_w + L_{\text{int}} + \frac{L_s}{2} \right) = 0.015 \cdot \text{m}^4$

$$\text{sumVW2} = V_w \frac{2}{3} + V_{\text{int}} + V_s \quad \text{sumVW2} = 0.034 \cdot \text{m}^3$$

$$\text{sumVGW2} = V_w \left( \frac{L_w}{6} \right) + V_{\text{int}} \left( \frac{1 \cdot L_w}{3} + \frac{L_{\text{int}}}{2} \right) + V_s \left( \frac{1}{3} \cdot L_w + L_{\text{int}} + \frac{L_s}{2} \right)$$

$$\text{sumVGW2} = 0.016 \cdot \text{m}^4$$

Location of center of gravity of maximum load, tool, interface, 1/3 of wrist from Joint W2.

$$G_{\text{jointW2}} = \frac{\text{sumVGW2}}{\text{sumVW2}} \quad G_{\text{jointW2}} = 0.475 \cdot \text{m}$$

Summation of analysis for center of gravity calculations.

$$G_{\text{base}} = 3.769 \cdot \text{m} \quad G_{\text{jointW1}} = 0.594 \cdot \text{m}$$

$$G_{\text{jointA}} = 2.291 \cdot \text{m} \quad G_{\text{jointC}} = 0.537 \cdot \text{m}$$

$$G_{\text{jointB}} = 0.965 \cdot \text{m} \quad G_{\text{jointW2}} = 0.475 \cdot \text{m}$$

## **Mass Moment of Inertia Calculations.**

Determination of the mass moments of inertia at base with respect to the y-axis for each member.

for section a of arm #1

$$I_a = \frac{1}{12} \cdot \text{mass}_a \cdot \left[ 3 \cdot (d_{a_o}^2 - d_{a_i}^2) + L_a^2 \right] + \text{mass}_a \cdot G_a^2$$

$$I_a = 0.4 \cdot \text{kg} \cdot \text{m}^2$$

for section b of arm #1

$$I_b = \frac{1}{12} \cdot \text{mass}_{\text{screw}} \cdot \left[ 3 \cdot (d_{s_o}^2 - d_{s_i}^2) + L_b^2 \right] + \text{mass}_b \cdot G_b^2$$

$$I_b = 16.73 \cdot \text{kg} \cdot \text{m}^2$$

for section c of arm #1

$$I_c = \frac{1}{12} \cdot \text{mass}_c \cdot \left[ 3 \cdot (d_{c_o}^2 - d_{c_i}^2) + L_c^2 \right] + \text{mass}_c \cdot G_c^2$$

$$I_c = 16.45 \cdot \text{kg} \cdot \text{m}^2$$

for arm #1 as a whole

$$I_1 = I_a + I_b + I_c$$

$$I_1 = 33.583 \cdot \text{kg} \cdot \text{m}^2$$

for arm #2

$$I_2 = \frac{1}{12} \cdot \text{mass}_2 \cdot \left[ 3 \cdot (d_{2_o}^2 - d_{2_i}^2) + (L_2)^2 \right] + \text{mass}_2 \cdot (G_2)^2$$

$$I_2 = 40.65 \cdot \text{kg} \cdot \text{m}^2$$

for arm #3

$$I_3 = \frac{1}{12} \cdot \text{mass}_3 \cdot \left[ 3 \cdot (d_{3_o}^2 - d_{3_i}^2) + (L_3)^2 \right] + \text{mass}_3 \cdot (G_3)^2$$

$$I_3 = 10.78 \cdot \text{kg} \cdot \text{m}^2$$

for wrist

$$I_w = \frac{1}{12} \cdot \text{mass}_w \cdot \left[ 3 \cdot (d_{w_o}^2) + (L_w)^2 \right] + \text{mass}_w \cdot (G_w)^2$$

$$I_w = 152.2 \cdot \text{kg} \cdot \text{m}^2$$

at Joint C in wrist

$$I_C = \frac{1}{12} \cdot \frac{2 \cdot \text{mass}_w}{3} \cdot \left[ 3 \cdot (d_{w_o}^2) + \left( \frac{2}{3} \cdot L_w \right)^2 \right] + \frac{2 \cdot \text{mass}_w}{3} \cdot \left( \frac{2}{3} \cdot G_w \right)^2$$

$$I_C = 45.1 \cdot \text{kg} \cdot \text{m}^2$$

at Joint W2 in wrist

$$I_{W2} = \frac{1}{12} \cdot \frac{1 \cdot \text{mass}_w}{3} \cdot \left[ 3 \cdot (d_{w_o}^2) + \left( \frac{1}{3} \cdot L_w \right)^2 \right] + \frac{1 \cdot \text{mass}_w}{3} \cdot \left( \frac{1}{3} \cdot G_w \right)^2$$

$$I_{W2} = 5.6 \cdot \text{kg} \cdot \text{m}^2$$

for interface

$$I_{int} = \frac{1}{12} \cdot \text{mass}_{int} \cdot [3 \cdot (d_4)^2] + (L_{int})^2 + \text{mass}_{int} \cdot G_{int}^2$$

$$I_{int} = 168.2 \cdot \text{kg} \cdot \text{m}^2$$

for wrist and interface combined

$$I_4 = I_w + I_{int}$$

$$I_4 = 320.3 \cdot \text{kg} \cdot \text{m}^2$$

for tool and load

$$I_{tool} = \frac{1}{12} \cdot \text{mass}_{tool} \cdot [((.5 \cdot \text{m})^2 + (.5 \cdot \text{m})^2) + (L_5)^2] + \text{mass}_{tool} \cdot G_5^2$$

$$I_{tool} = 198.7 \cdot \text{kg} \cdot \text{m}^2$$

$$I_{load} = \frac{1}{12} \cdot \text{mass}_{load} \cdot [((.5 \cdot \text{m})^2 + (.5 \cdot \text{m})^2) + (L_5)^2] + \text{mass}_{load} \cdot G_5^2$$

$$I_{load} = 993.3 \cdot \text{kg} \cdot \text{m}^2$$

$$I_5 = I_{load} + I_{tool}$$

$$I_5 = 1192 \cdot \text{kg} \cdot \text{m}^2$$

Mass moment of inertia for entire structure for use in calculating torque required at base of robotic arm.

$$i = 1..5 \quad I_{base} = \sum_i I_i \quad I_{base} = 1.597 \cdot 10^3 \cdot \text{kg} \cdot \text{m}^2$$

Mass moment of inertia for entire structure excluding arm #1 for use in calculating torque required at Joint A.

$$j = 2..5 \quad I_{jointA} = \sum_j I_j \quad I_{jointA} = 1.564 \cdot 10^3 \cdot \text{kg} \cdot \text{m}^2$$

Mass moment of inertia for entire structure excluding arm #1 and arm #2 for use in calculating torque required at Joint A.

$$I_{jointB} = I_3 + I_4 + I_5 \quad I_{jointB} = 1.523 \cdot 10^3 \cdot \text{kg} \cdot \text{m}^2$$

Mass moment of inertia for maximum load, tool, interface, and wrist for use in calculating torque at Joint W1 in wrist.

$$i = 4..5 \quad I_{jointW1} = I_4 + I_5 \quad I_{jointW1} = 1.512 \cdot 10^3 \cdot \text{kg} \cdot \text{m}^2$$

Mass moment of inertia for maximum load, tool, interface, and 2/3 of wrist for use in calculating torque at Joint C in wrist.

$$I_{jointC} = I_C - I_{int} + I_5 \quad I_{jointC} = 1.405 \cdot 10^3 \cdot \text{kg} \cdot \text{m}^2$$

Mass moment of inertia for maximum load, tool, interface, and 1/3 of wrist for use in calculating torque at Joint W2 in wrist.

$$I_{jointW2} = I_{W2} - I_{int} + I_5 \quad I_{jointW2} = 1.366 \cdot 10^3 \cdot \text{kg} \cdot \text{m}^2$$

**Summation of mass moment of inertia calculations.**

$$\begin{array}{llll}
 I_1 = 33.6 \cdot \text{kg} \cdot \text{m}^2 & I_a = 0.399 \cdot \text{kg} \cdot \text{m}^2 & I_C = 45.09 \cdot \text{kg} \cdot \text{m}^2 & I_{\text{jointW1}} = 1512 \cdot \text{kg} \cdot \text{m}^2 \\
 I_2 = 40.6 \cdot \text{kg} \cdot \text{m}^2 & I_b = 16.731 \cdot \text{kg} \cdot \text{m}^2 & I_{W2} = 5.64 \cdot \text{kg} \cdot \text{m}^2 & I_{\text{jointC}} = 1405 \cdot \text{kg} \cdot \text{m}^2 \\
 I_3 = 10.8 \cdot \text{kg} \cdot \text{m}^2 & I_c = 16.453 \cdot \text{kg} \cdot \text{m}^2 & I_{\text{base}} = 1597 \cdot \text{kg} \cdot \text{m}^2 & I_{\text{jointW2}} = 1366 \cdot \text{kg} \cdot \text{m}^2 \\
 I_4 = 320.3 \cdot \text{kg} \cdot \text{m}^2 & I_w = 152.2 \cdot \text{kg} \cdot \text{m}^2 & I_{\text{jointA}} = 1564 \cdot \text{kg} \cdot \text{m}^2 & \\
 I_5 = 1192 \cdot \text{kg} \cdot \text{m}^2 & I_{\text{int}} = 168.2 \cdot \text{kg} \cdot \text{m}^2 & I_{\text{jointB}} = 1523 \cdot \text{kg} \cdot \text{m}^2 & 
 \end{array}$$

**Dynamic Analysis**

As a result of the static analysis the geometric structure of the robotic arm is now known. From this, the masses of the members were calculated and then the center of gravity of each member was determined. The next step is to calculate the torque at each joint so that the proper motor selection can be carried out. For the calculation of the torques the accelerations of the members of the robotic arm must be determined. The accelerations are a function of the angles of the operating range of each joint and therefore the orientation of the robotic arm at which the maximum accelerations exist must be found. In addition, the mass moments of inertia for each member must be calculated.

**Range of motion of each joint** As specified by members of Mechanical Subsystem

- Joint at base  $\theta = 0 \dots 90$  in positive and negative directions, for a full range of 180 degrees
- Joint A  $\theta_a = 0 \dots 135$  in positive and negative directions, for a full range of 270 degrees
- Joint B  $\theta_b = 0 \dots 135$  in positive and negative directions, for a full range of 270 degrees
- Joint W1  $\theta_{w1} = 0 \dots 180$  in positive and negative directions, for a full range of 360 degrees
- Joint C  $\theta_c = 0 \dots 135$  in positive and negative directions, for a full range of 270 degrees
- Joint W2  $\theta_{w2} = 0 \dots 180$  in positive and negative directions, for a full range of 360 degrees

**Angular accelerations and angular velocities at each joint as specified by members of Mechanical Subsystem**

$$\begin{array}{lll}
 \text{Joint at base} & \omega_1 = 0.025 \cdot \text{sec}^{-1} & \alpha_1 = 0.005 \cdot \text{sec}^{-2} \\
 \text{Joint A} & \omega_2 = 0.025 \cdot \text{sec}^{-1} & \alpha_2 = 0.005 \cdot \text{sec}^{-2} \\
 \text{Joint B} & \omega_3 = 0.025 \cdot \text{sec}^{-1} & \alpha_3 = 0.005 \cdot \text{sec}^{-2} \\
 \text{Joint W1} & \omega_5 = 0.025 \cdot \text{sec}^{-1} & \alpha_5 = 0.005 \cdot \text{sec}^{-2} \\
 \text{Joint C} & \omega_4 = 0.025 \cdot \text{sec}^{-1} & \alpha_4 = 0.005 \cdot \text{sec}^{-2} \\
 \text{Joint W2} & \omega_6 = 0.025 \cdot \text{sec}^{-1} & \alpha_6 = 0.005 \cdot \text{sec}^{-2}
 \end{array}$$

**Center of gravity of arm #1; velocity and acceleration**

$$\begin{array}{lll}
 v_{x_1} = 0 \cdot \frac{\text{m}}{\text{sec}} & a_{x_1} = -(\omega_1)^2 \cdot G_1 & a_{x_1} = -5.88 \cdot 10^{-4} \cdot \frac{\text{m}}{\text{sec}^2} \\
 v_{y_1} = 0 \cdot \frac{\text{m}}{\text{sec}} & a_{y_1} = 0 \cdot \frac{\text{m}}{\text{sec}} & \\
 v_{z_1} = -\omega_1 \cdot G_1 & v_{z_1} = -0.024 \cdot \frac{\text{m}}{\text{sec}} & a_{z_1} = \alpha_1 \cdot G_1 & a_{z_1} = 0.0047 \cdot \frac{\text{m}}{\text{sec}^2}
 \end{array}$$

Joint A (end of arm #1): velocity and acceleration

$$\begin{aligned}
 v_{x,a} &= 0 \cdot \frac{m}{\text{sec}} & a_{xa} &= -\omega_1^2 \cdot L_1 & a_{xa} &= -0.001 \cdot \frac{m}{\text{sec}^2} \\
 v_{ya} &= 0 \cdot \frac{m}{\text{sec}} & a_{ya} &= 0 \cdot \frac{m}{\text{sec}} \\
 v_{za} &= -\omega_1 \cdot L_1 & v_{za} &= -0.05 \cdot \frac{m}{\text{sec}} & a_{za} &= \alpha_1 \cdot L_1 & a_{za} &= 0.01 \cdot \frac{m}{\text{sec}^2}
 \end{aligned}$$

Joint B (end of arm #2): velocity and acceleration

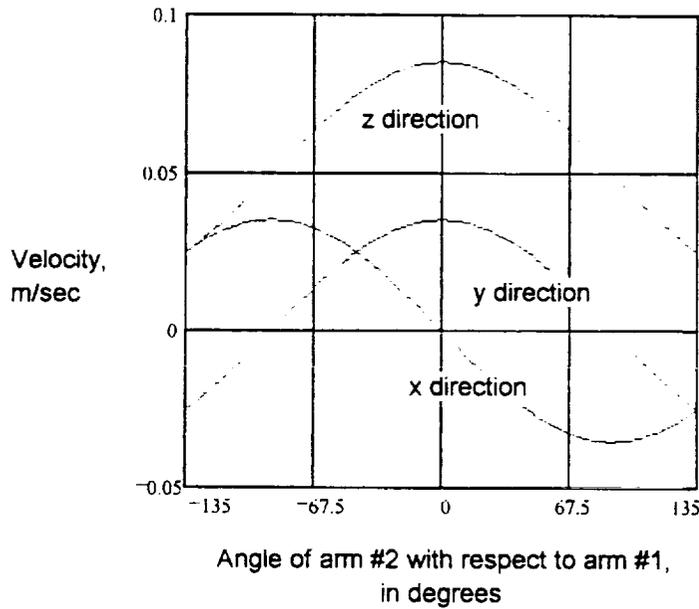
Velocity

in the x direction  $v_{xb_{\theta_a}} = -\omega_2 \cdot L_2 \cdot \sin\left(\theta_a \cdot \frac{\pi}{180}\right)$

in the y direction  $v_{yb_{\theta_a}} = \omega_2 \cdot L_2 \cdot \cos\left(\theta_a \cdot \frac{\pi}{180}\right)$

in the z direction  $v_{zb_{\theta_a}} = \omega_1 \cdot L_1 + L_2 \cdot \cos\left(\theta_a \cdot \frac{\pi}{180}\right)$

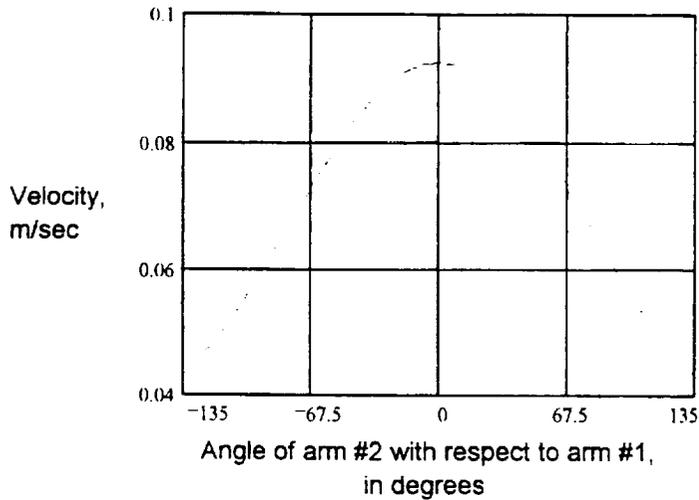
**Graph 4.3.0** Graph of directional velocities versus angle displacement for Joint B. The angle displacement is the angle between the axis of arm #2 and the axis of arm #1.



Magnitude of velocity for arm #2

$$v_{b_{\theta_a}} = \sqrt{(v_{xb_{\theta_a}})^2 + (v_{yb_{\theta_a}})^2 + (v_{zb_{\theta_a}})^2}$$

**Graph 4.3.1** Graph of velocity magnitude versus angle of displacement for Joint B



Graph 4.3.1 shows that the maximum velocity occurs when arm #2 is at 0 degrees displacement with respect to arm #1.

Acceleration

in the x direction

$$a_{xb_{\theta a}} = -(\omega_1)^2 \cdot \left( L_1 - L_2 \cdot \cos \left( \theta a \cdot \frac{\pi}{180} \right) \right) - (\omega_2)^2 \cdot L_2 \cdot \cos \left( \theta a \cdot \frac{\pi}{180} \right) - \alpha_2 \cdot L_2 \cdot \sin \left( \theta a \cdot \frac{\pi}{180} \right)$$

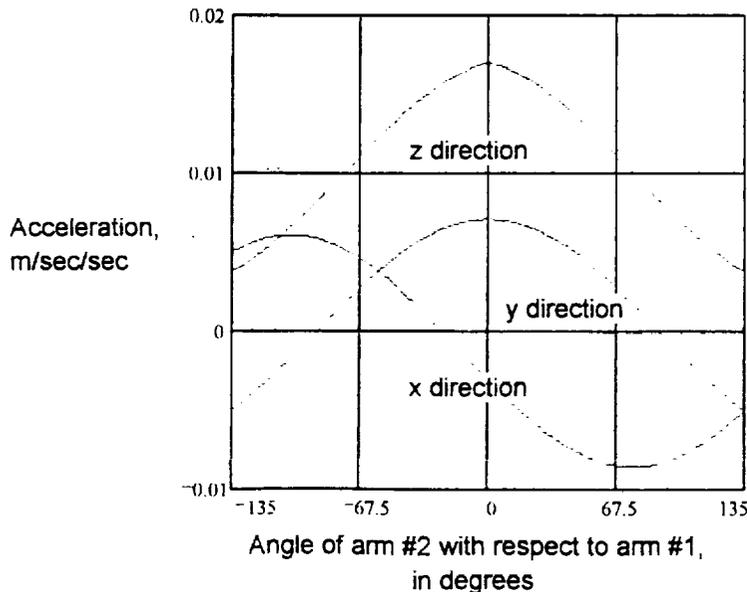
in the y direction

$$a_{yb_{\theta a}} = \alpha_2 \cdot L_2 \cdot \cos \left( \theta a \cdot \frac{\pi}{180} \right)$$

in the z direction

$$a_{zb_{\theta a}} = \alpha_1 \cdot \left( L_1 + L_2 \cdot \cos \left( \theta a \cdot \frac{\pi}{180} \right) \right) - 2 \cdot \omega_1 \cdot \omega_2 \cdot L_2 \cdot \sin \left( \theta a \cdot \frac{\pi}{180} \right)$$

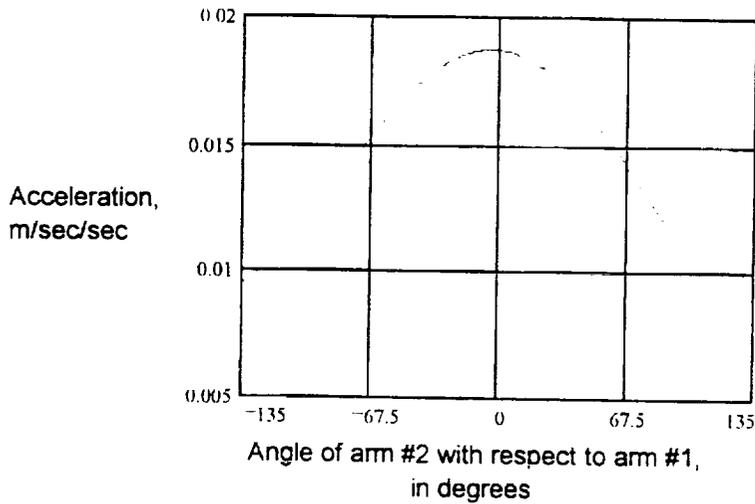
**Graph 4.3.2** Graph of directional accelerations versus angle displacement for Joint B. The angle displacement is the angle between the axis of arm #2 and the axis of arm #1.



Magnitude of acceleration for arm #2

$$a_{b_{\theta a}} = \sqrt{a_{x_{b_{\theta a}}}^2 + a_{y_{b_{\theta a}}}^2 + a_{z_{b_{\theta a}}}^2}$$

**Graph 4.3.3** Graph of acceleration magnitude versus angle of displacement for Joint B



Graph 4.3.3 shows that the maximum acceleration of arm #2 occurs when arm #2 is at 0 degrees displacement with respect to arm #1.

Joint C (located in wrist section): velocity and acceleration

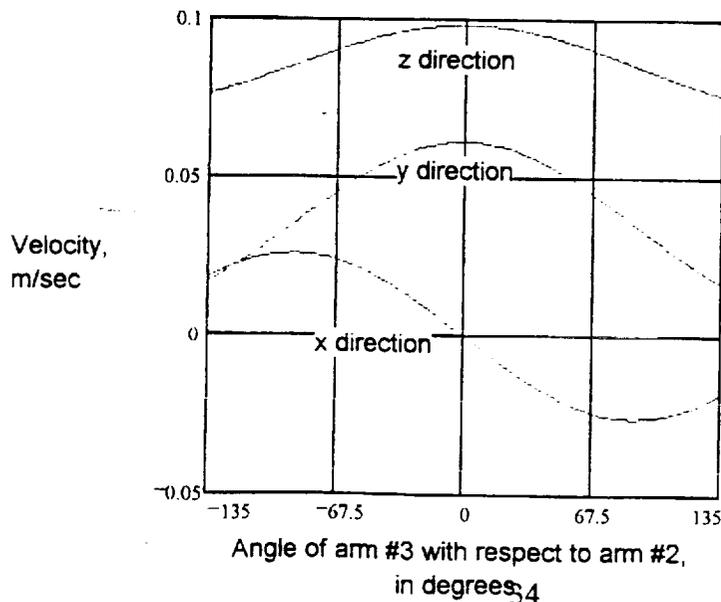
Velocity

in the x direction  $v_{x_{c_{\theta b}}} = -(\omega_2 + \omega_3) \cdot \left(L_3 + \frac{2}{3} \cdot L_w\right) \cdot \sin\left(\theta b \cdot \frac{\pi}{180}\right)$

in the y direction  $v_{y_{c_{\theta b}}} = \omega_2 \cdot L_2 + \omega_3 \cdot \left(L_3 + \frac{2}{3} \cdot L_w\right) \cdot \cos\left(\theta b \cdot \frac{\pi}{180}\right) + \omega_2 \cdot \left(L_3 + \frac{2}{3} \cdot L_w\right) \cdot \cos\left(\theta b \cdot \frac{\pi}{180}\right)$

in the z direction  $v_{z_{c_{\theta b}}} = \omega_1 \cdot (L_1 + L_2) + \omega_1 \cdot \left(L_3 + \frac{2}{3} \cdot L_w\right) \cdot \cos\left(\theta b \cdot \frac{\pi}{180}\right)$

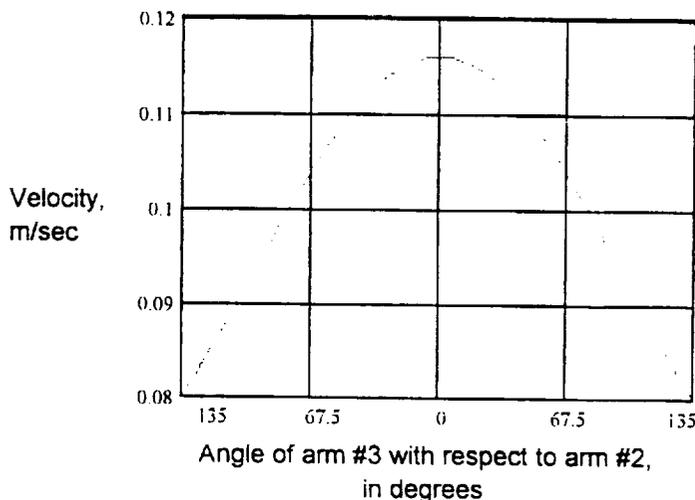
**Graph 4.3.4** Graph of directional velocities versus angle displacement for Joint C. The angle displacement is the angle between the axis of arm #3 and the axis of arm #2.



### Magnitude of velocity for arm #3

$$v_{c_{\theta b}} = \sqrt{v_{xc_{\theta b}}^2 + v_{yc_{\theta b}}^2 + v_{zc_{\theta b}}^2}$$

**Graph 4.3.5** Graph of velocity magnitude versus angle of displacement for Joint C



Graph 4.3.5 shows that the maximum velocity occurs when arm #3 is at 0 degrees displacement with respect to arm #2.

### Acceleration

in the x direction

$$X1_{\theta b} = -(\omega_1)^2 \left[ L_1 - L_2 + \left( L_3 + \frac{2}{3} L_w \right) \cdot \cos \left( \theta b \cdot \frac{\pi}{180} \right) \right]$$

$$X2_{\theta b} = -(\omega_2)^2 \left[ L_2 + \left( L_3 + \frac{2}{3} L_w \right) \cdot \cos \left( \theta b \cdot \frac{\pi}{180} \right) \right] - \alpha_2 \cdot \left( L_3 + \frac{2}{3} L_w \right) \cdot \sin \left( \theta b \cdot \frac{\pi}{180} \right)$$

$$X3_{\theta b} = -\alpha_3 \cdot \left( L_3 + \frac{2}{3} L_w \right) \cdot \sin \left( \theta b \cdot \frac{\pi}{180} \right) - (\omega_3)^2 \cdot \left( L_3 + \frac{2}{3} L_w \right) \cdot \cos \left( \theta b \cdot \frac{\pi}{180} \right)$$

$$a_{xc_{\theta b}} = X1_{\theta b} - X2_{\theta b} + X3_{\theta b}$$

in the y direction

$$Y1_{\theta b} = \alpha_2 \cdot \left[ L_2 + \left( L_3 + \frac{2}{3} L_w \right) \cdot \cos \left( \theta b \cdot \frac{\pi}{180} \right) \right] - (\omega_2)^2 \cdot \left( L_3 + \frac{2}{3} L_w \right) \cdot \sin \left( \theta b \cdot \frac{\pi}{180} \right)$$

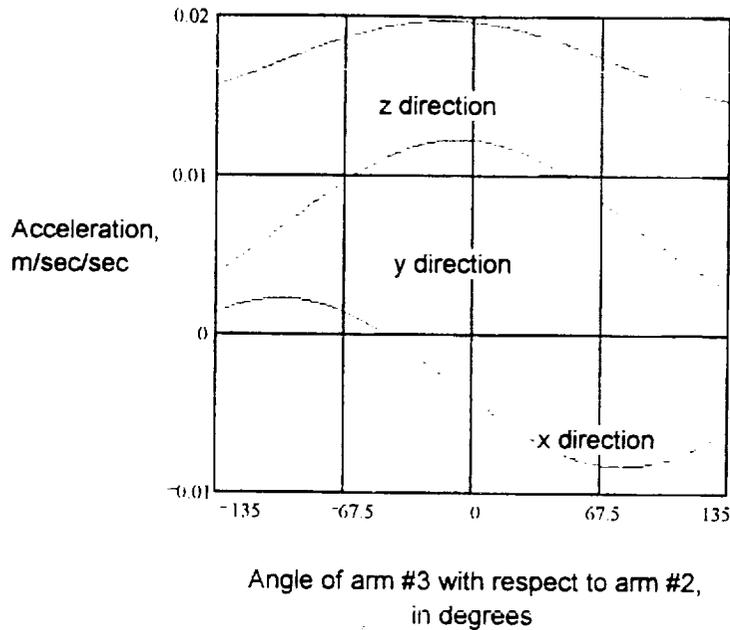
$$Y2_{\theta b} = \alpha_3 \cdot \left( L_3 + \frac{2}{3} L_w \right) \cdot \cos \left( \theta b \cdot \frac{\pi}{180} \right) - (\omega_3)^2 \cdot \left( L_3 + \frac{2}{3} L_w \right) \cdot \sin \left( \theta b \cdot \frac{\pi}{180} \right)$$

$$a_{yc_{\theta b}} = Y1_{\theta b} - Y2_{\theta b}$$

in the z direction

$$a_{zc_{\theta b}} = \alpha_1 \cdot \left[ L_1 - L_2 + \left( L_3 + \frac{2}{3} L_w \right) \cdot \cos \left( \theta b \cdot \frac{\pi}{180} \right) \right] - 2 \cdot \left[ \omega_1 \cdot \omega_3 \cdot \left( L_3 + \frac{2}{3} L_w \right) \cdot \sin \left( \theta b \cdot \frac{\pi}{180} \right) \right]$$

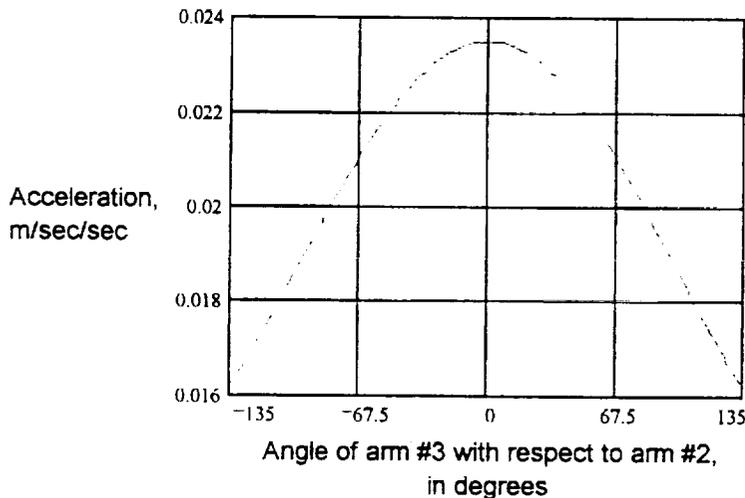
**Graph 4.3.6** Graph of directional accelerations versus angle displacement for Joint C. The angle displacement is the angle between the axis of arm #3 and the axis of arm #2.



