Optimization of Adaptive Intraply Hybrid Fiber Composites With Reliability Considerations

Michael C. Shiao  
NYMA, Inc.  
Engineering Services Division  
Brook Park, Ohio

and

Christos C. Chamis  
National Aeronautics and Space Administration  
Lewis Research Center  
Cleveland, Ohio

Prepared for the  
1994 North American Conference on Smart Structures and Materials  
sponsored by the Institute of Electrical and Electronics Engineers  
Orlando, Florida, February 13–18, 1994
OPTIMIZATION OF ADAPTIVE INTRAPLY HYBRID FIBER COMPOSITES WITH RELIABILITY CONSIDERATIONS

Michael C. Shiao
NYMA Inc.
Engineering Services Division
Brookpark, Ohio 44142

and

Christos C. Chamis
National Aeronautics and Space Administration
Lewis Research Center
Cleveland, Ohio 44135

SUMMARY

The reliability with bounded distribution parameters (mean and standard deviation) was maximized and the reliability-based cost was minimized for adaptive intraply hybrid fiber composites by using a probabilistic method. The probabilistic method accounts for all naturally occurring uncertainties including those in constituent material properties, fabrication variables, structure geometry, and control-related parameters. Probabilistic sensitivity factors were computed and used in the optimization procedures. For an actuated change in the angle of attack of an airfoil-like composite shell structure with an adaptive torque plate, the reliability was maximized to 0.9999 probability, with constraints on the mean and standard deviation of the actuation material volume ratio (percentage of actuation composite material in a ply) and the actuation strain coefficient. The reliability-based cost was minimized for an airfoil-like composite shell structure with an adaptive skin and a mean actuation material volume ratio as the design parameter. At a 0.91-mean actuation material volume ratio, the minimum cost was obtained.

INTRODUCTION

Aerospace structures are complex assemblages of structural components that operate under severe and often uncertain service environments. They require durability, high reliability, light weight, high performance, and affordable cost. Composite materials are potential candidates that meet these requirements because they possess outstanding mechanical properties with excellent fatigue strength and corrosion resistance. Their mechanical properties are derived from a wide variety of variables such as constituent material properties and laminate characteristics (fiber and void volume ratios, ply orientation, and ply thickness). These variables are known to be uncertain.

A new challenge is to further enhance the structural performance by investigating other advanced concepts. Recent developments in smart structure concepts that use actuation materials such as piezoelectric ceramics have the potential to improve structural performance, durability, and reliability (refs. 1 and 2). The control devices in smart structures consist of (1) a polarized material, (2) an electric field parallel to the direction of polarization, and (3) the expansion-contraction effects of the polarized material. When a control voltage is applied, the actuation material expands or contracts so that the structural behavior is altered by a desired amount and its reliability is changed. Present piezoelectric technology has been successfully applied to small-scale, low-stress structures. However, there are inevitable difficulties when current technology is applied to large-scale, high-stress composite
structures. These difficulties can be alleviated if the following concept is adopted: combine special piezoelectric fibers that have a fast actuation capability and relatively high-strength, high-modulus fiber to form an adaptive intraply hybrid composite. This composite can be readily integrated into a smart composite structure by using combinations of intraply and interply hybrid composites to ascertain that smart composite structures will operate in the design-specified range.

The adaptation of the intraply hybrid composite concept (refs. 3 and 4) to smart composite structures is depicted schematically in figure 1, which shows that the smart composite comprises (1) regular plies that consist of regular composite materials and (2) control plies that consist of regular strips of regular composite materials and strips of mixed regular and actuation materials. Actuators, containing actuation materials such as piezoelectric ceramics or piezoelectric fibers, are used to control the behavior of the composite structure by expanding or contracting the actuation strips to achieve the requisite design and operational goals. However, the strains induced by the actuator are affected by uncertainties in several factors that can only be quantified probabilistically. These factors include (1) inaccurate measurements made by sensors, (2) deviation from the intended electric field, (3) uncertain actuation strain–electric field strength relationship, (4) uncertain material properties for actuation materials, (5) uncertain electric field strength, and (6) improper location of sensor and/or control materials. Because of these factors, the use of control devices increases the uncertainty in already uncertain composite structural behavior.

