NASA Lewis Research Center Workshop on Forced Response in Turbomachinery

Proceedings of a conference sponsored by the NASA Lewis Research Center and held at the NASA Lewis Research Center Cleveland, Ohio August 11, 1993
PREFACE

This document serves as a summary of the NASA Lewis Research Center (LeRC) Workshop on Forced Response in Turbomachinery in August of 1993. The workshop was sponsored by the following NASA organizations: Structures, Space Propulsion Technology, and Propulsion Systems Divisions of the NASA Lewis Research Center and the Aeronautics and Advanced Concepts & Technology Offices of NASA Headquarters. In addition, the workshop was held in conjunction with the GIUde (Government/Industry/Universities) Consortium on Forced Response. The workshop was specifically designed to receive suggestions and comments from industry on current research at NASA LeRC in the area of forced vibratory response of turbomachinery blades which includes both computational and experimental approaches. There were eight presentations and a code demonstration. Major areas of research included aeroelastic response, steady and unsteady fluid dynamics, mistuning, and corresponding experimental work.

Forced response in turbomachinery blades is one of the most important issues in blade design. Controlling the forced response of turbomachinery blades is crucial for reliable operation of rocket and aircraft engines. Yet a major vacuum exists in predicting dynamic stresses in turbomachinery blades due to forced response. Furthermore, with the next generation engines being conceived for advanced aircraft, such as the high speed civil transport, forced response problems will have a major impact on their design and development. The acuteness of the problem is further emphasized by the willingness of all the engine companies to work together in the GIUde Consortium.

Representatives from the U.S. aerospace engine companies (Textron Lycoming, General Electric, Rocketdyne, Pratt & Whitney Rocket Engine Group, Pratt & Whitney Aircraft Engine Group, and United Technologies Research Center), numerous universities, the U.S. Air Force, and NASA LeRC attended the one-day workshop. In addition, the representatives from these companies, government agencies, and universities form the nucleus of the GIUde Consortium. The consortium was created to perform basic research on forced response of turbomachinery. This research work will improve the durability of turbomachinery and prevent engine failures.

An excellent dialogue took place in which many suggestions were given in a round-table discussion following each presentation. Action items from the participants were evaluated and incorporated into future work. An industry sub-committee was created to guide the FREPS (Forced REsponse Prediction System) development. Currently, FREPS is being modified to formulate a workstation version in order for industry to access the code for their design requirements. The industry representatives unanimously decided to participate in the Forced Response workshop every two years to evaluate NASA’s research efforts in this area.
# TABLE OF CONTENTS

WORKSHOP ORGANIZERS ......................................................... vii
ATTENDEES ............................................................................. ix
PRESENTATIONS ....................................................................... xiii

## PROGRAM OVERVIEW
G.L. Stefko, NASA Lewis Research Center ................................ 1

## FREPS—OBJECTIVES AND OVERVIEW
D.V. Murthy, NASA Lewis Research Center ................................ 5

## UNSTEADY AERODYNAMIC ANALYSES FOR TURBOMACHINERY
AEREOELASTIC PREDICTIONS
J.M. Verdon, M. Barnett, and T.C. Ayer, United Technologies Research Center .... 13

## STEADY POTENTIAL SOLVER FOR UNSTEADY AERODYNAMIC ANALYSES
D. Hoyniak, NASA Lewis Research Center .................................. 37

## FREPS—IMPLEMENTATION
D.V. Murthy, NASA Lewis Research Center .................................. 51

## MEASUREMENT OF GUST RESPONSE ON A TURBINE ANNULAR CASCADE
A.P. Kurkov and B.L. Lucci, NASA Lewis Research Center ............... 57

## EXPERIMENTAL INVESTIGATION OF AIRFOIL-GENERATED GUST FORCING FUNCTION
S. Fleeter, Purdue University .................................................. 73

## FORCED RESPONSE OF MISTUNED BLADED DISKS
C. Pierre, The University of Michigan ........................................ 95

## MISTUNING PATTERNS AND FORCED RESPONSE OF BLADED DISKS
B.C. Watson, Georgia Institute of Technology ............................. 107

## FREPS CODE DEMONSTRATION
M.R. Morel, NYMA, Inc. and D.V. Murthy, NASA Lewis Research Center ...... 121
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PRESENTATIONS

The following pages contain the presentations made at the Forced Response Workshop. Readers are encouraged to contact the researchers for further information. All researchers are listed in the Attendees list on pgs. iv-vi.
PROGRAM OVERVIEW

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NASA Lewis Research Center
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PROGRAM OVERVIEW

- FREPS SYSTEM
- EARTH-TO-ORBIT PROGRAM
- NASA LEWIS TEAM
- EXTERNAL TEAM MEMBERS
- FREPS PLANNED DEVELOPMENT

Turbomachinery
Blade Forced RESPONSE Prediction System

- Structural dynamic analysis
- Blade displacement response
- Fluid force
- Forcing function models
- Computational fluid dynamics analysis
- Alternating stress
- Mean stress
- Fatigue life

FREPS

Dynamic stress
Frequency
Blade dynamic stresses
EARTH-TO-ORBIT PROPULSION  
- TURBOMACHINERY SUBSYSTEMS -

TURBOMACHINERY SUBSYSTEMS

- MANUFACTURING
- SEALS
- STRUCTURAL DYNAMICS
- BEARINGS
- FATIGUE/FRACTURE/LIFE
- ROTORDYNAMICS
- COMPLEX FLOW PATHS
- TURBINE STAGES
- PUMP STAGES
- VALIDATED TURBOMACHINERY DESIGN TOOLS
- ADVANCED DESIGN METHODOLOGIES/CONCEPTS
- HIGH PERFORMANCE TURBOMACHINERY DATA BASES

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A. LIANG

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  - C. Pierre
  - G. Ottarsson

- NASA Lewis
  - Wake Measurements
  - Unsteady Aero

- Pratt & Whitney
  - M. Barnett

- General Electric
  - R. Kielb
  - D. Chiang

- UTRC
  - J. Verdon
  - T. Ayer
  - M. Barnett

- Georgia Tech
  - M. Kamat
  - B. Watson

FREPS PLANNED DEVELOPMENT

<table>
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<td>FREPS</td>
<td>WAKE MODEL</td>
<td>INVISCID/VISCID AERO</td>
<td>FREPS</td>
<td>VALIDATION</td>
<td>VALIDATION</td>
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3
Forced Vibratory Response in Turbomachinery

Consequences

- Decreases the Fatigue Life due to HCF Failures
- Increases Development and Maintenance Cost
- Imposes Operational Restrictions

Future Trends Point to Growing Problems

- Higher Power-to-Weight Ratio Goals
- Lower Damping Designs (e.g. Blisks)
- Increased Exposure to Resonances (e.g. Low AR Blades)

Rocket Engines vs Air Breathing Engines

<table>
<thead>
<tr>
<th>Rocket Engine Turbine</th>
<th>Air Breathing Engine Turbine</th>
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<tbody>
<tr>
<td>Turbine Inlet Temp, deg F</td>
<td>1540</td>
</tr>
<tr>
<td>Blades Cooled?</td>
<td>No</td>
</tr>
<tr>
<td>Blade Metal Temp, deg F</td>
<td>1500</td>
</tr>
<tr>
<td>Rotor Tip Speed, fps</td>
<td>1850</td>
</tr>
<tr>
<td>HP/Rotor Blade</td>
<td>500</td>
</tr>
<tr>
<td>Material</td>
<td>Super Alloy - MAR-M 246</td>
</tr>
<tr>
<td>Useful Life, hours</td>
<td>0.1 to 7.5</td>
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</tbody>
</table>

*NASA Resident Research Associate at Lewis Research Center.
Origins of Unsteady Aerodynamic Excitation

- Upstream Viscous Wake Shedding
- Downstream Potential Disturbances
- Inlet Flow Distortion and Turbulence
- Compressor Surge and Rotating Stall

Origins of Unsteady Mechanical Excitation

- Blade Tip-Casing Contact
- Shaft and Gear Meshing
- Disk Flexibility
- Cooling Jets
- Foreign Object Damage
Campbell Diagram

SHAFT SPEED (KRPM)

FREQUENCY (KHZ)

- 19E COOLING JETS
- No Quantitative Information
- Damping is Ignored
- No Accounting for Blade-to-Blade Variations

✓ FREPS Overcomes the Above Limitations and Provides an Efficient Tool for Design Environment

FREPS

Forced REsponse Prediction System

Objective

✓ To Develop a Design Tool for Predicting the Forced Vibratory Response of Turbomachinery Blades to Unsteady Excitations

✓ To Validate using Test/Experimental Data
Equations of Motion

Mechanical/Material Damping
Forces from Damping Model

External Forcing Function from
Mechanical Excitation Model

Elastic and Inertia Forces
from Structural Model
Unsteady Aerodynamic Forces
from Aerodynamic Model

Possible Models

<table>
<thead>
<tr>
<th>ROTOR</th>
<th>BLADE</th>
<th>AERO DYNAMIC</th>
<th>DAMPING</th>
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<tbody>
<tr>
<td>Tuned</td>
<td>Typical Section</td>
<td>Fully Linear</td>
<td>Structural</td>
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<td>Identical Blades</td>
<td>Lumped Parameter</td>
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<td>Empirical</td>
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<tr>
<td>Mistuned</td>
<td>Beam</td>
<td>Linearized</td>
<td>Aerodynamic</td>
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<tr>
<td>Blade-to-Blade Variations</td>
<td>1d variation</td>
<td>Potential</td>
<td>Friction</td>
</tr>
<tr>
<td>Rigid Disk</td>
<td>Shell/Plate</td>
<td>Nonlinear</td>
<td>Impact</td>
</tr>
<tr>
<td>Flexible Disk</td>
<td>2d variation</td>
<td>Potential</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Finite Element</td>
<td>Euler</td>
<td></td>
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</tbody>
</table>
Accurate Predictions

Detailed Models

Rotor
Steady Flow

Blade
Unsteady Flow

Rotor Structural Model: Tuned or Mistuned?