**Magnitude of acceleration for arm #3**

$$a_{c_{0b}} = \sqrt{(a_{xc_{0b}})^2 + (a_{yc_{0b}})^2 + (a_{zc_{0b}})^2}$$

**Graph 4.3.7** Graph of acceleration magnitude versus angle of displacement for Joint C



Graph 4.3.7 shows that the maximum acceleration of arm #3 occurs when arm #3 is at 0 degrees displacement with respect to arm #2.

**Center of gravity of tool and load: velocity and acceleration**

**Velocity**

in the x direction

$$v_{x5_{\theta c}} = (\omega_2 - \omega_3) \cdot \left[ L_3 - \frac{1}{3} \cdot L_w + L_{int} \right] \cdot \sin \left( \theta c \cdot \frac{\pi}{180} \right) - (\omega_2 - \omega_3) \cdot \left[ \frac{2}{3} \cdot L_w + G_5 \right] \cdot \sin \left( \theta c \cdot \frac{\pi}{180} \right)$$

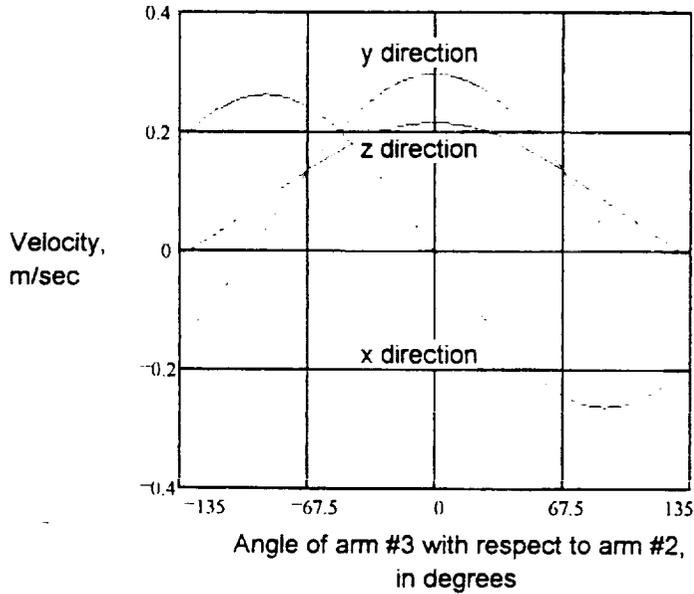
in the y direction

$$v_{y5_{\theta c}} = \omega_2 \cdot L_2 + \omega_3 \cdot (L_3 + L_4 + G_5) \cdot \cos \left( \theta c \cdot \frac{\pi}{180} \right) + \omega_2 \cdot (L_3 + L_4 + G_5) \cdot \cos \left( \theta c \cdot \frac{\pi}{180} \right)$$

in the z direction

$$v_{zS_{\theta c}} = \omega_1 \cdot (L_1 + L_2) - \omega_1 \cdot L_3 + L_4 - G_5 \cdot \cos\left(\theta c \cdot \frac{\pi}{180}\right)$$

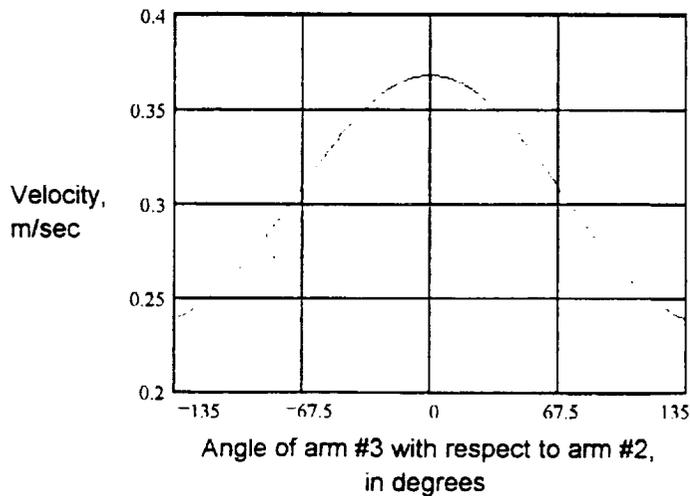
**Graph 4.3.8** Graph of directional velocities versus angle displacement for tool. The angle displacement is the angle between the axis of the interface and the axis of arm #3.



**Magnitude of velocity at center of gravity of tool and load**

$$v_{S_{\theta b}} = \sqrt{(v_{xS_{\theta b}})^2 + (v_{yS_{\theta b}})^2 + (v_{zS_{\theta b}})^2}$$

**Graph 4.3.9** Graph of velocity magnitude versus angle of displacement for tool and load



Graph 4.3.9 shows that the maximum velocity occurs when tool and load are at 0 degrees displacement with respect to arm #2.

### Acceleration

in the x direction

$$X1_{\theta c} = -\omega_1^2 \cdot (L_1 + L_2 + L_3 + L_4 + G_5) \cdot \cos \theta c \cdot \frac{\pi}{180}$$

$$X2_{\theta c} = \omega_2^2 \cdot (L_2 + L_3 + L_4 + G_5) \cdot \cos \theta c \cdot \frac{\pi}{180} - \alpha_2 \cdot (L_3 + L_4 + G_5) \cdot \sin \theta c \cdot \frac{\pi}{180}$$

$$X3_{\theta c} = -\alpha_3 \cdot (L_3 + L_4 + G_5) \cdot \sin \theta c \cdot \frac{\pi}{180} - \omega_3^2 \cdot (L_3 + L_4 + G_5) \cdot \cos \theta c \cdot \frac{\pi}{180}$$

$$a_{x5_{\theta c}} = X1_{\theta c} + X2_{\theta c} + X3_{\theta c}$$

in the y direction

$$Y1_{\theta c} = \alpha_2 \cdot \left[ L_2 + (L_3 + L_4 + G_5) \cdot \cos \theta c \cdot \frac{\pi}{180} \right] - \omega_2^2 \cdot (L_3 + L_4 + G_5) \cdot \sin \theta c \cdot \frac{\pi}{180}$$

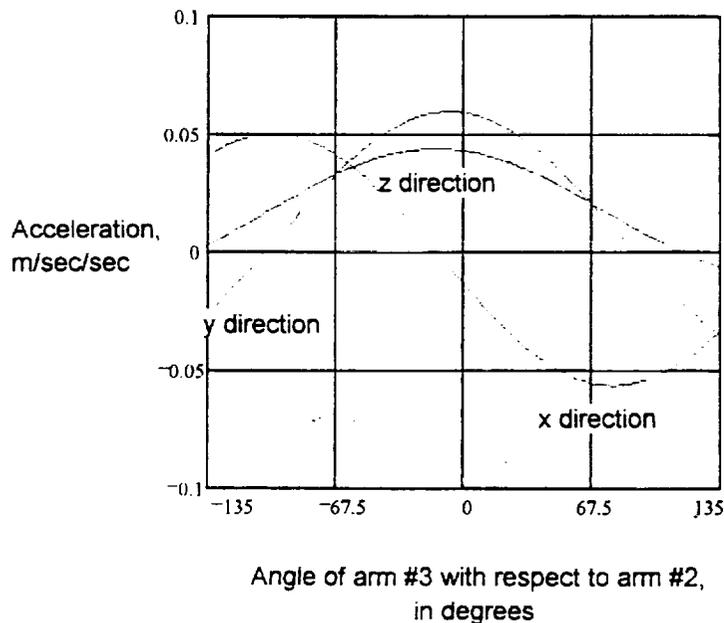
$$Y2_{\theta c} = \alpha_3 \cdot (L_3 + L_4 + G_5) \cdot \cos \theta c \cdot \frac{\pi}{180} - \omega_3^2 \cdot (L_3 + L_4 + G_5) \cdot \sin \theta c \cdot \frac{\pi}{180}$$

$$a_{y5_{\theta c}} = Y1_{\theta c} + Y2_{\theta c}$$

in the z direction

$$a_{z5_{\theta c}} = \alpha_1 \cdot \left[ L_1 + L_2 + (L_3 + L_4 + G_5) \cdot \cos \theta c \cdot \frac{\pi}{180} \right] - 2 \cdot \omega_1 \cdot \omega_3 \cdot (L_3 + L_4 + G_5) \cdot \sin \theta c \cdot \frac{\pi}{180}$$

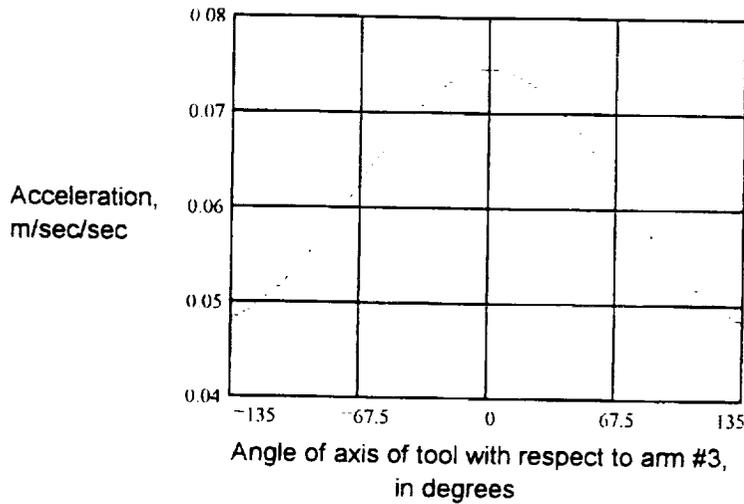
**Graph 4.3.10** Graph of directional accelerations versus angle displacement for tool and load. The angle displacement is the angle between the axis of the tool and the axis of arm #3.



### Magnitude of acceleration for center of gravity for tool and load

$$a_{5_{\theta c}} = \sqrt{a_{x5_{\theta c}}^2 + a_{y5_{\theta c}}^2 + a_{z5_{\theta c}}^2}$$

**Graph 4.3.11** Graph of acceleration magnitude versus angle of displacement for tool and load



Graph 4.3.11 shows that the maximum acceleration of tool and load occurs when axis of tool is at 0 degrees displacement with respect to arm #3.

So far the dynamic analysis has proven that the maximum acceleration at each member of the arm occurs when each member of the arm is in the same horizontal plane as the base. Or in other words, the displacement angle at each joint is 0 degrees. This is the expected result. The following calculations are intended to calculate the torques required at each joint so that the appropriate motors can be chosen.

### Torque Calculations.

#### For motor at the base of the robotic arm.

Tangential acceleration (z direction) at center of gravity of entire structure.

$$a_{\text{base}} = \alpha_1 \cdot G_{\text{base}} \qquad a_{\text{base}} = 0.0188 \cdot \frac{\text{m}}{\text{sec}^2}$$

Torque required for motor at base of robotic arm.

$$T_{\text{base}} = I_{\text{base}} \cdot \alpha_1 + G_{\text{base}} \cdot \text{mass}_{\text{total}} \cdot a_{\text{base}}$$

$$T_{\text{base}} = 20.1 \cdot \text{N} \cdot \text{m}$$

#### For motor at Joint A.

Tangential acceleration (y direction) at center of gravity of structure excluding arm #1.

$$a_{\text{jointA}} = \alpha_1 \cdot G_{\text{jointA}} \qquad a_{\text{jointA}} = 0.0115 \cdot \frac{\text{m}}{\text{sec}^2}$$

Mass of entire structure minus arm #1.

$$\text{mass}_{\text{jointA}} = \text{mass}_{\text{total}} - \text{mass}_1 \qquad \text{mass}_{\text{jointA}} = 136.25 \cdot \text{kg}$$

Torque required for motor at joint A.

$$T_{\text{jointA}} = I_{\text{jointA}} \cdot \alpha_1 + G_{\text{jointA}} \cdot \text{mass}_{\text{jointA}} \cdot a_{\text{jointA}} + G_{\text{jointA}} \cdot \text{mass}_{\text{jointA}} \cdot g$$

$$T_{\text{jointA}} = 521.6 \cdot \text{N} \cdot \text{m}$$

**For motor at Joint B.**

Tangential acceleration (y direction) at center of gravity of structure excluding arms #1 and #2.

$$a_{\text{jointB}} = \alpha_1 \cdot G_{\text{jointB}} \qquad a_{\text{jointB}} = 0.0048 \cdot \frac{\text{m}}{\text{sec}^2}$$

Mass of entire structure minus arms #1 and #2.

$$\text{mass}_{\text{jointB}} = \text{mass}_{\text{jointA}} - \text{mass}_2 \qquad \text{mass}_{\text{jointB}} = 130.83 \cdot \text{kg}$$

Torque required for motor at Joint B.

$$T_{\text{jointB}} = I_{\text{jointB}} \cdot \alpha_1 + G_{\text{jointB}} \cdot \text{mass}_{\text{jointB}} \cdot a_{\text{jointB}} + G_{\text{jointB}} \cdot \text{mass}_{\text{jointB}} \cdot g$$

$$T_{\text{jointB}} = 214.7 \cdot \text{N} \cdot \text{m}$$

**For motor at Joint W1.**

Tangential acceleration (z direction) at center of gravity of structure excluding arms #1, #2 and #3.

$$a_{\text{jointW1}} = \alpha_1 \cdot G_{\text{jointW1}} \qquad a_{\text{jointW1}} = 2.97 \cdot 10^{-3} \cdot \frac{\text{m}}{\text{sec}^2}$$

Mass of entire structure minus arms #1, #2, and #3.

$$\text{mass}_{\text{jointW1}} = \text{mass}_{\text{jointB}} - \text{mass}_3 \qquad \text{mass}_{\text{jointW1}} = 130 \cdot \text{kg}$$

Torque required for motor at Joint W1. Weight does play a part because Joint W is rotated 90 degrees.

$$T_{\text{jointW1}} = I_{\text{jointW1}} \cdot \alpha_1 + G_{\text{jointW1}} \cdot \text{mass}_{\text{jointW1}} \cdot a_{\text{jointW1}} + G_{\text{jointW1}} \cdot \text{mass}_{\text{jointW1}} \cdot g$$

$$T_{\text{jointW1}} = 134.1 \cdot \text{N} \cdot \text{m}$$

**For motor at Joint C.**

Tangential acceleration (y direction) at center of gravity of structure including tool, interface, and wrist.

$$a_{\text{jointC}} = \alpha_1 \cdot G_{\text{jointC}} \qquad a_{\text{jointC}} = 2.69 \cdot 10^{-3} \cdot \frac{\text{m}}{\text{sec}^2}$$

Mass of tool, load, interface, Joints C and W2.

$$\text{mass}_{\text{jointC}} = \text{mass}_{\text{tool}} + \text{mass}_{\text{int}} + \frac{2}{3} \cdot \text{mass}_w \qquad \text{mass}_{\text{jointC}} = 26.7 \cdot \text{kg}$$

Torque required for motor at Joint C.

$$T_{\text{jointC}} = I_{\text{jointC}} \cdot \alpha_1 + G_{\text{jointC}} \cdot \text{mass}_{\text{jointC}} \cdot a_{\text{jointC}} + G_{\text{jointC}} \cdot \text{mass}_{\text{jointC}} \cdot g$$

$$T_{\text{jointC}} = 30.5 \cdot \text{N} \cdot \text{m}$$

**For motor at Joint W2.**

Tangential acceleration (z direction) at center of gravity of structure including tool, load, interface, and joint W2.

$$a_{\text{jointW2}} = \alpha_1 \cdot G_{\text{jointW2}} \qquad a_{\text{jointW2}} = 2.38 \cdot 10^{-3} \cdot \frac{\text{m}}{\text{sec}^2}$$

Mass of tool, load, interface, and Joint W2.

$$\text{mass}_{\text{jointW2}} = \text{mass}_{\text{jointC}} + \frac{1}{3} \cdot \text{mass}_w \quad \text{mass}_{\text{jointW2}} = 30 \cdot \text{kg}$$

Torque required for motor at Joint W2. Weight does not play a part because axis of rotation is the same as centerline axis for members.

$$T_{\text{jointW2}} = I_{\text{jointW2}} \cdot \alpha_1 + G_{\text{jointW2}} \cdot \text{mass}_{\text{jointW2}} \cdot a_{\text{jointW2}}$$

$$T_{\text{jointW2}} = 6.9 \cdot \text{N} \cdot \text{m}$$

## Units, dimensions and other constants

$$\text{MPa} = 1 \cdot 10^6 \cdot \text{Pa} \quad \text{GPa} = 1 \cdot 10^9 \cdot \text{Pa} \quad \rho = \frac{\text{kg}}{\text{m}^3} \cdot 1 \quad \alpha = 1 \cdot 10^{-6} \quad g_{\text{moon}} = 1.634 \frac{\text{m}}{\text{sec}^2}$$

Material of the structure is aluminum 2014-T6

$$\rho_{\text{al}} = 2800 \cdot \rho \quad \text{Density:}$$

$$\text{UST}_{\text{al}} = 480 \cdot \text{MPa} \quad \text{Ultimate Strength (Tension):}$$

$$\text{USS}_{\text{al}} = 290 \cdot \text{MPa} \quad \text{Ultimate Strength (Shear)}$$

$$\text{YST}_{\text{al}} = 410 \cdot \text{MPa} \quad \text{Yield Strength (Tension)}$$

$$\text{YSS}_{\text{al}} = 220 \cdot \text{MPa} \quad \text{Yield Strength (Shear)}$$

$$E_{\text{al}} = 72 \cdot \text{GPa} \quad \text{Modulus of Elasticity:}$$

$$G_{\text{al}} = 27 \cdot \text{GPa} \quad \text{Modulus of Rigidity:}$$

$$\alpha_{\text{al}} = 23 \cdot \alpha \quad \text{Coefficient of linear thermal expansion:}$$

$$\text{Duc}_{\text{al}} = 13 \quad \text{Ductility, percent elongation:}$$

Material of the ball and screw is Cold Rolled Stainless Steel (302)

$$\rho_{\text{st}} = 7920 \cdot \rho \quad \text{Density:}$$

$$\text{UST}_{\text{st}} = 860 \cdot \text{MPa} \quad \text{Ultimate Strength (Tension):}$$

$$\text{USS}_{\text{st}} = 430 \cdot \text{MPa} \quad \text{Ultimate Strength (Shear, assumed)}$$

$$\text{YST}_{\text{st}} = 520 \cdot \text{MPa} \quad \text{Yield Strength (Tension)}$$

$$\text{YSS}_{\text{st}} = 260 \cdot \text{MPa} \quad \text{Yield Strength (Shear, assumed)}$$

$$E_{\text{st}} = 190 \cdot \text{GPa} \quad \text{Modulus of Elasticity:}$$

$$G_{\text{st}} = 73 \cdot \text{GPa} \quad \text{Modulus of Rigidity:}$$

$$\alpha_{\text{st}} = 17.3 \cdot \alpha \quad \text{Coefficient of linear thermal expansion:}$$

$$\text{Duc}_{\text{st}} = 12 \quad \text{Ductility, percent elongation:}$$

The analysis for the mechanical structure of the robotic arm section #1 (Translational Mechanism) is shown below. For the analysis, the worst case scenario was taken to ensure proper performance.

The analysis of this portion of the arm consists of 6 sections. They are:

1. Material analysis (due to the stresses and strains on the structure, bending and fatigue)
2. Dynamic analysis of the structure and the translating mechanism (velocities, accelerations Forces and torques)
3. Motor selection (based on torques derived in dynamic analysis)
4. Machine analysis of the translating mechanism (wear and material selection of the mechanism and its components)
5. Thermal considerations (different materials exhibit different coefficients of linear thermal expansion)
6. Radiation considerations (different materials exhibit different reactions to prolonged radiation)

## Material Analysis

The following is the Material Analysis of the first link of the robotic arm (the translational link). Just as all subsequent components of the analysis, this analysis is performed on each of three sections of the first link. The following analysis is performed on section b of the link arm #1 (the middle section).

The assigned lengths of of each member of the robotic arm.

Arm #1: arm #1 is composed of three sections: a, b, and c. Sections a, b, and c are each hollow cylinders. The following analysis is for section b of the arm #1 link (the screw part of the ball and screw translational mechanism)

Entire length of arm #1:	$L_1 = 2 \cdot m$	Entire length of arm #3:	$L_3 = 0.386 \cdot m$
Length of section a:	$L_a = 0.5 \cdot m$	Entire length of wrist:	$L_w = 0.2 \cdot m$
Length of section b:	$L_b = 0.667 \cdot m$	Length of interface:	$L_{int} = 0.2 \cdot m$
Length of section c:	$L_c = 0.833 \cdot m$	Length of combined wrist and interface:	$L_4 = L_w + L_{int}$
Entire length of arm #2:	$L_2 = 1.414 \cdot m$		

Tool: The length of the tool used is for the maximum size tool as stated by the Tool Subsystem.

Entire length of tool:  $L_5 = 0.5 \cdot m$

Assigned masses and weights for each member of the robotic arm

For arms one, two and three, the masses are to be computed by static analysis:

Mass and weight of tool

$$\text{mass}_{\text{tool}} = 10 \cdot \text{kg} \quad w_{\text{tool}} = \text{mass}_{\text{tool}} \cdot g_{\text{moon}} \quad w_{\text{tool}} = 16.34 \cdot \text{N}$$

Mass and weight of maximum load:

$$\text{mass}_{\text{load}} = 50 \cdot \text{kg} \quad w_{\text{load}} = \text{mass}_{\text{load}} \cdot g_{\text{moon}} \quad w_{\text{load}} = 81.7 \cdot \text{N}$$

Combined mass and weight of tool and maximum load (for simplified calculations):

$$\text{mass}_5 = \text{mass}_{\text{tool}} + \text{mass}_{\text{load}} \quad \text{mass}_5 = 60 \cdot \text{kg}$$

$$w_5 = w_{\text{tool}} + w_{\text{load}} \quad w_5 = 98.04 \cdot \text{N}$$

Mass and weight of interface:

$$\text{mass}_{\text{int}} = 10 \cdot \text{kg} \quad w_{\text{int}} = \text{mass}_{\text{int}} \cdot g_{\text{moon}} \quad w_{\text{int}} = 16.34 \cdot \text{N}$$

Mass and weight of wrist:

$$\text{mass}_w = 10 \cdot \text{kg} \quad w_w = \text{mass}_w \cdot g_{\text{moon}} \quad w_w = 16.34 \cdot \text{N}$$

Combined mass and weight of wrist and interface:

$$\text{mass}_4 = \text{mass}_{\text{int}} + \text{mass}_w \quad \text{mass}_4 = 20 \cdot \text{kg}$$

$$w_4 = w_w + w_{\text{int}} \quad w_4 = 32.68 \cdot \text{N}$$

In addition to the masses and weights of the robotic arm members, a miscellaneous weight is added at each joint to account for the added weight of the motor, gears, casing, axle, etc. at each joint. This weight is given as a value larger than that actually expected in keeping with the worst case scenario.

$$\text{mass}_{\text{misc}} = 5 \cdot \text{kg} \quad w_{\text{misc}} = \text{mass}_{\text{misc}} \cdot g_{\text{moon}} \quad w_{\text{misc}} = 8.17 \cdot \text{N}$$

### Distances from base of robotic arm to centers of gravity of all members

Since each member of the robotic arm is symmetric in the yz and xy planes, the only direction that is going to affect the center of gravity of each member is the distance in x-direction from the base of the arm to the center of gravity of each member.

Center of gravity for each section of arm #1

$$\text{Section a:} \quad G_a = \frac{L_a}{2} \quad G_a = 0.25 \cdot \text{m}$$

$$\text{Section b:} \quad G_b = L_a + \frac{L_b}{2} \quad G_b = 0.834 \cdot \text{m}$$

$$\text{Section c:} \quad G_c = L_a + L_b + \frac{L_c}{2} \quad G_c = 1.584 \cdot \text{m}$$

Center of gravity for arm #2:

$$G_2 = L_1 + \frac{L_2}{2} \quad G_2 = 2.707 \cdot \text{m}$$

Center of gravity for arm #3:

$$G_3 = L_1 + L_2 + \frac{L_3}{2} \quad G_3 = 3.607 \cdot \text{m}$$

Center of gravity for wrist (assuming constant cross section):

$$G_w = L_1 + L_2 + L_3 + \frac{L_w}{2} \quad G_w = 3.9 \cdot \text{m}$$

Center of gravity for interface (assuming constant cross section):

$$G_{\text{int}} = L_1 + L_2 + L_3 + L_w + \frac{L_{\text{int}}}{2} \quad G_{\text{int}} = 4.1 \cdot \text{m}$$

Center of gravity for wrist and interface (as a whole to simplify calculations)

$$G_4 = L_1 + L_2 + L_3 + \frac{L_4}{2} \quad G_4 = 4 \cdot \text{m}$$

Center of gravity for tool with maximum load:

$$G_5 = L_1 \cdot L_2 \cdot L_3 \cdot L_4 \cdot \frac{L_5}{2} \quad G_5 = 4.45 \cdot \text{m}$$

Condensed static analysis numbers for the other sections of the arm. Note, these numbers were computed in another Mathcad document so the documentation is not included here.

At Joint C:

$$R_{yc} = w_5 + w_{int} + \frac{1}{3} \cdot w_w + 2 \cdot w_{misc} \quad R_{yc} = 136.167 \cdot \text{N}$$

$$M_c = \left[ \frac{1}{3} \cdot L_w + L_{int} + \frac{1}{2} \cdot L_5 \right] \cdot w_5 + \left[ \frac{1}{3} \cdot L_w + \frac{L_{int}}{2} \right] \cdot w_{int} + w_{misc} + \left[ \frac{1}{3} \cdot L_w + \frac{1}{6} \cdot w_w + w_{misc} \right]$$

$$M_c = 55.465 \cdot \text{N} \cdot \text{m}$$

At Joint B:

$$R_{yb} = R_{yc} + \frac{2}{3} \cdot w_w + 2 \cdot w_{misc} + w_3 \quad R_{yb} = 163.4 \cdot \text{N}$$

$$M_b = M_c + \left[ L_3 + \frac{1}{2} \cdot \left( \frac{2}{3} \cdot L_w \right) \right] \cdot \left( \frac{2}{3} \cdot w_w + 2 \cdot w_{misc} \right) + \frac{L_3}{2} \cdot w_3 \quad M_b = 67.793 \cdot \text{N} \cdot \text{m}$$

For Torque Load

$$T = \frac{1}{2} \cdot L_w \cdot w_w + \left( L_w + L_{int} \right) \cdot w_{int} + \left( L_w + L_{int} + \frac{1}{2} \cdot L_5 \right) \cdot w_5 \quad T = 71.896 \cdot \text{N} \cdot \text{m}$$

**BEGIN iterative loop**

The desired outer diameter of arm #3 is:  $d3_o = 100 \cdot \text{mm}$

Guess for the inner diameter of arm #3:  $d3_i = 95 \cdot \text{mm}$

Area at predetermined cross section of arm:

$$A_3 = \frac{\pi}{4} \cdot (d3_o^2 - d3_i^2) \quad A_3 = 7.658 \cdot 10^{-4} \cdot \text{m}^2$$

Centroidal moment of inertia for hollow cross section at specified inner and outer diameters:

$$Iz_3 = \frac{\pi}{64} \cdot (d3_o^4 - d3_i^4) \quad Iz_3 = 9.105 \cdot 10^{-7} \cdot \text{m}^4$$

Polar moment of inertia:

$$Iy_3 = Iz_3 \quad J_3 = Iy_3 + Iz_3 \quad J_3 = 1.821 \cdot 10^{-6} \cdot \text{m}^4$$

Distance from neutral axis to point of maximum tension (top surface) and compression (bottom surface) are the same:

$$c_3 = \frac{d3_o}{2} \quad c_3 = 0.05 \cdot \text{m}$$

Maximum magnitude of tensile and compressive stress (when arm is fully extended):

$$\sigma_3 = \frac{M_b \cdot c_3}{I_{z_3}} \quad \sigma_3 = 3.723 \cdot 10^6 \cdot \text{Pa}$$

Maximum shearing stress:  $Q = \frac{A_3}{2} \frac{d_3^2 - d_3^2}{d_3 + d_3} \pi$   $t_{t_3} = d_3 - d_3$

$$\tau_3 = \frac{R_{yb} \cdot Q}{I_{z_3} \cdot t_{t_3}} \quad \tau_3 = 4.266 \cdot 10^5 \cdot \text{Pa}$$

Maximum torsional stress (when joint W1 is rotated so that the rest of the arm past Joint C is positioned at a 90 degree angle with respect to the axis of the robotic arm resulting in a torsional stress):

$$\tau_{\text{twist},3} = \frac{T \cdot c_3}{J_3} \quad \tau_{\text{twist},3} = 1.974 \cdot 10^6 \cdot \text{Pa}$$

The maximum total stress in preselected cross section of arm #3 is:

$$\tau_{3,\text{max}} = \sqrt{\left(\frac{\sigma_3}{2}\right)^2 + \tau_3^2 + \tau_{\text{twist},3}^2} \quad \tau_{3,\text{max}} = 2.746 \cdot 10^6 \cdot \text{Pa}$$

The resulting factor of safety is:

$$n = \frac{0.4 \cdot YSS_{al}}{\tau_{3,\text{max}}} \quad n = 32.041$$

### END iterative loop

When the resulting F.S. equals the desired F.S., then the guess for the needed diameter is appropriate. Therefore the inner diameter for arm #3 is:

$$d_{3_i} = 95 \cdot \text{mm}$$

The inner and outer diameters of arm #3 are now known, therefore the volume, mass, weight, and wall thickness of arm #3 can now be determined.