To account for various uncertainties and to satisfy design requirements, knockdown (safety) factors are used extensively. These knockdown factors result in a substantial weight increase but without a quantifiable measure of reliability. This paper describes an alternate method to determine structural reliability. This method is embedded in the computer code IPACS (Integrated Probabilistic Assessment of Composite Structures, ref. 2) for a comprehensive probabilistic assessment of composite structures. The schematic of IPACS is shown in figure 2.

Because actuation materials are more expensive than regular composite materials, optimization techniques should be sought to achieve the balance between maximum reliability and minimum cost. In this paper, two types of optimization issues with reliability considerations are discussed: reliability maximization with parameter constraints and reliability-based cost optimization.

**SYMBOLS**

- \( C_F \) cost due to failure
- \( C_I \) manufacturing cost
- \( C_{i\{P\}} \) manufacturing cost for \( j^{th} \) distribution parameter
- \( C_0 \) constant cost
- \( C_T \) total cost
- \( \bar{C} \) a constant
- \( d_{31} \) actuation strain coefficient
- \( E_{f11} \) fiber modulus in longitudinal direction
- \( E_{f22} \) fiber modulus in transverse direction
- \( E_m \) matrix elastic modulus
- \( G_{f12} \) in-plane fiber shear modulus
- \( G_{f23} \) out-of-plane fiber shear modulus
- \( G_m \) matrix shear modulus
- \( m_i \) mean of \( i^{th} \) random variable failure
- \( m_{0_i} \) \( i^{th} \) reference mean
The IPACS computer code (ref. 5) has evolved from extensive research activities at the NASA Lewis Research Center to integrate probabilistic structural analysis methods (ref. 6) and computational composite mechanics (ref. 3). The composite micromechanics, macromechanics, and laminate theory (including interply and intraply hybrids) are embodied in ICAN (Integrated Composite ANalyzer, ref. 3). IPACS consists of two stand-alone computer modules: PICAN (Probabilistic Integrated Composite ANalyzer) and NESSUS (Numerical Evaluation of Stochastic Structures Under Stress). PICAN simulates probabilistic composite mechanics (ref. 7); NESSUS uses the information from PICAN to simulate probabilistic structural responses (ref. 8). Direct coupling of these two modules makes it possible to simulate uncertainties in all inherent scales of the composite—from constituent materials to the composite structure, including its boundary and loading conditions as well as environmental effects. It is worth noting that the special reliability algorithm FPI (fast probability integrator, ref. 9) is used instead of the conventional Monte Carlo simulation (ref. 10) to achieve substantial computational efficiencies that are acceptable for practical applications. Therefore, a probabilistic composite structural
analysis which cannot be done traditionally becomes feasible, especially for composite structures that have a large number of uncertain variables. The results from the IPACS analysis include a probability distribution function of the structural response, the reliability for a design criterion, and the probabilistic sensitivity factors of the primitive variables to the structural response and structural reliability.

In IPACS computational simulation for smart composite structures, electrically induced in-plane strain $\lambda$ is defined by

$$\lambda = d_{31} V$$

where $d_{31}$ is the actuation strain coefficient, and $V$ is the electric field strength. Because of the similarity between thermal strain and strain induced in the actuation material, the actuation strain is simulated using the thermal strain computed from a temperature field and the thermal expansion coefficient:

$$\lambda = d_{31} V = \alpha \Delta T$$

where $\alpha$ is the thermal expansion coefficient, and $\Delta T$ is the temperature difference.