Tuned Model is More Suitable to Design Procedures:

✔ Model One Blade Only
✔ Smaller Problem Size
✔ Usually Designer's Preference

Mistuned Model Difficult to Incorporate in Design Procedures:

☆ Much Larger Problem Size
☆ Greater Complexity
☆ Mistuning Pattern / Strength Unknown at Design Time
☆ Mistuning Pattern / Strength Vary from Rotor to Rotor
☆ Monte Carlo Simulations are Very Expensive
Blade Structural Model

(a) typical section model

(b) beam model with rigid disk

(c) finite element model

Aerodynamic Model

Steady Aerodynamic Model

2D Steady Nonlinear Potential
(SFLOW - Dr. Hoyniak, NASA Lewis)
Accounts for:
- Incidence
- Camber
- Thickness

Unsteady Aerodynamic Model

2D Unsteady Linearized Euler
(LINFLO - Dr. Verdon, UTRC)
- Compressible
- Distorting gust
- Deforming Airfoil
- Vortical, Entropic and Acoustic Excitations
- Downstream Potential Disturbances
- Viscid / Inviscid Interaction
Accurate Predictions

Computational Efficiency

Solution Methods

Linearized Dynamic Analysis

Modal Solution

Statistical Treatment of Mistuning

Goals
Accurate Predictions

Computational Efficiency

Solution Methods

Linearized Dynamic Analysis

Modal Solution

Statistical Treatment of Mistuning

Goals

FEWER DEGREES OF FREEDOM
LITTLE LOSS IN ACCURACY

TUNED SYSTEM SIMPLICITY
USEFUL TREATMENT OF MISTUNING
UNSTEADY AERODYNAMIC ANALYSES FOR TURBOMACHINERY
AEROELASTIC PREDICTIONS

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UNSTEADY AERODYNAMIC ANALYSES

• Applications
  – Aeroelastic: blade flutter and forced vibration
  – Aeroacoustic: noise generation
  – Vibration and noise control
  – Effects of unsteadiness on performance

• Requirements
  – Accuracy/efficiency
    * Realistic operating conditions
    * Arbitrary modes of unsteady excitation

• Approaches
  – Numerical simulation/analytical modeling

ASSUMPTIONS

• Turbulence and transition can be modeled
  ⇒ Reynolds averaged, Navier-Stokes equations

• High Reynolds number, “attached” flow
  ⇒ Thin-layer Navier-Stokes equations, or
    Inviscid/viscid interaction analyses

• Small-amplitude unsteady excitations
  ⇒ Nonlinear steady + linearized unsteady analyses

• \( \text{Re} \to \infty \) ⇒ inviscid flow
  – Potential steady background flow ⇒ LINFLO
  – Uniform steady background flow ⇒ CLT
CONTRACT NAS3-25425

NASA Program Managers: J. Gauntner, G. Stefko

- Linearized inviscid unsteady aerodynamic analysis: LINFLO
- Unsteady viscous layer analysis: UNSVIS
- Steady, inviscid/viscid interaction analysis: SFLOW-IVI
- Coupled SFLOW-IVI/LINFLO analysis

EXAMPLE

CONFIGURATION
UNSTEADY EXCITATIONS

\[
\begin{align*}
\hat{s}_{-\infty}(x,t) & \\
\hat{\bar{p}}_{l,-\infty}(x,t) & \\
\hat{\zeta}_{-\infty}(x,t) & \\
\hat{\bar{p}}_{l,+\infty}(x,t) & \\
\end{align*}
\]

- Far-field conditions (uniform mean flow)
  \[
  \hat{s}(x,t) = Re\{ s_{-\infty} \exp[i (\kappa_{-\infty} \cdot x + \omega t)] \}, \quad \xi < \xi_-
  \]
  \[
  \hat{\zeta}(x,t) = Re\{ \zeta_{-\infty} \exp[i (\kappa_{-\infty} \cdot x + \omega t)] \}, \quad \xi < \xi_-
  \]
  \[
  \hat{\bar{p}}_{l,-\infty}(x,t) = Re\{ p_{l,-\infty} \exp[-\beta_{-\infty} \xi + i (\kappa_{-\infty} \cdot x + \omega t)] \}, \quad \xi < \xi_-
  \]

LINEARIZED INVISCID ANALYSES

- Linearization
  \[
  \hat{P}(x,t) = P(x) + Re\{ p(x) \exp(i \omega t) \} + \ldots
  \]

- Nonlinear BVP for steady background flow
- Linear variable-coefficient problem for each Fourier component of first-order unsteady flow
  - Time independent
  - Surface conditions imposed at mean surfaces
  - Analytic far-field solutions for \( s, \zeta, \) and \( p \)
  - Single extended blade-passage solution domain
    \[
    \hat{P}(x + mG, t) = P(x) + Re\{ p(x) \exp[i (\omega t + m \sigma)] \} + \ldots
    \]

- Prescribed quantities:
  \( \omega, \sigma, \gamma_B, s_{-\infty}, \zeta_{-\infty}, \) and \( p_{l,-\infty} \)
LINFLO

• Unsteady perturbation of a potential mean flow

• Steady flow: \( \nabla \cdot (\bar{\rho} \nabla \Phi) = 0 \)

• Unsteady velocity decomposition: \( \mathbf{v} = \nabla (\phi + \phi_*) + \mathbf{v}_R \)
  
  \(- p = -\bar{\rho} \frac{D\phi}{Dt} \)
  
  \(- \nabla \cdot \mathbf{v}_R = 0 \) far upstream
  
  \(- \frac{D\phi}{Dt} = 0; (\nabla \phi_* + \mathbf{v}_R) \cdot \mathbf{n} = 0 \) on \( B_m \) & \( W_m \)

• Entropy & rotational velocity: \( \mathbf{X} = \Delta \mathbf{e}_T + \Psi \mathbf{e}_N \rightarrow \mathbf{x} \) as \( \xi \rightarrow -\infty \)

\[ s(x) = s_{-\infty} \exp(i\kappa_{-\infty} \cdot \mathbf{x}) \]

\[ \mathbf{v}_R(x) = [\nabla (\mathbf{X} \cdot \mathbf{A}_{-\infty}) + s_{-\infty} \nabla \Phi/2] \times \exp(i\kappa_{-\infty} \cdot \mathbf{x}) \]

• Unsteady velocity potential

\[ \bar{\nabla} \left( A^{-2} \frac{D\phi}{Dt}/Dt - \bar{\rho}^{-1} \nabla \cdot (\bar{\rho} \nabla \phi) \right) = \frac{\rho}{2} - \nabla [\phi_* \mathbf{A}_T] \]

where \( \phi_* = F(\mathbf{A}_{-\infty}, \Psi) \exp(i\kappa_{-\infty} \cdot \mathbf{x}) \)

• Surface conditions:
  
  - Blades: \( \nabla \phi \cdot \mathbf{n} = f(r_B) \)
  
  - Wakes: \( [D\phi/Dt] = 0 \) and \( [\nabla \phi] \cdot \mathbf{n} = 0 \)
  
  - Shocks: \( [\rho \nabla \phi + \rho \nabla \Phi] \cdot \mathbf{n} = f(r_{SB} \cdot \mathbf{n}, \nabla \Phi); r_{SB} \cdot \mathbf{n} = -[\phi]/[\Phi] \)

• Far field conditions:
  
  - \( \phi_{1,\infty} \) prescribed; \( \phi_{R,\infty} \) must be determined
  
  - Analytic far-field solutions for \( \phi = \phi_1 + \phi_R \)
NUMERICAL SOLUTION DOMAIN

- Extended blade-passage region of finite extent in axial-flow direction

NUMERICAL APPROXIMATION

- Implicit, least-squares, finite-difference model

\[ (L\phi)_0 \approx (L\phi)_0 = q^0 \phi_0 + \sum_{m=1}^{n} \beta_m (\phi_m - \phi_0) \]

- Transonic differencing strategies
- Cascade, local and composite mesh solutions
- Direct solution procedure
  - Block tridiagonal system of algebraic equations for subsonic flow
  - Block pentadiagonal system for transonic flow with fitted shocks
AERODYNAMIC RESPONSE AT A BLADE SURFACE

