Solution of Matrix Equations Using Sparse Techniques

by Majdi Baddourah, (majdi@sunny.larc.nasa.gov or 804-864-2913)

The solution of large systems of matrix equations is key to the solution of a large number of scientific and engineering problems.

Tradition has it that iterative methods persist for CFD and direct methods for Structures applications. With the increase in computational power (over 3 orders of magnitude this decade) problem sizes with full detail that could not have even been considered tractable are now solved routinely. The equation solvers used for structures applications have advanced from the use of full matrix (LINPACK, LAPACK BLAS-3) to band solvers to variable band and skyline solvers to sparse matrix solvers with corresponding increases in performance. It appears that for large-scale structural analysis applications sparse matrix methods have a significant performance advantage over other methods. This talk will describe the latest sparse matrix solver developed at Langley which if not the fastest in the world is among the best. It can routinely solve in excess of 263,000 equations in 40 seconds on one Cray C-90 processor.

Dr. Majdi Baddourah received the Ph D. in the Department of Civil Engineering at Old Dominion University in 1991. He has been employed by Lockheed Engineering and Sciences Company since then in support of the Computational Structures Branch at NASA Langley Research Center. Dr. Baddourah is widely recognized for contributing to the development of software to exploit scalable high-performance computers for structural analysis applications including the solution of large systems of equations (approaching 1 million) by both direct and iterative methods.
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Majdi A. Baddourah
Lockheed Engineering and Sciences Co.

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Outline

- Matrix Storage
- Reordering
- Factoring
- Results (Computational Structures and Fluids)
- Conclusion
Original Matrix (No reordering)

1 Node = 6 DOF = 6 Equations

Mach 2.4 High Speed Civil Transport

Reordering Methods

Method = 0 (ND)
Method = 1 (MD)
Method = 2 (RCM)
Method = 3 (GPS)
Matrix After Factoring

Matrix Storage Memory Requirement

[Bar chart showing memory requirement for different methods]
Reordering Time

![Bar Graph]

Equation Reordering Reduces Solution Time

Typical Node Reordering

- Maximum Band = 1266
- Average Band = 770

Equation Reordering

- Maximum Band = 609
- Average Band = 347
Factoring Matrix

- Banded or full:
  - easy to vectorize.
  - problem size limit.
- General sparse:
  - difficult to vectorize.
  - fewer operations.
  - indirect addressing.

Results

- High Speed Civil Transport
- Space Station
- CFD Application
- Automotive Application
Mach 2.4 HSCT Results

- Only VSS solves 172,400 equation HSCT on Convex C240

Space Station Application

111893 Equations
1564964 Non-zero terms
97 solution secs*

* Using 1 Cray Y-MP processor and Solid State Disk at NAS

Beam Elements
Triangular Elements
Quadrilateral Elements
Banded solver took 2200 sec for full static analysis. CRAY sparse solver took 102 sec for full static analysis.

(22 sec reading, 45 sec factors and 1 sec F/B)
- Fastest solver known to date
- On 1 Cray C-90 Processor

NASA solution took 78 sec for full static analysis
- 263,774 Equations
- 46,994 Elements
- 44,188 Nodes

Automotive Application

1 Cray C-90 Solution Time = 6.7 seconds

Number of Equations = 25779
Number of Collisions = 106

After Reordering with Hill

Before Reordering

CFD Application
Conclusion

- Sparse solvers are preferred for large-scale structures.
- Sparse Solver outperforms iterative solver which can have convergence problems.
- Sparse Solver can be used for CFD applications
- Sparse solvers uses minimum memory.