**PROBABILISTIC SENSITIVITY INFORMATION FOR OPTIMIZATION**

The commonly used sensitivity in a deterministic analysis is the performance sensitivity $\partial Z / \partial X_i$, which measures the change in the performance $Z$ due to the change in a design parameter $X_i$. This concept is extended to a probabilistic analysis to define the probabilistic sensitivity that measures the change in reliability relative to the change in each random variable. The failure probability for a given performance is defined in the following equation (ref. 11):

$$P_f = \Phi(-\beta)$$

where $\Phi$ is the cumulative distribution function of a normally distributed random variable, and $\beta$ is the reliability index. The probabilistic sensitivity factor (PSF$_i$) for $i$th random variable is defined as

$$\text{PSF}_i = \frac{\partial \beta}{\partial X_i} = \frac{u_i^*}{\beta}$$

where $u^*$ is the most probable failure point of a limit state function in a unit normal probability space. The sensitivity of design parameters to structural reliability is another bit of useful information for controlling and adjusting design parameters from manufacturing to obtain the best benefit with minimum alteration. The sensitivity of the reliability to the mean of a normally distributed random variable $X_i$ can be represented by the following equation (ref. 11)

$$\frac{\partial \beta}{\partial m_i} = -\frac{\text{SF}_i}{\sigma_i}$$
where \( m_i \) and \( \sigma_i \) are the mean and standard deviation of random variable \( X_i \), respectively. Similarly, the sensitivity of the reliability to the standard deviation of a normally distributed random variable \( X_i \) can be computed from

\[
\frac{\partial \beta}{\partial \sigma_i} = -\frac{SF_i \ u_i^*}{\sigma_i} = -\frac{u_i^2}{\beta \sigma_i}
\]  

(6)

With this information, optimization with reliability considerations can be achieved as demonstrated in the following sections.

**OPTIMIZATION WITH RELIABILITY CONSIDERATIONS**

Two optimization problems with reliability considerations are addressed and discussed in the following sections.

Maximization of Structural Reliability with Bounded Distribution Parameters

The constraints are applied to distribution parameters such as mean and standard deviation for each random variable as shown in the following equation:

\[
P_i^l \leq P_i \leq P_i^u
\]  

(7)

where \( P_i \) is the \( i \)th distribution parameter and \( P_i^l \) and \( P_i^u \) are the lower and upper bounds for \( P_i \), respectively. From equation (5), it can be seen that the reliability can be increased by increasing the mean of the random variable with a negative probabilistic sensitivity factor and vice versa. In equation (6), it is found that \( \frac{\partial \beta}{\partial \sigma_i} \) is always negative. Therefore, reliability can be increased only by scatter reduction regardless of the sign of the sensitivity factor. With these guidelines, reliability maximization can be easily accomplished.

Minimization of Reliability-Based Cost

A major design goal is to achieve the balance between maximum reliability and minimum cost. The criterion that simultaneously addresses both reliable components and cost is the cost function:

\[
C_T = C_1 + P_f C_F
\]  

(8)

where \( C_T \) is the total cost, \( C_1 \) is the manufacturing cost, \( P_f \) is the failure probability, and \( C_F \) is the cost incurred from structural failure. The manufacturing cost is represented by
where $N$ is the number of distribution parameters, $C_j(P_j)$ is the manufacturing cost for the $j$th distribution parameter, and $C_0$ is constant cost. The total cost can be minimized when

$$\frac{\partial C_T}{\partial P_j} = 0 \quad j=1, N$$

Substituting equation (10) into equation (9) achieves optimization by solving a system of nonlinear algebraic equations:

$$\frac{\partial C_j(P_j)}{\partial P_j} + C_F \frac{\partial \Phi(-\beta)}{\partial \beta} \frac{\partial \beta}{\partial P_j} = 0 \quad j=1, N$$

For a normally distributed random variable, $\frac{\partial \beta}{\partial P_j}$ can be calculated by equations (5) and (6).

