THE PROBABILITY OF FLAW DETECTION AND THE PROBABILITY OF FALSE CALLS IN NONDESTRUCTIVE EVALUATION EQUIPMENT

Prepared By: Enoch C. Temple, Ph.D.
Academic Rank: Associate Professor
Institution and Department: Alabama A&M University Mathematics Department

NASA/MSFC:
Laboratory: Materials and Processes
Division: Engineering Physics
Branch: Nondestructive Evaluation

MSFC Colleague: Sam Russell, Ph.D.
1. INTRODUCTION

The space industry has developed many composite materials that have been designed to have high durability in proportion to their weights. Many of these materials have a likelihood for flaws that is higher than in traditional metals. There are also material coverings (such as paint) that develop flaws that may adversely affect the performance of the system in which they are used. Therefore, there is a need to monitor the soundness of composite structures. To meet this monitoring need, many nondestructive evaluation (NDE) systems have been developed. An NDE system is designed to detect material flaws and make flaw measurements without destroying the inspected item. Also, the detection operation is expected to be performed in a rapid manner in a field or production environment.

Within the last few years, several video-based optical NDE methodologies have been introduced. Some of the most recent of these methodologies are shearography, holography, thermography, and video image correlation. A detailed description of these may be found in Chu et al. (1985), Hung (1982), and Russell and Sutton (1989).

The NDE Branch of the Materials and Processes Laboratory at Marshall Space Flight Center has contracted to purchase video optical NDE equipment. Therefore, that Branch is now interested in qualifying the performance capability of this equipment. Qualification standards are found in MIL-STD (1989, draft No. 2) which gives the probability of detection (POD) curve as the primary qualification measurement. This curve summarizes the POD of a particular equipment type for a wide range of flaw sizes.

Section 2 will discuss what techniques are now available to estimate the POD curve from sample data. Section 3 will develop needed extensions for shortcomings in POD analysis. Section 4 shows how these extensions may be used to solve problems in the NDE Branch.

2. METHODS FOR COMPUTING POD

Figure 1 shows an example of a POD curve along with a lower 95-percent confidence boundary. The horizontal axis represents flaw size \( a \) and the vertical axis expresses probability as a percentage. As expected, the POD increases with flaw size. The darker curve is labeled POD and is sometimes called the 50-percent confidence boundary or the estimated mean POD. The lower lighter curve is called (in this figure), the lower 95-percent confidence boundary.
When qualifying an NDE equipment system, two major tasks must be performed. They are: (a) use sample data (perhaps from a demonstration test) to estimate the POD curve, and (b) determine a confidence boundary on the estimated POD curve. MIL-STD specifies a 95-percent confidence boundary.

Researchers have always classified POD(a) as a random variable that depends on $a$. This classification means that if the probability distribution of POD(a) is known, then mean POD and confidence boundaries are easily computed. Direct approaches used to identify the probability distribution of POD(a) have met with varying degrees of success. At least three different direct approaches are discussed by Berens and Hovey (1984). They also described alternative approaches that avoid the difficulties normally encountered with direct POD(a) distribution identification procedures. One is the pass/fail method and the other is labeled as the $a$-hat method.

**Pass/Fail Method**

The first method used to estimate the POD curve is described in Packman et al. (1976) and is based on the binomial distribution. To describe this method, let us assume that there are $N_i$ identically fabricated flaws for each fixed flaw size $a_i$. An NDE system examines the $N_i$ flaws and detects $n_i$ of them. Binomial theory tells us that $P_i = \frac{n_i}{N_i}$ is an estimate of POD for flaw size $a_i$. Packman uses a slightly different definition for $P_i$ and uses a collection of $P_i$'s, $i = 1, 2, ..., k$, to estimate a POD curve and its confidence boundary. A NASA supported study (Yee et al. (1976)) improves Packman's method by using a smoothing procedure. The improved method appears in the USAF POD software system as subroutine FF. The NDE Branch at MSFC uses this software system (see Berens et al. (1988)).

**A-hat Method**

An alternative to the direct characterization of the distribution of POD(a) is to find a probability distribution for the output signal of the NDE equipment. In fact, if we let $\hat{a}$ denote equipment response to the examination of a flaw of size $a$, experimentation by Berens et al. (1984) has shown that $\hat{a}$ and $a$ are related by

\[ \ln \hat{a} = \alpha_0 + \alpha_1 \ln a + \varepsilon, \]  

(2.1)

where $\varepsilon$ denotes a random error term.