- Surface pressure (transonic flow):
  \[ \hat{P}(r_B, t) = P(r_B) + \text{Re}\{p_b(r_B)\exp(i\omega t)\} + \sum_n \hat{P}_{\delta n}(r_B, t) + \ldots \]

- Blade motion: \( r_B(x) = \sum_{i=1}^{l} \delta_i R_i(x) \)

- Unsteady airloads:
  \[ q_i = \Phi q_i, \ldots = -\Phi \left\{ P \frac{\partial \theta}{\partial r} \times e_z + p_{in} - \sum r_{sh} \right\} \cdot R_i \] 

- Work per cycle/pressure-displacement function
  \[ W_C = \Phi \frac{dW}{dt} = \Phi w(\tau) d\tau = \pi \Phi \text{Im} \left( \delta_i q_i, \right) d\tau = \pi \text{Im} \left( \sum_{i=1}^{l} \delta_i q_i \right) \]

EXAMPLE RESPONSE PREDICTIONS

- Compressor exit guide vane (EGV): \( \Theta = 15 \text{ deg}, \ G = 0.6 \)
  - Thick, highly-cambered NACA 0012 airfoils
  - Subsonic flow: \( M_{\infty} = 0.3, \ \Omega_{\infty} = 40 \text{ deg} \)
  - Vortical excitation: \( \omega = 10, \ \sigma = -2\pi \)
  - Acoustic excitation from downstream:
    \( \omega = 10, \ \sigma = 0 \)
- High speed compressor cascade: \( \Theta = 45 \text{ deg}, \ G = 1 \)
  - Cambered NACA 0006 airfoils
  - Subsonic flow: \( M_{\infty} = 0.7, \ \Omega_{\infty} = 58 \text{ deg} \)
  - Transonic flow: \( M_{\infty} = 0.8, \ \Omega_{\infty} = 55 \text{ deg} \)
  - SDOF blade motions: \( \delta_i = (1, 0), \ \omega = 1 \)
- Linear/nonlinear result comparisons
  - NGUST analysis (Navier-Stokes)
  - NPHASE analysis (Euler)
COMPRESSOR EXIT GUIDE VANE

NGUST Computational Grid

Steady surface pressure coefficient

![Graph showing steady surface pressure coefficient for CASPOF (Full potential), NGUST (Euler), and NGUST (Navier-Stokes).]
Unsteady vorticity, \( \vec{v}_{R,-\infty} = (0.05\bar{q}, 0), \quad \sigma = -2\pi, \quad \omega = 10.0 \)

Unsteady pressure, \( \vec{v}_{R,-\infty} = (0.05\bar{q}, 0), \quad \sigma = -2\pi, \quad \omega = 10.0 \)
VORTICITY WAVE IN AN EGV CASCADE

FIRST HARMONIC UNSTEADY PRESSURE DIFFERENCE
\[ \bar{\psi}_{R, -\infty} = (0.05q, 0), \quad \sigma = -2\pi, \quad \omega = 10.0 \]

\[ \bar{\psi}_{R, -\infty} = \{ \begin{array}{ll} 0.05q & \\
0.10q & \\
0.25q & \\
0.50q & 
\end{array} \]

\[ \frac{\Delta \bar{p}}{\bar{\psi}_{R, -\infty}} \]

REAL

IMAGINARY

VORTICITY WAVE IN AN EGV CASCADE

FIRST HARMONIC UNSTEADY PRESSURE DIFFERENCE
\[ \bar{\psi}_{R, -\infty} = (0.05q, 0), \quad \sigma = -2\pi, \quad \omega = 10.0 \]

\[ \bar{\psi}_{R, -\infty} = \{ \begin{array}{ll} 0.05q & \\
0.10q & \\
0.25q & \\
0.50q & 
\end{array} \]

\[ \frac{\Delta \bar{p}}{\bar{\psi}_{R, -\infty}} \]

REAL

IMAGINARY
COMPRESSOR EXIT GUIDE VANE

Unsteady Pressure Response

\[ p_{+\infty} = (0.04, 0), \quad \omega = 10.0, \quad \sigma = 0.0 \]

Linearized Inviscid (LINFLO)  Navier-Stokes (NGUST)

EXIT ACOUSTIC WAVE IN AN EGV CASCADE

FIRST HARMONIC UNSTEADY PRESSURE DIFFERENCE

\[ p_{1,+\infty} = (0.04P, 0), \quad \sigma = 0, \quad \omega = 10.0 \]

\[ \frac{\Delta p}{p_{1,+\infty}} \]

REAL  IMAGINARY
EXIT ACOUSTIC WAVE IN AN EGV CASCADE

FIRST HARMONIC UNSTEADY PRESSURE DIFFERENCE
\[ p_{I,+\infty} = (0.04\bar{P}, 0), \quad \sigma = 0, \quad \omega = 10.0 \]

- \( p_{I,+\infty} = 0.04\bar{P} \)
- \( p_{I,+\infty} = 0.12\bar{P} \)
- \( p_{I,+\infty} = 0.20\bar{P} \)

NACA 0006 CASCADE
NPHASE Computational Grid
HIGH SPEED COMPRESSOR CASCADE

Surface Mach Number Distributions

--- Potential, - - - - Euler

Subsonic flow

<table>
<thead>
<tr>
<th>x</th>
<th>0</th>
<th>0.2</th>
<th>0.4</th>
<th>0.6</th>
<th>0.8</th>
<th>1.0</th>
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<tbody>
<tr>
<td>M</td>
<td>0</td>
<td>0.4</td>
<td>0.8</td>
<td>1.2</td>
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Transonic flow

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</table>

Suction surface

Pressure surface

HIGH SPEED COMPRESSOR CASCADE

Pressure Displacement Function Distributions for Torsional Blade Vibrations at $\alpha = 2\,\text{deg}$, $\omega = 1$

--- Linearized Analysis (LINFLO)

--- Nonlinear Euler Analysis (NPHASE)

Subsonic Flow

<table>
<thead>
<tr>
<th>x</th>
<th>0</th>
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<tr>
<td>$x^{-2}\omega$</td>
<td>-10</td>
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Transonic Flow

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</tbody>
</table>

Suction surface

Pressure surface

$0.23(\tau - \tau_{sh})$
HIGH SPEED COMPRESSOR CASCADE

Work per Cycle versus Interblade Phase Angle for
torsional blade vibrations at $\alpha = 2 \text{ deg}$, $\omega = 1$

- Linearized Analysis (LINFLO)
- Nonlinear Euler Analysis (NPHASE)

Subsonic Flow

Transonic Flow

INVISCID/VISCID INTERACTION ANALYSES

- High Reynolds Number Flow
- Inviscid region: Euler or potential flow equations
  - Surface conditions modified to account for viscous displacement effects
- Viscous region: Prandtl’s equations
  - Direct solution: $P \rightarrow \delta$
  - Inverse solution: $\delta \rightarrow P$
- Inviscid viscous interaction law
  - Weak interaction $\Rightarrow$ sequential solution, pressure determined by inviscid flow
  - Strong interaction $\Rightarrow$ simultaneous solution, pressure determined by inviscid and viscous flows
CASCADE FLOW WITH LOCAL REGIONS OF STRONG INTERACTION

SFLOW-IVI: INVISCID REGION

- Field equation
  - \( \rho \nabla \Phi = 0 \) or \( A^2 \nabla^2 \Phi = \nabla \Phi \cdot \nabla(\nabla \Phi)^2 / 2 \)

- Surface b.c.'s account for viscous displacement effects; i.e.,
  - Blades: \( \nabla \Phi \cdot \vec{n}_s = \rho_e^{-1} d(\rho_e u_e \delta) / ds \)
  - Wakes: \( [\nabla \Phi] \cdot \vec{n}_w = \rho_e^{-1} d(\rho_e u_e \delta_w) / ds \)

- Inlet flow conditions prescribed
- Exit flow conditions determined by Kutta cond. & global mass conservation
SFLOW-IVI: VISCOUS REGION

• Classical Viscous-Layer Eqs. (Boundary layers & Wakes)
  – Weak interaction:
    specify $du_x/ds \rightarrow$ calc. $\delta'$ (direct)
  – Strong interaction:
    specify $\bar{m} = \rho_c u_x \delta' \rightarrow$ calc. $u_x$ (inverse)

• Turbulence and transition
  – Algebraic eddy-viscosity model
    • Blade: Cebeci-Smith w/separation modification
    • Wake: Chang, et al
  – Instantaneous transition

• Solutions in terms of Levy Lees variables

SFLOW-IVI: COUPLING PROCEDURE

\[
\begin{align*}
n &= n + 1 \\
\bar{m}^n & \rightarrow \text{Inviscid Solver} \\
\bar{u}_{eI} & \\
\text{Max} |\bar{u}_{eI} - u_{eI}|/u_{eI} & < \varepsilon \\
\text{Stop} & \\
\bar{m}^{n+1} & = \bar{m}^n [1 + \omega(u_{eV}^n/u_{eI}^n - 1)] \\
\end{align*}
\]
NUMERICAL RESULTS