**DEMONSTRATION FOR OPTIMIZATION OF SMART COMPOSITE STRUCTURES**

**Maximization of Structural Reliability with Bounded Distribution Parameters**

Reliability maximization is demonstrated using an airfoil-like composite shell structure with an adaptive torque plate (fig. 3). The structure consists of skin, spar, and a torque plate. The skin and spar are a graphite-epoxy composite. The torque plate consists of five layers with the configuration (45/-45/steel/-45/45) in which the ±45° plies are control plies. In each control ply, both control (hybridizing actuation) and regular strips exist. However, in this paper, the control strip is assigned throughout the control ply for computational simplicity. The actuation material volume ratio is the percentage of actuation composite material in a ply. The percentage of the actuation fiber in a composite is denoted by the actuation fiber volume ratio. The constituent material properties for regular plies and their assumed probabilistic distribution and coefficient of variation (representing the range of the scatter) are summarized in table I. The corresponding fabrication variables used to make the smart composite wing are summarized in table II, and those for the control are summarized in table III.

The torque plate is fixed at the root. When control plies are electrically actuated, the angle of attack varies about its reference position. The assumed design criterion requires that the actuated change in the angle of attack be greater than 3°. The mean value $m_i$ of $i$th random variable is assumed to be bounded by

$$0.97 m_{0i} \leq m_i \leq 1.03 m_{0i}$$

where $m_{0i}$ is $i$th reference mean. The standard deviation $\sigma_i$ of the $i$th random variable is assumed to be bounded by
where \( \sigma_{0i} \) is the \( i \)th reference standard deviation. The actuated change in the angle of attack due to control is first probabilistically evaluated at the reference mean and standard deviation. The sensitivity factors for the six most sensitive random variables are presented in figure 4, which shows that the actuation material volume ratio has the largest (absolute) sensitivity factor (-0.568), followed by the actuation strain coefficient, the actuation material fiber modulus, the expansion and contraction electric field strengths, and the actuation material fiber volume ratio (-0.540, -0.409, -0.220, -0.217, -0.179, respectively). The reliability can be improved by increasing the mean value of these six random variables as indicated by equation (5). Also, from equation (6), the smaller the standard deviation of the random variable, the larger the reliability of the structure. Therefore, the largest mean value and the smallest standard deviation should be used to achieve maximum reliability. For the purpose of demonstration, only mean values and standard deviations of the actuation material volume ratio and actuation strain coefficient are chosen to be distribution parameters. The probability density function (pdf) of the actuated change in the angle of attack for a 3-percent increase in the mean of either the actuation material volume ratio or the actuation strain coefficient is shown in figure 5. Note that the mean value of the actuated angle increases whereas the scatter remains the same. The reliability can be improved by moving the pdf away from the design critical value. The pdf of the actuated angle for a 40-percent reduction in the scatter of either random variable is also shown in figure 5. The mean response (a change in the angle of attack) remains the same. The scatter of the response is reduced the most with a 40-percent reduction in the scatter of the random variable with the largest sensitivity factor. The increase in the reliability index \( \beta \) for a 3-percent increase in the values of the actuation material volume ratio and the actuation strain coefficient are estimated using equation (5). The increase in \( \beta \) due to a 40-percent reduction in the scatter is estimated by equation (6). The estimated values of \( \beta \) shown in table IV are verified by reanalyzing the smart composite structure using IPACS computer code. From the analysis, the maximum reliability index under the distribution parameter constraints is 3.712, which corresponds to a 0.9999-reliability.

**Minimization of Reliability-Based Cost**

The cost minimization is demonstrated by using an airfoil-like composite shell structure with an adaptive skin (fig. 6). The structure consists of skin and spar. The spar is a graphite-epoxy composite. The skin is a ±45° adaptive intraply hybrid fiber composite for control. The upper and lower skins are electrically actuated to generate a change in the angle of attack. The reliability of the structure is calculated for an assumed performance criterion which requires that the actuated change in the angle of attack be greater than 1.2°.