Volume of arm #3:  $V_3 = A_3 \cdot L_3$   $V_3 = 295.585 \cdot \text{cm}^3$

Mass of arm #3:  $\text{mass}_3 = V_3 \cdot \rho_{al}$   $\text{mass}_3 = 0.828 \cdot \text{kg}$

Weight of arm #3:  $w_3 = \text{mass}_3 \cdot g_{\text{moon}}$   $w_3 = 1.352 \cdot \text{N}$

Wall thickness of arm #3:  $t_3 = \frac{d_3^2 - d_3^2}{2}$   $t_3 = 2.5 \cdot \text{mm}$

At joint B:

$$R_{yb} = R_{yc} + \frac{2}{3} \cdot w_w + 2 \cdot w_{\text{misc}} - w_3 \quad R_{yb} = 164.752 \cdot \text{N}$$

$$M_b = M_c + \left[ L_3 + \frac{1}{2} \left( \frac{2}{3} \cdot L_w \right) \right] \cdot \left[ \frac{2}{3} \cdot w_w + 2 \cdot w_{\text{misc}} \right] + \frac{L_3}{2} \cdot w_3 \quad M_b = 68.054 \cdot \text{N} \cdot \text{m}$$

$$T = \frac{1}{2} \cdot L_w \cdot w_w + (L_w + L_{int}) \cdot w_{int} + \left( L_w + L_{int} + \frac{1}{2} \cdot L_5 \right) \cdot w_5$$

$$T = 71.896 \cdot \text{N} \cdot \text{m}$$

At Joint A:

$$R_{ya} = R_{yb} + w_{misc} + w_2 \quad R_{ya} = 172.922 \cdot \text{N}$$

$$M_a = \frac{L_2}{2} \cdot w_2 + L_2 \cdot (R_{yb} + w_{misc}) + M_b \quad M_a = 312.566 \cdot \text{N} \cdot \text{m}$$

$$T = \frac{1}{2} \cdot L_w \cdot w_w + (L_w + L_{int}) \cdot w_{int} + \left( L_w + L_{int} + \frac{1}{2} \cdot L_5 \right) \cdot w_5$$

$$T = 71.896 \cdot \text{N} \cdot \text{m}$$

### BEGIN Iterative loop

The desired inner diameter of arm #2 is:  $d_{2i} = 105 \cdot \text{mm}$

Guess for the outer diameter of arm #2 is:  $d_{2o} = 113 \cdot \text{mm}$

Area at predetermined cross section of arm:

$$A_2 = \frac{\pi}{4} \cdot (d_{2o}^2 - d_{2i}^2) \quad A_2 = 0.001 \cdot \text{m}^2$$

Centroidal moment of inertia for hollow cross section at specified inner and outer diameters:

$$I_{z2} = \frac{\pi}{64} \cdot (d_{2o}^4 - d_{2i}^4) \quad I_{z2} = 2.037 \cdot 10^{-6} \cdot \text{m}^4$$

Polar moment of inertia:

$$I_{y2} = I_{z2} \quad J_2 = I_{y2} + I_{z2} \quad J_2 = 4.074 \cdot 10^{-6} \cdot \text{m}^4$$

Distance from neutral axis to point of maximum tension (top surface) and compression (bottom surface) are the same:

$$c_2 = \frac{d_{2o}}{2} \quad c_2 = 0.057 \cdot \text{m}$$

Maximum magnitude of tensile and compressive stress:

$$\sigma_2 = \frac{M_a \cdot c_2}{I_{z2}} \quad \sigma_2 = 8.67 \cdot 10^6 \cdot \text{Pa}$$

Maximum shearing stress:  $Q_2 = \frac{A_2}{2} \cdot \left[ \frac{d_{2o}^2 + d_{2i} \cdot d_{2o} - d_{2i}^2}{d_{2o} - d_{2i}} \right] \cdot \pi \quad t_{t2} = d_{2o} - d_{2i}$

$$\tau_2 = \frac{R_{ya} \cdot Q_2}{I_{z2} \cdot t_{t2}} \quad \tau_2 = 2.523 \cdot 10^5 \cdot \text{Pa}$$

Maximum torsional stress (when Joint W1 is rotated so that rest of arm past Joint C is positioned at a 90 degree angle with respect to axis of the robotic arm resulting in a torsional stress):

$$\tau_{\text{twist},2} = \frac{T \cdot c_2}{J_2} \quad \tau_{\text{twist},2} = 9.971 \cdot 10^5 \cdot \text{Pa}$$

The maximum total stress in preselected cross section of arm #2 is:

$$\tau_{2 \text{ max}} = \sqrt{\left(\frac{\sigma_2}{2}\right)^2 + \tau_2^2 + \tau_{\text{twist},2}^2} \quad \tau_{2 \text{ max}} = 4.455 \cdot 10^6 \cdot \text{Pa}$$

The resulting factor of safety is:

$$n = \frac{0.4 \cdot YSS_{al}}{\tau_{2 \text{ max}}} \quad n = 19.752$$

### END iterative loop

When the resulting F.S. equals the desired F.S, then the guess for the needed diameter is appropriate. Therefore, the outer diameter for arm #2 is:

$$d_{2o} = 113 \cdot \text{mm}$$

The inner and outer diameters of arm #2 are now known. Therefore, the volume, mass weight, and wall thickness of arm #2 can be determined

$$\text{Volume of arm \#2:} \quad V_2 = A_2 \cdot L_2 \quad V_2 = 1.937 \cdot 10^3 \cdot \text{cm}^3$$

$$\text{Mass of arm \#2:} \quad \text{mass}_2 = V_2 \cdot \rho_{al} \quad \text{mass}_2 = 5.423 \cdot \text{kg}$$

$$\text{Weight of arm \#2:} \quad w_2 = \text{mass}_2 \cdot g_{\text{moon}} \quad w_2 = 8.861 \cdot \text{N}$$

$$\text{Wall thickness of arm \#2:} \quad t_2 = \frac{d_{2o} - d_{2i}}{2} \quad t_2 = 4 \cdot \text{mm}$$

At Joint A:

$$R_{ya} = R_{yb} + w_{\text{misc}} + w_2 \quad R_{ya} = 172.922 \cdot \text{N}$$

$$M_a = \frac{L_2}{2} \cdot w_2 + L_2 \cdot (R_{yb} + w_{\text{misc}}) + M_b \quad M_a = 312.566 \cdot \text{N} \cdot \text{m}$$

$$T = \frac{1}{2} \cdot L_w \cdot w_w + (L_w + L_{\text{int}}) \cdot w_{\text{int}} + \left[ (L_w + L_{\text{int}} + \frac{1}{2} \cdot L_5) \cdot w_5 \right] \quad T = 71.896 \cdot \text{N} \cdot \text{m}$$

At cross section M-M of section c of arm #1:

$$R_{\text{sectionc}} = w_c + w_{\text{misc}} + R_{ya} \quad R_{\text{sectionc}} = 190.954 \cdot \text{N}$$

$$M_{\text{sectionc}} = \frac{L_c}{2} \cdot w_c + L_c \cdot (w_{\text{misc}} + R_{ya}) + M_a \quad M_{\text{sectionc}} = 477.479 \cdot \text{N} \cdot \text{m}$$

$$T = \frac{1}{2} \cdot L_w \cdot w_w + (L_w + L_{\text{int}}) \cdot w_{\text{int}} + \left[ L_w + L_{\text{int}} + \frac{1}{2} \cdot L_5 \right] \cdot w_5 \quad T = 71.896 \cdot \text{N} \cdot \text{m}$$

### BEGIN iterative loop

The desired inner diameter of section c of arm #1 is:

$$dc_i = 118 \text{ mm}$$

Guess for the outer diameter of section c of arm #1:

$$dc_o = 132 \text{ mm}$$

Area at predetermined cross section of arm:

$$A_c = \frac{\pi}{4} \cdot (dc_o^2 - dc_i^2) \quad A_c = 0.003 \cdot m^2$$

Centroidal moment of inertia for hollow cross section at specified inner and outer diameters:

$$I_{z_c} = \frac{\pi}{64} \cdot (dc_o^4 - dc_i^4) \quad I_{z_c} = 5.386 \cdot 10^{-6} \cdot m^4$$

Polar moment of inertia:

$$I_{y_c} = I_{z_c} \quad J_c = I_{y_c} + I_{z_c} \quad J_c = 1.077 \cdot 10^{-5} \cdot m^4$$

Distance from neutral axis to point of maximum tension (top surface) and compression (bottom surface) are the same:

$$c_c = \frac{dc_o}{2} \quad c_c = 0.066 \cdot m$$

Maximum magnitude of tensile and compressive stress:

$$\sigma_c = \frac{M_{\text{sectionc}} \cdot c_c}{I_{z_c}} \quad \sigma_c = 5.851 \cdot 10^6 \cdot Pa$$

Maximum shearing stress:

$$Q_c = \frac{A_c}{2} \cdot \frac{2}{3} \cdot \left[ \frac{dc_o^2 + dc_i \cdot dc_o + dc_i^2}{(dc_o + dc_i) \cdot \pi} \right] \quad tt_c = dc_o - dc_i$$

$$\tau_c = \frac{R_{\text{sectionc}} \cdot Q_c}{I_{z_c} \cdot tt_c} \quad \tau_c = 1.386 \cdot 10^5 \cdot Pa$$

Maximum torsional stress (when joint W1 is rotated so that the rest of arm past Joint C is positioned at a 90 degree angle with respect to axis of the robotic arm resulting in a torsional stress):

$$\tau_{\text{twistc}} = \frac{T \cdot c_c}{J_c} \quad \tau_{\text{twistc}} = 4.405 \cdot 10^5 \cdot Pa$$

$$\tau_{c \text{ max}} = \sqrt{\left(\frac{\sigma_c}{2}\right)^2 + \tau_c^2 + \tau_{\text{twistc}}^2} \quad \tau_{c \text{ max}} = 2.962 \cdot 10^6 \cdot Pa$$

The resulting factor of safety is:

$$n = 0.4 \cdot \frac{YSS_{al}}{\tau_{c \text{ max}}} \quad n = 29.711$$

END iterative loop

When the resulting F.S. equals the desired F.S then the guess for the needed diameter is appropriate. Therefore, the outer diameter for section c of arm #1 is:

$$d_{c_o} = 132 \cdot \text{mm}$$

The inner and outer diameter for section c of arm #1 is now known. Therefore, the volume, mass, weight, and wall thickness of section c of arm #1 can be determined.

$$\text{Volume of section c of arm \#1} \quad V_c = A_c \cdot L_c \quad V_c = 2.29 \cdot 10^3 \cdot \text{cm}^3$$

$$\text{Mass of section c of arm \#1} \quad \text{mass}_c = V_c \cdot \rho_{al} \quad \text{mass}_c = 6.412 \cdot \text{kg}$$

$$\text{Weight of section c of arm \#1} \quad w_c = \text{mass}_c \cdot g_{\text{moon}} \quad w_c = 10.476 \cdot \text{N}$$

$$\text{Wall thickness of section c of arm \#1} \quad t_c = \frac{d_{c_o} - d_{c_i}}{2} \quad t_c = 7 \cdot \text{mm}$$

At cross section M-M of section c of arm #1:

$$R_{\text{sectionc}} = w_c + w_{\text{misc}} + R_{ya} \quad R_{\text{sectionc}} = 200.43 \cdot \text{N}$$

$$M_{\text{sectionc}} = \frac{L_c}{2} \cdot w_c - L_c \cdot (w_{\text{misc}} + R_{ya}) + M_a \quad M_{\text{sectionc}} = 481.426 \cdot \text{N} \cdot \text{m}$$

$$T = \frac{1}{2} \cdot L_w \cdot w_w + (L_w + L_{\text{int}}) \cdot w_{\text{int}} + \left( L_w + L_{\text{int}} + \frac{1}{2} \cdot L_5 \right) \cdot w_5 \quad T = 71.896 \cdot \text{N} \cdot \text{m}$$

At cross section N-N of section b of arm #1:

$$R_{\text{ysb}} = R_{\text{sectionc}} + w_b + w_{\text{misc}} \quad R_{\text{ysb}} = 209.6 \cdot \text{N}$$

$$M_{\text{sb}} = \frac{L_b}{2} \cdot w_b + L_b \cdot (w_{\text{misc}} + R_{\text{sectionc}}) + M_{\text{sectionc}} \quad M_{\text{sb}} = 620.896 \cdot \text{N} \cdot \text{m}$$

Arm #1 section b has the cross section of a hollow cylinder. By giving the desired outer diameter, the inner diameter can be determined adesired factor of safety. The iterative loop used for this calculation is given below.

$$\text{The desired inner diameter of arm \#1 section b is:} \quad d_{si} = 30 \cdot \text{mm}$$

### BEGIN Iterative Loop

$$\text{Guess for the outer diameter of arm \#1 section b:} \quad d_{so} = 80 \cdot \text{mm}$$

Area at predetermined cross section of arm:

$$A_{sb} = \frac{\pi}{4} \cdot (d_{so}^2 - d_{si}^2) \quad A_{sb} = 0.004 \cdot \text{m}^2$$

Centriodal moment of inertia for hollow cross section at specified inner and outer diameters

$$I_{z_{sb}} = \frac{\pi}{64} \cdot (d_{so}^4 - d_{si}^4) \quad I_{z_{sb}} = 1.971 \cdot 10^{-6} \cdot \text{m}^4$$

Polar moment of inertia:

$$J_{sb} = I_{y_{sb}} + I_{z_{sb}} = 3.942 \cdot 10^{-6} \cdot \text{m}^4$$

Distance from Neutral Axis to point of maximum tension (top surface) and compression (bottom surface) are the same:

$$c_{sb} = \frac{d_{so}}{2} = 0.04 \cdot \text{m}$$

Maximum magnitude of tensile and compressive stress (when arm is fully extended):

$$\sigma_{sb} = \frac{M_{sb} \cdot c_{sb}}{I_{z_{sb}}} = 1.26 \cdot 10^7 \cdot \text{Pa}$$

Maximum shearing stress:

$$\tau_{sb} = \frac{R_{y_{sb}} \cdot Q}{I_{z_{sb}} \cdot t_{sb}} = 8.597 \cdot 10^4 \cdot \text{Pa}$$

$$Q = \frac{A_{sb}}{2} \cdot \frac{d_{so}^2 - d_{si} \cdot d_{so} - d_{si}^2}{d_{so} + d_{si}} \cdot \pi \quad t_{sb} = d_{so} - d_{si}$$

Maximum torsional stress (occurs when joint W1 is rotated so that the rest of arm past Joint c is positioned at a 90 degree angle with respect to longitudinal axis of the robotic arm structure)

$$\tau_{\text{twist},sb} = \frac{T \cdot c_{sb}}{J_{sb}} = 7.296 \cdot 10^5 \cdot \text{Pa}$$

The maximum total stress in preselected cross section of arm #3 is:

$$\tau_{sb,\text{max}} = \sqrt{\left(\frac{\sigma_{sb}}{2}\right)^2 + \tau_{sb}^2 + \tau_{\text{twist},sb}^2} = 6.343 \cdot 10^6 \cdot \text{Pa}$$

The resulting factor of safety is:

$$n = \frac{0.4 \cdot YSS_{st}}{\tau_{sb,\text{max}}} = 16.395$$

### END of iterative loop

When the resulting F.S. equals the desired F.S. then the guess for the needed diameter is appropriate. Therefore, the outer diameter for arm #1 section b is:

$$d_{so} = 0.08 \cdot \text{m}$$

The inner and outer diameters of arm #1 section b are now known, therefore, the volume, mass, weight, and wall thickness of arm #1 section b can be determined.

Volume of arm #1 section b:  $V_{sb} = A_{sb} \cdot L_b = 2.881 \cdot 10^3 \cdot \text{cm}^3$

Mass of arm #1 section b:  $\text{mass}_{sb} = V_{sb} \cdot \rho_{st} = 22.819 \cdot \text{kg}$

Weight of arm #1 section b:  $w_{sb} = \text{mass}_{sb} \cdot g_{\text{moon}} = 37.287 \cdot \text{N}$

Wall thickness of arm #1 section b:  $t_{sb} = \frac{d_{so} - d_{si}}{2} = 0.025 \cdot \text{m}$

## Calculations for Joint A.

### GIVEN:

Angular velocity of gear:  $\omega = 0.025 \cdot \text{sec}^{-1}$

Angular acceleration of gear:  $\alpha = 0.005 \cdot \text{sec}^{-2}$

Torque on each gear at Joint A as given by dynamic analysis of robotic arm:  $T = 521.6 \text{ N} \cdot \text{m}$

Shear force exerted on each axle at Joint A as given by static analysis of robotic arm:  $R_{ya} = 181.8 \text{ N}$

Bending moment applied on each axle at Joint A as given by static analysis of robotic arm:  $M_a = 318.9 \text{ N} \cdot \text{m}$

Torsion on each axle at Joint A as given by static analysis of robotic arm:  $T_{\text{twist}} = 71.92 \text{ N} \cdot \text{m}$

Wall thickness of arm #1:  $t_{\text{wall}} = 7 \text{ mm}$

Clearance between arms #1 and #2:  $t_{\text{armtol}} = 2.5 \text{ mm}$

Clearance between arm #1 and gear:  $t_{\text{gtol}} = 2 \text{ mm}$

### STATED values for gears:

material: Cold rolled stainless steel (302)

Pitch diameter of gear at Joint B:  $d_p = 212 \text{ mm}$

Pitch radius of gears:  $r_p = \frac{d_p}{2} \quad r_p = 106 \text{ mm}$

Force on gear resulting from chosen pitch diameter:  $F = \frac{T}{r_p} \quad F = 4.921 \text{ kN}$

Thickness of gears:  $t_g = 5 \text{ mm}$

### STATED values for axle which gears are connected to:

material: Aluminum 2014-T6

Yield strength in shear:  $S_{ys} = 220 \cdot 10^6 \text{ Pa}$

Length of each axle:  $l_a = t_{\text{armtol}} + t_{\text{wall}} + t_{\text{gtol}} + t_g$

$l_a = 16.5 \text{ mm}$

**STATIC ANALYSIS** of axle at Joint A to determine thickness using Maximum Shear Stress Theory.

Assume a diameter for axle:  $d_a = 15.8 \cdot \text{mm}$

**BEGIN iterative loop**

Area of cross section at assumed diameter:  $A = \frac{\pi}{4} \cdot d_a^2 \quad A = 196.067 \cdot \text{mm}^2$

Moment of inertia about y axis for circular cross section:  $I_y = \frac{\pi}{4} \cdot d_a^4 \quad I_y = 4.895 \cdot 10^4 \cdot \text{mm}^4$

Moment of inertia about z axis for circular cross section:  $I_z = \frac{\pi}{4} \cdot d_a^4 \quad I_z = 4.895 \cdot 10^4 \cdot \text{mm}^4$

Polar moment of inertia for circular cross section:  $J_a = I_y + I_z \quad J_a = 9.789 \cdot 10^4 \cdot \text{mm}^4$

Thickness at Neutral Axis of circular cross section:  $t = d_a \quad t = 15.8 \cdot \text{mm}$

Shear stress on axle:  $\tau = \frac{0.5 \cdot R_{ya} \cdot F}{A} \tau = 2.556 \cdot 10^7 \cdot \text{Pa}$

Torsional stress on axle:  $\tau_{\text{torsion}} = \frac{T + M_a \cdot (0.5 \cdot d_a)}{J_a}$

$$\tau_{\text{torsion}} = 6.783 \cdot 10^7 \cdot \text{Pa}$$

Bending moment stress on axle:  $\sigma = \frac{T_{\text{twist}} \cdot (0.5 \cdot d_a)}{I_z}$

$$\sigma = 1.161 \cdot 10^7 \cdot \text{Pa}$$

Maximum shear stress:  $\tau_{\text{max}} = \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2 + \tau_{\text{torsion}}^2}$

$$\tau_{\text{max}} = 7.272 \cdot 10^7 \cdot \text{Pa}$$

Factor of safety resulting from chosen diameter:  $n = \frac{0.4 \cdot S_{ys}}{\tau_{\text{max}}} \quad n = 1.2$

**END iterative loop**

For a factor of safety of  $n = 1.2$  a diameter of  $d_a = 15.8 \cdot \text{mm}$  was reached for the axle at Joint B.

**STATIC ANALYSIS** of arm #1 to determine whether wall thickness of hinge is adequate for given diameter of axle.

Nominal bearing area:  $A_{\text{bearing}} = t_{\text{wall}} \cdot d_a \quad A_{\text{bearing}} = 110.6 \cdot \text{mm}^2$

Bearing stress in hinge  
at location of axle:

$$\sigma_s = \frac{0.5 \cdot R_{ya} + F}{A_{\text{bearing}}}$$

$$\sigma_s = 4.531 \cdot 10^7 \cdot \text{Pa}$$

Factor of safety:

$$n_{\text{bearing}} = \frac{0.4 \cdot S_{ys}}{\sigma_s}$$

$$n_{\text{bearing}} = 1.9$$

With a wall thickness of  $t_{\text{wall}} = 7 \cdot \text{mm}$  a factor of safety of  $n_{\text{bearing}} = 1.9$  was reached at the bearing location in the hinge of arm #2.

STATIC ANALYSIS of arm #2 to determine if hinge width is adequate.

Since the diameter of the axle is  $d_a = 15.8 \cdot \text{mm}$  then the width in the hinge on arm #1 must be greater than 15.8 mm. Therefore, a width will be chosen and the analysis will be carried out to determine the factor of safety on that width.

Chosen width of  
hinge on arm #2:

$$w = 3.0 \cdot d_a \quad w = 47.4 \cdot \text{mm}$$

Cross sectional area at  
location of axle on hinge:

$$A_h = t_{\text{wall}} \cdot w - A_{\text{bearing}} \quad A_h = 221.2 \cdot \text{mm}^2$$

Shear stress on hinge  
at location of axle:

$$\sigma_h = \frac{0.5 \cdot R_{ya} + F}{A_h} \quad \sigma_h = 2.266 \cdot 10^7 \cdot \text{Pa}$$

Resulting factor of safety:

$$n_h = \frac{0.4 \cdot S_{ys}}{\sigma_h} \quad n_h = 3.9$$

With a hinge width of  $w = 47.4 \cdot \text{mm}$  on arm #1 the resulting factor of safety is  $n_h = 3.9$

Now that the hinge width and thickness are chosen, the hinge must be analyzed to ensure that the hinge will not fail at the point of maximum bending.

Choose a length for the hinge  
from the base of arm #1 to  
center of axle:

$$l_{\text{hinge}} = 0.5 \cdot d_a + 3 \cdot \text{mm} \quad l_{\text{hinge}} = 10.9 \cdot \text{mm}$$

Bending stress on hinge (where  
hinge and arm #1 meet):

$$\sigma_{\text{hinge}} = \frac{F + 0.5 \cdot R_{ya} \cdot l_{\text{hinge}} \cdot 0.5 \cdot d_a}{\frac{1}{3} \cdot t_{\text{wall}}^3 \cdot w} \quad \sigma_{\text{hinge}} = 1.265 \cdot 10^8 \cdot \text{Pa}$$

Shear stress on hinge (where  
hinge and arm #1 meet):

$$\tau_{\text{hinge}} = \frac{0.5 \cdot R_{ya}}{t_{\text{wall}} \cdot w} \quad \tau_{\text{hinge}} = 3.196 \cdot 10^5 \cdot \text{Pa}$$

Maximum stress on hinge  
at base of arm #1:

$$\tau_{\text{maxhinge}} = \sqrt{\left(\frac{\sigma_{\text{hinge}}}{2}\right)^2 + \tau_{\text{hinge}}^2} \quad \tau_{\text{maxhinge}} = 6.323 \cdot 10^7 \cdot \text{Pa}$$

Factor of safety:  $n_{\text{hinge}} = \frac{0.4 S_{ys}}{\tau_{\text{maxhinge}}} \quad n_{\text{hinge}} = 1.4$

With a hinge length of  $l_{\text{hinge}} = 10.9 \cdot \text{mm}$  for the hinge on arm #1 the resulting factor of safety is  $n_{\text{hinge}} = 1.4$  at the base of the hinge.

**Determination of motor specification:**

Voltage:  $\text{voltage} = 12 \cdot \text{volt}$

Torque on gear at axle:  $T = 521.6 \cdot \text{N} \cdot \text{m}$

Pitch diameter of gear attached to motor:  $d_{\text{pinion}} = 20 \cdot \text{mm}$

Pinion radius:  $r_{\text{pinion}} = \frac{d_{\text{pinion}}}{2} \quad r_{\text{pinion}} = 10 \cdot \text{mm}$

Gear ratio of pinion to output gear:  $\text{gear}_{\text{ratio}} = \frac{d_p}{d_{\text{pinion}}} \quad \text{gear}_{\text{ratio}} = 10.6$

Angular velocity of pinion:  $\omega_{\text{pinion}} = \omega \cdot \frac{r_p}{r_{\text{pinion}}} \quad \omega_{\text{pinion}} = 0.265 \cdot \text{sec}^{-1}$

Revolutions per minute of motor:  $\text{rpm} = 60 \cdot \omega_{\text{pinion}} \cdot \frac{180}{\pi} \quad \text{rpm} = 911 \cdot \frac{\text{rev}}{\text{min}}$

Number of stages in gearhead:  $n = 1$

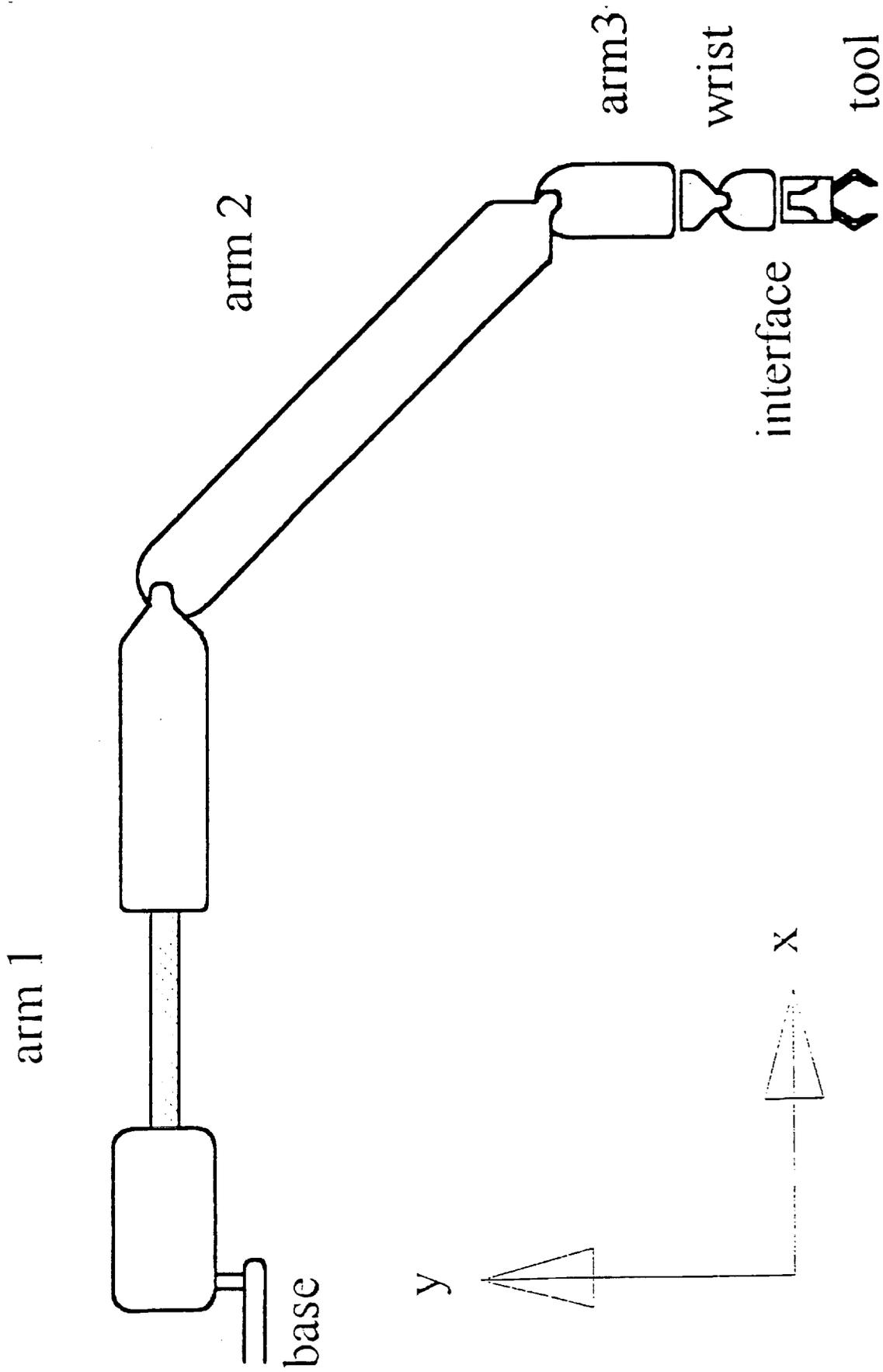
Torque of the motor to be chosen:  $T_{\text{motor}} = \frac{T}{0.85^n \cdot \text{gear}_{\text{ratio}}} \quad T_{\text{motor}} = 57.9 \cdot \text{N} \cdot \text{m}$