• Two Cascade Configurations
  - Compressor exit guide vane (EGV)
  - High-speed compressor cascade

• Effect of Varying Re

• Comparison with Navier-Stokes solutions

• Incidence Angle Study (EGV)

COMPRESSOR EXIT GUIDE VANE

Effect of Varying Re: --- --- \( Re = 10^5 \)
--- --- \( Re = 10^6 \)
--- --- Inviscid
COMPRESSOR EXIT GUIDE VANE

Comparison with Navier-Stokes Solution: $Re = 10^6$

--- IVI
--- N-S

![Graph 1: Pressure Distribution](image1)

![Graph 2: Boundary Layer Thickness](image2)

29
COMPRESSOR EXIT GUIDE VANE

Loss Parameter, $\omega$, & separation point location, $x_{sep}$, versus Inlet Flow Angle:

$Re = 10^6$, $M_{\infty} = 0.3$

COMPRESSOR EXIT GUIDE VANE

Streamlines in Trailing-Edge Region: $Re = 10^6$

$\Omega_{\infty} = 36^\circ$

$\Omega_{\infty} = 45^\circ$

$\Omega_{\infty} = 54^\circ$
HIGH SPEED COMPRESSOR CASCADE

Comparison with Navier-Stokes Solution: $Re = 10^6$

--- IVI
--- --- N-S

--- IVI
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GOAL: UNSTEADY IVI ANALYSIS
FOR AEROELASTIC APPLICATIONS

- High Re unsteady cascade flows
- Inviscid region
  - Nonlinear steady (SFLOW) ⇒ \( \Phi \)
  - Linearized unsteady (LINFLO) ⇒ \( s, v_R, \phi \)
    note: \( \tilde{V} = \nabla \Phi + Re\{[\nabla(\phi + \phi') + v_R\exp(i\omega t)] \}
  - Surface conditions
    * Blades: \( (\tilde{V} - \hat{K}) \cdot n = f_B(\tilde{\delta}) \)
    * Wakes: \( [\tilde{V}] \cdot n = f_W(\tilde{\delta}) \)
- Viscous region
  - Unsteady viscous layer analysis UNSVIS
  - UNSVIS is a direct, time marching solution procedure
- Inviscid/viscid coupling
  - Procedure must be developed for unsteady flows
- Issues
  - Must modify UNSVIS to deal with moving blades
  - Matching of inviscid and viscous solutions for \( s \) and \( v_R \) excitations
  - Need inverse unsteady viscous layer calculation
  - Inviscid/viscid coupling ⇒ long computer run times; unless
    * \( \tilde{\delta} \approx \tilde{\delta} + \delta \exp(i\omega t) \), i.e., linearization, or
    * Integral boundary layer calculation
INTERMEDIATE STEP: COUPLED SFLOW-IVI/LINFLO

- Effects of strong steady interactions on unsteady pressure response

- Assumptions
  - $\bar{\delta}(x,t) = \delta(x) + \delta(x,t)$
  - Strong steady inviscid/viscid interaction
  - Weak unsteady interaction

- SFLOW-IVI will provide steady background flow information for LINFLO calculation
  - Unsteady surface pressure determined by linearized inviscid calculation
  - Unsteady viscous layer determined by direct solution procedure
Figure 1: LINFLO results for EGV cascade undergoing torsional vibration
(\(\alpha = (1, 0)\), \(\sigma = 0\) deg, \(\omega = 1\)); \(\Omega_{-\infty} = 40\) deg, \(M_{-\infty} = 0.30\): (---) inviscid; (----) viscous, \(Re = 10^6\).

Figure 2: LINFLO results for EGV cascade undergoing torsional vibration
(\(\alpha = (1, 0)\), \(\sigma = 0\) deg, \(\omega = 1\)); \(\Omega_{-\infty} = 54\) deg, \(M_{-\infty} = 0.30\): (---) inviscid; (----) viscous, \(Re = 10^6\).
Figure 1: LINFLO results for HSC cascade undergoing torsional vibration 
\( [\alpha = (1,0), \sigma = 0 \text{ deg}, \omega = 1]; \Omega_{-\infty} = 55 \text{ deg}, M_{-\infty} = 0.70: \) (---) inviscid; (-----) viscous, \( Re = 10^6 \).

Figure 2: LINFLO results for HSC cascade undergoing torsional vibration 
\( [\alpha = (1,0), \sigma = 180 \text{ deg}, \omega = 1]; \Omega_{-\infty} = 55 \text{ deg}, M_{-\infty} = 0.70: \) (---) inviscid; (-----) viscous, \( Re = 10^6 \).
CONCLUDING REMARKS

• Linearized unsteady aerodynamic analysis: LINFLO
  – Realistic 2D flow configurations
  – Arbitrary modes and frequencies of excitation
  – Efficient prediction of unsteady pressure response

• Steady inviscid/viscid interaction analysis: SFLOW-IVI
  – 2D cascade flows
  – Local strong inviscid/viscid interactions
  – Efficient: CPU < 5 min
  – Robust: wide range of operating conditions

• Future work
  – Transonic/supersonic gust response analysis
  – SFLOW-IVI/LINFLO coupling
  – Unsteady IVI analysis
STEADY POTENTIAL SOLVER FOR UNSTEADY AERODYNAMIC ANALYSES

D. Hoyniak
NASA Lewis Research Center
Cleveland, Ohio 44135

Presentation Outline

- Description of flow solver, SFLOW
- Subsonic Calculations (Steady & Unsteady)
  - Compressor Cascade (10th Standard Configuration)
  - Turbine Cascade (4th Standard Configuration)
  - GE Low Speed Research Compressor
  - GE Low Speed Research Turbine
- Transonic Calculations (Steady)
  - Compressor Cascade (10th Standard Configuration)

Objective

Develop steady flow solver for use with LINFLO
- Compatible with LINFLO
- Composite Mesh
- Transonic Capability

Approach

- Steady flow potential equation written in nonconservative form
- Newton's Method
- Implicit, Least-Squares, Interpolation Method used to obtain finite difference expressions
- Matrix inversion routines from LINFLO
Differential Equations

Steady Flow

\[ A^2 \nabla^2 \phi - (\gamma - 1) \nabla^2 \phi \frac{D\phi}{Dt} - \frac{D^2 \phi}{Dt^2} - \nabla \phi \cdot \frac{\nabla (\nabla \phi)^2}{2} \]

= \[ - A^2 \nabla^2 \phi + \nabla \phi \cdot \frac{\nabla (\nabla \phi)^2}{2} \]

\[ \frac{D\phi}{Dt} = \nabla \phi \cdot \nabla \]

Unsteady Flow

\[ A^2 \nabla^2 \phi - (\gamma - 1) \nabla^2 \phi \frac{D\phi}{Dt} - \frac{D^2 \phi}{Dt^2} - \nabla \phi \cdot \frac{\nabla (\nabla \phi)^2}{2} = 0 \]

\[ \frac{D\phi}{Dt} = i\omega + \nabla \phi \cdot \nabla \]

Newton's Method

\[ [A(\Phi)] \{\phi\} = \{b(\Phi)\} \]

\[ \Phi(\bar{x})^{n+1} = \Phi(\bar{x})^n + \phi(\bar{x})^n \]

Convergence Criterion

\[ |\phi(\bar{x})^n| < \epsilon \]

38
Composite Mesh

Cascade Geometry
10th Standard Configuration, Subsonic Flow Conditions
Steady Mach Number Distribution
$M_\infty = 0.7, \, \Omega_\infty = 55 \, \text{deg}$

10th Standard Configuration, Subsonic Flow Conditions
Unsteady Torsion Mode Response
$\alpha = 1.0, \, \omega = 0.24, \, \sigma = 30 \, \text{deg}$
10th Standard Configuration, Subsonic Flow Conditions
Unsteady Torsion Mode Response

\[ \alpha = 1.0, \quad \omega = 0.24 \quad \sigma = 180 \, \text{deg} \]

Standard Configuration Number 4
Turbine Cascade Composite Mesh
Standard Configuration Number 4
Steady Surface Mach Number Distribution

Standard Configuration Number 4
Unsteady Aerodynamic Response

\[ h = (0.0016, 0.0029), \, \omega = 0.24, \, \sigma = -90 \text{ Deg} \]
Standard Configuration Number 4
Unsteady Aerodynamic Response

\( h = (0.0016, 0.0029), \omega = 0.24, \sigma = 90 \text{ Deg} \)

GE Low Speed Research Compressor & Turbine Configurations

Compressor Test Rig

Turbine Test Rig
GE Low Speed Research Compressor
Steady Blade Loading

a) Design Point (DP)

b) High Loading (HL)

GE Low Speed Research Compressor
Design Point, Suction Surface
GE Low Speed Research Compressor
Design Point, Pressure Surface

GE Low Speed Research Turbine
Steady Blade Loading

45
GE Low Speed Research Turbine
Design Point, Pressure Surface

GE Low Speed Research Turbine
Design Point, Suction Surface
Transonic Flow Calculations

• Artificial viscosity added using rotated difference scheme of Jameson
• Dissipation coefficient based on local Mach number
• Modified Newton's method used to solve resulting equations

Modified Newton' Method for Transonic Flow Calculations

\[
[A(\Phi)] \{\phi\} = \{b(\Phi)\}
\]

\[
\Phi (\bar{x})^{n+1} = \Phi (\bar{x})^n + \omega\phi (\bar{x})^n
\]

Convergence Criterion

\[
|\phi (\bar{x})^n| < \varepsilon
\]
10th Standard Configuration, Transonic Flow Conditions
$M_{\infty} = 0.8$, $\Omega_{\infty} = 58$ deg.