To simplify the optimization procedure, only the mean value of the actuation material volume ratio for the upper skin is chosen to be the design parameter. The initial cost \( C_1(P_1) \) is assumed to be

\[
C_1(P_1) = P_1 \bar{C} \tag{14}
\]

where \( P_1 \) is the mean value of the actuation material volume ratio, and \( \bar{C} \) is a constant; \( \bar{C} \) and \( C_F \) are assumed to be 1 and 20, respectively. Substituting equation (14) into equation (11) achieves optimization by solving the following equation:
Equation (15) is used to find the minimum cost at a 0.91-actuation material volume ratio. The result is verified by plotting the total cost and failure probability versus the mean actuation material volume ratio shown in figure 7. As can be seen, increasing the mean actuation material volume ratio results in a lower failure probability. The expected cost for failure $P_f C_F$ is also reduced. At the optimum mean actuation material volume ratio (0.91), the manufacturing cost and expected cost due to failure are balanced. However, the reduction in the expected cost due to failure is less than the increase in the manufacturing cost when the mean actuation material volume ratio is greater than 0.91. This study demonstrates that a selection among possible arrangements for the optimum cost with a reliability consideration can be achieved using the probabilistic method adopted in the computer code IPACS.

CONCLUSIONS

Optimization problems with reliability considerations for adaptive intraply hybrid fiber composite structures were studied and demonstrated. The study was accomplished by integrating probabilistic methods and smart composite concepts into the IPACS (Integrated Probabilistic Assessment of Composite Structures) computer code. Probabilistic sensitivity factors from the probabilistic assessment of composite structures were used in the optimization procedure. The reliability for actuated change in the angle of attack of an airfoil-like composite shell structure with an adaptive torque plate was maximized to 0.9999 probability with constraints on the mean and standard deviation of the actuation material volume ratio (percentage of actuation composite material in a ply) and the actuation strain coefficient. This demonstrated that structural reliability with constraints on the distribution parameters could be maximized. The reliability-based cost for an airfoil-like composite shell structure with adaptive skin was also minimized with the mean actuation material volume ratio as the design parameter. The minimum cost was found when the mean actuation material volume ratio was equal to 0.91. Therefore, the balance between the maximum reliability and the minimum cost was achieved.

REFERENCES


<table>
<thead>
<tr>
<th>TABLE I.—STATISTICS OF FIBER AND MATRIX PROPERTIES FOR GRAPHITE-EPOXY COMPOSITE</th>
</tr>
</thead>
<tbody>
<tr>
<td>[Assumed distribution type, normal; assumed coefficient of variation 0.03.]</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Property</th>
<th>Assumed mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fiber modulus direction, Mpsi</td>
<td></td>
</tr>
<tr>
<td>Longitudinal, E_{11}</td>
<td>31.0</td>
</tr>
<tr>
<td>Transverse, E_{22}</td>
<td>2.0</td>
</tr>
<tr>
<td>Fiber shear modulus, Mpsi</td>
<td></td>
</tr>
<tr>
<td>IN-plane, G_{12}</td>
<td>2.0</td>
</tr>
<tr>
<td>Out-of-plane, G_{23}</td>
<td>1.0</td>
</tr>
<tr>
<td>Fiber Poisson's ratio</td>
<td></td>
</tr>
<tr>
<td>In-plane, \nu_{12}</td>
<td>.2</td>
</tr>
<tr>
<td>Out-of-plane, \nu_{23}</td>
<td>.25</td>
</tr>
<tr>
<td>Matrix</td>
<td></td>
</tr>
<tr>
<td>Elastic modulus, E_{ex}, Mpsi</td>
<td>.5</td>
</tr>
<tr>
<td>Shear modulus, G_{ex}, Mpsi</td>
<td>.185</td>
</tr>
<tr>
<td>Poisson's ratio, \nu_{ex}</td>
<td>.35</td>
</tr>
</tbody>
</table>

| TABLE II.—STATISTICS OF FABRICATION VARIABLES |
| [Assumed distribution type, normal; assumed coefficient of variation, 0.03.] |

<table>
<thead>
<tr>
<th>Variable</th>
<th>Assumed mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>Volume ratio</td>
<td></td>
</tr>
<tr>
<td>Fiber</td>
<td>0.60</td>
</tr>
<tr>
<td>Void</td>
<td>0.02</td>
</tr>
<tr>
<td>Ply</td>
<td></td>
</tr>
<tr>
<td>Misalignment angle, deg</td>
<td>0</td>
</tr>
<tr>
<td>Thickness, in.</td>
<td>.005</td>
</tr>
</tbody>
</table>

*Assumed coefficient of variation, 1 (stdv).
### TABLE III. — STATISTICS OF CONTROL-RELATED VARIABLES

[Assumed distribution type, normal; assumed coefficient of variation, 0.03.]