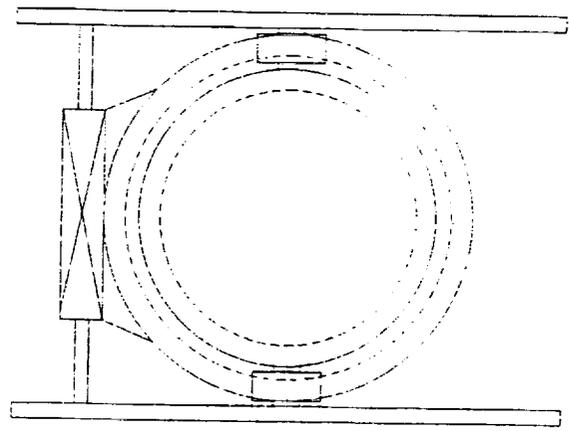
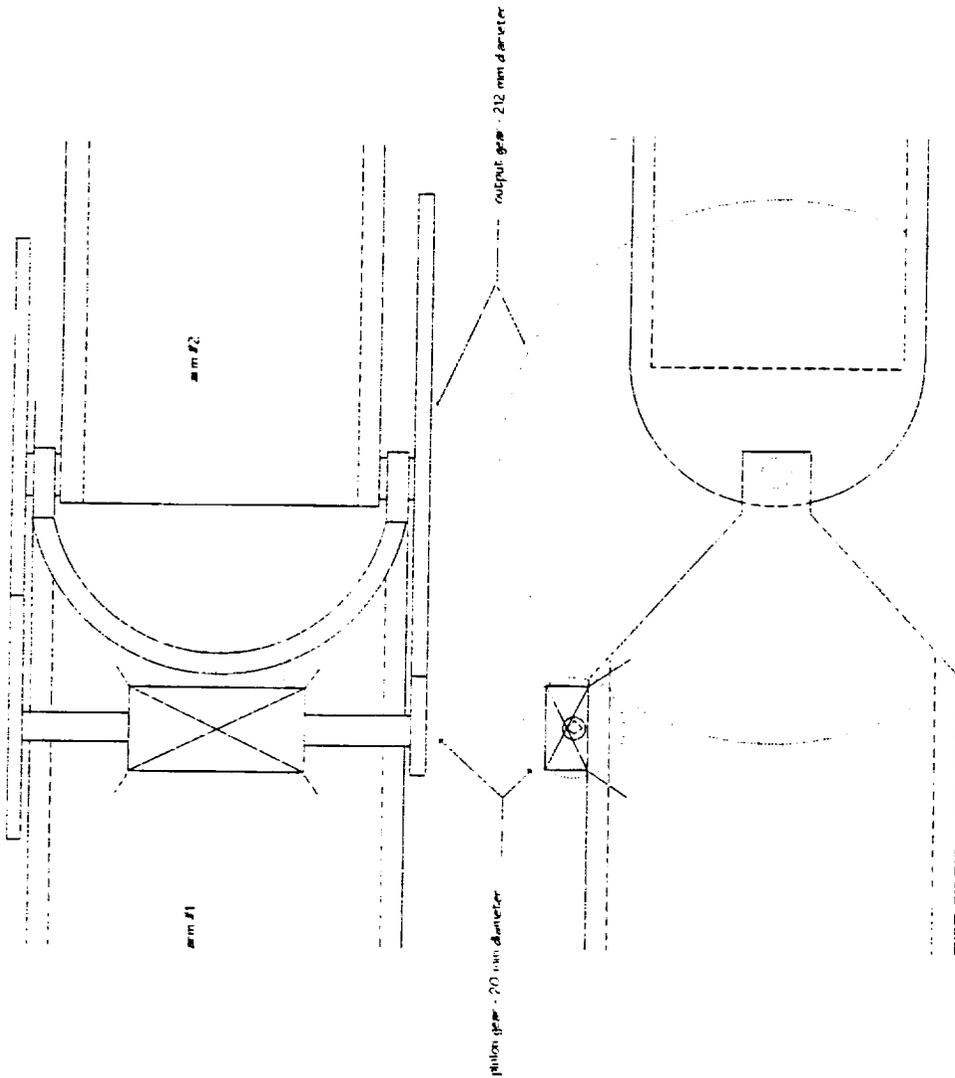
For Joint A a motor capable of delivering a torque of  $T_{\text{motor}} = 57.9 \cdot \text{N} \cdot \text{m}$  at a speed of  $\text{rpm} = 911 \cdot \frac{\text{rev}}{\text{min}}$  is needed. The ratio of the pinion to the output gear is 1:10.6.

With the data known for the specification of the motor at joint A the process of choosing a motor was begun. All available motors are not space rated so therefore a decision was made to base our selection on modern available motors. It is assumed that by the time frame of this robot that an equivalent space rated motor will be available as instructed by Dr. Hollis.

The Compumotor Digiplan model Z-940 fits to the needed torque and rpm requirements. The Z-940 model is a brushless, three phase motor which provides 63.7 Nm of continuous stall torque and 127.5 Nm of peak torque at speeds up to 1500 rpm. The mass of the Z-940 motor is stated as 51.0 kg and the operating temperature range is from -40 degrees Celsius to 125 degrees Celsius. As stated it is assumed that technological advances in the years leading to the development of the robotic arm will lead to a greater operating temperature range and decrease in mass. In Appendix A is photocopied information on the Z-940 motor.

protective boot, gears, and motors not shown shown for clarity





April, 1985

Joint A - The revolute joint connecting arm #1 and arm #2

Figure 4.2

## Calculations for Joint B.

### GIVEN:

Angular velocity of gear:  $\omega = 0.025 \cdot \text{sec}^{-1}$

Angular acceleration of gear:  $\alpha = 0.005 \cdot \text{sec}^{-2}$

Torque on each gear at Joint b as given by dynamic analysis of robotic arm:  $T = 214.7 \cdot \text{N} \cdot \text{m}$

Shear force exerted on each axle at Joint B as given by static analysis of robotic arm:  $R_{yb} = 164.8 \cdot \text{N}$

Bending moment applied on each axle at Joint B as given by static analysis of robotic arm:  $M_b = 68.07 \cdot \text{N} \cdot \text{m}$

Torsion on each axle at Joint B as given by static analysis of robotic arm:  $T_{\text{twist}} = 71.92 \cdot \text{N} \cdot \text{m}$

Wall thickness of arm #2:  $t_{\text{wall}} = 5 \cdot \text{mm}$

Clearance between arms #2 and #3:  $t_{\text{armtol}} = 2.5 \cdot \text{mm}$

Clearance between arm #2 and gear:  $t_{\text{gtol}} = 2 \cdot \text{mm}$

### STATED values for gears:

material: Cold rolled stainless steel (302)

Pitch diameter of gear at Joint B:  $d_p = 195 \cdot \text{mm}$

Pitch radius of gears:  $r_p = \frac{d_p}{2} \quad r_p = 97.5 \cdot \text{mm}$

Force on gear resulting from chosen pitch diameter:  $F = \frac{T}{r_p} \quad F = 2.202 \cdot \text{kN}$

Thickness of gears:  $t_g = 5 \cdot \text{mm}$

### STATED values for axle which gears are connected to:

material: Aluminum 2014-T6

Yield strength in shear:  $S_{ys} = 220 \cdot 10^6 \cdot \text{Pa}$

Length of each axle:  $l_a = t_{\text{armtol}} + t_{\text{wall}} + t_{\text{gtol}} + t_g$   
 $l_a = 14.5 \cdot \text{mm}$

STATIC ANALYSIS of axle at Joint A to determine thickness using Maximum Shear Stress Theory.

Assume a diameter for axle:  $d_a = 12.1 \cdot \text{mm}$

### BEGIN iterative loop

Area of cross section at assumed diameter:

$$A = \frac{\pi}{4} \cdot d_a^2 \quad A = 114.99 \cdot \text{mm}^2$$

Moment of inertia about y axis for circular cross section:

$$I_y = \frac{\pi}{4} \cdot d_a^4 \quad I_y = 1.684 \cdot 10^4 \cdot \text{mm}^4$$

Moment of inertia about z axis for circular cross section:

$$I_z = \frac{\pi}{4} \cdot d_a^4 \quad I_z = 1.684 \cdot 10^4 \cdot \text{mm}^4$$

Polar moment of inertia for circular cross section:

$$J_a = I_y + I_z \quad J_a = 3.367 \cdot 10^4 \cdot \text{mm}^4$$

Thickness at Neutral Axis of circular cross section:

$$t = d_a \quad t = 12.1 \cdot \text{mm}$$

Shear stress on axle:

$$\tau = \frac{0.5 \cdot R_{yb} \cdot F}{A} \quad \tau = 1.987 \cdot 10^7 \cdot \text{Pa}$$

Torsional stress on axle:

$$\tau_{\text{torsion}} = \frac{T \cdot M_b \cdot 0.5 \cdot d_a}{J_a} \quad \tau_{\text{torsion}} = 5.081 \cdot 10^7 \cdot \text{Pa}$$

Bending moment stress on axle:

$$\sigma = \frac{T_{\text{twist}} \cdot 0.5 \cdot d_a}{I_z} \quad \sigma = 2.584 \cdot 10^7 \cdot \text{Pa}$$

Maximum shear stress:

$$\tau_{\text{max}} = \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2 + \tau_{\text{torsion}}^2} \quad \tau_{\text{max}} = 5.606 \cdot 10^7 \cdot \text{Pa}$$

Factor of safety resulting from chosen diameter:

$$n = \frac{0.4 \cdot S_{ys}}{\tau_{\text{max}}} \quad n = 1.6$$

### END iterative loop

For a factor of safety of  $n = 1.6$  a diameter of  $d_a = 12.1 \cdot \text{mm}$  was reached for the axle at Joint B.

STATIC ANALYSIS of arm #2 to determine whether wall thickness of hinge is adequate for given diameter of axle.

Nominal bearing area:

$$A_{\text{bearing}} = t_{\text{wall}} \cdot d_a \quad A_{\text{bearing}} = 60.5 \cdot \text{mm}^2$$

Bearing stress in hinge at location of axle:

$$\sigma_s = \frac{0.5 \cdot R_{yb} \cdot F}{A_{\text{bearing}}} \quad \sigma_s = 3.776 \cdot 10^7 \cdot \text{Pa}$$

Factor of safety:

$$n_{\text{bearing}} = \frac{0.4 \cdot S_{ys}}{\sigma_s} \quad n_{\text{bearing}} = 2.3$$

With a wall thickness of  $t_{\text{wall}} = 5 \cdot \text{mm}$  a factor of safety of  $n_{\text{bearing}} = 2.3$  was reached at the bearing location in the hinge of arm #2.

**STATIC ANALYSIS** of arm #2 to determine if hinge width is adequate.

Since the diameter of the axle is  $d_a = 12.1 \text{ mm}$  then the width in the hinge on arm #2 must be greater than 16.5 mm. Therefore, a width will be chosen and the analysis will be carried out to determine the factor of safety on that width.

Chosen width of hinge on arm #2:

$$w = 2.0 \cdot d_a \quad w = 24.2 \text{ mm}$$

Cross sectional area at location of axle on hinge:

$$A_h = t_{\text{wall}} \cdot w - A_{\text{bearing}} \quad A_h = 60.5 \text{ mm}^2$$

Shear stress on hinge at location of axle:

$$\sigma_h = \frac{0.5 \cdot R_{yb} + F}{A_h} \quad \sigma_h = 3.776 \cdot 10^7 \text{ Pa}$$

Resulting factor of safety:

$$n_h = \frac{0.4 \cdot S_{ys}}{\sigma_h} \quad n_h = 2.3$$

With a hinge width of  $w = 24.2 \text{ mm}$  on arm #2 the resulting factor of safety is  $n_h = 2.3$

Now that the hinge width and thickness are chosen the hinge must be analyzed to ensure that the hinge will not fail at the point of maximum bending.

Choose a length for the hinge from the base of arm #2 to center of axle:

$$l_{\text{hinge}} = 0.5 \cdot d_a + 3 \text{ mm} \quad l_{\text{hinge}} = 9.05 \text{ mm}$$

Bending stress on hinge (where hinge and arm #2 meet):

$$\sigma_{\text{hinge}} = \frac{(F + 0.5 \cdot R_{yb}) \cdot l_{\text{hinge}} \cdot 0.5 \cdot d_a}{\frac{1}{3} \cdot t_{\text{wall}}^3 \cdot w} \quad \sigma_{\text{hinge}} = 7.179 \cdot 10^7 \text{ Pa}$$

Shear stress on hinge (where hinge and arm #2 meet):

$$\tau_{\text{hinge}} = \frac{0.5 \cdot R_{yb}}{t_{\text{wall}} \cdot w} \quad \tau_{\text{hinge}} = 5.675 \cdot 10^5 \text{ Pa}$$

Maximum stress on hinge at base of arm #2:

$$\tau_{\text{maxhinge}} = \sqrt{\left(\frac{\sigma_{\text{hinge}}}{2}\right)^2 + \tau_{\text{hinge}}^2} \quad \tau_{\text{maxhinge}} = 3.59 \cdot 10^7 \text{ Pa}$$

Factor of safety:

$$n_{\text{hinge}} = \frac{0.4 \cdot S_{ys}}{\tau_{\text{maxhinge}}} \quad n_{\text{hinge}} = 2.5$$

With a hinge length of  $l_{\text{hinge}} = 9.05 \text{ mm}$  for the hinge on arm #1 the resulting factor of safety is  $n_{\text{hinge}} = 2.5$  at the base of the hinge.

**Determination of motor specification:**

Voltage: voltage = 12·volt

Torque on gear at axle:  $T = 214.7 \cdot \text{N} \cdot \text{m}$

Pitch diameter of gear attached to motor:  $d_{\text{pinion}} = 17 \cdot \text{mm}$

Pinion radius:  $r_{\text{pinion}} = \frac{d_{\text{pinion}}}{2}$   $r_{\text{pinion}} = 8.5 \cdot \text{mm}$

Gear ratio of pinion to output gear:  $\text{gear ratio} = \frac{d_p}{d_{\text{pinion}}}$   $\text{gear ratio} = 11.5$

Angular velocity of pinion:  $\omega_{\text{pinion}} = \omega \cdot \frac{r_p}{r_{\text{pinion}}}$   $\omega_{\text{pinion}} = 0.287 \cdot \text{sec}^{-1}$

Revolutions per minute of motor:  $\text{rpm} = 60 \cdot \omega_{\text{pinion}} \cdot \frac{180}{\pi}$   $\text{rpm} = 985.8 \cdot \frac{\text{rev}}{\text{min}}$

Number of stages in gearhead:  $n = 1$

Torque of the motor to be chosen:  $T_{\text{motor}} = \frac{T}{0.85^n \cdot \text{gear ratio}}$   $T_{\text{motor}} = 22 \cdot \text{N} \cdot \text{m}$

For Joint A a motor capable of delivering a torque of  $T_{\text{motor}} = 22 \cdot \text{N} \cdot \text{m}$  at a speed of  $\text{rpm} = 985.8 \cdot \frac{\text{rev}}{\text{min}}$  is needed. The ratio of the pinion to the output gear is 1:11.5.

With the data known for the specification of the motor at joint A the process of choosing a motor was begun. All available motors are not space rated so therefore a decision was made to base our selection on modern available motors. It is assumed that by the time frame of this robot that an equivalent space rated motor will be available as instructed by Dr. Hollis.

The Compumotor Digiplan model Z-640 fits to the needed torque and rpm requirements. The Z-640 model is a brushless, three phase motor which provides 29 Nm of continuous stall torque and 58 Nm of peak torque at speeds up to 1600 rpm. The mass of the Z-640 motor is stated as 23.2 kg and the operating temperature range is from -40 degrees Celsius to 125 degrees Celsius. As stated it is assumed that technological advances in the years leading to the development of the robotic arm will lead to a greater operating temperature range and decrease in mass. In Appendix A is photocopied information on the Z-640 motor.

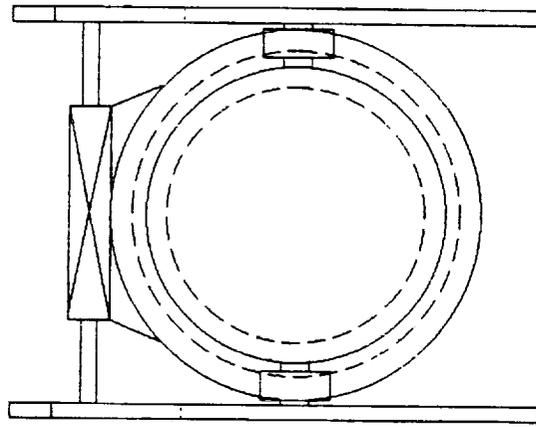
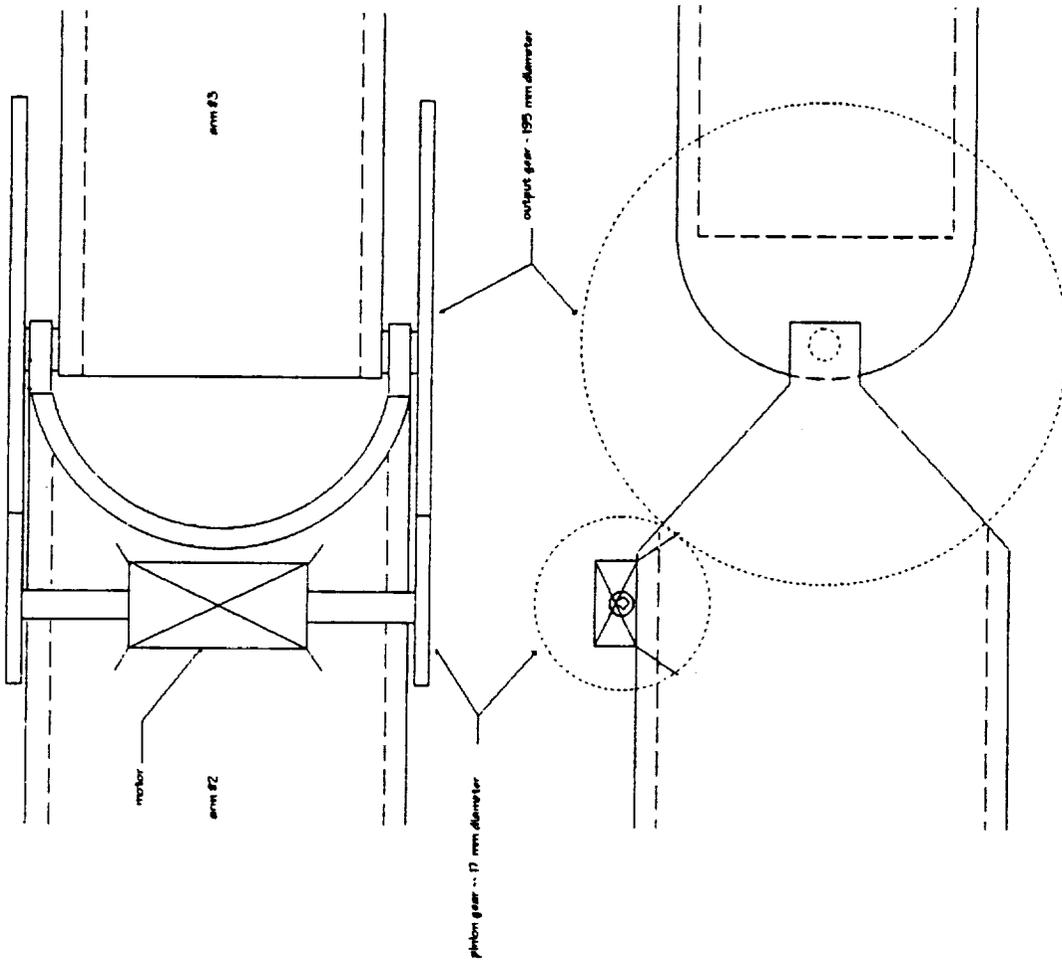


Figure 8 AF

Joint B The revolute joint connecting arm #2 and arm #3

April, 1993

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## 5. MODIFIED WRIST

### 5.1 Introduction

In a continued effort to satisfy the stated performance objectives, which together comprise the mission of the robotic arm, a modified wrist is required. The primary purpose of this subsystem is to provide crucial degrees of freedom to the system. Located between the primary mechanical structure and the structure-to-end effector interface, the modified wrist enhances the overall dexterity.

### 5.2 Operation

The way in which the modified wrist joint operates is centered on the fact that it adds three (3) degrees of freedom to the robotic arm. Its operation will, therefore, be explained by detailing the three (3) mechanisms that each contribute one degree of freedom.

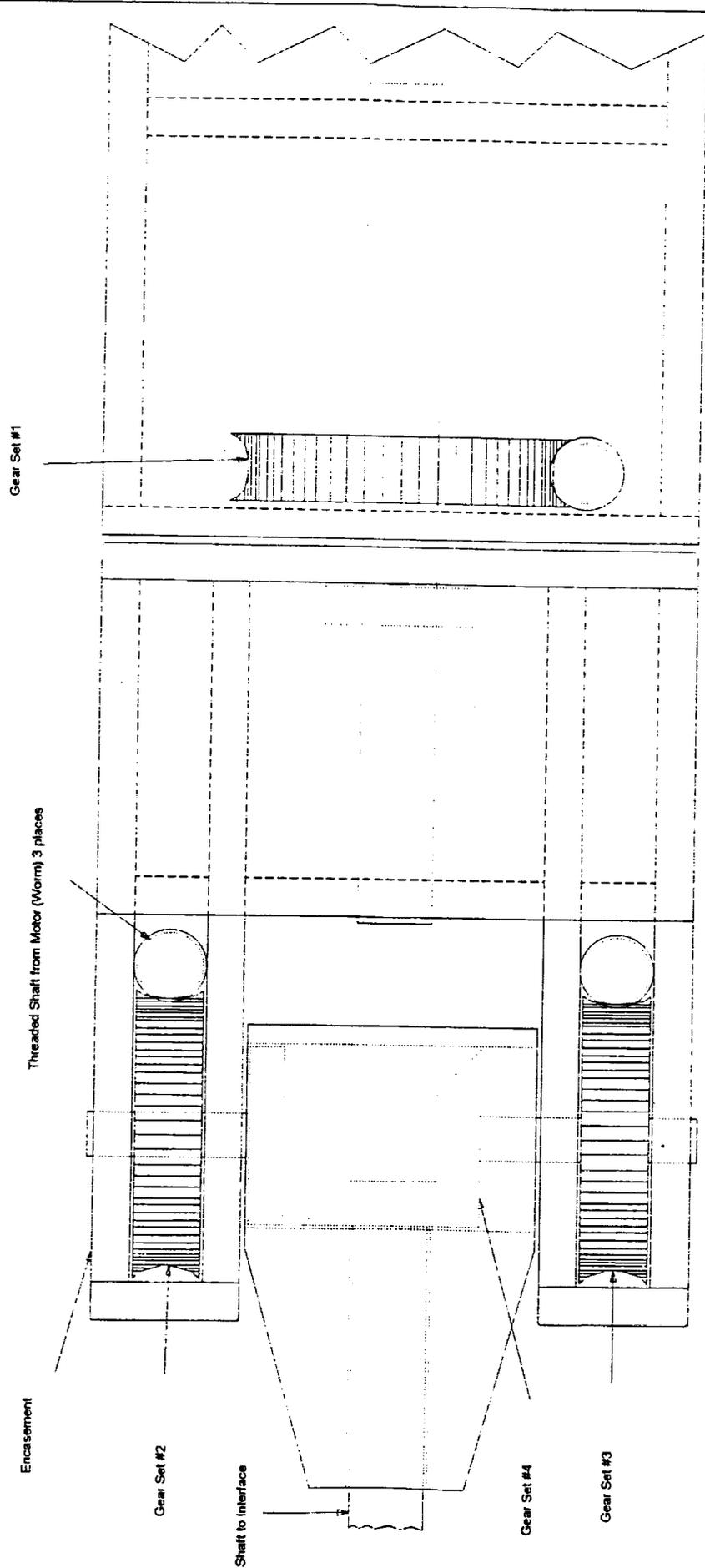
In general, each mechanism consists of a motor, one or two gear sets, and its object of manipulation.

Gear set #1 of the modified wrist joint is specifically a worm gear set. The worm gear is welded to its axle which rigidly connects the modified wrist joint to the mechanical structure. The worm is shaft-attached to the motor and oriented to provide a torque through the page (or about the y-axis). The purpose of gear set #1 is to rotate the higher level members of the arm one hundred eighty (180) degrees in both directions.

Gear set #2 is also a worm gear which is rigidly attached to the encasement of the higher level mechanisms. This worm gear rotates about an axle which partially traverses the cross-section of the encasement. The worm and motor are situated similarly to that of the gear set #1, providing a torque along the y-axis also. The purpose of this gear set is to provide a pitch range of one hundred thirty-five (135) degrees in both directions.

The third mechanism for the modified wrist is a gear chain. It consists of a worm gear set (gear set #3) and a bevel gear set (gear set #4). The worm gear rotates freely about an axle which partially traverses the cross-section of the encasement. The worm and motor are again oriented such that the torque is out of or in the page. Additionally, the bevel gear set is used to redirect the output about the x-axis. This output rotates the interface and the installed end effector.

### 5.3 Drawings



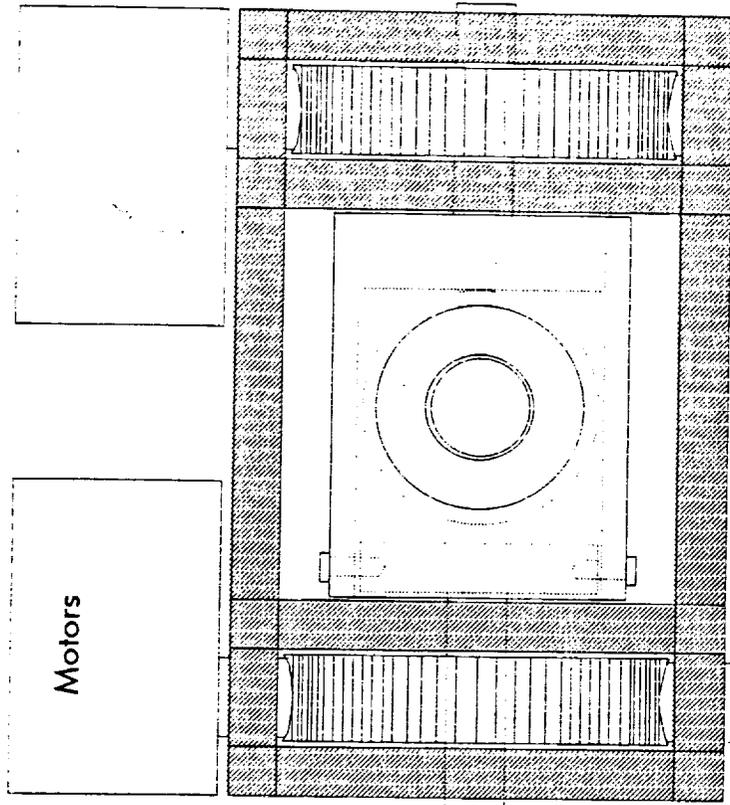
Top View of Modified Wrist Joint

Figure 5.1

Date : April, 1993

Title : Modified Wrist Joint

Note : Generalized Motor Features



Front View of Modified Wrist Joint

Title Modified Wrist Joint

Date April, 1993

Figure 5.2

## 5.4 Materials

### 5.4.1 Make-up

Space-qualified materials are required. From correspondences with NASA engineers, it has been advised to use an aluminum alloy. Aluminum 2014-T6 has been selected to make-up the encasement for the mechanisms. The axles, gears, worms, and fasteners are composed of stainless steel. This was chosen because of its strength and resilience, which is crucial in areas of gear teeth interfacing.

### 5.4.2 Motors

Motor specifications are a result of the analyses performed. The requirements were exacted and the specific motor specifications were gathered from company publications.<sup>2</sup>

Type	Digital Brushless Servo Motors
Peak Duty Torque	160 oz. in.
Continuous Duty Torque	105 oz. in.
Step Resolution	0.07 degrees
Weight	1.95 kg

Due to present design conventions, standard motors do not satisfy the torque requirements without violating the weight/size parameters for proper integration of the components. The problem arises with motors being commonly designed to carry a relatively constant torque over an extensive range of velocities, revolutions per seconds (rps). The motors for these systems require similar torques, but at much smaller velocities. Research has shown a correlation in large velocities and large motors. Although a dependence has not been determined, an assumption has been made based on this correlation.

It is assumed that the torque requirements can be satisfied without violating weight/size parameters using present technology. The solution to this problem involves custom designing smaller motors to provide only the needed torques and velocities.

### 5.4.3 Bearing

Bearings are used to minimize friction at all gear-to-axis interfaces.

## 5.5 Calculations

The following calculations are to determine the motor sizes at the wrist joints.