Comparison with NPHASE & CASPOF Results
$M_{\infty} = 0.8$, $\Omega_{\infty} = 58$ deg.
Summary

• 10th standard configuration predictions show good agreement with other flow solvers
• 4th standard configuration turbine predictions show good agreement with the magnitude of measured data, however there are some problems with phase near trailing edge on suction surface
• GE low speed research compressor and turbine predictions show reasonable agreement with magnitude and phase measurements
• Transonic solution progressing, needs better model for artificial viscosity near shock, and mesh clustering capability
Coupling Among Disciplines

- Sequential Coupling
- Strip Theory
- Interpolation/Extrapolation
- Coordinate Transformations

Sequential Coupling

Geometry, Operating Conditions

Steady Aerodynamic Analysis

Aeroelastic Forced Response Analysis

Unsteady Aerodynamic Analysis

Structural Dynamic Analysis

Dynamic Stresses

*NASA Resident Research Associate at Lewis Research Center.
Equations of Motion

Mechanical/Material Damping
Forces from Damping Model

External Forcing Function from
Mechanical Excitation Model

Elastic and Inertia Forces
from Structural Model

Unsteady Aerodynamic Forces
from Aerodynamic Model

Tuned System Assumption

CONSTANT INTERBLADE PHASE ANGLE MODES - UNCOUPLE THE EQUATIONS OF MOTION

\[ u_l(t) = \sum_{r=1}^{n} u_r(t) e^{i\beta_r}, \quad \beta_r = \frac{2\pi r}{n} \]

LEADS TO

\[ M\ddot{u}_r + C\dot{u}_r + Ku_r = F_A(u_r(t),\dot{u}_r(t),t) + F_A^r(t) + F_M(t) \]

VALID FOR MOTION IN THE INTERBLADE PHASE ANGLE \( \beta_r \)
Modal Transformation

\[ q(t) = \phi^T u(t) \]

\[
\begin{align*}
M_\text{c}\ddot{q} + C_\text{c}\dot{q} + K_\text{c}q &= Q_\text{A}(q(t), \dot{q}(t), t) + Q_\text{M}(t) + \text{Q}_\text{t}(t) \\
Q_\text{A}(t) &= Q_\text{A} e^{\text{i}\omega t} \\
Q_\text{M}(t) &= Q_\text{M} e^{\text{i}\omega t} \\
q(t) &= \bar{q} e^{\text{i}\omega t}
\end{align*}
\]

\[
[ -\omega^2 I + (1 + \text{i}2\zeta_j \omega_j ) \begin{bmatrix} \omega_j^2 & \cdot \\ \cdot & -A(\omega) \end{bmatrix} ] \bar{q} = 0
\]

Discipline Communication through Databases

F. E. DATABASE

GRID POINT COORDINATES, ELEMENT CONNECTIVITIES,
NATURAL FREQUENCIES, NATURAL MODE SHAPES, NATURAL MODAL STRESSES

STEADY AERODYNAMIC DATABASE

AIRFOIL DEFINITION, STEADY MESH, STEADY POTENTIAL FIELD

UNSTEADY AERODYNAMIC DATABASE

UNSTEADY MESH
Input Interface

FINITE ELEMENT ANALYSIS
STEADY AERODYNAMIC ANALYSIS
UNSTEADY AERODYNAMIC MESH GENERATION
FREPS AEROELASTIC ANALYSIS

EXAMPLES

STRIP 1 235,236,237,238,239,240,241,242,243,244,245,246,247,248,250,
SFLUID 100,1520.0,4635.0,0.0000,5618.0,1.367
ROTOR 78 29000.0 4.790 5.290

Output Interface

INPUT ECHO
GEOMETRY ECHO
STEADY FLOW RESULTS ECHO
FINITE ELEMENT ANALYSIS RESULTS ECHO
UNSTEADY FLOW RESULTS FOR EACH STRIP

GENERALIZED FORCES
AEROELASTIC EIGENVALUES, DAMPING RATIOS, EIGENVECTORS
FORCED DISPLACEMENT AMPLITUDES
FORCED STRESS AMPLITUDES
PRINTER PLOTS
PATRAN NEUTRAL FILE

54
Current Capabilities

- Incidence, Camber and Thickness Effects
- Solid/Shell Elements
- Aerodynamic Damping
- Distorting Gust
- Vortical, Entropic, Acoustic Excitations
- Subsonic Flow
- Mistuning Sensitivity Measure
- MSC/NASTRAN Interface
- Printer Plots
- Patran Post-Processing

Features in Coming Versions

- Tighter Integration of Aerodynamic/Aeroelastic Modules
- Transonic Flow Capability
- Disk Flexibility / Cyclic Symmetry Model Support
- Other FEA Program Support
- Wake Modeling
- Viscous Effects (Inviscid/Viscid Interactions)
- Improved Mistuning Sensitivity Measure
- Graphical User Interface
OUTLINE

○ Test facility
○ Instrumentation
○ Wake measurements
○ Steady-state blade-surface-pressure measurements
○ Unsteady blade-surface-pressure measurements
○ Concluding Remarks

INSTRUMENTATION

UNSTEADY PRESSURE BLADE
HOT-WIRE MEASUREMENTS

U Component, M=0.27, Far Wake

\[
\begin{align*}
\omega &= 10 \\
i &= 2.98
\end{align*}
\]

\[
\begin{align*}
\omega &= 5 \\
i &= 3.52
\end{align*}
\]

\[
\begin{align*}
\omega &= 2.5 \\
i &= 5.17
\end{align*}
\]

Circumferential position, degrees
V Component, M=0.27, Far Wake

\[ \frac{V}{(\text{Free stream velocity})}, \% \]

- \( \omega = 10 \)
  - \( i = 2.98 \)

- \( \omega = 5 \)
  - \( i = 3.52 \)

- \( \omega = 2.5 \)
  - \( i = 5.17 \)

Circumferential position, degrees
U Component, M=0.27, Far Wake

Frequency, engine orders

$\omega = 10$

$\omega = 5$

$\omega = 2.5$
$V$ Component, $M=0.27$, Far Wake

Frequency, engine orders

$\Omega = 10$
$\beta = 2.98$

$\Omega = 5$
$\beta = 3.52$

$\Omega = 2.5$
$\beta = 5.17$
M = 0.2, Positive incidence

M = 0.27, Positive incidence
M = 0.27, Negative incidence

M = 0.27, Positive incidence, Reduced frequency 5
$M = 0.27, \omega = 10, i < 0$

Pressure Surface

Suction Surface

Unsteady pressure coefficient

Frequency, engine order

Port 1

Port 2

Port 3

Port 4

Port 5

Port 6

Port 7

Port 8

Port 9

Port 10

Port 11

Port 12
$M = 0.27, \omega = 5, i < 0$

Pressure Surface

Suction Surface

Port 1

Port 2

Port 3

Port 4

Port 5

Port 6

Port 7

Port 8

Port 9

Port 10

Port 11

Port 12

Unsteady pressure coefficient

Frequency, engine order
For Wake, \( M = 0.27, \omega = 2.5, i < 0 \)

Pressure Surface

Port 1

Port 2

Port 3

Port 4

Port 5

Port 6

Suction Surface

Port 7

Port 8

Port 9

Port 10

Port 11

Port 12

Frequency, engine order

Unsteady pressure coefficient
$C_p : \text{RMS Power, Far Wake, } M=0.27, \omega=10, i < 0$

Pressure Surface

Suction Surface

Unsteady pressure coefficient

Frequency, engine order
$C_p$: RMS Power, For Wake, $M=0.27$, $\omega = 5$, $i < 0$

Pressure Surface

Suction Surface

Frequency, engine order

Port 1

Port 2

Port 3

Port 4

Port 5

Port 6

Port 7

Port 8

Port 9

Port 10

Port 11

Port 12
$C_p : \text{RMS Power, For Wake, } M=0.27, \omega = 10, i > 0$

**Pressure Surface**

- Port 1
- Port 2
- Port 3
- Port 4
- Port 5
- Port 6

**Suction Surface**

- Port 7
- Port 8
- Port 9
- Port 10
- Port 11
- Port 12

**Unsteady pressure coefficient vs Frequency, engine order**

70
$C_p$ : RMS Power, Far Wake, $M=0.2, \omega=10, i < 0$

Pressure Surface

Suction Surface

Frequency, engine order

Unsteady pressure coefficient

Port 1

Port 2

Port 3

Port 4

Port 5

Port 6

Port 7

Port 8

Port 9

Port 10

Port 11

Port 12

0 20 40 60
CONCLUDING REMARKS

- At the higher Mach number, steady-state blade pressure distribution is to some extent dependent on the reduced frequency of the gust.