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>Actuation material fiber modulus, Mpsi</td>
<td>12.4</td>
</tr>
<tr>
<td>Actuation fiber volume ratio</td>
<td>0.60</td>
</tr>
<tr>
<td>Actuation material volume ratio</td>
<td>0.80</td>
</tr>
<tr>
<td>Actuation strain coefficient, in/(\text{N})</td>
<td>(10^{-8})</td>
</tr>
<tr>
<td>Electric field strength, V/in.</td>
<td>(10^5)</td>
</tr>
</tbody>
</table>

### TABLE IV. — ENHANCED RELIABILITY BY MANUFACTURING CONTROLLING RANDOM VARIABLES

<table>
<thead>
<tr>
<th>Random variable being controlled (actuation material)</th>
<th>Sensitivity factor</th>
<th>Reliability index, (\beta)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(\Delta \beta^b)</td>
<td>(\beta)</td>
</tr>
<tr>
<td>Actuation material Volume ratio</td>
<td>-0.568</td>
<td>0.568</td>
</tr>
<tr>
<td>Strain coefficient</td>
<td>-0.540</td>
<td>0.540</td>
</tr>
<tr>
<td>Fiber volume ratio</td>
<td>-0.179</td>
<td>0.179</td>
</tr>
<tr>
<td>Fiber modulus</td>
<td>-0.409</td>
<td>0.409</td>
</tr>
<tr>
<td>Electric field strength</td>
<td>-0.220</td>
<td>.220</td>
</tr>
<tr>
<td>Expansion</td>
<td>-0.217</td>
<td>.217</td>
</tr>
<tr>
<td>Contraction</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Initial (unchanged)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Final (combined)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Estimated using sensitivity factors and recalculated by IPACS.

\[\Delta \beta^b \equiv \frac{\partial \beta}{\partial m_i} \Delta m_i.\]

\[\Delta \beta^c \equiv \frac{\partial \beta}{\partial \sigma_i} \Delta \sigma_i.\]
Figure 1.—Adaptation of intraply hybrid to smart composite system. (a) Intraply hybrid composite system. (b) Structural control using sensor/control material.

Figure 2.—Concept of probabilistic assessment of composite structures.
Figure 3.—Airfoil-type composite shell structure with adaptive torque plate. All dimensions are in inches.

Figure 4.—Sensitivity factors of uncertain variables to reliability for lower bound of change in angle of attack.
Figure 5.—Actuated change in angle of attack.

Figure 6.—Airfoil-type composite shell structure with adaptive skin. All dimensions are in inches.
Figure 7.—Reliability-based total cost as function of mean control volume ratio.
The reliability with bounded distribution parameters (mean, standard deviation) was maximized and the reliability-based cost was minimized for adaptive intraply hybrid fiber composites by using a probabilistic method. The probabilistic method accounts for all naturally occurring uncertainties including those in constituent material properties, fabrication variables, structure geometry, and control-related parameters. Probabilistic sensitivity factors were computed and used in the optimization procedures. For actuated change in the angle of attack of an airfoil-like composite shell structure with an adaptive torque plate, the reliability was maximized to 0.9999 probability, with constraints on the mean and standard deviation of the actuation material volume ratio (percentage of actuation composite material in a ply) and the actuation strain coefficient. The reliability-based cost was minimized for an airfoil-like composite shell structure with an adaptive skin and a mean actuation material volume ratio as the design parameter. At a 0.9-mean actuation material volume ratio, the minimum cost was obtained.