RESIDUAL STRESSES IN WELDED PLATES

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INTRODUCTION

Residual stresses are stresses that remain in a body after all external stresses are removed. They may either lower the usable strength, or may be beneficial and raise the strength. Residual stresses can arise from manufacturing processes or from non-uniform temperature changes. They result from stress levels causing plastic deformation of the material, which is then elastically unloaded, but prevented from returning to its initial state by interior or exterior structural restraints. The welding process, in particular, creates residual stresses. These stresses have been related to cracking in repair welds which has been observed in Al 2195, the lightweight aluminum-lithium alloy to be used in the Light Weight Space Shuttle Tank.

The mechanism that results in residual stresses in the welding process starts with the deposition of molten weld metal which heats the immediately adjacent material. After the solidification of weld material, normal thermal shrinkage is resisted by the adjacent, cooler material. When the thermal strain exceeds the elastic strain corresponding to the yield point stress, the stress level is limited by this value, which decreases with increasing temperature. Cooling then causes elastic unloading which is restrained by the adjoining material. Permanent plastic strain occurs, and tension is caused in the region immediately adjacent to the weld material. Compression arises in the metal farther from the weld in order to maintain overall static equilibrium. Subsequent repair welds may add to the level of residual stresses.

The level of residual stress is related to the onset of fracture during welding. Thus, it is of great importance to be able to predict the level of residual stresses remaining after a weld procedure, and to determine the factors, such as weld speed, temperature, direction, and number of passes, which may affect the magnitude of remaining residual stress. The purpose of this project was to develop a simple model which could be used to study residual stress. It was hoped to use traditional analytical modeling techniques so that it would be easier to comprehend the effect of these variables on the resulting stress. This approach was chosen in place of finite element methods so as to facilitate the understanding of the physical processes. The accuracy of the results was to be checked with some existing experimental studies giving residual stress levels found from X-ray diffraction measurements.

THE MODEL

The structure which was modeled was a thin plate of infinite width. It was considered to have a weld of uniform thickness deposited along a central line. The plate was assumed to have uniform thickness and uniform physical properties, although the yield stress, elastic constants, and coefficient of thermal expansion vary with temperature. The coordinate system is shown in Figure 1. The x-coordinate is along the width of the plate perpendicular from the weld, y is the weld direction, and z is the thickness direction.

The analysis of the mathematical model was composed of a thermal analysis to determine the temperature distribution, and a subsequent stress analysis, using the resulting temperature distribution.

THERMAL ANALYSIS

The heat supplied by a welding arc produces complex thermal cycles in the weld zone. It was assumed that the heat source was a point source, and all losses were by conduction into the plate. The temperature was assumed to be unchanged at a large distance from the weld. The basic equation is:

$$\frac{\partial T}{\partial t} = \kappa \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right)$$

V-1
where

\[ \kappa = \text{thermal diffusivity} = \frac{\lambda}{cp}, \lambda = \text{thermal conductivity}, c = \text{specific heat} \]

It was assumed that the temperature distribution varied only in the x-direction so that the equation could be further simplified to

\[ \frac{\partial T}{\partial t} = \kappa \left( \frac{\partial^2 T}{\partial x^2} \right) \]

This equation was solved by a finite difference formulation:

\[ \frac{T_{ij+1} - T_{ij}}{\Delta t} = \frac{\kappa}{\Delta x} \left( \frac{T_{i+1j} - T_{ij}}{\Delta x} - \frac{T_{ij} - T_{i-1j}}{\Delta x} \right) \]

or

\[ T_{ij+1} = \kappa \frac{\Delta t}{\Delta x^2} (T_{i+1j} - 2T_{ij} + T_{i-1j}) + T_{ij} \]

where the subscript j represents a time increment and i represents a spatial nodal increment in the x-direction. The model is shown in Figure 1.

Initially, the temperature at the central node was set at 600° C, the approximate melting temperature of aluminum, and the other nodes set to 20° C ambient temperature. The above equation was used to calculate heat flow over enough time intervals until an approximately uniform distribution was reached across the plate. This formulation will approximate the temperature states passed through by a welded plate. A total of 27 nodes were used, representing symmetric nodes on each side of the center weld. The nodes were spaced at \( x = \text{weld width} = \text{thickness of plate} = \frac{1}{4}'' \).