- Unsteady blade pressures are strongly dependent on reduced frequency and incidence. Mach number dependence is weaker.

- Strong pressure variation was noticed on the suction side of the forward part of the blade at lower reduced frequencies and negative incidence.

- At the higher Mach number, a high-frequency narrow-band excitation on the suction surface near the trailing edge was observed.
Forcing Function Modeling For Flow Induced Vibration
INTRODUCTION

* Blade Row-Wake Interactions - Most Common Unsteady Aerodynamic Excitation Causing High-Cycle Blade Fatigue

* Forced Response Design Systems Require Valid Wake Forcing Function Model

* Based on Steady Performance Wake Data
FORCED RESPONSE DESIGN SYSTEMS

* Wake - Unsteady Aerodynamic Forcing Function

* Harmonic Wake - Gust Response

1st Harmonic Vortical Gust

Downstream Airfoil Response
PROBLEM

* Unique Requirements of Forced Response Design Systems
  * Vortical & Potential Gust Forcing Functions

Vortical Gust

Airfoil Wake-Gust
RESEARCH OBJECTIVE

* Develop Complete Model of Unsteady Aerodynamic Forcing Functions Within Linear Theory Framework

TECHNICAL APPROACH

* Perform Series of Fundamental Experiments to Investigate Unsteady Aerodynamic Forcing Functions

  * Vortical and Potential Gusts
* Periodicity in Axial-Tangential Coordinate System $\overline{W}_2 \perp \vec{k}$
LINEAR THEORY GUST MODEL

\( \frac{1}{c_0^2} \frac{Dp}{Dt} + \rho \nabla \cdot \vec{w} = 0 \) (Continuity)

\( \frac{\rho D\vec{w}}{Dt} + \nabla p = 0 \) (Momentum)

Vortical & Potential Splitting

\( \vec{w} = \vec{w}_v + \vec{w}_p \)

Vortical Gust \( \vec{w}_v \)

\( \nabla \cdot \vec{w}_v = 0 \)

\( \frac{D\vec{w}_v}{Dt} = 0 \)

Potential Gust \( \vec{w}_p \)

\( \vec{w}_p = \nabla \phi_p \)

\( p = -\rho \frac{D\phi_p}{Dt} \)
Vortical Gust Propagation

\[ w_v = w_v^+ \exp[-ik \cdot (x - \overline{W}t)] \]

\[ k \cdot \overline{W} = 0 \]

\[ k \cdot w_v^+ = 0 \quad w_v \perp \overline{k} \]

\[ w_v^+ \parallel \overline{W} \]

\[ w_v = D\overline{W}\exp(-ik \cdot \overline{x}) \]

- 2 Constraints
  - \( \phi_w = 0^\circ \) or \( 180^\circ \)
  - \( \beta_w = \beta_2 \)

Both Satisfied

1st Violated

2nd Violated

Both Violated
Potential Gust Propagation

$$\tilde{w}_p = \nabla \phi_p$$

$$(1-M^2) \frac{\partial^2 \phi_p}{\partial x^2} + \frac{\partial^2 \phi_p}{\partial y^2} = 0$$

$$\phi_p = A \exp[-ik_\eta \eta + \chi \xi]$$

$$\chi = \frac{iM_\xi M_\eta k_\eta - \sqrt{(1-M^2)k_\eta^2}}{1-M_\xi^2}$$
Combined Vortical and Potential Gust

\[ \bar{w}_v = \sum_n D_n \bar{w} \exp[-i(k\eta_n + k \xi_n \zeta)] \]

\[ \phi_p = \sum_n A_n \exp[-i\eta_n \eta + \chi_n \xi] \]

Vortical Combined Potential

Relating Measurements to Theory

- 2 Unknowns - A & D
- Experimentally Measure 3 Quantities - u, v, p
- Solution: Weighted Least Squares Approach
System of Equations

\[
\begin{bmatrix}
\chi & \bar{W}_\xi \\
-ik_\eta & \bar{W}_\eta \\
\bar{\rho}(\bar{W}_\xi \chi + \bar{W}_\eta ik_\eta) & 0
\end{bmatrix}
\begin{bmatrix}
A \\
D
\end{bmatrix} =
\begin{bmatrix}
 u \\
v \\
p
\end{bmatrix}
\]

\[[T] \{c\} = \{b\}\]

Weighted Least Squares

\[[T]^T[W][T]{\{c\}} = [T]^T[W]{\{b\}}\]

where

\[
[W] =
\begin{bmatrix}
W_v & 0 & 0 \\
0 & W_v & 0 \\
0 & 0 & W_p
\end{bmatrix}
\]
ORATED PLATE

NACA 0024 AIRFOIL

AIRFOIL MOUNTING LOCATIONS

UNSTEADY TOTAL PRESSURE PROBE

45°

HOTWIRE PROBE

TRACING MECHANISM

ROTOM

AIRFOIL

WAKE GENERATOR

PERFORATED PLATE

ROTOR

NACA 0024 AIRFOIL

ROTOR

84
Perforated Plate Gust

\[ \bar{\beta}_1 = 24^\circ \quad \bar{\beta}_2 = 23^\circ \]

Unsteady Static Pressure

\[ N_h = 25 \]

Unsteady Velocity Vectors

First Harmonic

\[ \phi_w = 182^\circ \quad \beta_w = 23^\circ \]
Perforated Plate

$N_h = 25$

First Harmonic

**Measured Gust**
- ID*1 = 0.204608

**Vertical Component**

**Potential Component**
- IA*1 = 0.002683

**Recombined Gust**
Neutrally Loaded Airfoil
Method V

$N_h = 25$

First Harmonic

Measured Gust
$|D^*| = 0.003894$

Vortical Component

Potential Component
$|A^*| = 0.019303$

Recombined Gust
Neutrally Loaded Airfoil
Method P

First Harmonic

$N_h = 25$

Measured Gust
$\text{ID}^* l = 0.005984$

Vortical Component

Potential Component
$|A^* l| = 0.015976$

Recombined Gust
Compressor-Loaded Airfoil
Decayed Potential Field

\[ \beta_1 = 25^\circ \]
\[ \alpha_2 = 2^\circ \]
\[ \beta_2 = 24^\circ \]

\[ N_h = 25 \]

First Harmonic
\[ \phi_w = 273^\circ \]
\[ \beta_w = 93^\circ \]
Method V
Decayed Potential Field
Compressor-Loaded Airfoil
Perforated Plate
Vortical Component

![Graph showing data and method comparison for Perforated Plate vortical component.]

Compressor-Loaded Airfoil
Vortical Component

![Graph showing data and method comparison for Compressor-Loaded Airfoil vortical component.]

92
Compressor-Loaded Airfoil
Potential Component

![Compressor-Loaded Airfoil Potential Component Graph]

Turbine-Loaded Airfoil
Vortical Component

![Turbine-Loaded Airfoil Vortical Component Graph]
CONCLUSIONS

* Vortical Gust Constraints Do Not Apply to Combined Vortical/Potential Gusts

* Gust Should Not Be Defined Utilizing Only Unsteady Velocity Data
  - Forcing Functions Defined by Unsteady Velocity & Pressure

* Least Squares Velocity-Pressure Method Produces Results Which Conform to Linear Theory
Mistuning

- Manufacturing tolerances, material non-uniformities, non-identical root fixtures, and in-service degradation result in blade-to-blade differences that destroy cyclic symmetry.

- Small mistuning can cause large, catastrophic changes in blade vibrational response.

  - Amplitudes of vibration of some blades may increase by several hundred percent, producing "rogue" blades and HCF failure.

  - Free and forced responses may be highly sensitive to mistuning.

  - Tuned system predictions may be qualitatively in error and grossly underestimate blade forced response and overestimate fatigue life.

- A credible forced response prediction system for turbomachinery vibration must take mistuning into account.
An Example of Mistuning Effects on the Free Aeroelastic Response

(Mistuning causes vibration localization
→ much larger amplitudes for some blades
→ high stresses
→ blade fatigue

If unaccounted for, mistuning could cause cracks and catastrophic blade failures.

Effect of mistuning on forced response for a common blade assembly model

\[
\text{\textbf{Effect of mistuning on forced response}} \quad \text{\textbf{for a common blade assembly model}}
\]
Obstacles

- Mistuned assembly analyses are very expensive. Parametric studies cannot be performed
  → need for accurate reduced-order models

- Mistuning is random by nature
  - mistuning pattern (and sometimes mistuning strength) is typically not available
  - mistuning differs from rotor to rotor
  - mistuning that results from in-service degradation cannot be modeled deterministically
  → calls for statistical and parametric tools

- Studies of mistuning by Afolabi, Bendiksen, Ewins, Griffin, Kaza, Kielb, Pierre, Sinha, Srinivasan, Mignolet, etc., have led to general conclusions:
  - helps flutter
  - increases forced response amplitudes

- However — quantitatively and even qualitatively different findings regarding other issues
  - blade with largest amplitude
  - forced response amplitude increase over tuned system

- A new perspective of the mistuning problem (Bendiksen, Pierre):
  - Mistuning belongs to the broader topic of repetitive structures with periodicity-breaking irregularities
  - identification of the basic physical mechanism governing mistuning effects: sensitivity of aeroelastic eigensolution to mistuning is inverse proportional to the distance between the eigenvalues
    \[
    \delta^2 \lambda_j \propto \frac{1}{\lambda_{ej} - \lambda_{ek}}
    \]
- The closeness of eigenvalues is governed by the *interblade coupling* and number of blades.
  - Weakly coupled assemblies are highly sensitive to mistuning (interblade coupling depends on frequency).
  - Assemblies with many blades are more sensitive.
  - Mistuning effects increase with frequency (tip modes).