Distance to maximum estimated load on wrist:	$l_{\text{load}} = 1.5 \cdot m$
Acceleration of load due to gravity of moon and arm:	$a_A = 1.639 \cdot \frac{m}{\text{sec}^2}$
Mass of load and wrist estimated at distance above:	$m_{\text{load}} = 50 \cdot \text{kg}$
Coefficient of friction for worm-gear assembly:	$\mu_s = 0.58$
Pitch of worm-gear and worm:	$p = 5 \cdot \text{mm}$
Efficiency of steel to steel gear assembly:	$e_s = 0.9$
Mean diameter of gear:	$D_{\text{gear}} = 9.5 \cdot \text{cm}$
Maximum desired rotation of wrist joints:	$\omega = 0.05 \cdot \frac{\text{rad}}{\text{sec}}$
Force of load on gearset:	$F = m_{\text{load}} \cdot a_A \cdot \frac{l_{\text{load}}}{D_{\text{gear}}}$ $F = 290.891 \cdot \text{lbf}$
Polar moment of inertia for load rotating about wrist joints:	$J_{\text{load}} = m_{\text{load}} \cdot l_{\text{load}}^2 \cdot a_A$
Rotational velocity of worm screw:	$\omega_s = \omega \cdot \frac{\pi \cdot D_{\text{gear}}}{p}$ $\omega_s = 2.985 \cdot \frac{\text{rad}}{\text{sec}}$ $\omega_s = 28.5 \cdot \text{rpm}$
Gear ratio (power ratio) of worm-gear assembly:	$\text{GearRatio} = \frac{\omega_s}{\omega}$ $\text{GearRatio} = 59.69$
Torque of load due to accelerations:	$T_{\text{accel}} = m_{\text{load}} \cdot a_A \cdot l_{\text{load}} \cdot \text{GearRatio}$ $T_{\text{accel}} = 291.633 \cdot \text{oz} \cdot \text{in}$
Torque required to overcome friction:	$T_{\text{friction}} = \frac{\mu_s \cdot F \cdot p}{2 \cdot \pi \cdot e_s}$ $T_{\text{friction}} = 93.971 \cdot \text{oz} \cdot \text{in}$
Total torque required by motor:	$T_{\text{total}} = T_{\text{friction}} + T_{\text{accel}}$ $T_{\text{total}} = 385.603 \cdot \text{oz} \cdot \text{in}$ $T_{\text{total}} = 2.723 \cdot \text{N} \cdot \text{m}$

## 6. STRUCTURE-TO-END EFFECTOR INTERFACE

### 6.1 Introduction

In a continued effort to satisfy the stated performance objectives, which together comprise the mission of the robotic arm, a structure-to-end effector interface subsystem is required for a number of reasons. The primary purpose of such an interface involves supporting a variety of interchangeable end effectors individually and actuating the innate functions of each end effector.

### 6.2 Operation

In its most basic sense, the interface consists of a cylindrical shell which encloses the fit-locking mechanism, the end effector actuation mechanism, and the motor. The interface also operates as an electric outlet, channeling power to the end effectors that require it.

#### 6.2.1 Fit-locking Mechanism

This mechanism is responsible for securing the fit between the end effector's outer mating surface and the inner mating surface of the shell. The fit-lock consists of the actuating link, the followers (2), and the clamps (2) shown in Figure 6.1.

Each clamp is spring loaded and grounded at one revolute joint (hashed). The clamps are spring loaded in such a way, that the flange portion of the clamps are restored to the 'closed' position when no other force is applied at its follower-attached revolute joint. The closed position is achieved when the flange of each clamp protrudes from the shell window to the extent that the end effector (if present) would be adequately secured to perform the most demanding of system operations.

The actuating link is connected to the clamps via the followers. This link slides along the unthreaded portion of the motor shaft and transmits the force needed to restrain the clamps in the 'open' position. This force originates from the rotational motion of the motor shaft, which is then converted into linear motion at the threaded actuating rod. This rod in turn translates toward the motor, sliding the actuating link backward and establishing the open position of the clamps.

The open position is established prior to engagement of the end effector and constitutes disengagement of the same end effector.

### 6.2.2 End Effector Actuation Mechanism

This mechanism is responsible for the actuation of innate end effector functions. It consists of the actuating rod (of square cross-section) and the motor-driven threaded shaft.

Since end effectors, such as the small and large grippers are actuated by the displacement of their respective piston, the actuation mechanism must deliver the force necessary to induce such a displacement. This task is made possible by the conversion of the shaft's rotational energy into the translational energy of the rod. This translating actuating rod in turn performs the required work on the end effector's piston.

Controlling the precision of this rod motion and its consequential effect on the end effector involves understanding pertinent motor characteristics (i.e. step size, running torque, etc.) and programming supporting controls hardware accordingly.

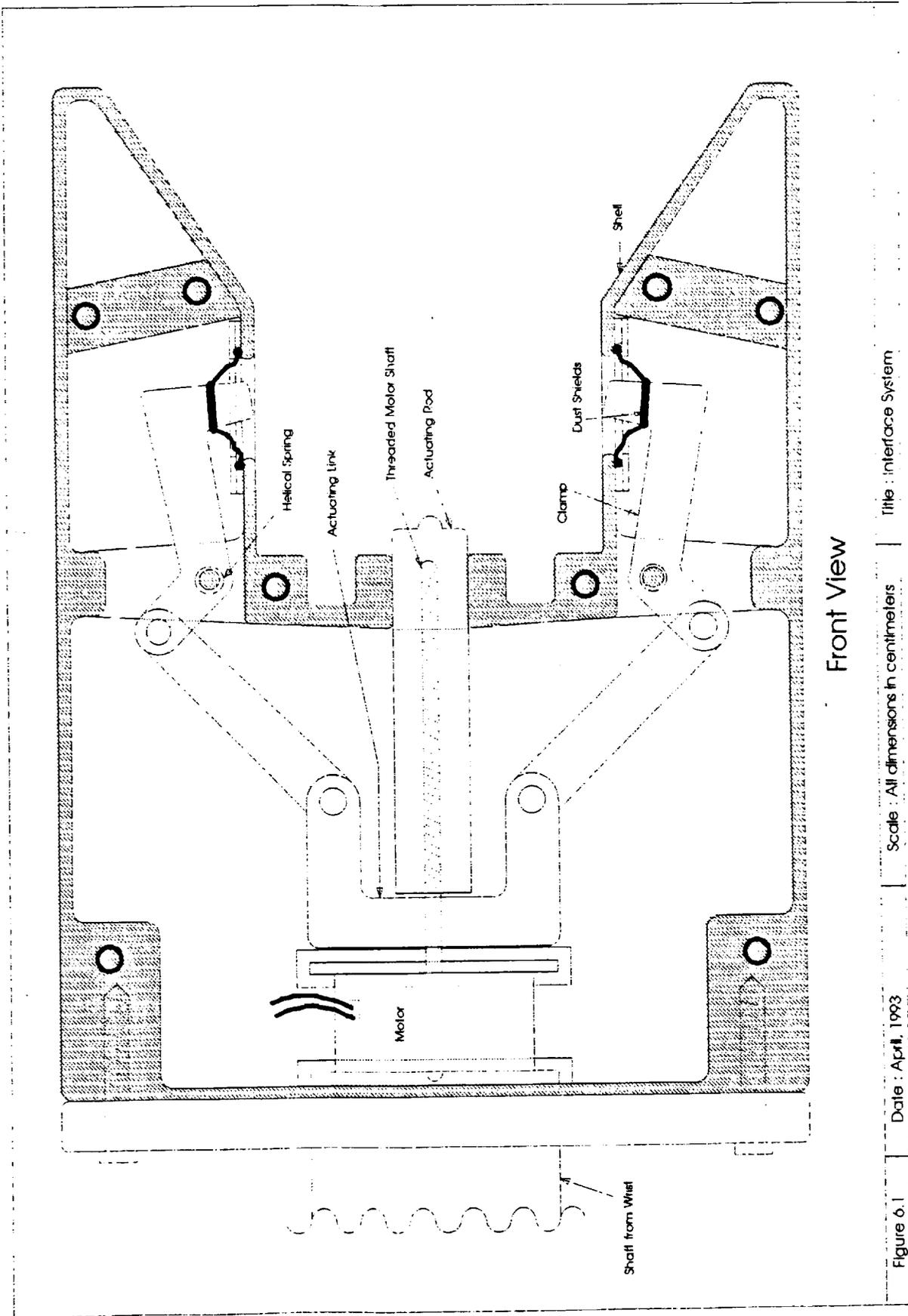
### 6.2.3 Motor

The motor provides the mechanical power needed to perform major subsystem functions. By operating in both directions, the motor powers the fit-locking mechanism to its open position and powers end effector actuation as well.

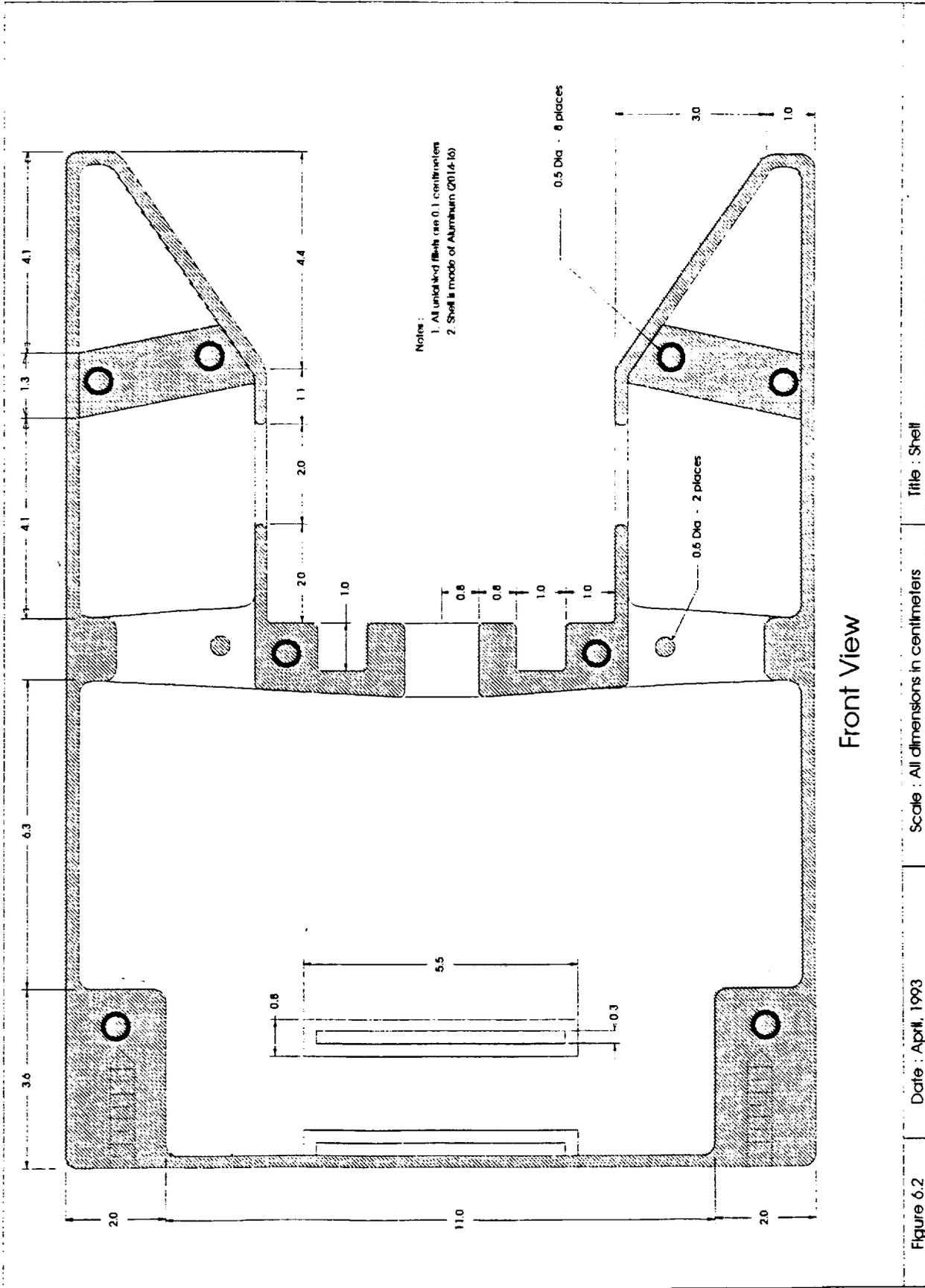
### 6.2.4 Electric Power

Electric power, if required, is channeled to the end effector through the ports (2) located in the vertical mating surface of the shell. These ports also serve to align the end effector during engagement and absorb torques during operation.

## 6.3 Drawings



Front View



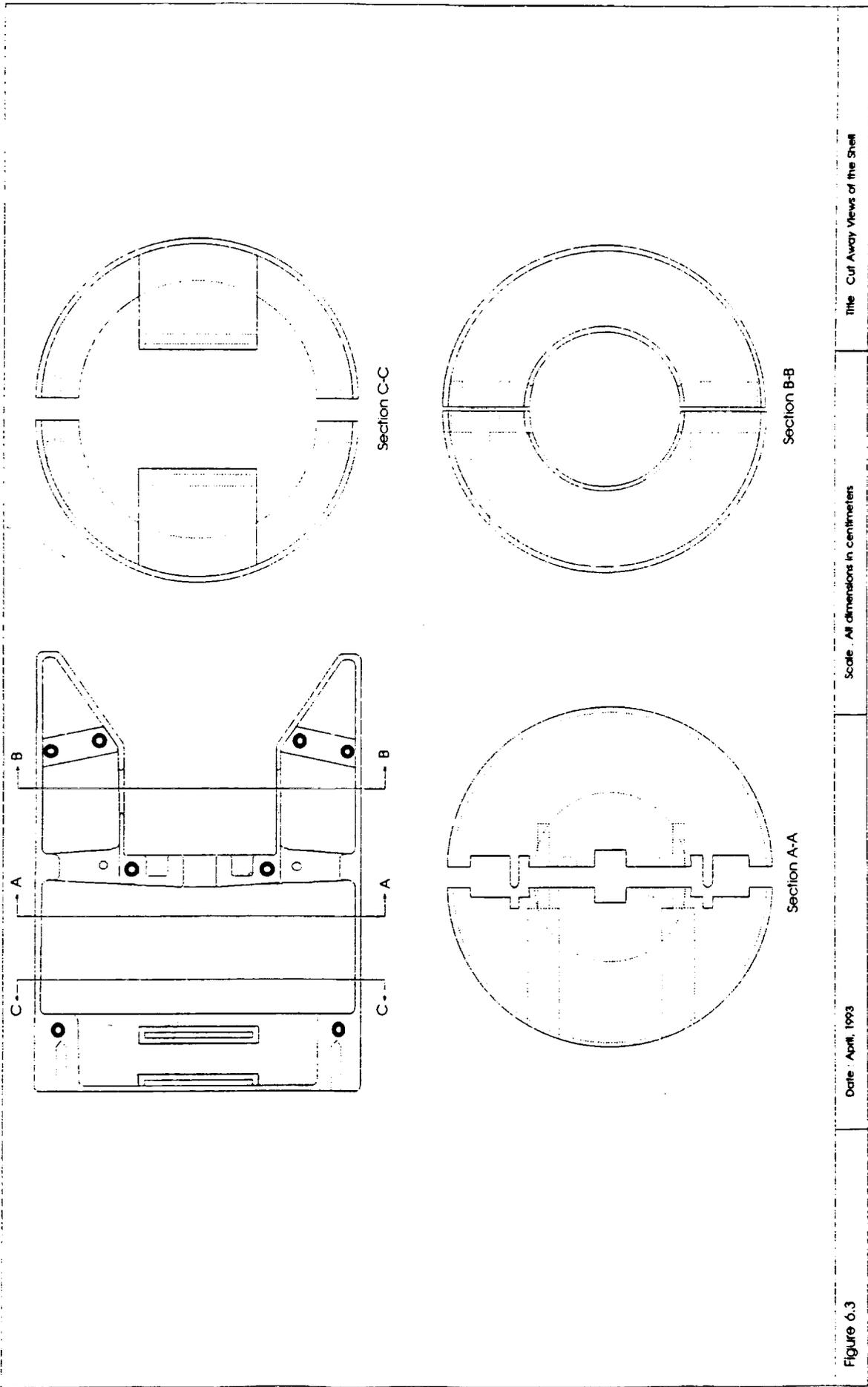
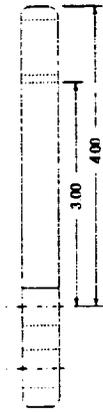
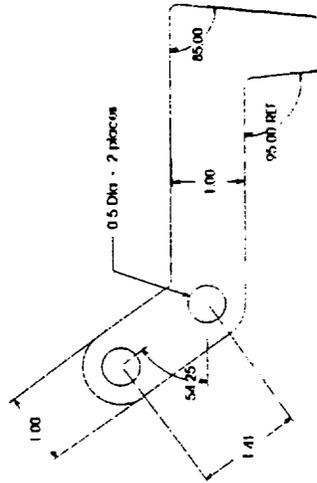


Figure 6.3 Date: April, 1993 Scale: All dimensions in centimeters Title: Cut Away Views of the Shell

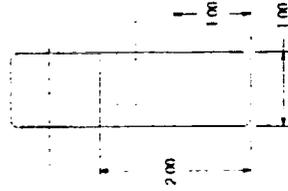


Top View

- Notes
1. Clamp is made of Aluminum (2014 T6)
  2. All unlabelled fillets are 0.1 centimeters



Front View



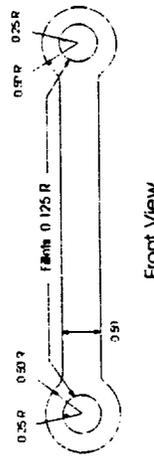
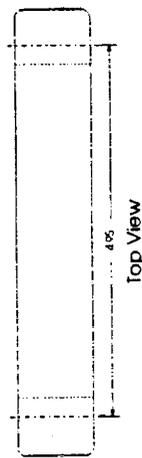
Side View

Figure 6.4

Date : April, 1993

Scale : All dimensions in centimeters

Title : Clamp



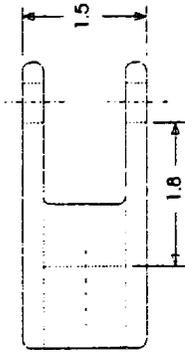
- Notes
1. All uncalled fillets are 0.1 centimeters
  2. Follower is made of Aluminum (2014-T6)

Title : Follower

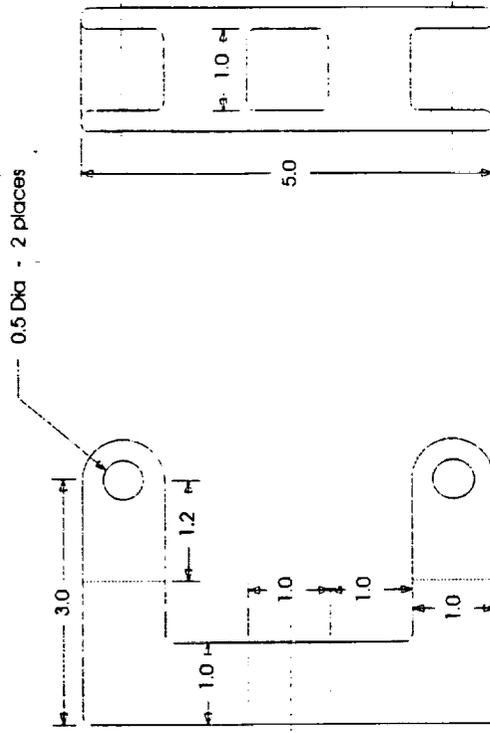
Scale : All dimensions in centimeters

Date : April, 1993

Figure 6.5



Top View

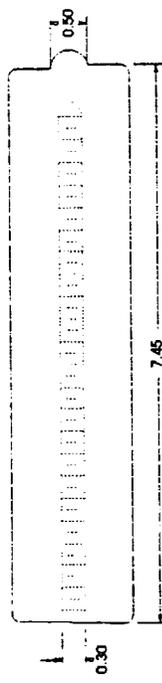
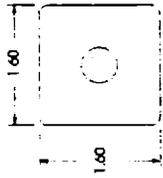


Front View

Side View

Notes :

1. Link is symmetrical
2. Link is made of Aluminum (2014 T6)
3. All unlabelled fillets are 0.1 centimeters



Notes:

1. All unlabelled fillets are 0.1 centimeters
2. Actuating Rod is made of Stainless Steel

Title : Actuating Rod

Scale : All dimensions in centimeters

Date : April, 1993

Figure 6.7

## 6.4 Materials

### 6.4.1 Make-up

Like the entire design, space-qualified materials are required. Aluminum 2014-T6 was selected as the primary material for this subsystem for a number of reasons. In addition to the fundamental demands on weight, strength, and cost, two more characteristics were considered: resistance to thermal expansion and resistance to outgassing.

From correspondences with NASA engineers, it is understood that an aluminum alloy is commonly used in similar applications. Interface designers decided to follow the agency's example, composing the entire shell of aluminum 2014-T6. This particular alloy was selected because of its high strength and low coefficient of thermal expansion among aluminum alloys.

The links, and other components of the interface mechanisms are also composed of aluminum 2014-T6.

The effect of thermal expansion is critical at surfaces such as the thread-to-thread interfacing of the motor shaft and the actuating rod. To minimize the threat of locking at extreme temperatures, the shaft and rod are made of the same material: stainless steel.

### 6.4.2 Motor

Motor specifications are highly dependent on the requirements of the interface-based mechanisms in addition to those of the end effectors. As a result of analyses performed on these subsystems, the requirements were exacted and the consequential motor specifications are as follows:

Type	Digital Brushless Servo Motors
Peak Duty Torque	160 oz. in.
Continuous Duty Torque	105 oz. in.
Step Resolution	0.07 degrees
Weight	1.95 kg

Due to present design conventions, standard motors do not satisfy the torque requirements without violating the weight/size parameters for proper integration of the

components. The problem arises with motors being commonly designed to carry a relatively constant torque over an extensive range of velocities, revolutions per seconds (rps). The motors for these systems require similar torques, but at much smaller velocities. Research has shown a correlation in large velocities and large motors. Although a dependence has not been determined, an assumption has been made based on this correlation.

It is assumed that the torque requirements can be satisfied without violating weight/size parameters using present technology. The solution to this problem involves custom designing smaller motors to provide only the needed torques and velocities.

#### 6.4.3 Fasteners

Fasteners are used in two instances: to secure the shell halves and to attach the interface to the modified wrist subsystem. For the assembly of the shell, eight (8) stainless steel fillister head cap screws are used. For the interface attachment to the wrist, four (4) stainless steel fillister head cap screws are used. Additional details on these fasteners are included in the Calculations section of this document.

## 6.5 Calculations

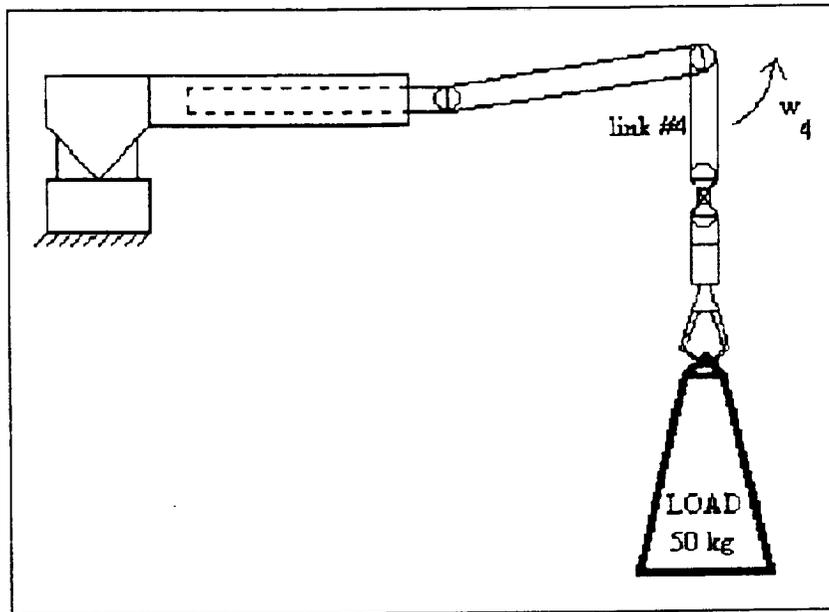
The following calculations are for the interface system of the robotic arm.  
The proposed weight of the interface system is not to exceed 10 kg.

The calculations below are for each part are documented and describe the concerns and/or weaknesses of each part in order to determine the size of the whole interface and its individual parts.

**Properties of Aluminum 2014-T6 (4.4% Cu):** *Aluminum 2014-T6 will be used until better or necessary material changes are made.*

Gravity on Moon	$g = \frac{1}{6} g$	$g = 1.634 \cdot \frac{\text{m}}{\text{sec}^2}$
Density of Aluminum	$\rho = 2800 \cdot \frac{\text{kg}}{\text{m}^3}$	
Modulus of Elasticity	$E = 72 \cdot 10^9 \cdot \text{Pa}$	
Modulus of Rigidity	$G = 27 \cdot 10^9 \cdot \text{Pa}$	
Coefficient of Thermal Expansion	$\alpha = 23 \cdot \frac{10^{-6}}{\text{K}}$	
Ultimate Strength under Tension	$S_{\text{utT}} = 480 \cdot 10^6 \cdot \text{Pa}$	
Ultimate Strength under Shear	$S_{\text{utS}} = 290 \cdot 10^6 \cdot \text{Pa}$	
Yield Strength under Tension	$S_{\text{yT}} = 410 \cdot 10^6 \cdot \text{Pa}$	
Yield Strength under Shear	$S_{\text{yS}} = 220 \cdot 10^6 \cdot \text{Pa}$	
Factor of Safety for all parts	$\eta = 2.5$	

The following calculations are to determine the maximum force acting on each clamp. The figure below shows the proposed orientation of the Robotic Arm which should produce the forces of the worst-case scenario acting on the interface and its components.

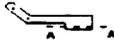


An assumption that a 1 meter maximum distance may exist from the end of the tool to the center of gravity of the load. Link #4 can rotate at a maximum angular velocity in the worst-case scenario.

Length of link #4:	$L_4 = 0.558 \cdot m$
Length of wrist and interface:	$L_{int} = 22 \cdot cm$
Radius of rotation from joint 4 to load:	$r_4 = L_4 + L_{int} + 1 \cdot m$
Maximum rotational velocity of joint 4:	$\omega_4 = 0.05 \frac{rad}{sec}$
Normal acceleration of load due to rotation of joint 4:	$a_{4,n} = \omega_4^2 \cdot r_4$
Maximum mass of load:	$m_{load} = 50 \cdot kg$
Total acceleration of load:	$a_A = g + a_{4,n}$
Maximum force on holding clamps due to load:	$F_{max} = m_{load} \cdot a_A$ $F_{max} = 81.944 \cdot N$
Force acting on one side of clamp:	$F_{maxA} = \frac{F_{max}}{2}$ $F_{maxA} = 40.972 \cdot N$

The following calculations are to determine the cross-sectional area of section A-A.

The size of the clamp face (section A-A) is tested below and found to be under no danger of shearing.

Figure of section A-A: 

Maximum expected stress through face: 
$$\sigma_{\max S} = \frac{0.4 \cdot S_y S}{\eta} \quad \sigma_{\max S} = 3.52 \cdot 10^7 \text{ Pa}$$

Minimum area of clamp surface: 
$$\text{area}_{C.F.} = \frac{F_{\max A}}{\sigma_{\max S}} \quad \text{area}_{C.F.} = 1.164 \cdot \text{mm}^2$$

Based on the calculations and results found, the cross-section A-A of the clamp face can be virtually any realistic size in order to satisfy our needs.

The following calculations are to determine the thickness (b) of the clamp at Point B to prevent the part from failing.

Point B shown is tested for failure under the load conditions shown:



Maximum Yield Stress under Tension:

$$\sigma_{\max T} = \frac{0.45 \cdot S_y T}{\eta}$$

Stress due to bending:

Distance from pin to Force vector:

$$r_M = 2 \text{ cm}$$

Moment due to force:

$$M = F_{\max A} \cdot r_M$$

Height of small arm of clamp:

$$h = 1 \text{ cm}$$

Distance to Point B from neutral axis:

$$c = \frac{h}{2}$$

Moment of inertia for section containing Point B:

$$I = \frac{b \cdot h^3}{12}$$

General Stress Form for bending:

$$\sigma_{\text{bend}} = \frac{M \cdot c}{I} = \frac{12 \cdot M \cdot c}{b \cdot h^3}$$

Stress due to tension:

General Stress Form for tension:

$$\sigma_{\text{tension}} = \frac{F}{A} = \frac{F_{\max A}}{b \cdot h}$$

The thickness (b) of the part at Point B is found to be less than 1.0 cm and is calculated in the following fashion:

Maximum total expected stress at Point B:

$$\sigma_{\max T} = \sigma_{\text{tension}} + \sigma_{\text{bending}}$$

Substituting general stress equations for bending and tension:

$$\sigma_{\max T} = \frac{12 \cdot M \cdot c}{b \cdot h^3} + \frac{F_{\max A}}{b \cdot h}$$

Therefore: 
$$b = \frac{12 \cdot M \cdot c - F_{\max A} \cdot h^2}{\sigma_{\max T} \cdot h^3} \quad b = 0.722 \cdot \text{mm}$$

Based on the calculations and results found for the thickness (b) of the part at Point B, virtually any thickness (b) can be used. Keeping in mind the weight constraint, a thickness (b) of 1.0 cm has been chosen.

The following calculations are to determine the diameter of Pin A which the clamp pivots around.

The required cross-sectional area of Pin A was found to be extremely small. Therefore a diameter of 0.5 cm was chosen as the diameter to be used for the design.

A diagram of the position of Pin A on the clamp and the force acting on it is shown below:



Minimum cross-sectional area of Pin A: 
$$\text{area}_{\text{pinA}} = \frac{F_{\max A}}{\sigma_{\max S}} \quad \text{area}_{\text{pinA}} = 0.582 \cdot \text{mm}^2$$

Minimum diameter of Pin A: 
$$d_{\text{pinA}} = \sqrt{\frac{4 \cdot \text{area}_{\text{pinA}}}{\pi}} \quad d_{\text{pinA}} = 0.861 \cdot \text{mm}$$

Because such a small force is being applied, the diameter of Pin A was found to be extremely small. Therefore, the diameter of the pin to be used will be 0.5 cm. This diameter was chosen since virtually any realistic size could be used and weight constraints exist.