A time increment was chosen corresponding to a weld speed of \( v = 8 \text{ in/min} \). Then \( \Delta t = \Delta y/v = \frac{1}{8''/8 \text{ in/min} = 1/64 \text{ min} \approx 1 \text{ second} } \). However, it was found that a time interval of 0.1 second was required for convergence. A value of \( \kappa = 53 \text{mm}^2/\text{sec} = 0.08 \text{ in}^2/\text{sec} \) for 2219 Al was used. The MATLAB program was used to solve the equations, and it was found that an approximately uniform distribution was reached after 76 time iterations or 7.1 seconds when the change in temperature at the middle of plate from one time interval to the next was less than 5%. The temperature distribution for eight time intervals is shown in Figure 2. These values were used for the initial values in the following stress analysis.
STRESS ANALYSIS

Residual stresses were calculated using a model which represents the plate as a series of unconnected bars, undergoing only axial strain. Eight time intervals were selected, and each bar was assigned a temperature distribution based on the thermal analysis above. The mechanical properties were assumed to vary with temperature, and values for yield stress, coefficient of thermal expansion, and modulus of elasticity were used for 2219 aluminum alloy based on Ref. 2. It was assumed that an idealized stress-strain relationship holds, with a constant yield stress for each temperature level. Time intervals were taken close together near the onset of welding and further apart as the time approached the state of uniform temperature distribution. The effects of transverse stress were ignored and uniform longitudinal strain across the plate was assumed.

The solution procedure was based on one developed in Ref. 1 and is as follows: At time 0 the thermal stress for the i-th axial element is calculated from $\sigma_{yi} = E_i \alpha_i \Delta T_i$, where $E_i$ is the modulus of elasticity, $\alpha_i$ is the coefficient of thermal expansion, and $\Delta T_i$ is the temperature change from the previous time interval (or the initial temperature for the first time interval). The axial force over the entire width of the plate is then calculated and averaged to produce an average stress in each element. In order to maintain static equilibrium, the stress is then subtracted from the thermal stress to produce a net stress level. If this value exceeds the yield stress, it is lowered to that value. A new axial force is then calculated over the width, and an check for static equilibrium performed. If static equilibrium is not satisfied to within a predetermined tolerance, a new average axial stress is calculated and the process repeated until static equilibrium is attained. Then, the temperature distribution for the next time interval is applied, and the calculation process began again, adding the stresses resulting from the previous time increment, before matching them against the yield stress level. At the conclusion of the process, the final stress level at the last time increment is the residual stress in the plate.

The results of the analysis are shown in Figure 3.

A maximum value of 48.5 ksi tensile is found at the weld centerline, with a compressive value of -6.0 ksi seen at the edge of the weld. The compressive stress then decreases as the distance from the weld increases.

Comparison with Experimental Results

These results were compared with X-ray diffraction residual stress measurements previously made on 2195 aluminum alloy. These results are shown in Figure 4. They reveal a maximum tensile stress of 39.7 ksi at a distance of 0.42 in from the weld centerline, and a maximum compressive stress of -16.7 ksi at a distance of 1.08 in from the centerline. They also show a reduction of tensile stress within the weld to almost zero at the centerline. The analytical model gives a...
maximum tensile stress of 48.7 ksi at the centerline and a maximum compressive stress of -6.0 ksi at a distance of 0.5 inches from the centerline. These results compare reasonably well in magnitude, although the experimental results show stress maxima to be located farther away from the centerline. The analytical results show the same shape of stress distribution.

While this correlation is encouraging for the simple model used, it was hypothesized that an improved model, incorporating the effects of transverse stress, might show the decrease in tensile stress within the weld seen in the experimental study. It also would provide a value for the transverse stress, which may be related to the onset of fracture observed in repair welds. The supposition was that as the weld area shrinks, a restraint to transverse contraction is provided by the adjacent cooler material. This restraint causes a transverse tensile stress which is associated with a longitudinal compressive stress, through the Poisson effect. This stress would tend to lower the tensile longitudinal stress previously modeled.