- Highly sensitive mistuned assemblies feature localized responses.

- **Objectives**

  Provide the designer with tools for predicting the forced response amplitudes of real (i.e., mistuned) bladed disks. Incorporate a mistuning analysis capability into forced response prediction system (FREPS).

  - Develop low-dimensional reduced-order models.
  - Evaluate the significance of mistuning effects in terms of system parameters. Identify key parameters governing sensitivity to mistuning.
  - Predict the sensitivity of the system dynamics to blade mistuning.
  - Determine true response amplitudes for typical mistuned bladed disks.
  - Obtain confidence intervals for amplitudes and stresses and estimates of fatigue life.
NASA research program thrusts

- Aeroelastic characteristics of mistuned assemblies: mode localization and root locus scattering
- Stochastic measures of sensitivity to mistuning
  - transfer matrix based
  - eigenvalue perturbation based
  - localization factors
  - composite sensitivity measure for structurally and aerodynamically coupled rotors
- Dynamics of mistuned assemblies with several component modes per blade. Effect of close blade modes on tuned and mistuned system dynamics.
- Design for low sensitivity to mistuning: formulation of an optimization constraint.
- Forced response of mistuned assemblies:
  - physical mechanisms governing mistuning effects
  - efficient statistical computational methods
- Mistuned bladed disk formulation via component mode analysis and validation of simple models
\[ \epsilon = 0 \]

\[ \epsilon = 4.76\% \]

Amplitude, [\mu m]

Blade number, \( i \)

Re \( \lambda \)

Im \( \lambda \)

Tuned, \( \epsilon = 0 \)

Mistuned, \( \epsilon = 4.76\% \)
Practical Significance of the Localization Factor

\[ S_{mid} = 25 \]
\[ \gamma = 0.2 \]
90% amplitude decay by the 11th blade

For \( \gamma = 0.1 \), amplitude decays by a factor \( e^{-0.1} \approx 0.9 \) from one bay to the next (on average).

56% of the energy is transmitted to the 3rd bay.

For \( \gamma = 1.0 \), average energy transmitted to next bay is 13.5% and less than 0.25% of the energy reaches the 3rd bay.

\( \gamma \) is an average quantity and specific realizations of mistuned systems may exhibit different behavior.

\( \gamma \) can be calculated in terms of a universal sensitivity measure for simple models.

Use in design:
Maximum allowable localization strength \( \gamma \approx S \) corresponding permitted regions in parameter space.
Forced Response of Mistuned Assemblies

\[ u = \frac{\text{Maximum blade amplitude in mistuned system}}{\text{Blade amplitude in tuned system}} \]
Closing

- Because of its potentially catastrophic effects such as single blade failure, mistuning must be accounted for in the design and analysis of blade assemblies.
- Simple and effective mistuning capability must be incorporated into FREPS.
- Underlying physical mechanisms must be understood to generate proper reduced-order models.

Future work:

- Forced response: develop physical understanding and associated efficient computational techniques.
- Mistuning experiment: corroborate occurrence of localization and high sensitivity in nonrotating/rotating conditions.
- Beneficial mistuning patterns: practical only if mistuning can be controlled.
RESEARCH OBJECTIVES

- BETTER UNDERSTANDING OF MISTUNED BLADED DISK ASSEMBLY RESPONSE CHARACTERISTICS

- RELATIONSHIPS BETWEEN MISTUNE PATTERN, EXCITATION MODE, AND RESPONSE AMPLITUDE

- OPTIMIZATION OF MISTUNING PATTERNS SUCH THAT FATIGUE DAMAGE IS MINIMIZED
BLADE/DISK MODEL

Blade '0' connects to blade 'N' for cyclic periodicity

AERODYNAMIC MODEL
Mistune Modes

\[ \gamma_j^2 = \delta_0 + \sum_{i=0}^{\left\lceil \frac{N}{2} \right\rceil - 1} \delta_i \cos(\beta_{ij}) + \frac{1}{\sqrt{2}} \delta_{N/2} \cos(\beta_{N/2} j) + \sum_{i=\frac{N}{2}+1}^{N-1} \delta_i \sin(\beta_{i-N/2} j) \]

Mean = \delta_0

Variance = \[ s^2 = \frac{1}{2} \sum_{i=1}^{N-1} \delta_i^2 \]
GOVERNING EQUATIONS

\[
\begin{bmatrix}
B_{00} & B_{01} & 0 \\
B_{10} & B_{11} & B_{12} \\
0 & B_{21} & B_{22}
\end{bmatrix}
\begin{bmatrix}
y_a^* \\
h_a^* \\
\alpha_a
\end{bmatrix}
=
\begin{bmatrix}
0 \\
R_i \\
R_2
\end{bmatrix}
\]

\( y_a \) is the vector of disk element modal amplitudes
\( h_a \) is the vector of blade translational modal amplitudes
\( \alpha_a \) is the vector of blade torsional modal amplitudes

\[
B_{00} = \left[ -(\gamma^2 - \gamma_0^2)I + \frac{\mu_1}{\gamma_h^2} \gamma_h^2[I + [C_h]] + 2\gamma_h^2[I - \cos \beta_{1,j}] \right]
\]

\[
B_{01} = \frac{\mu_1}{\gamma_h^2} \gamma_h^2[I + [C_h]]
\]

\[
B_{10} = -\mu_1 \gamma_h^2[I + [C_h]]
\]

\[
B_{11} = \left[ -\mu_1 \gamma^2 I + \mu_1 \gamma_h^2[I + [C_h]] + \gamma^2[C_h] \right]
\]

\[
B_{12} = \gamma^2[C_h]
\]

\[
B_{21} = -\gamma^2[m_h]
\]

\[
B_{22} = \left[ -\mu_1 r_\alpha \gamma^2 I + \mu_1 r_\alpha \gamma^2[C_h] - \gamma^2[m_\alpha] \right]
\]

110
STRUCTURE OF THE CIRCULANT MISTUNE MATRIX

\[
[C_a] = \frac{1}{\delta_0} \text{circ} \left[ 0 \quad \frac{1}{2} \left( \delta_1 - i\delta_{\frac{N}{2}} \right) \quad \frac{1}{2} \left( \delta_2 - i\delta_{\frac{N}{2}} \right) \quad \cdots \quad \frac{1}{2} \left( \delta_2 + i\delta_{\frac{N}{2}} \right) \quad \frac{1}{2} \left( \delta_1 + i\delta_{\frac{N}{2}} \right) \right]
\]

SMALL MISTUNE PARAMETER

\[
[C_b] = \epsilon [\tilde{C}_b]
\]

\[
[C_a] = \epsilon [\tilde{C}_a]
\]

\[
B_{00} = \hat{B}_{00} + \epsilon \tilde{B}_{00}
\]

\[
B_{01} = \hat{B}_{01} + \epsilon \tilde{B}_{01}
\]

\[ \text{etc.} \]

\[\hat{B}_{ij} \text{ are diagonal matrices} \]

\[\tilde{B}_{ij} \text{ are circulant matrices} \]
RESPONSE PERTURBATION

\[ y_a^* = y_{a0}^* + \varepsilon y_{a1}^* + \varepsilon^2 y_{a2}^* + \cdots \]

\[ h_a^* = h_{a0}^* + \varepsilon h_{a1}^* + \varepsilon^2 h_{a2}^* + \cdots \]

\[ \alpha_a = \alpha_{a0} + \varepsilon \alpha_{a1} + \varepsilon^2 \alpha_{a2} + \cdots \]