The following calculations are to determine the thickness (b) of the clamp at the location of Pin A.

The cross-section of the clamp at Pin A is shown. The necessary thickness of this part at Pin A was calculated to be extremely small. Therefore, again, a thickness (b) of 1.0 cm will be used in the design.

The figure below shows the force being applied and the location where the thickness (b) is to be found:



Maximum expected actual stress: 
$$\sigma_{\max T} = \frac{0.45 \cdot S_{yT}}{\eta}$$

General Stress Form under tension:

$$\sigma_{\max T} = \frac{F}{A} = \frac{F_{\max A}}{b \cdot (h - d_{\text{pinA}})}$$

Therefore, the thickness (b) of clamp can be solved for at location of Pin A as:

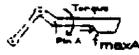
$$b = \frac{F_{\max A}}{(h - d_{\text{pinA}}) \cdot \sigma_{\max T}} \quad b = 0.061 \cdot \text{mm}$$

Because such a small force is being applied, the diameter of pin A was found to be extremely small. Therefore, the thickness (b) of the clamp at the location of Pin A will be set to a distance of 1.0 cm. This distance was chosen since virtually any realistic size could be used and weight constraints exist.

The following calculations are to determine the minimum spring torque that would hold the tool in place with the maximum angular velocities and the maximum load of the robotic arm being applied.

The clamps are needed to hold the tool within the interface and to keep the tool from spinning within the interface. The clamps will be spring loaded at Pin A. These springs must produce at least the following calculated torque:

The figure below shows where the torque is being applied:



Slope of the sides measured  
CCW from the vertical:

$$\theta = 5 \cdot \text{deg}$$

Distance of lever arm  
(horizontal distance from Pin A  
to location of applied force):

$$L = 3 \cdot \text{cm}$$

Force acting on each clamp:

$$F_{\max A} = 40.972 \cdot \text{N}$$

Minimum spring force needed to  
keep the clamp in place:

$$f_{\text{spring}} = F_{\max A} \cdot \sin(\theta) \quad f_{\text{spring}} = 3.571 \cdot \text{N}$$

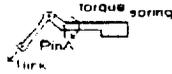
Minimum torque needed to  
keep the clamp in place:

$$\text{Torque}_{\text{spring}} = f_{\text{spring}} \cdot L \quad \text{Torque}_{\text{spring}} = 0.107 \cdot \text{N} \cdot \text{m}$$

From the calculations above, the spring must exert a minimum of 0.107 Nm to maintain the clamp in the locked position. The effect of friction was neglected for the calculations above. By neglecting the effect of friction, a larger spring torque would be calculated. Therefore, this would ensure that the spring would hold.

The following calculations are to determine the force in the link of the clamping system.

The Figure below shows the torque being applied and the force being determined:



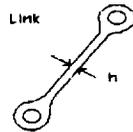
Force in the link due to the spring torque:

$$f_{\text{link}} = \frac{\text{Torque spring}}{1.414 \cdot \text{cm}} \quad f_{\text{link}} = 7.576 \cdot \text{N}$$

From the calculations above, the force in the link was found to be 7.576 N. From knowing this force, the thickness (b) of the link can be determined.

The following calculations are to determine the thickness (b) of the link within the clamping system.

The Figure below helps to understand the calculations being made:



Force in the link due to the spring torque:

$$f_{\text{link}} = 7.576 \cdot \text{N}$$

Height of link:

$$h = 0.25 \cdot \text{cm}$$

Cross-sectional area of the link as a function of the thickness:

$$\text{area}(b) = b \cdot h$$

Tensile stress throughout length of link as a function of the thickness:

$$\sigma_t(b) = \frac{f_{\text{link}}}{\text{area}(b)}$$

Factor of safety as a function of the thickness:

$$\eta(b) = \frac{0.45 \cdot S_{yT}}{\sigma_t(b)}$$

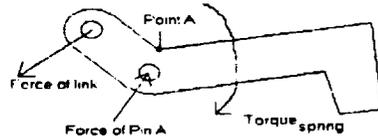
Using a root function to solve for the thickness (b) at a factor of safety of 2.5:

$$b = \text{root}(\eta(b) - 2.5, b) \quad b = 0.004 \cdot \text{cm}$$

Therefore, the thickness of the part will be set to  $b=1.0 \text{ cm}$  and will encounter no potential problems.

The following calculations are made to test the inside of the clamp curve (Point A) as shown. The situation occurs while disengaging the clamp from the tool.

The following Figure below aids in the understanding of the calculations to follow. Point A shown experiences tearing from bending and tensile stresses:



- Distance from Pin A to Force vector of link:  $r = 1.414 \cdot \text{cm}$
- Height from top of Pin A to Point A:  $h_1 = 0.25 \cdot \text{cm}$
- Height from bottom of Pin A to bottom of clamp:  $h_2 = 0.25 \cdot \text{cm}$
- Angle that the the short end of the clamp makes with the horizontal (refer to detailed description of clamp):  $\phi = 27.469 \cdot \text{deg}$
- Tensile force in the short end of the clamp:  $f_{\text{ten}} = f_{\text{link}} \cdot \sin(\phi) \quad f_{\text{ten}} = 3.495 \cdot \text{N}$
- Shear force in the short end of the clamp:  $f_{\text{shear}} = f_{\text{link}} \cdot \cos(\phi) \quad f_{\text{shear}} = 6.722 \cdot \text{N}$
- Moment due to tangential force in link:  $M = f_{\text{shear}} \cdot r \quad M = 0.095 \cdot \text{N} \cdot \text{m}$
- Diameter of Pin A:  $d_{\text{pin}} = 0.5 \cdot \text{cm}$

Distance of Point A from the neutral axis as a function of the thickness (b):

$$c(b) = \frac{h_1 \cdot b \cdot \frac{h_1}{2} + h_2 \cdot b \cdot \left( h_1 - d_{\text{pin}} + \frac{h_2}{2} \right)}{(h_1 + h_2) \cdot b}$$

Moment of inertia for the cross-sectional area as a function of the thickness (b):

$$I(b) = \frac{1}{12} \cdot b \cdot h_1^3 + \frac{1}{12} \cdot b \cdot h_2^3 + b \cdot h_1 \left( c(b) - \frac{h_1}{2} \right)^2 + b \cdot h_2 \left( h_1 + d_{\text{pin}} + \frac{h_2}{2} - c(b) \right)^2$$

Stress due to tensile force caused by link as a function of the thickness (b):

$$\sigma_{\text{ten}}(b) = \frac{f_{\text{ten}}}{b \cdot (h_1 + h_2)}$$

Stress caused by the bending moment as a function of the thickness (b):

$$\sigma_M(b) = \frac{M \cdot c(b)}{I(b)}$$

Stress due to shear force as a function of the thickness (b):

$$\tau_{\text{shear}}(b) = \frac{f_{\text{shear}}}{b \cdot (h_1 - h_2)}$$

Maximum combined stress at Point A as a function of the thickness (b):

$$\sigma'(b) = \sqrt{[\sigma_{\text{ten}}(b) + \sigma_M(b)]^2 + \tau_{\text{shear}}(b)^2}$$

Factor of safety as a function of the thickness (b):

$$\eta(b) = \frac{0.40 \cdot S_{yT}}{\sigma'(b)}$$

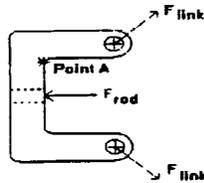
Using a root function to solve for the thickness (b) at a factor of safety of 2.5:

$$b = \text{root}(\eta(b) - 2.5, b) \quad b = 0.011 \cdot \text{cm}$$

Based on the calculations, the thickness (b) was found to be 0.011 cm. The design thickness is 1.0 cm. Due to small forces applied, the calculated thickness was extremely small.

The following calculations are to determine the thickness (b) of the actuating component.

This Figure shows the appropriate forces being applied on the component and the location of the point where the maximum stress occurs:



Width of the actuating link:

$$w = 0.25 \cdot \text{cm}$$

Vertical distance of the actuating link:

$$L = 4 \cdot \text{cm}$$

Vertical distance from where force is being applied to center of pin:

$$r = \frac{L}{2}$$

Distance from neutral axis to edge of actuating link:

$$c = \frac{w}{2}$$

Moment about Point A caused by force in link as a function of the thickness (b):

$$M = 2 \cdot \text{cm} \cdot f_{\text{link}} \cdot \cos(45 \cdot \text{deg}) - 0.5 \cdot \text{cm} \cdot f_{\text{link}} \cdot \sin(45 \cdot \text{deg})$$

Moment of inertia for actuating link as a function of the thickness (b):

$$I(b) = \frac{b \cdot w^3}{12}$$

Stress due to tensile force in link as a function of the thickness (b):

$$\sigma_t(b) = \frac{f_{\text{link}} \cdot \cos(45 \cdot \text{deg})}{b \cdot w}$$

Stress due to shear force in link as a function of the thickness (b):

$$\tau(b) = \frac{f_{\text{link}} \cdot \sin(45 \cdot \text{deg})}{b \cdot w}$$

Stress due to bending moments caused by the force in the link as a function of the thickness (b):

$$\sigma_b(b) = M \cdot \frac{c}{I(b)}$$

Maximum stress in actuating link at Point A as a function of the thickness (b):

$$\sigma_{\text{actual}}(b) = \sqrt{\sigma_t(b) + \sigma_b(b)^2 + \tau(b)^2}$$

Factor of safety as a function of the thickness (b):

$$\eta(b) = \frac{0.40 \cdot S_{yT}}{\sigma_{\text{actual}}(b)}$$

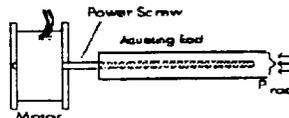
Using a root function to solve for the thickness (b) at a factor of safety of 2.5:

$$b = \text{root}(\eta(b) = 2.5, b) \quad b = 0.121 \cdot \text{cm}$$

Based on the calculations above, it was again found that the required thickness was much less than the specified thickness (2.0 cm) of the actuating link.

The following calculations are to determine the velocity and torque required by the motor.

The following Figure below aids in the understanding of the calculations to follow. The Figure shows the motor, power screw (motor shaft), actuating rod, and appropriate forces being applied:



Force acting on end of actuating rod:  $P_{\text{rod}} = 200 \cdot \text{N}$

Major diameter of the power screw:  $d = 5 \cdot \text{mm}$

Pitch of the power screw:  $p = 3 \cdot \text{mm}$

Mean diameter of the power screw:  $d_m = d - \frac{p}{2} \quad d_m = 3.5 \cdot \text{mm}$

Number of threads (single threaded):  $n = 1$

Lead angle of power screw:  $l = n \cdot p$

Coefficient of friction between power screw and actuating rod:  $\mu = 0.58$

Coefficient of friction of motor:  $\mu_c = 0.58$

Torque required by the motor:

$$T = \frac{P_{rod} \cdot d_m}{2} \frac{1 - \pi \mu d_m}{\pi d_m - \mu l} + \frac{P_{rod} \mu_c d}{2}$$

$$T = 0.645 \cdot N \cdot m$$

Translation distance of actuating rod:

$$\text{Dist} = 4 \text{ cm}$$

Time taken to fully engage actuating rod and subsequently the tool:

$$\text{Time} = 4 \text{ sec}$$

Work done by the actuating rod:

$$\text{Work}_{rod} = P_{rod} \cdot \text{Dist}$$

Power required by a motor with 100% efficiency:

$$\text{Power}_{rod} = \frac{\text{Work}_{rod}}{\text{Time}} \quad \text{Power}_{rod} = 2 \text{ watt}$$

Maximum desired angular velocity of the power screw:

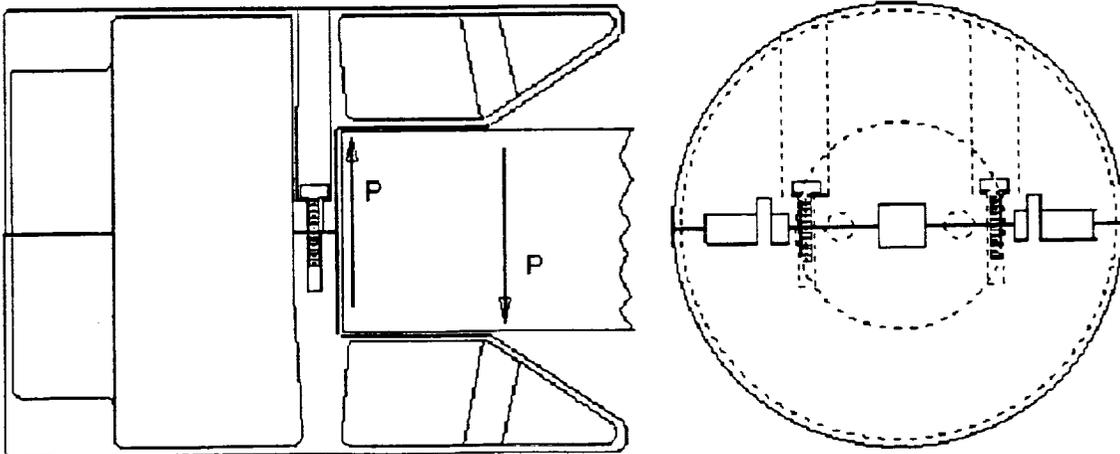
$$\omega = \frac{2 \cdot \pi \cdot \text{rad} \cdot \text{Dist}}{p \cdot \text{Time}} \quad \omega = 20.944 \cdot \frac{\text{rad}}{\text{sec}}$$

Maximum desired RPM's of the power screw:

$$\text{RPM's} = \frac{\omega}{2 \cdot \pi \cdot \text{rad}} \quad \text{RPM's} = 3.333 \cdot \frac{1}{\text{sec}}$$

The following calculations are to determine the necessary screw size and pre-load required to hold the shell of the interface system together while a maximum load is applied to the tool.

The following figure shows the front half of the shell of the interface system and the estimated maximum loads to cause failure of the securing screws.



Modulus of elasticity of aluminum (2014-T6) shell:

$$E = 7.2 \cdot 10^{10} \text{ Pa}$$

Screw type used:

Fillister head cap screws:  
M5 X 0.8

Maximum diameter of screws:

$$d = 5 \text{ mm}$$

Major diameter of cap screws:	$A_d = \pi \cdot \frac{d^2}{4}$
Tensile stress area of screws:	$A_t = 14.2 \cdot \text{mm}^2$
Pitch length of screws:	$p = 0.8 \text{ mm}$
Thread angle:	$\alpha = 30 \text{ deg}$
Modulus of elasticity of stainless steel screws:	$E_s = 190 \cdot 10^9 \cdot \text{Pa}$
Minimum proof strength of cap screws:	$S_p = 830 \cdot 10^6 \cdot \text{Pa}$
Endurance limit for screws with rolled threads:	$S_e = 162 \cdot 10^6 \cdot \text{Pa}$
Ultimate strength of stainless steel screws:	$S_{\text{uts}} = 1040 \cdot 10^6 \cdot \text{Pa}$
Thickness of top component:	$t_1 = 1 \text{ cm}$
Reach of screw into bottom component:	$t_2 = 1 \text{ cm}$
Length of threaded portion of screws:	$l_t = t_1 + t_2 = 1 \text{ mm}$
Length of unthreaded portion of screws:	$l_d = 1 \text{ mm}$
Effective grip length:	$l = t_1 + \frac{d}{2}$ for diameter of screw $< t_2$
Maximum width of frustra:	$D_1 = 1.5 \cdot d + 0.577 \cdot l$
Minimum width of frustra:	$D_2 = 1.5 \cdot d$
Minimum diameter of frustra:	$D = D_2$
Stiffness of shell material being compressed:	$k_m = \frac{0.577 \cdot \pi \cdot E \cdot d}{\ln \left[ \frac{(1.15 \cdot (0.5 \cdot l) + D - d) \cdot (D + d)}{((1.15 \cdot (0.5 \cdot l) + D) + d) \cdot (D - d)} \right]}$
Effective stiffness of the cap screws:	$k_b = \frac{A_d \cdot A_t \cdot E_s}{A_d \cdot l_t + A_t \cdot l_d}$
Joint constant:	$C = \frac{k_b}{k_b + k_m}$
Pre-load for screws:	$f_i = 0.75 \cdot A_t \cdot S_p$
Number of screws that load is applied to:	$n_s = 2$ may possibly be distributed over 6 screws.
Load of screws due to tool bending moment:	$P = \frac{l_m}{n_s \cdot 5.1 \cdot \text{cm}} \cdot m_{\text{load}} \cdot A$
Factor of safety for proof load:	$\eta_p = \frac{S_p \cdot A_t - f_i}{C \cdot P} \quad \eta_p = 23.102$

Factor of safety for separation:  $\eta_s = \frac{f_i}{P \cdot (1 - C)}$   $\eta_s = 13.079$

Strengths for Goodman's factor of safety:  $S_a = \frac{S_{uts} \cdot \left(\frac{f_i}{A_t}\right)}{1 + \frac{S_{uts}}{S_e}}$   $S_m = S_{uts} \cdot \left(1 - \frac{S_a}{S_e}\right)$

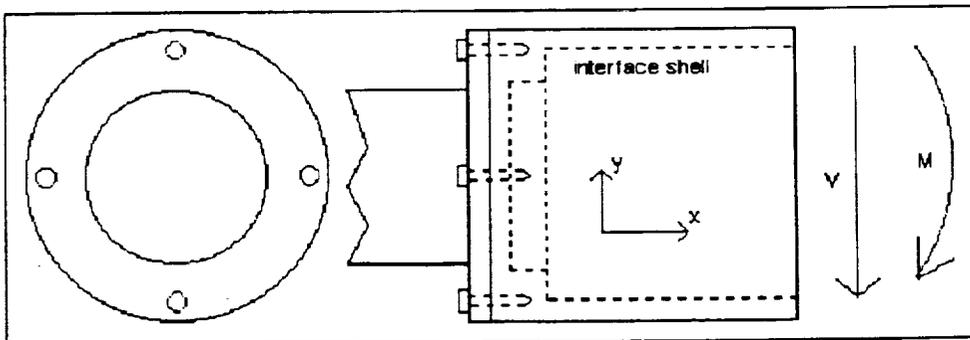
Estimated stress in the shell around screws:  $\sigma_a = \frac{C \cdot P}{2 \cdot A_t}$

Goodman's Factor of Safety for the shell:  $\eta_g = \frac{S_a}{\sigma_a}$   $\eta_g = 12.529$

The results from the factor of safeties above show that the interface shell screws will all be safe from failure due to the proof load, the separation strength, and the Goodman Criterion.

The following calculations are to determine the necessary screw size and pre-load required to hold the shell of the interface system attached to the wrist sub-system while a maximum load is applied to the tool.

The following figure shows the back half of the shell of the interface system and connection plate for the wrist sub system.



Modulus of elasticity of aluminum (2014-T6) shell:  $E = 7.2 \cdot 10^{10} \cdot \text{Pa}$

Screw type used: Fillister head cap screws: M5 X 0.8

Maximum diameter of screws:  $d = 5 \cdot \text{mm}$

Major diameter of cap screws:  $A_d = \pi \cdot \frac{d^2}{4}$

Tensile stress area of screws:	$A_t = 14.2 \cdot \text{mm}^2$
Pitch length of screws:	$p = 0.8 \cdot \text{mm}$
Thread angle:	$\alpha = 30 \cdot \text{deg}$
Modulus of elasticity of stainless steel screws:	$E_s = 190 \cdot 10^9 \cdot \text{Pa}$
Minimum proof strength of cap screws:	$S_p = 830 \cdot 10^6 \cdot \text{Pa}$
Endurance limit for screws with rolled threads:	$S_e = 162 \cdot 10^6 \cdot \text{Pa}$
Ultimate strength of stainless steel screws:	$S_{uts} = 1040 \cdot 10^6 \cdot \text{Pa}$
Thickness of top component:	$t_1 = 1 \text{ cm}$
Reach of screw into bottom component:	$t_2 = 1 \text{ cm}$
Length of threaded portion of screws:	$l_t = t_1 + t_2 = 1 \text{ mm}$
Length of unthreaded portion of screws:	$l_d = 1 \text{ mm}$
Effective grip length:	$l = t_1 + \frac{d}{2}$ for diameter of screw $< t_2$
Maximum width of frustra:	$D_1 = 1.5 \cdot d + 0.577 \cdot l$
Minimum width of frustra:	$D_2 = 1.5 \cdot d$
Minimum diameter of frustra:	$D = D_2$
Stiffness of shell material being compressed:	$k_m = \frac{0.577 \cdot \pi \cdot E \cdot d}{\ln \left[ \frac{(1.15 \cdot (0.5 \cdot l) + D - d) \cdot (D - d)}{((1.15 \cdot (0.5 \cdot l) + D) + d) \cdot (D - d)} \right]}$
Effective stiffness of the cap screws:	$k_b = \frac{A_d \cdot A_t \cdot E_s}{A_d \cdot l_t + A_t \cdot l_d}$
Joint constant:	$C = \frac{k_b}{k_b + k_m}$
Pre-load for screws:	$f_i = 0.75 \cdot A_t \cdot S_p$
Number of screws that load is applied to:	$n_s = 1$
Load of screw caused by bending from load at end of tool:	$P = \frac{1.2 \cdot m}{n_s \cdot 14 \cdot \text{cm}} \cdot m_{\text{load}} \cdot a \cdot A$
Factor of safety for proof load:	$\eta_p = \frac{S_p \cdot A_t - f_i}{C \cdot P} \quad \eta_p = 26.424$

Factor of safety for separation:  $\eta_s = \frac{f_i}{P \cdot (1 - C)}$   $\eta_s = 14.96$

Strengths for Goodman's factor of safety:  $S_a = \frac{S_{uts} \cdot \left(\frac{f_i}{A_t}\right)}{1 + \frac{S_{uts}}{S_e}}$   $S_m = S_{uts} \cdot \left(1 - \frac{S_a}{S_e}\right)$

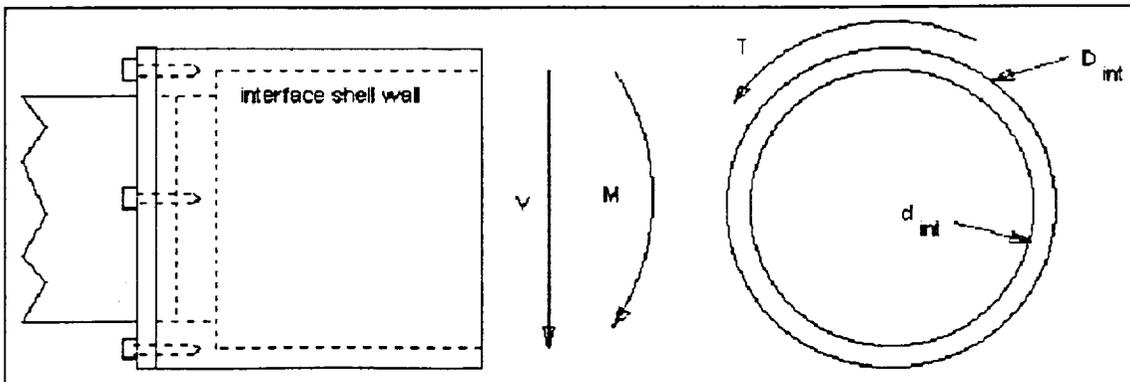
Estimated stress in the shell around screws:  $\sigma_a = \frac{C \cdot P}{2 \cdot A_t}$

Goodman's Factor of Safety for the shell:  $\eta_g = \frac{S_a}{\sigma_a}$   $\eta_g = 14.331$

The results from the factor of safeties above show that the screws holding the interface shell to the wrist sub-system are all safe from failure due to the proof load, the separation strength, and the Goodman Criterion.

The following calculations are made in order to determine the minimum acceptable thickness of the shell walls.

This figure shows the interface shell, the location of interest for failure in the shell wall, and the worst-case scenario for a load on the tool that will cause bending, torque, and shear throughout the shell walls.



The following section calculates the factors of safety for the worst-case scenario in which the interface is located in a horizontal position relative to the ground.

Outer diameter of interface shell:  $D_{int} = 15\text{-cm}$

Inner diameter of interface shell:  $d_{int} = 14.5\text{-cm}$

Moment of inertia of interface shell about z-axis as shown in figure:

$$I_{zWall} = \frac{\pi}{64} \cdot (D_{int}^4 - d_{int}^4)$$

Polar moment of inertia for the interface shell:

$$J_{Wall} = 2 \cdot I_{zWall}$$

Force of load located at distance shown:

$$V = m_{load} \cdot a \cdot A$$

Moment caused by load at interface about z-axis:

$$M = (1.2 \cdot m) \cdot V$$

Distance from center of shell to outer edge element of interest:

$$c = \frac{D_{int}}{2}$$

Torque exerted on shell due to load shown:

$$T = V \cdot (25 \cdot \text{cm})$$

Distance from center to centroid of top half of wall:

$$a_{top} = \frac{\pi}{2 \cdot 4} \cdot (D_{int}^2 - d_{int}^2)$$

Shear flow parameter needed for shear at point of interest:

$$Q = a_{top} \cdot \frac{2}{3} \cdot \frac{D_{int}^2 + d_{int} \cdot D_{int} - d_{int}^2}{(D_{int} + d_{int}) \cdot \pi}$$

Thickness of shell cross-section at equator:

$$t = D_{int} - d_{int}$$

Maximum shear stress for shell wall:

$$\tau_{max} = \sqrt{\left(\frac{M \cdot c}{2 \cdot I_{zWall}}\right)^2 + \left(\frac{T \cdot c}{J_{Wall}}\right)^2 + \left(\frac{V \cdot Q}{I_{zWall} \cdot t}\right)^2}$$

Factor of safety for point of interest:

$$\eta = \frac{0.4 \cdot S_{yS}}{\tau_{max}} \quad \eta = 73.115$$

## 7. END EFFECTORS

Tools are located at the end of the robotic arm and will perform their functions by immediately touching the equipment, cargo, etc. that needs to be moved or activated. These interchangeable end effectors will be connected to the robotic arm by an interface and will be stored in an area accessible to the arm. The interface will contain an activation rod to supply both mechanical and electrical power to the tool. Several end effector designs will be presented in this section; however the versatility of the interface allows for additional tools to be designed and implemented as the need arises.

### 7.1 Functions, Sub-system Requirements

Given the system tasks to be performed, the end effectors must perform the following functions:

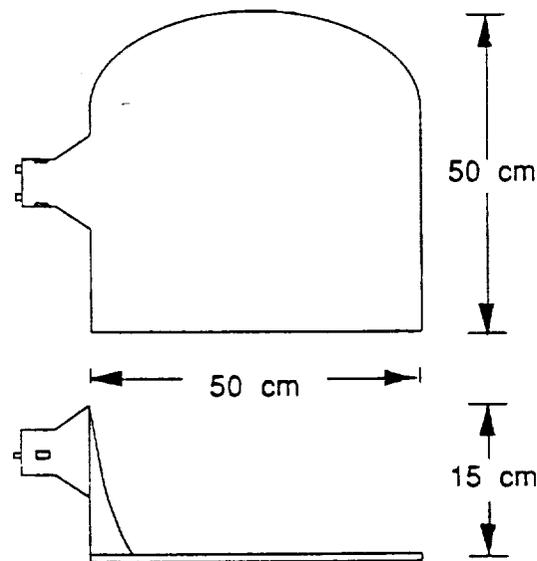
- Shovel regolith
- Level a 2 m X 2 m area by distributing regolith
- Extend antennae, push buttons, pull levers and/or turn dials on specially adapted equipment
- Move lunar rocks
- Grasp specialized handles on cargo
- Drill into lunar surface for the purpose of sampling

In light of these functions, there are many sub-system requirements that must be met. Each end effector must retain its shape while performing a task. The tools need to contain a connection to the interface and must stay connected until ejected by the interface. Tools with moving parts should have emergency positioning capabilities in case of failure and these emergency positions need to have manual releases. In addition, the mass of the end effector must be such that the combined total mass of the robot arm does not exceed 100 kg. A mass limit of 7 kg is proposed for each end effector.

## 7.2 End Effector Design

### 7.2.1 Shovel

The shovel is flat with two different sides, as shown in Figure 7.1. Its purpose is to push aside large moon rocks as well as to shovel piles of regolith. The material selected was aluminum (Al 2014-T6) due to its tensile strength and low density. The surface area of the shovel is such that a relatively thin layer of regolith can be handled. A thickness of 1 cm, which is greater than the thickness required to overcome stress, was chosen to insure the prevention of deformation due to unforeseen collisions. The shovel will not require any mechanical or electrical power to be supplied to it from the interface and will possess only a handle located at the vertical mass center. The shovel is oriented in a manner such that the interface does not come in contact with objects during operation of the shovel.



**Figure 7.1 Shovel**

### 7.2.2 Large Gripper

The large gripper, shown in Figure 7.2, is a three-pronged device used for moving large rocks and cargo. The large gripper is capable of carrying a spherical lunar rock with a mass of 50 kg. Cargo is adapted with handles, shown in Figure 7.3, designed for use with the large gripper. A cage-like enclosure on the end of the prongs is provided to insure that objects are completely encased by the gripper while being transported. The interface provides mechanical power that activates the piston to open the gripper. Springs are placed between each of the prongs to restore the gripper to the closed

position when the piston is not activated. Steel (ASTM-A514) was used due to its strength. The 2 cm by 2 cm prong cross-sectional area was selected to provide maximum strength while remaining under the mass constraint for the end effector.