Berens also showed that $\ln \hat{a}$ has an approximate normal probability distribution. Hence, to summarize the probability behavior of equation (2.1), we say that $\varepsilon \sim N(0, \sigma^2)$. $\ln \hat{a} \sim N(\mu(a), \sigma^2)$ where $\mu(a) = \alpha_0 + \alpha_1 \ln a$ and $N(\cdot)$ denotes the normal distribution. The value of POD(a) is computed by

\[ \text{POD}(a) = P(\ln \hat{a} > a_{th}), \]  

(2.2)

where $a_{th}$ is called a threshold value and
\[ a_{th} = \mu(a) + Z_p \sigma. \]  

Here, \( Z_p \) is the \( p \)th percentile of the standard normal distribution.

From sample to sample, sample estimates \( \hat{\mu}(a) \) and \( \hat{\sigma} \) of parameters in equation (2.3) change in a random manner. The random behavior of \( \hat{\mu}(a) \) and \( \hat{\sigma} \) makes POD(a) a random variable. (It appears that \( a_{th} \) changes from sample to sample. Actually, the probability distribution changes relative to \( a_{th} \) where \( a_{th} \) is fixed.) Hence, POD(a) as defined in equation (2.2) has a probability distribution which, if known, may be used to determine a confidence boundary for POD(a). However, Cheng and Iles (1983) showed that if \( \mu(a) \) and \( \sigma \) are estimated and their confidence region determined, a corresponding confidence boundary for POD(a) may be determined without actually knowing the probability distribution of POD(a). Their approach used Lagrange multipliers to find maximum and minimum values of \( a_{th} \) where \( \alpha_0, \alpha_1, \) and \( \sigma \) are restricted to some confidence region. The maximum and minimum values of \( a_{th} \) are used to find corresponding confidence boundaries for POD(a).

3. SHORTCOMING OF POD

The probability of false calls (POF) is the probability that an NDE system will indicate the presence of a flaw when a flaw does not exist. The POD(a) curve does not assess this situation. In fact, MIL-STD (1989) left POF evaluation as an open question. The next section uses the relationship in equation (2.1) to recommend a computation procedure for POD and POF.

4. GENERAL SOLUTION FOR POD, POF

Let \( y = \ln \hat{a} \) where \( \hat{a} \) is the output of NDE equipment when a flaw exists. Let \( X_1 = \ln a \) where \( a \) is actual flaw size. Assume 
\[ y = f(X_1, \alpha_1) + \varepsilon_1, \]  
where \( y \) is normal, \( X_1 \) is a vector that contains \( x_1 \) as a first component, and \( \varepsilon_1 \sim N(0, \sigma) \). In a similar manner, let \( w \) denote the logarithm of the output of NDE equipment when flaw does not exist. Assume \( w = g(X_1, \alpha_2) + \varepsilon_2 \) where \( w \) has normal distribution and \( \varepsilon_2 \sim N(0, \sigma^2) \). In functions \( f \) and \( g \), parameters \( \alpha_1, \alpha_2, \) and \( \sigma \) are to be estimated from sample data. POD and POF are based on equation (2.2). That is 
\[ \text{POD}(a) = P(y > a_{th}), \]  
and 
\[ \text{POF}(a) = P(w > a_{th}). \]  

After parameters are estimated, confidence bounds for POD(a) and POF(a) may be determined by procedures outlined in section 2.
Problem

Qualification standards require that the camera used in video optical equipment be qualified at a fixed distance between the camera lens and the object being examined. However, the NDE Branch of the Materials and Processes Laboratory is interested in the POD performance of this equipment at the tested distance as well as at untested distances. The solution to this problem requires the use of equation (4.2) which has the ability to predict POD performance of NDE equipment.

5. RECOMMENDATION

A solution to the problem posed above may be obtained by using equations (4.1) and (4.2). In this discussion, original variables of equation (2.1) will be used instead of the general vector variables of equation (4.1). That is, using equation (2.1) along with $d$ which represents the distance of the camera lens from the examined object, $\hat{a}$ may be related to $a$ and $d$ by

$$\ln \hat{a} = \alpha_0 + \alpha_1 \ln a + \alpha_2 d + \epsilon_1 = f_1(a,d) + \epsilon_1$$  \hspace{1cm} (5.1)

or

$$\ln \hat{a} = \alpha_0 + C_1 d \ln a + \epsilon_2 = f_2(a,d) + \epsilon_2$$  \hspace{1cm} (5.2)

Data inspection will provide a more realistic relationship. Furthermore, analysis of experimental data will determine if $d$ in equations (5.1) or (5.2) is a significant predictor of $\ln \hat{a}$. If $d$ is judged to be a significant variable, then POD($a$) and its confidence bounds may be determined for any fixed distance $d$. Confidence boundaries are determined as discussed in section 2. However, if the functions $f_1$ or $f_2$ are nonlinear in their parameters, the confidence boundary computation procedure will depend on the nonlinear function. Actual data for this problem will be analyzed at a later date.

REFERENCES


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