**STRESS ANALYSIS INCLUDING TRANSVERSE STRESS**

Each of the elements in Figure 1 is assumed to be in a state of plane stress. It is assumed that the longitudinal strain $e_{yi}$ and the transverse stress $\sigma_{xi}$ is constant across all elements $i$. The stress strain relations in the $i$-th element are (Ref. 3)

$$
e_{xi} = \frac{1}{E_i} [\sigma_x - \nu_i \sigma_{yi}] + \alpha_i \xi_i + \varepsilon_{xi}^p + \Delta \varepsilon_{xi}^p$$

$$\varepsilon_y = \frac{1}{E_i} [\sigma_x - \nu_i \sigma_{yi}] + \alpha_i \xi_i + \varepsilon_{yi}^p$$

where $e_{xi}$ is the transverse strain, $\varepsilon_y$ is the constant longitudinal strain, $\varepsilon_{xi}^p$ and $\varepsilon_{yi}^p$ are the plastic strains which accumulated prior to the current time interval, $\Delta \varepsilon_{xi}^p$ and $\Delta \varepsilon_{yi}^p$ are the incremental plastic strains developed in the current time interval, $\sigma_x$ is the constant transverse stress, $\sigma_{yi}$ is the constant longitudinal stress, and $\nu_i$ is Poisson's ratio. Static equilibrium across the width of the plate is given by the equation

$$\Sigma \sigma_{yi} \Delta x_i = 0$$

and the condition restricting the total transverse displacement to zero is given as

$$\Sigma e_{xi} \Delta x_i = 0$$

where the summations are over all the elements, and $\Delta x_i$ is the width of the $i$-th element. These equations can be written as a series of linear equations in the unknown quantities $\sigma_x$, $\sigma_{yi}$, $e_{xi}$, $\varepsilon_y$. At the first time interval, $t = 0$ the right-hand side of these equations contains the known quantities $\alpha_i \xi_i$, where $\xi_i$ are again taken from the thermal stress analysis. Elastic strains are assumed, so that the plastic strain terms $\varepsilon_{xi}^p$ and $\Delta \varepsilon_{xi}^p$ are zero. The equations are then solved, and if a stress level exceeds the yield stress, it is equated to the yield stress. The equations are then solved again, but the yield stress for that element is now known, and a plastic strain is now introduced as an unknown, replacing the role of the elastic stress component. The solution of this set of equations yields the plastic strain increment for the first time interval. At the next time period the previous plastic strain increment becomes the $\varepsilon_{xi}^p$ term and is placed on the right hand side of the equations. The procedure is repeated for each time interval. The solution at the final time interval provides the residual stresses in the plate. The results are shown in Figure 5.
They show a similar distribution to the previous analysis. The maximum tensile stress was reduced about 30% to 34 ksi and compares more closely to the experimental results. The compressive stress level of -6 ksi is comparable to the previous result. A transverse compressive stress of -4.5 ksi was calculated, and can be compared to an experimental result varying from -4.6 to -13.1 ksi located from 1 in to 3 1/2 inches from the weld centerline.

![Figure 5. Results of Transverse Stress Analysis](image)

**Future Directions**

This project did succeed in reproducing qualitative residual stress characteristics similar to expected and observed results. The maximum value of tensile residual stress agreed within 12% of the experimental values, while the compressive stress levels were approximately 1/2 of the experimental results. The dip in tensile values in the interior of the weld was not reflected in the model. The following areas for further research in this area should be pursued:

1. The thermal analysis can easily be coupled with the stress analysis for greater computational efficiency. In this way a solution would flow directly from the assumed temperature distribution to the final residual stresses.

2. Refinement of the model within the weld area should be made to include smaller elements and changed properties reflecting the changed metallurgical characteristics of the solidified weld material.

3. A more realistic thermal model reflecting the continuous heat input of a moving line heat source would could have a large effect on the residual stresses.

4. The transverse stress model should be improved. The plastic strain portion of the solution should be analyzed more carefully, possibly using the Prandtl-Reuss flow rule.

5. Critical stress values should be correlated to fracture of welded plates and to further experimental results which should be tied to analytical models.

**References**