PERTURBATION EQUATIONS

\[
\begin{bmatrix}
\dot{B}
\end{bmatrix}
\begin{bmatrix}
y_{a0}^* \\
h_{a0}^* \\
\alpha_{a0}
\end{bmatrix}
= \begin{bmatrix} 0 \\ R_1 \\ R_2 \end{bmatrix} + \varepsilon \begin{bmatrix}
\dot{B}
\end{bmatrix}
\begin{bmatrix}
y_{a1}^* \\
h_{a1}^* \\
\alpha_{a1}
\end{bmatrix}
+ \varepsilon^2 \begin{bmatrix}
\dot{B}
\end{bmatrix}
\begin{bmatrix}
y_{a2}^* \\
h_{a2}^* \\
\alpha_{a2}
\end{bmatrix}
+ \cdots
\begin{bmatrix}
\dot{B}
\end{bmatrix}
\begin{bmatrix}
y_{a1}^* \\
h_{a1}^* \\
\alpha_{a1}
\end{bmatrix}
= \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}
\]

\[
\begin{bmatrix}
\dot{B}
\end{bmatrix} =
\begin{bmatrix}
\hat{B}_{00} & \hat{B}_{01} & 0 \\
\hat{B}_{10} & \hat{B}_{11} & \hat{B}_{12} \\
0 & \hat{B}_{21} & \hat{B}_{22}
\end{bmatrix}
\]

\[
\begin{bmatrix}
\dot{B}
\end{bmatrix} =
\begin{bmatrix}
\hat{\bar{B}}_{00} & \hat{\bar{B}}_{01} & 0 \\
\hat{\bar{B}}_{10} & \hat{\bar{B}}_{11} & 0 \\
0 & 0 & \hat{\bar{B}}_{22}
\end{bmatrix}
\]
FIRST ORDER EXCITATION MODES

FOR EXCITATION MODE \( p \)
and
SINGLE MISTUNE MODE \( r \)

\[
p_1 = (p-r) \mod N
\]
\[
p_2 = (p+r) \mod N
\]

POSSIBLE RESPONSE MODES

FOR MISTUNE MODE \( S \) WHICH DIVIDES \( N \)

\[
(p + j N/S) \mod N; \quad j = 0, 1, 2, ..., S-1
\]
MODE 11 MISTUNE OFF RESONANCE

MODE 11 MISTUNE NEAR RESONANCE
DESIGN PROBLEM

Given a set of (mistuned) blades, find an arrangement (blade - slot pairing) such that the forced response amplitudes are minimized.

Note: For an assembly with N blades, there are (N-1)! possible arrangements.

TWO PHASE SYSTEM

• Phase One
  Continuous optimization to find Mistune Mode Parameters with a constraint on mistune strength

• Phase Two
  Find the arrangement that best matches the pattern described by mistune mode parameters
CONSTRAINTS

FOR A GIVEN SET OF BLADES, THE MEAN AND THE STANDARD DEVIATION ARE FIXED

• FIXED MEAN --> DETERMINES $\delta_0$

• FIXED STANDARD DEVIATION --> DETERMINES $\sqrt{\sum_{r=1}^{N-1} \delta_r^2}$

Optimal mistune mode parameters for $\gamma = 0.99$
Objective function values for single mode mistune cases for $\gamma = 0.99$

Optimal blade natural frequencies for $\gamma = 0.99$
BLADE-SLOT ASSIGNMENT

- Find \( \{ \delta \} \) such that \( \| \delta - \delta' \| \) is minimized

- Classic Linear Sum Assignment Problem

Minimize \( \sum_{j=0}^{N-1} \sum_{k=0}^{N-1} a_{jk} c_k \) subject to: \( \sum_{j=0}^{N-1} a_{jk} = 1 \) and \( \sum_{k=0}^{N-1} a_{jk} = 1 \)

\[ c_k = (\gamma_{ij} - \gamma_{ik}^2)^2 \]

Where:
\[ a_{jk} = \begin{cases} 1 & \text{if blade } j \text{ is at slot } k \\ 0 & \text{otherwise} \end{cases} \]

![Graph showing blade natural frequency vs. slot number]

Best blade arrangement for optimal mistune pattern
Best blade arrangement for mode 1 mistune pattern

Best blade arrangement for alternate mistune pattern
Response amplitudes of best fit to mistune patterns

LOCALIZATION

- Localized physical blade amplitudes can imply non-localized modal amplitudes
- Localized modal amplitudes implies non-localized physical blade amplitudes
FREPS CODE DEMONSTRATION

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and

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FREPS Analysis Options

Flutter Analysis

Forced Response Analysis

- Mechanical Loading
- Gust Loading

*NASA Resident Research Associate at Lewis Research Center.
Turbine Blade Aeroelastic Strip Definitions

Strip Definition
- Aerodynamic Properties
- Fluid and Thermodynamic Properties
- Database for SFLOW and LINFLO Results

Description of Rotor

Define Aerodynamic Matrix and Structural Damping

Type of Analysis and Corresponding Parameters
- Flutter
  Flutter Frequency, Range and Iteration Parameters
- Forced Response
  Finite Element Modal Stresses; Aerodynamic/Mechanical Excitation

Output Request
- Steady and Unsteady Results
- Line Plots
- Raw Data
- PATRAN Neutral File

Turbine Blade Aeroelastic Strip Definitions

Strip Number
1
2
3
4
5
6

Radius (in.)
5.17
5.08
5.00
4.93
4.87
4.83
FREPS Sample Input

1 .......................................................... 80

TITLE TURBINE BLADE -- FORCED RESPONSE ANALYSIS

$ $ STRIP DEFINITION
$$
$ nodal points defining locus

STRIP 1 3 27 75 159 243 327 411
STRIP 1 495 579 663 747 831 915 999
STRIP 1 1083 1206 1321 1394 1397 1401 1405
STRIP 1 1409 1413 1417 1421 1424 1427 1428
STRIP 1 1426 1422 1419 1414 1410 1406 1402
STRIP 1 1398 1318 1203 1180 996 912 828
STRIP 1 744 660 576 492 408 324 240
STRIP 1 156 72 26 3

$ $ FLUID AND AERO DESCRIPTION FOR STRIPS
$$
$ fluidID T°R) p(psi) \rho(lbm/in^3) a(fps) \gamma

SFLUID 100 518.69, 14.69, 0.0000, 1116.4, 1.4

$ $ stripID \theta

SROTATE 1 0.0

$ $ aeroID W \beta M H

SAERO 100, 0.0, 114.0, 0.3800, 0.225

$ $ groupID stripID aeroID fluidID

SGROUP 100 1 100 100

$ $ STRIP DATABASE DEFINITIONS
$$
$ stripID filename

DATABASE 1 airfoil_1
FREPS Sample Input

$ ROTOR DESCRIPTION
$ NofBLADES Ω(rpm) hubRADIUS tipRADIUS
ROTOR 50 6000.0 4.79 5.29
$

$ AEROELASTIC SET DEFINITIONS
$ groupIDs
AESET 100
$

$ INTERBLADE PHASE ANGLE
$ n, σ
AESIGMA 1 72.0
$

$ GUST LOADS
$ stripID V_g
GUST 1 25.0
$

$ FREQUENCY RANGE
$ f_{low} f_{high} Δf
FREQUENCY 9700.0 10100.0 20.00

FREPS Sample Input

$ NODAL DISPLACEMENTS
$ nodes
NODEOUT 7851 7860
$

$ ELEMENT STRESSES
$ elements
ELEMOUT 2886 2917 2892
$

$ MSC/NASTRAN PUNCH FILE OF STRESSES
$ iounit filename
STRFILE 1 hp104.data
$

$ FORCED RESPONSE ANALYSES REQUESTED
$ RESPONSE
$ END

125
Space Shuttle Main Engine (SSME) Blade

10,014 Nodal Points
7758 3-D Elements
Root Locus of the Second Blade Mode (Edgewise)

0% Modal Damping for the SSME HPOTP Blade

Real Part of Eigenvalue

-0.002 -0.001 0 0.001 0.002 0.003 0.004

Imaginary Part of Eigenvalue

-0.002 -0.001 0 0.001 0.002

Stable

Unstable

Numbers Denote Interblade Phase Angles

Root Locus of the Second Blade Mode (Edgewise)

1% Modal Damping for the SSME HPOTP Blade

Real Part of Eigenvalue

-0.012 -0.011 -0.01 -0.009 -0.008 -0.007 -0.006

Imaginary Part of Eigenvalue

-0.002 -0.001 0 0.001 0.002

Stable

Numbers Denote Interblade Phase Angles

127
Forced Dynamic Stresses Due to an Assumed Gust Load
SSME HPOTP Blade

Forced Dynamic Stresses Due to Cooling Jet Excitation
SSME HPOTP Blade

Max Principal Stress (ksi)

Frequency (Hz)
NASA Lewis Research Center Workshop on Forced Response in Turbomachinery

G.L. Stekof, D.V. Murthy, M. Morel, D. Hoyniak, and J.W. Gauntner, compilers

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Subject Category 07

Forced response; Turbomachinery; Blades; Computational fluid dynamics

This document serves as a summary of the NASA Lewis Research Center (LeRC) Workshop on Forced Response in Turbomachinery in August of 1993. The workshop was sponsored by the following NASA organizations: Structures, Space Propulsion Technology, and Propulsion Systems Divisions of NASA LeRC and the Aeronautics and Advanced Concepts & Technology Offices of NASA Headquarters. In addition, the workshop was held in conjunction with the GUIDE (Government/Industry/Universities) Consortium on Forced Response. The workshop was specifically designed to receive suggestions and comments from industry on current research at NASA LeRC in the area of forced vibratory response of turbomachinery blades which includes both computational and experimental approaches. There were eight presentations and a code demonstration. Major areas of research included aerelastic response, steady and unsteady fluid dynamics, mistuning, and corresponding experimental work.