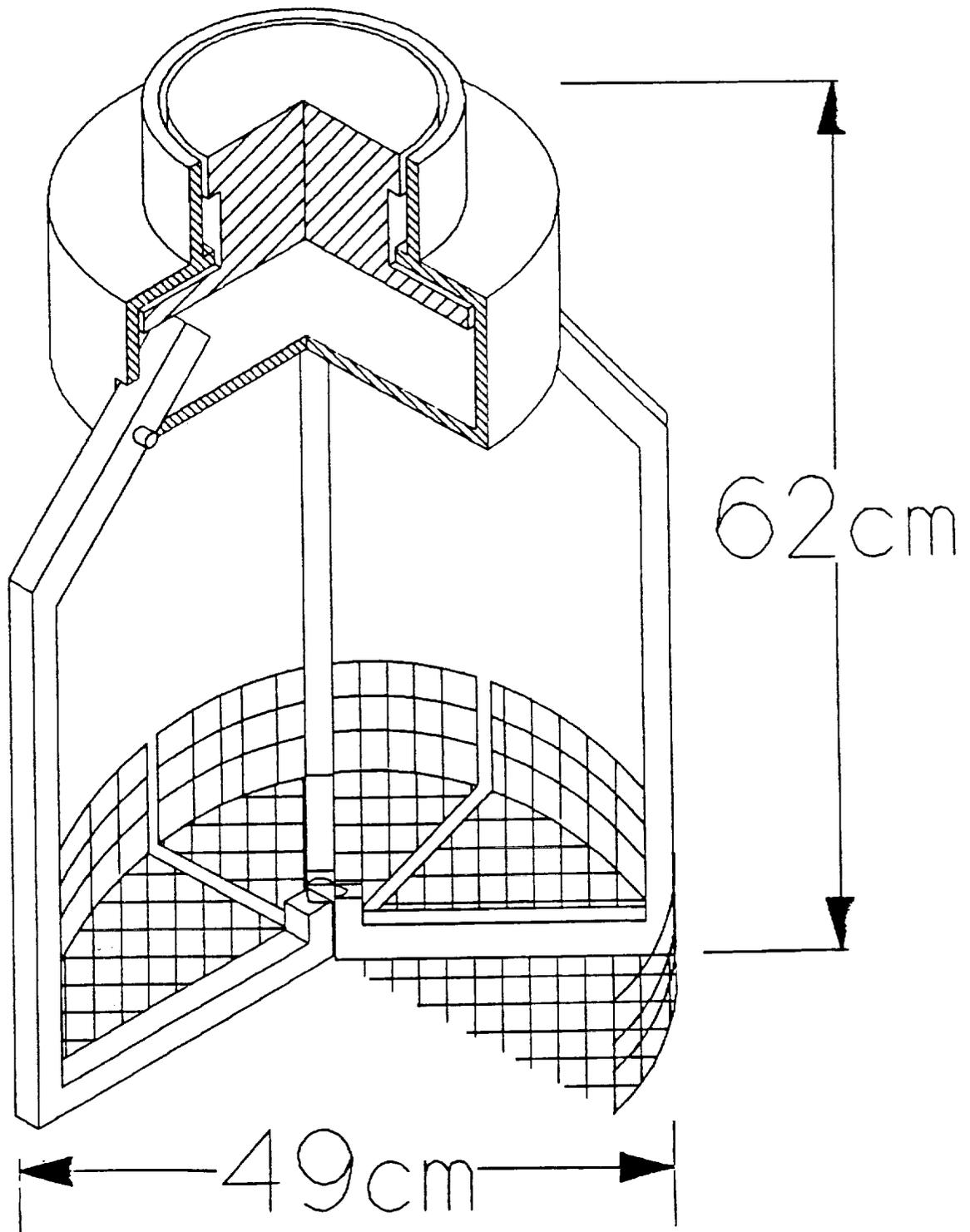
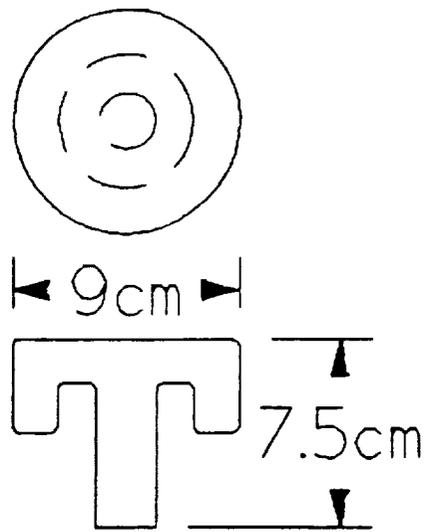


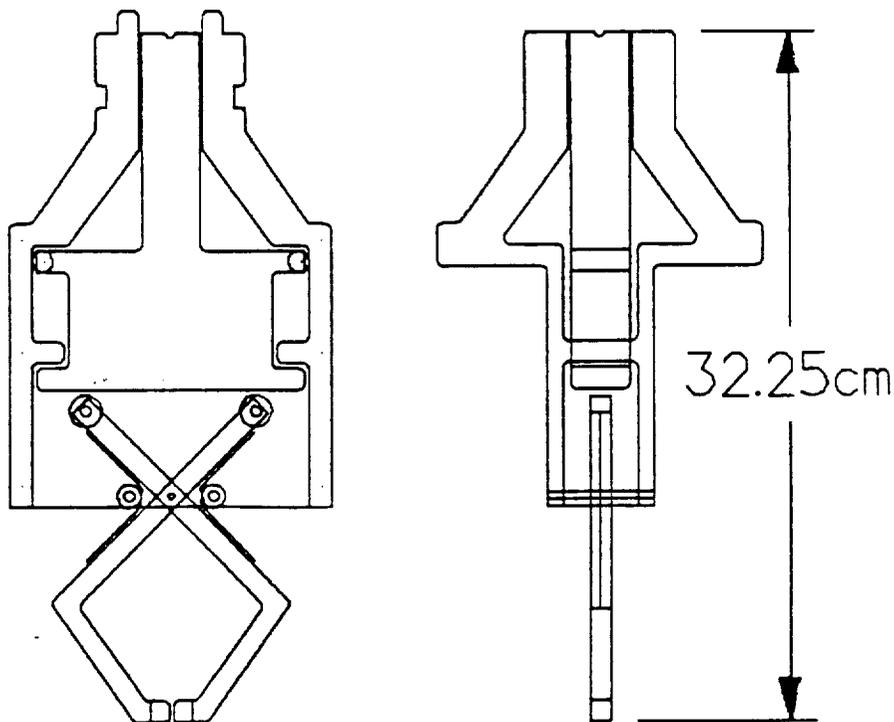
Figure 7.2 Large Gripper



**Figure 7.3 Cargo Handle**

### 7.2.3 Small Gripper

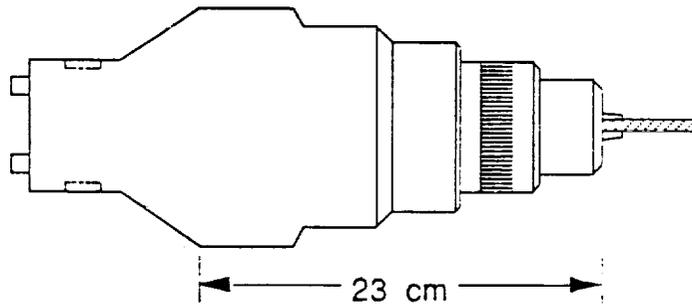
The small gripper, shown in Figure 7.4, is a two-pronged device requiring mechanical power from the interface. The gripper is able to activate specially adapted equipment as well as being able to grasp small objects. Springs keep the gripper in a closed position until the piston activates the prongs. Steel (ASTM-A514) was selected as a material for construction due to its strength. The tips of the prongs are fitted with a space-rated rubber-like material to provide a large coefficient of friction.



**Figure 7.4 Small Gripper**

## 7.2.4 Drill

A drill, shown in Figure 7.5, is provided as an example of the interface's capability to support power tools. This drill is used for obtaining mine samples from the lunar surface. The design depicted is based on a Black & Decker #7014 electric drill. A comparable space-rated motor possessing the same specifications will be used in actual construction



**Figure 7.5 Drill**

## 7.3 Calculations

### 7.3.1 Shovel

mass of maximum allowable load  $m_{\max} = 50 \cdot \text{kg}$

gravity on the moon  $g_m = 1.62 \cdot \frac{\text{m}}{\text{sec}^2}$

thickness of the shovel  $t = .01 \cdot \text{m}$

length of the shovel  $l = .5 \cdot \text{m}$

Calculating the stress:

$$\sigma = \frac{g_m \cdot m_{\max}}{l \cdot t} \quad \sigma = 1.62 \cdot 10^4 \cdot \text{Pa}$$

This stress is much lower than the tensile yield strength of 410 MPa.

Finding the center of gravity

Portion	Area	y	yA
rectangle	$A_r = (0.5 \cdot \text{m}) \cdot (0.35 \cdot \text{m})$	$y_r = \frac{1}{2} \cdot (0.35 \cdot \text{m})$	$A_r \cdot y_r = 0.031 \cdot \text{m}^3$
parabola	$A_p = \frac{4}{3} \cdot (0.15 \cdot \text{m}) \cdot \left\{ \frac{0.5}{2} \cdot \text{m} \right\}$	$y_p = 0.5 \cdot \text{m} - \left\{ \frac{3}{5} \cdot 0.15 \cdot \text{m} \right\}$	$A_p \cdot y_p = 0.021 \cdot \text{m}^3$
sum	$A_s = A_r + A_p$ $A_s = 0.225 \cdot \text{m}^2$	$y_s = y_r + y_p$ $y_s = 0.585 \cdot \text{m}$	$yA = A_r \cdot y_r + A_p \cdot y_p$ $yA = 0.051 \cdot \text{m}^3$

Distance to center of gravity

$$Y = \frac{yA}{A_s} \quad Y = 0.227 \cdot \text{m}$$

Calculating the weight:

Volume

$$V_s = A_s \cdot t \quad V_s = 0.002 \cdot \text{m}^3$$

Density of 2014-T6 Al  $\rho = 2800 \cdot \frac{\text{kg}}{\text{m}^3}$

Mass of Shovel:  $\text{Mass}_s = V_s \cdot \rho \quad \text{Mass}_s = 6.3 \cdot \text{kg}$

The actual mass of the shovel is within the given mass for each end effector.

### 7.3.2 Large Gripper

number of prongs  $p = 3$

thickness of prongs  $t_p = 0.2 \cdot m$

density of lunar rocks  $\rho_r = 1400 \cdot \frac{kg}{m^3}$

for a spherical rock:  $V = \frac{m_{max}}{\rho_r} \quad V = 0.036 \cdot m^3 \quad r = \left( \frac{3 \cdot V}{4 \cdot \pi} \right)^{\frac{1}{3}} \quad r = 0.204 \cdot m$

A spherical rock with a mass of 50 kg has a radius of 20.4 cm. This is the maximum size rock the arm would be required to lift.

Calculating the stress on each prong:

$$\sigma = \frac{g \cdot m \cdot m_{max}}{t_p^2 \cdot p} \quad \sigma = 6.75 \cdot 10^4 \cdot Pa$$

This stress is much lower than the tensile yield strength of 690 MPa.

Volume of each part

length of each bar  $l_A = .205 \cdot m$

$l_B = .250 \cdot m$

$l_C = .1131 \cdot m$

$l_D = .050 \cdot m$

total lengths  $l_t = l_A + l_B + l_C + l_D$

$$V = l_t \cdot t_p^2 \quad V = 2.472 \cdot 10^{-4} \cdot m^3$$

Density of Steel ASTM-AS14  $\rho = 7860 \cdot \frac{kg}{m^3}$

Mass of Large Gripper:  $Mass_{lg} = V \cdot \rho \quad Mass_{lg} = 5.83 \cdot kg$

The actual mass of the large gripper is within the given mass for each end effector.

Calculate the force required to open the gripper. N = newton

	$F_A = l_A \cdot \rho \cdot t_p^2 \cdot g_m$	$F_A = 1.044 \cdot N$	$r_A = .2881 \cdot m$
forces	$F_B = l_B \cdot \rho \cdot t_p^2 \cdot g_m$	$F_B = 1.273 \cdot N$	moment arms $r_B = .2015 \cdot m$
	$F_C = l_C \cdot \rho \cdot t_p^2 \cdot g_m$	$F_C = 0.576 \cdot N$	$r_C = .05655 \cdot m$
	$F_D = l_D \cdot \rho \cdot t_p^2 \cdot g_m$	$F_D = 0.255 \cdot N$	$r_D = .025 \cdot m$
			$r_F = .05 \cdot m$

sum of the moments about the pin while the prong is in an open position.

$$F = \frac{F_A \cdot r_A + F_B \cdot r_B + F_C \cdot r_C + F_D \cdot r_D}{r_F} \quad F = 11.672 \cdot N \text{ on each prong}$$

Total force needed to open prongs:  $p \cdot F = 35.016 \cdot N$

The interface actuating rod supplies a much greater amount of force.

### 7.3.3 Small Gripper

number of prongs  $p = 2$   
 thickness of prongs  $t_p = .01 \cdot m$

Volume of each part

length of each bar  $l_A = .0796 \cdot m$   
 $l_B = .0762 \cdot m$   
 $l_C = .0584 \cdot m$

total lengths  $l_t = l_A + l_B + l_C$

$$V = l_t \cdot t_p^2 \quad V = 2.142 \cdot 10^{-5} \cdot m^3$$

Density of Steel ASTM-A514  $\rho = 7860 \cdot \frac{kg}{m^3}$

Mass of Small Gripper:  $Mass_{sm} = V \cdot \rho \quad Mass_{sm} = 0.337 \cdot kg$

The actual mass of the small gripper is within the given mass for each end effector.

Calculating the stress on each prong:

$$\sigma = \frac{g_{m \cdot m \max}}{t_p^2 \cdot p} \quad \sigma = 4.05 \cdot 10^5 \cdot Pa$$

This stress is much lower than the tensile yield strength of 690 MPa.

Calculate the spring needed.

Coefficient of friction for rubber on concrete  $\mu = .75$

Weight of object  $W = m_{\max} \cdot g_m \quad W = 81 \cdot N$

Frictional force  $F_f = \frac{W}{p} \quad F_f = 40.5 \cdot N$

Normal force  $N_f = \frac{F_f}{\mu} \quad N_f = 54 \cdot N$

Calculate the force required to open the gripper. N = newton

$F_A = l_A \cdot p \cdot t_p^2 \cdot g_m$	$F_A = 0.101 \cdot N$		$r_A = .0762 \cdot m$
$F_B = l_B \cdot p \cdot t_p^2 \cdot g_m$	$F_B = 0.097 \cdot N$	moment arms	$r_B = .0381 \cdot m$
$F_C = l_C \cdot p \cdot t_p^2 \cdot g_m$	$F_C = 0.074 \cdot N$		$r_C = .0292 \cdot m$
			$r_s = .0584 \cdot m$
			$r_N = .0796 \cdot m$
			$r_p = .0584 \cdot m$

sum of the moments about the pin while the prong is in an open position and holding an object.

$$F_s = \frac{F_A \cdot r_A + F_B \cdot r_B - F_C \cdot r_C + N_f \cdot r_N}{r_s} \quad F_s = 73.761 \cdot \text{N} \quad \text{spring force on each prong}$$

calculate the spring constant

maximum displacement  $x = .04 \cdot \text{m}$

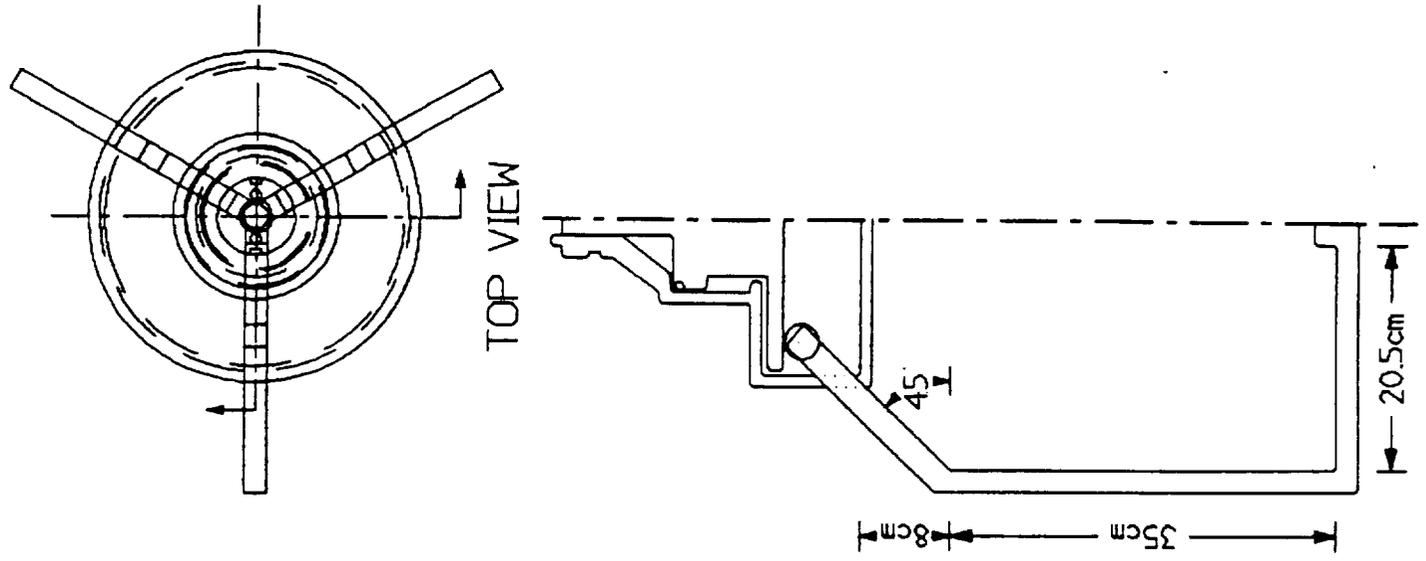
$$k = \frac{F_s}{x} \quad k = 1.844 \cdot 10^3 \cdot \frac{\text{N}}{\text{m}}$$

sum of the moments about the pin while the prong is in an open position.

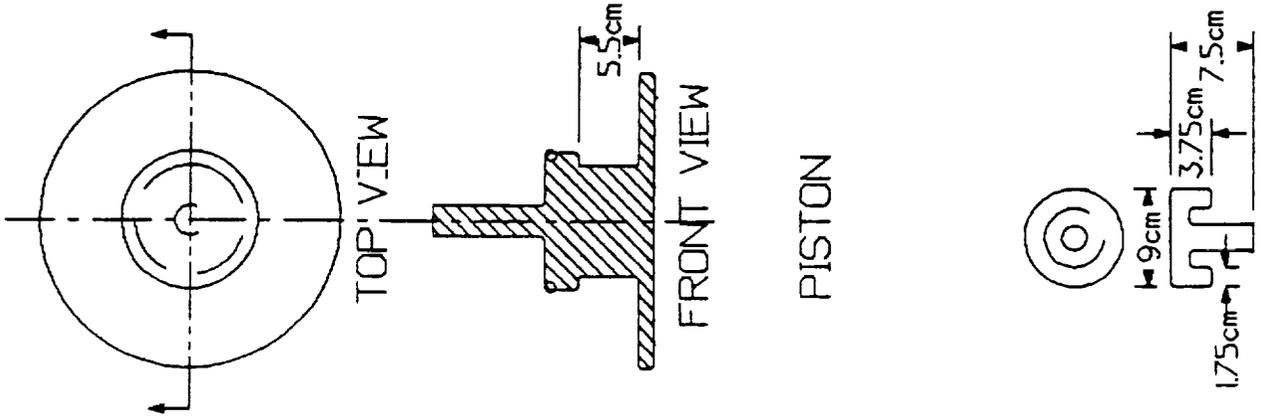
$$F_p = \frac{F_A \cdot r_A + F_B \cdot r_B - F_C \cdot r_C + F_s \cdot r_s}{r_p} \quad F_p = 73.919 \cdot \text{N} \quad \text{piston force on each prong}$$

Total force needed to open prongs:  $p \cdot F_p = 147.839 \cdot \text{N}$

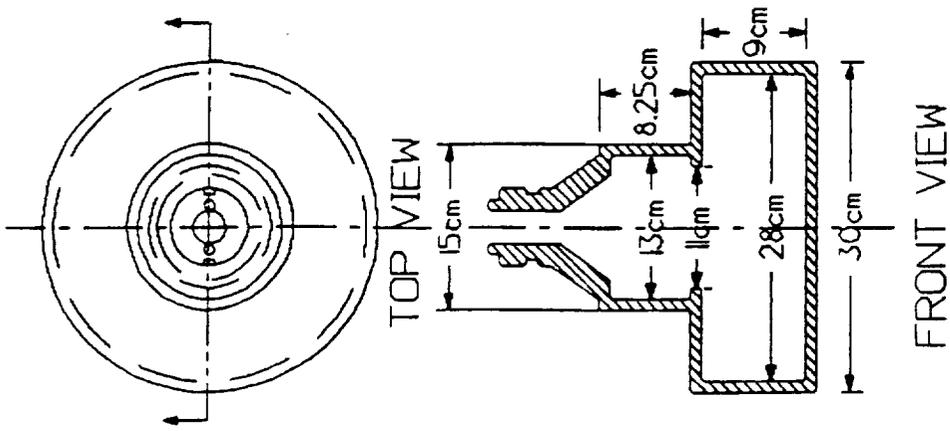
The interface actuating rod supplies a greater amount of force.



PRONG

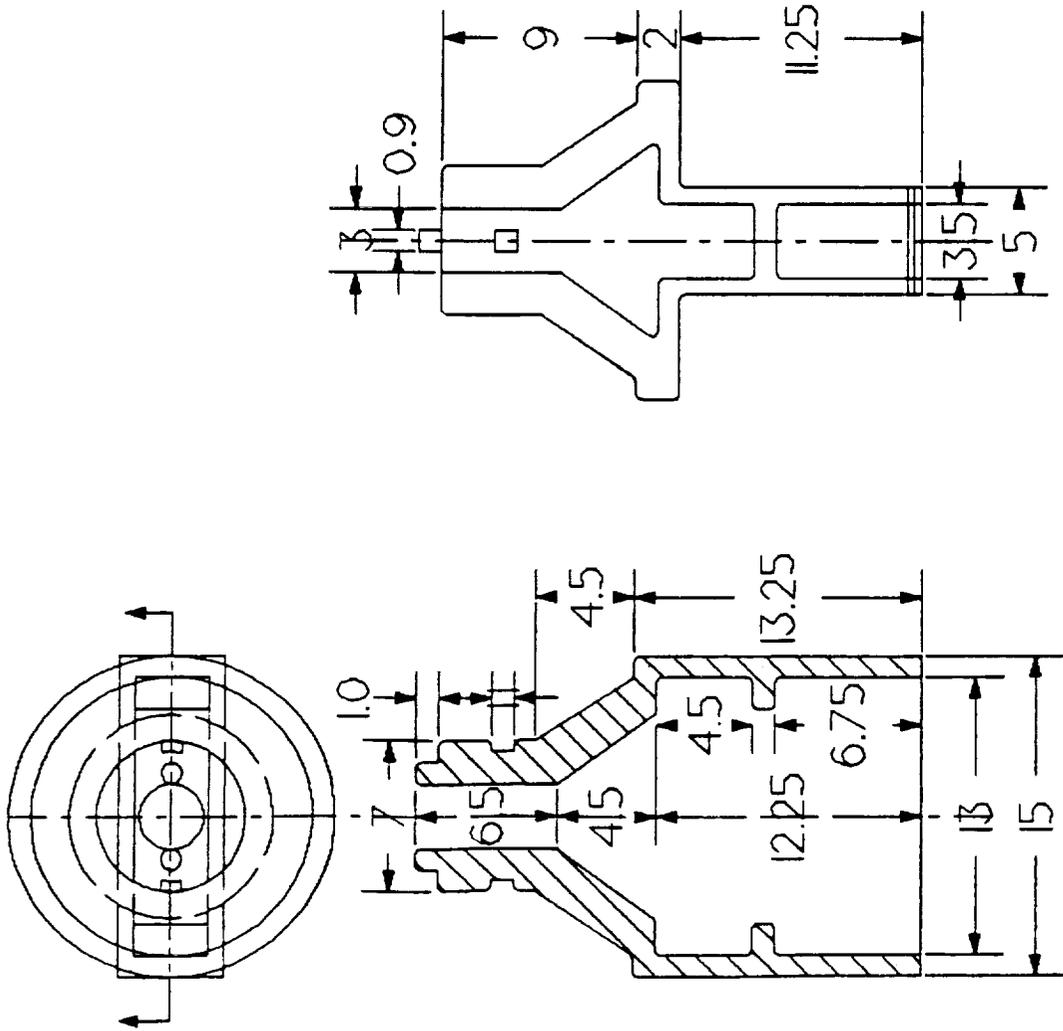
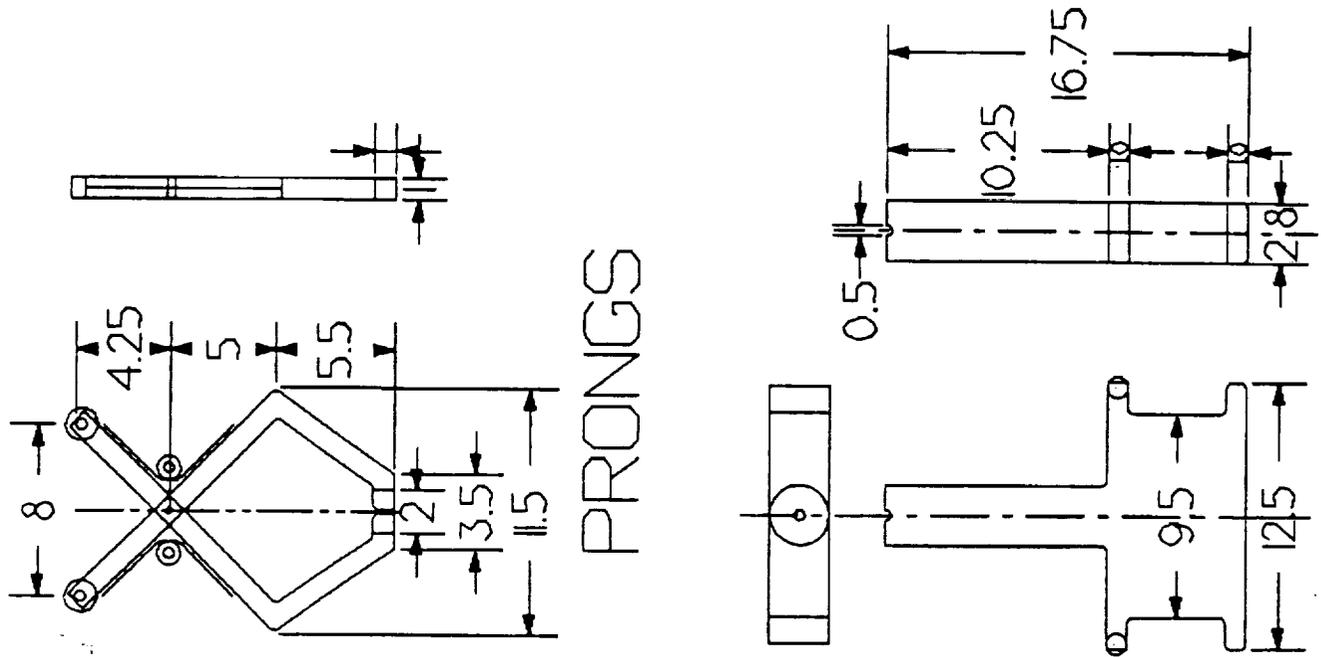


CARGO HANDLE



OUTER CASING

Figure 7.6 Large Gripper Dimensions



ALL DIMENSIONS IN CENTIMETERS

Figure 7.6 Small Gripper Dimensions

## 8. SYSTEM CONTROLS

### 8.1 Introduction

Proper operation of the mechanical systems requires the integration of supportive hardware. Such hardware must serve a number of functions which help satisfy the performance objectives of the robotic arm. These functions include providing a means of controlling system processes.

### 8.2 Controls System Selection

The controls system places the functionality of the entire robotic arm in the hands of the user. With the agitation of hand held joysticks at a computer console the input is received, processed and delivered to the output devices by this system.

The selection of this system involved many important considerations. Compumotor and Digiplan, Incorporated offers motion control systems based on the following technologies.<sup>3</sup>

#### 8.2.1 Position Control

"The position of the motor is controlled digitally by the indexer. The incremental nature of a causes the amount of motor movement to be equal to the number of pulses applied. For example, a motor with a resolution of 25,000 steps/rev will move 2 revolutions upon receipt of 50,000 pulses" (E5).

Note: An Indexer is a programmable motion controller use for single or multi-axis motion control with I/O as an auxiliary function (A78).

#### 8.2.2 Velocity

Incremental motion systems characteristically have discrete increments of motion which respond to each pulse generated by the indexer. "A string of pulses of a given frequency will cause the motor to move at a velocity proportional to that frequency. For example, an applied 25 kHz pulse train will cause a motor with a resolution of 25000 steps/rev to rotate at exactly one rev/sec. Several axes of motion can be ratioed synchronously with simple frequency control" (E5).

### 8.2.3 Acceleration/Deceleration Control

A typical indexer controlled motion profile (velocity vs. time) is trapezoidal. The motor accelerates then continues to run at a constant velocity for a period of time, then decelerates to a stop.

All Compumotor and Digiplan indexers automatically select the optimum move profile based on the commanded acceleration, deceleration, velocity and distance parameters.

### 8.2.4 Indexer

All Compumotor indexers provide programmable acceleration, velocity, and position control. The right indexer for a given application can be determined by considering the following factors:

#### 8.2.4.1 Interface Type

"The source of motion control commands may be a computer programmable controller, custom logic or an operator" (E5).

#### 8.2.4.2 Packaging

"It is important to consider how the indexer's physical configuration fits with other equipment" (E5).

#### 8.2.4.3 Front Panel Control

Some indexers offer front panel thumbwheels for selecting acceleration, velocity, and position. These controls are useful for testing and operator setup. Dedicated computer or programmable controller interfaces often eliminate the need for front control panels.

#### 8.2.4.4 Open vs. Closed Loop Control

"Critical positioning applications may require constant motor monitoring or positioning capabilities in excess of the microstepping motor/drive's normal capabilities. Some indexers provide the electronic circuitry necessary to utilize a position sensor--typically an optical encoder--to detect stalls and correct positional errors. Servo-based motors/drives--like those used throughout this design--include an integral brushless resolver feedback element, eliminating the need for encoder or tachometer circuitry in the indexer" (E5).

### 8.3 Selected Control System

The controller selected for the robotic arm is that of a ZXF Series for brushless series systems offered by the Compumotor Digiplan Corporation. Features of the ZXF Series is listed below:

- Digital brushless servo system
- RS-232C command interface; up to 16 devices per port
- Optical isolation for high noise immunity
- Battery backed RAM for storage of up to 99 sequences
- User definable resolutions up to 65,536 steps per revolution
- Built-in power supply
- Diagnostic display
- Extended edition X programming language
- Motion Profiling - allows change of velocity, distance or output base on distance travelled without stopping
- Conditional branching commands: IF/THEN/ELSE, WHILE, REPEAT/UNTIL, GOTO, GOSUB
- Complex evaluations such as input states, boolean logic, and mathematical comparisons for program branching
- BCD thumbwheel interface for entering motion or program parameters
- Program debugging tools - single step, trace and I/O simulation
- Separate acceleration and deceleration commands
- 50 user defined variables
- Math functions - add, subtract, mutliply and divide
- Registration inputs - high level interrupt for repeatable registration sensing
- Control of speed base at a ratio of a master axis speed
- Makes preset moves at a velocity ratio of a master axis
- Synchronize speed to a master axis based on registration marks
- Jog in the following mode of assist set up
- Incrementally change the following ratio using the front panel pushbuttons or thumbwheels
- Change following ratios on the fly without affecting distance move accuracy
- Motion profiling capabilities based on the primary encoder position

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<sup>2</sup>Compumotor Digiplan: Positioning Control Systems and Drives, Parker Hannifin Corp., 1992.

<sup>3</sup>Ibid.

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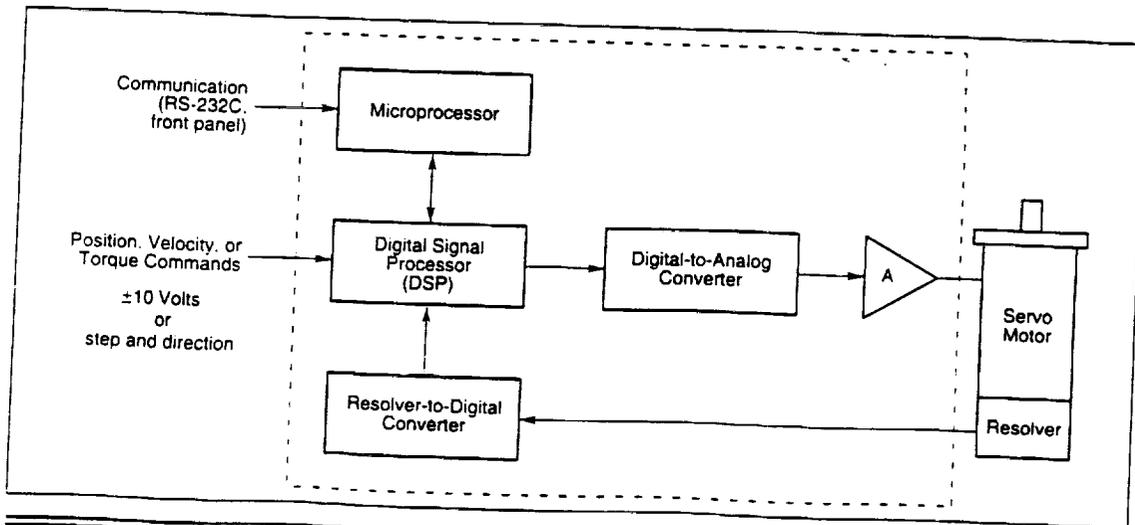
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Appendix A:  
Motor Specifications

**Series**  
Brushless Servo  
Systems

**Specifications-Z Drive**

Parameter	Value
<b>Performance</b>	
Repeatability	±0.088°, unloaded
Resolver accuracy	±7 arc min (range 200-65,536 steps/rev)
R/D converter accuracy	±8 arc min
Speed/torque	Performance curves for each model on page B38 and B39
<b>Resolution</b>	
	5,000 steps/rev factory default- 65,536 steps/rev (max)
<b>Input Power</b>	
Voltage nominal	208 to 252 VAC 3-phase
Frequency	47 to 66 Hz
Current	15 amps max continuous (RMS)—Z 600 series 30 amps max continuous (RMS)—Z 900 series
<b>Command interface</b>	
Step input	Low going pulse. Minimum pulse width is 200 nSec. Maximum pulse rate is 2.5 MHz.
Direction input	Logic high = CW rotation Logic low = CCW rotation Can be reversed through software (SSM Command)
Shutdown	Logic high = Amplifier disable Logic low = Normal operation
Analog input	+/-10V differential signals
All inputs are optically isolated and require TTL level signals to operate	Voltage low = 0.4V maximum Voltage high = 2.5-5.0V
<b>Outputs</b>	
CHA, CHB, CHZ RTO	Differential, optically isolated signals
Output voltages	Voltage low = 0.5V max. referenced to isolated ground Voltage high = 2.5-5.0V referenced to isolated ground
<b>Interface RS-232C</b>	
Baud	9,600 Baud (configurable)
Data bits	8
Stop bits	1
Parity	None
<b>Environmental</b>	
Operating Driver	32° to 122° F max (0° to 50°C) with adequate air flow (10 cfm) Maximum heatsink temperature is 160°F (71.1°C)
<b>Motor</b>	
Storage	32°F to 104°F max (0°C to 40°C). Max motor case temperature is 257°F (125°).
Humidity	-40° to 185°F (-40° to 85°C)
Technical data	0-95% non-condensing Complete motor data on each model on pages B41 and B42.

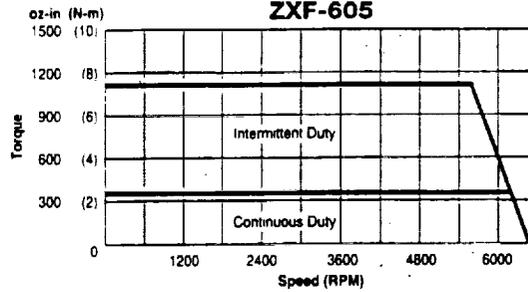


# Z, ZX & ZXF Series

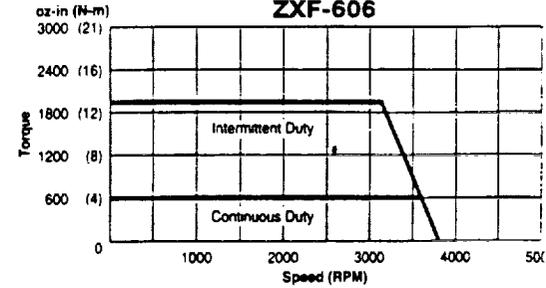
Brushless Servo Systems

## Torque/Speed Curves

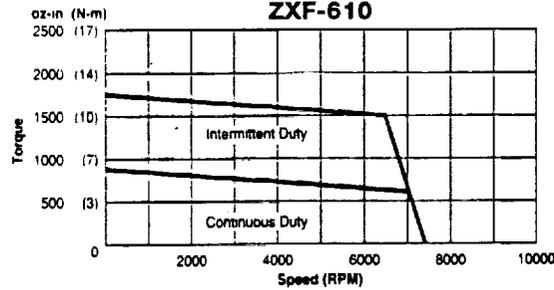
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ZX-605  
ZXF-605**



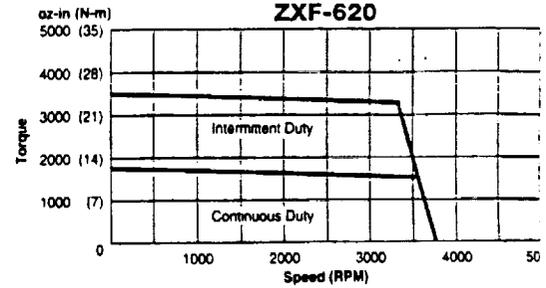
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ZX-606  
ZXF-606**



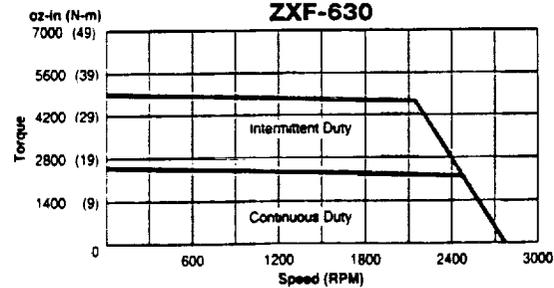
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ZX-610  
ZXF-610**



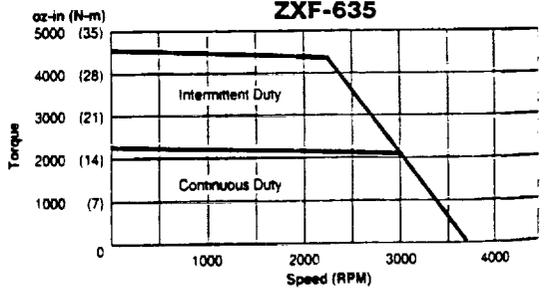
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ZX-620  
ZXF-620**



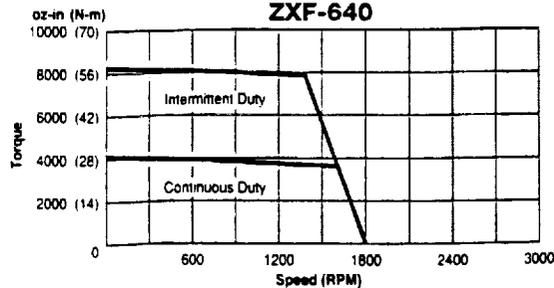
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ZXF-630**



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ZX-635  
ZXF-635**



**Z-640  
ZX-640  
ZXF-640**



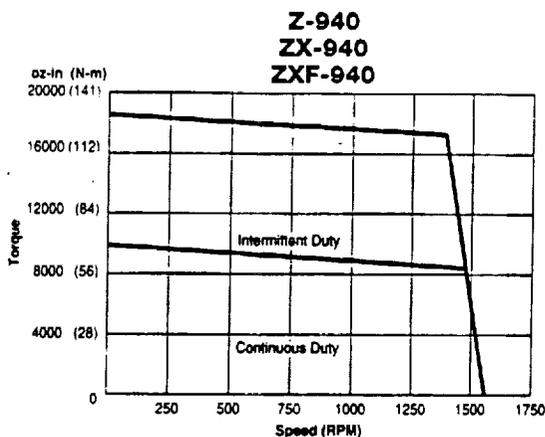
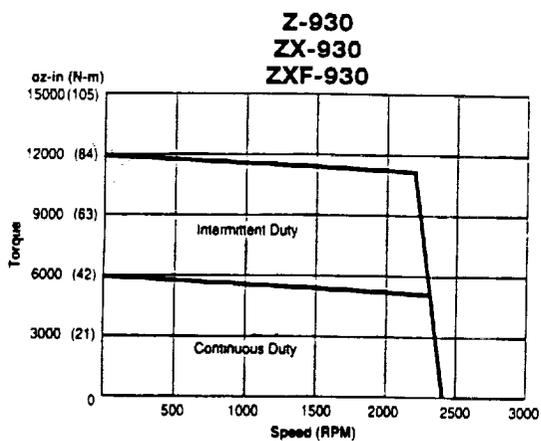
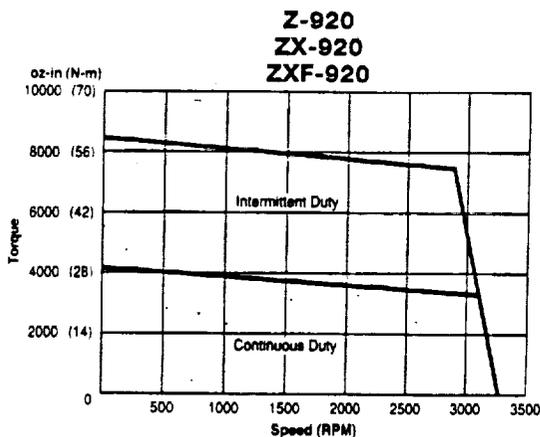
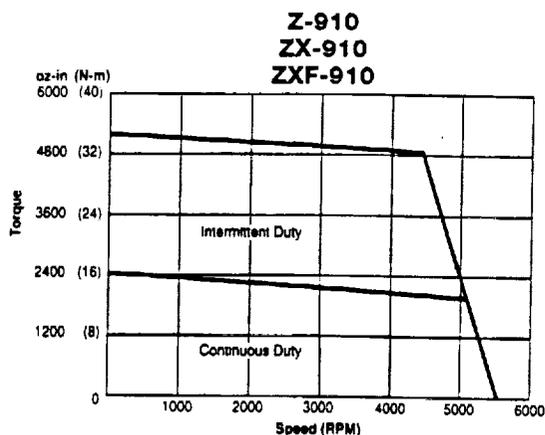
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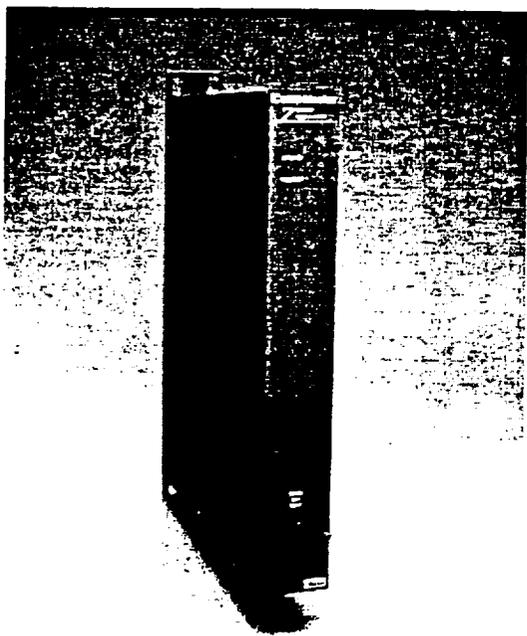
# Z, ZX & ZXF Series

Brushless Servo Systems

## Torque/Speed Curves



B Servo Systems



### Z Series Shunt Regulator

The Z shunt regulator monitors the Z Drive's internal DC bus voltage. If the bus voltage rises above a preset value, the regulator dumps some of the excess power into resistors to reduce the bus voltage and prevent an overvoltage fault. Rapidly decelerating high inertial loads from high velocities can produce these conditions. Shunt regulators are not required for most applications; they simply enhance the system allowing higher deceleration rates than are otherwise possible.

There are two sizes of shunt regulators, a 400-watt (Z-shunt-400W H14" x W2" x D11.5") and an 800-watt (Z-shunt-800W H14" x W4" x D11.5") version. The higher wattage shunt regulator dumps more regenerated energy and generally allows lower move times. Multiple shunt regulators may be connected to one drive or multiple drives to a single shunt regulator. The Z-drive and shunt regulator combination give the highest system performance possible with the Z Series brushless servo systems. To determine if your application can benefit from the addition of a shunt regulator, contact your local Automation Technology Center or call our application engineering department at 1-800-358-9070 and ask for technical bulletin #169.



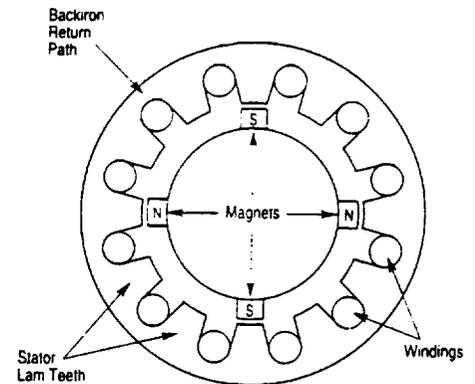
# Series

## Brushless Servo Systems

### Features of Compumotor Servo Motors

The Z motor family consists of brushless, 3-phase, AC motors. The basic construction of a Z motor is shown to the right. The permanent magnets are securely held in place by metal bands to allow high speed performance. The rotors are precision balanced, resulting in both low and high speed smoothness. The windings are located in the outer portion of the motor (stator). This "inside-out" construction allows better heat dissipation than conventional brush type motors. As a result, higher continuous torque and horsepower ratings are achieved for a given motor size.

### Basic Elements of a Brushless Motor



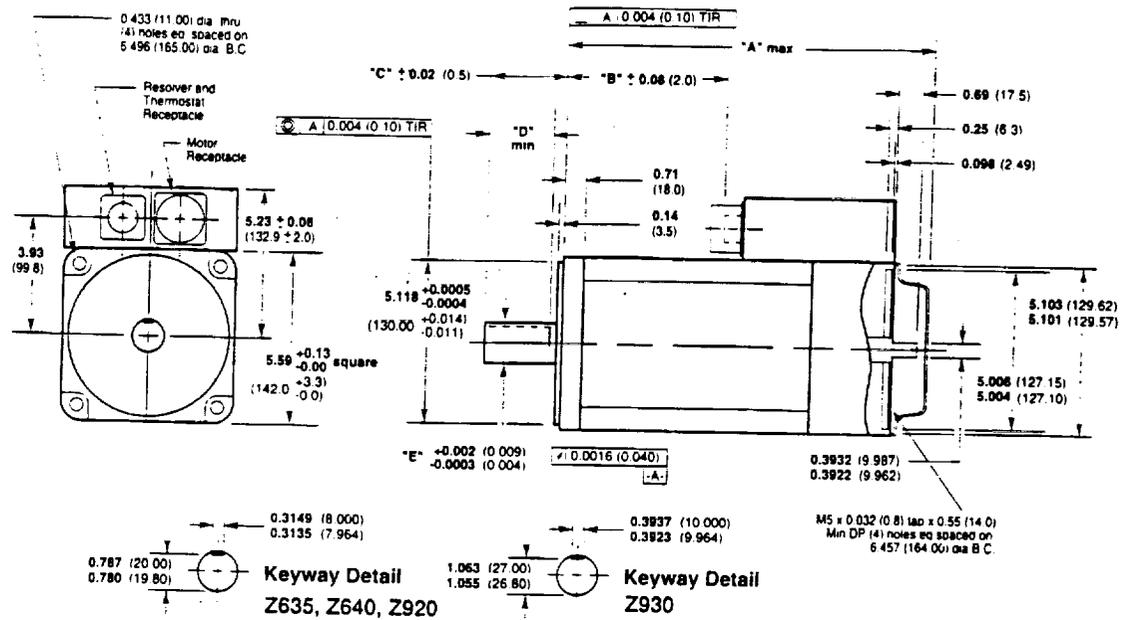
Technical Data	Z-605	Z-606	Z-610	Z-620	Z-630	Z-635	Z-640	Z-910	Z-920	Z-930	Z-94
	ZX-605 ZXF-605	ZX-606 ZXF-606	ZX-610 ZXF-610	ZX-620 ZXF-620	ZX-630 ZXF-630	ZX-635 ZXF-635	ZX-640 ZXF-640	ZX-910 ZXF-910	ZX-920 ZXF-920	ZX-930 ZXF-930	ZX-94 ZXF-94
Continuous stall torque											
oz-in	346	633	867	1,743	2,475	2,319	4,114	2,407	4,263	5,990	9,020
lb-in	22	40	54	109	155	145	257	150	266	374	56.2
Nm	2.44	4.47	6.12	12.31	17.48	16.38	29.05	17.0	30.1	42.3	63.0
Peak torque											
oz-in	1,083	1,954	1,733	3,486	4,951	4,638	8,228	5,205	8,525	11,980	18,040
lb-in	68	122	108	218	309	290	514	325	533	749	1,120
Nm	7.65	13.80	12.24	24.62	34.96	32.75	58.10	35.4	61.5	84.6	127.0
Rated power											
hp	2	2.1	4.2	5.6	5.4	5.4	5.9	9.6	10.4	11.0	11.0
k Watts	1.49	1.57	3.13	4.18	4.03	4.24	4.40	7.2	7.8	8.2	8.0
Rated speed											
rpm	6,200	3,600	7,000	3,700	2,500	3,000	1,600	5,000	3,150	2,300	1,500
rps	103	60	117	62	42	50	27	83.3	52.5	38.3	25.0
Rated current (line)	5	5.3	14.1	14.1	14.1	14.1	14.1	27.2	27.7	28.3	28.3
Peak current (3.3 sec max)	16.6	17.2	28.2	28.2	28.2	28.2	28.2	56.6	56.6	56.6	56.6
Max cont AC input power (3 phase 240 VAC)	6	6	15	15	15	15	15	30	30	30	30
Rotor inertia											
oz-in <sup>2</sup> (mass)	5.45	9.45	13.73	35.87	50.79	56.21	111.21	50.79	111.21	166.21	459.45
oz-in-sec <sup>2</sup>	0.01	0.02	0.04	0.09	0.13	0.15	0.29	0.132	0.288	0.431	1.190
kg m <sup>2</sup> x 10 <sup>-6</sup>	99.6	172.9	251.2	656	929	1,028	2,034	929	2,034	3,040	8,400
Motor weight											
lbs	10.0	14.0	17.0	29.0	32.0	37	51.0	32.0	57.0	65.0	112.0
kg	4.5	6.4	7.7	13.2	14.5	16.8	23.2	15.0	26.0	29.0	51.0
Shipping weight											
lbs	52.0	55.0	58.0	71.0	74.0	79.0	93.0	89.0	114.0	122.0	169.0
kg	23.6	25.0	26.4	32.3	33.6	35.9	42.3	40.0	52.0	55.0	77.0

# Series

Brushless Servo  
Systems

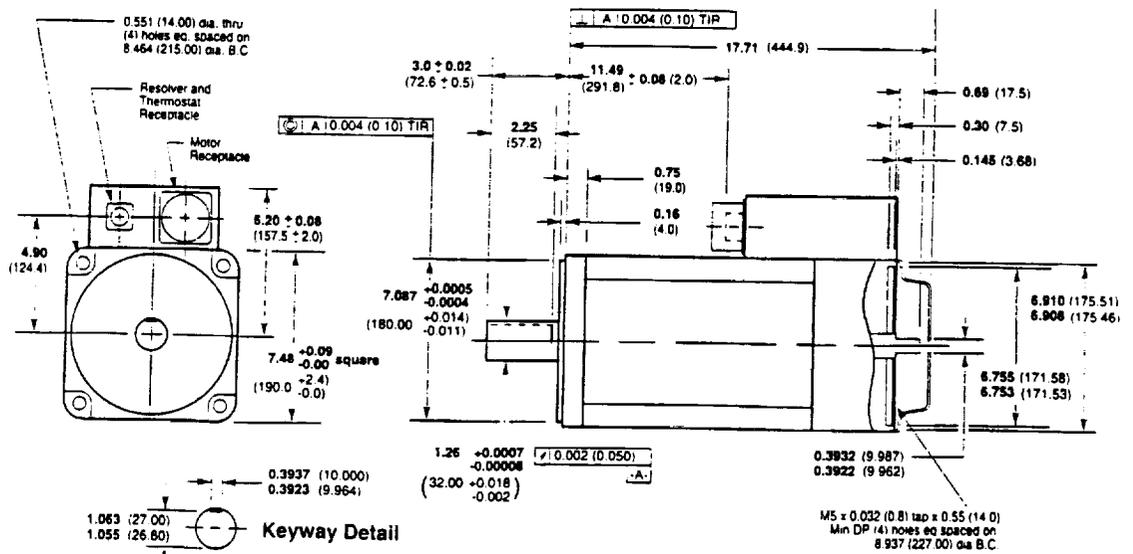
## Dimensions (—) denotes millimeters

**Z635**  
**Z640**  
**Z920**  
**Z930**



Model	A	B	C	D	E
Z635	11.78 (299.2)	6.70 (170.2)	1.967 (49.96)	1.45 (37.01)	0.945 (24.03)
Z640	14.48 (367.8)	9.49 (241.0)	1.967 (49.96)	1.457 (37.01)	0.945 (24.03)
Z920	14.48 (367.8)	9.49 (241.0)	1.967 (49.96)	1.457 (37.01)	0.945 (24.03)
Z930	17.18 (436.4)	10.87 (276.2)	3.00 (76.20)	2.25 (57.15)	1.260 (32.00)

## Z940

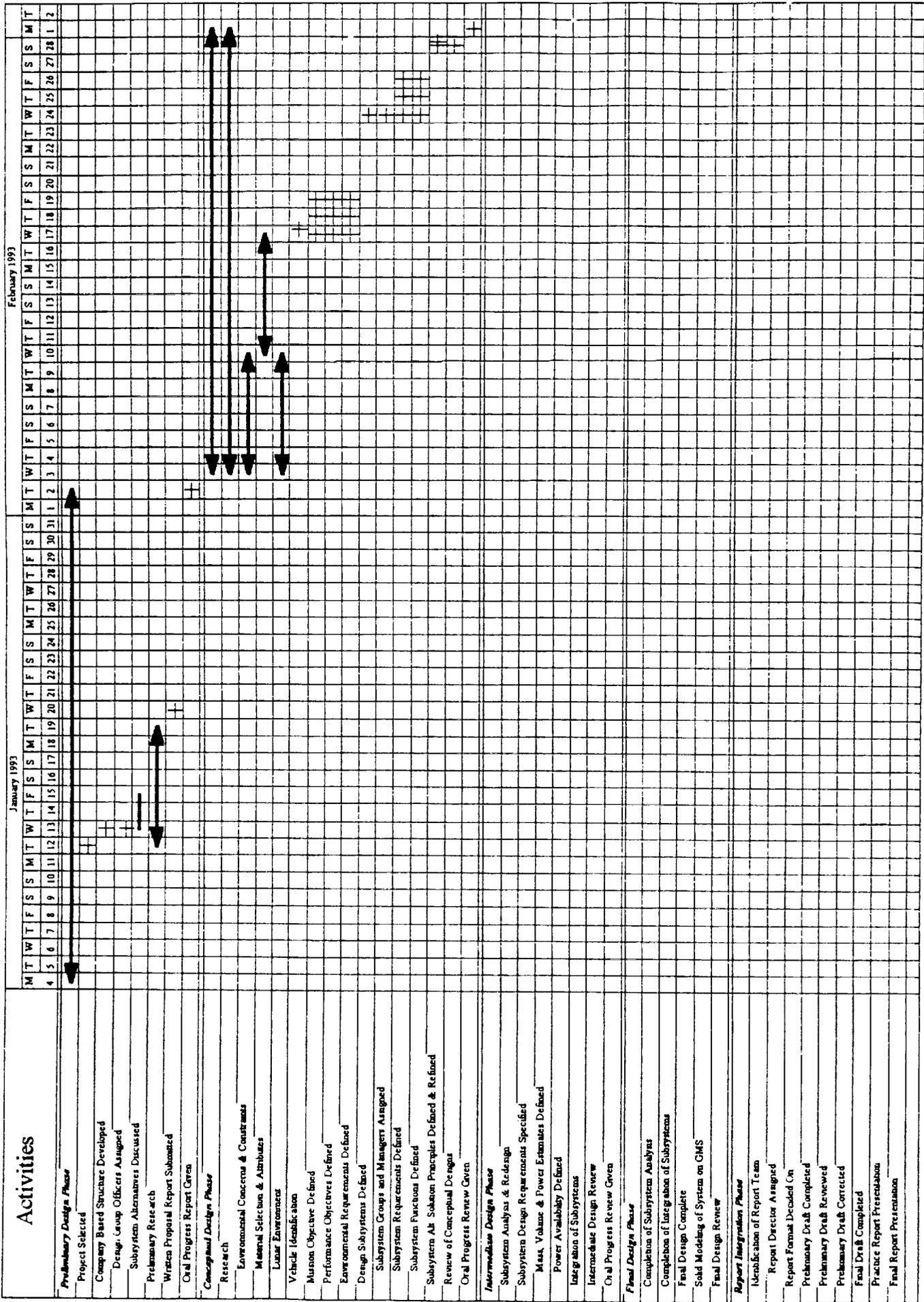


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Appendix B:  
Time Line

February 1993

January 1993



April 1993

March 1993

# Activities

## Preliminary Design Phase

- Project Selected
  - Company Based Structure Developed
  - Design Group Officers Assigned
  - Subsystem Alternatives Discussed
  - Preliminary Research
  - Written Proposal Report Submitted
  - Oral Progress Report Given
- ## Conceptual Design Phase
- ### Research
- Environmental Concerns & Constraints
  - Material Selection & Attributes
  - Linear Environment
  - Vehicle Identification
  - Mission Objective Defined
  - Performance Objectives Defined
  - Environmental Requirements Defined
  - Design Subsystems Defined
  - Subsystem Groups and Managers Assigned
  - Subsystem Requirements Defined
  - Subsystem Functions Defined
  - Subsystem All Solution Principles Defined & Refined
  - Review of Conceptual Designs
  - Oral Progress Review Given

## Intermediate Design Phase

- Subsystem Analysis & Redesign
- Subsystem Design Requirements Specified
- Mass, Volume & Power Estimates Defined
- Power Availability Defined
- Integration of Subsystems
- Intermediate Design Review
- Oral Progress Review Given

## Final Design Phase

- Completion of Subsystem Analysis
- Completion of Integration of Subsystems
- Final Design Complete
- Solid Modeling of System on GMS
- Final Design Review

## Report Integration Phase

- Identification of Report Team
- Report Director Assigned
- Report Format Decided On
- Preliminary Draft Completed
- Preliminary Draft Reviewed
- Preliminary Draft Corrected
- Final Draft Completed
- Practice Report Presentation
- Final Report Presentation

