UNSTEADY AERODYNAMIC ANALYSES FOR TURBOMACHINERY
AEROELASTIC PREDICTIONS
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UNSTEADY AERODYNAMIC ANALYSES

- Applications
  - Aeroelastic: blade flutter and forced vibration
  - Aeroacoustic: noise generation
  - Vibration and noise control
  - Effects of unsteadiness on performance

- Requirements
  - Accuracy/efficiency
    * Realistic operating conditions
    * Arbitrary modes of unsteady excitation

- Approaches
  - Numerical simulation/analytical modeling

ASSUMPTIONS

- Turbulence and transition can be modeled
  \( \Rightarrow \) Reynolds averaged, Navier-Stokes equations

- High Reynolds number, “attached” flow
  \( \Rightarrow \) Thin-layer Navier-Stokes equations, or
  Inviscid/viscid interaction analyses

- Small-amplitude unsteady excitations
  \( \Rightarrow \) Nonlinear steady + linearized unsteady analyses

- \( Re \rightarrow \infty \Rightarrow \) inviscid flow
  - Potential steady background flow \( \Rightarrow \) LINFLO
  - Uniform steady background flow \( \Rightarrow \) CLT
CONTRACT NAS3-25425

NASA Program Managers: J. Gauntner, G. Stefko

- Linearized inviscid unsteady aerodynamic analysis: LINFLO
- Unsteady viscous layer analysis: UNSVIS
- Steady, inviscid/viscid interaction analysis: SFLOW-IVI
- Coupled SFLOW-IVI/LINFLO analysis

EXAMPLE CONFIGURATION

\[ \begin{align*}
\Delta \Omega_{-\infty} & \quad m = 0 \\
\Delta \Omega_{+\infty} & \quad m = 1 \\
\Delta \Omega_{+\infty} & \quad m = 2
\end{align*} \]

\[ \begin{align*}
W_1 & \quad m = 2 \\
\Omega_{+\infty} & \quad m = 1 \\
\Omega_{-\infty} & \quad m = 0
\end{align*} \]
UNSTEADY EXCITATIONS

\[ \ddot{s}_{-\infty}(x, t) \]

\[ \ddot{p}_{l, -\infty}(x, t) \]

\[ \ddot{\zeta}_{-\infty}(x, t) \]

- Far-field conditions (uniform mean flow)

\[ \dot{s}(x, t) = \text{Re}\{s_{-\infty}\exp[i(\kappa_{-\infty} \cdot x + \omega t)]\}, \quad \xi < \xi_- \]

\[ \dot{\zeta}(x, t) = \text{Re}\{\zeta_{-\infty}\exp[i(\kappa_{-\infty} \cdot x + \omega t)]\}, \quad \xi < \xi_- \]

\[ \dot{p}_{l, -\infty}(x, t) = \text{Re}\{p_{l, -\infty}\exp[-\beta_{-\infty}\xi + i(\kappa_{-\infty} \cdot x + \omega t)]\}, \quad \xi < \xi_+ \]

LINEARIZED INVISCID ANALYSES

- Linearization

\[ \ddot{P}(x, t) = P(x) + \text{Re}\{p(x)\exp(i\omega t)\} + \ldots \]

- Nonlinear BVP for steady background flow

- Linear variable-coefficient problem for each Fourier component of first-order unsteady flow
  - Time independent
  - Surface conditions imposed at mean surfaces
  - Analytic far-field solutions for \( s, \zeta, \) and \( p \)
  - Single extended blade-passage solution domain

\[ \ddot{P}(x + mG, t) = P(x) + \text{Re}\{p(x)\exp[i(\omega t + m\sigma)]\} + \ldots \]

- Prescribed quantities:
  \( \omega, \sigma, r_B, s_{-\infty}, \zeta_{-\infty}, \) and \( p_{l, -\infty} \)
LINFLO

- Unsteady perturbation of a potential mean flow
- Steady flow: \( \nabla \cdot (\hat{\rho} \nabla \Phi) = 0 \)
- Unsteady velocity decomposition: \( v = \nabla(\phi + \phi_\star) + v_R \)
  - \( p = -\hat{\rho} \frac{\partial \phi}{\partial t} \)
  - \( \nabla \cdot v_R = 0 \) far upstream
  - \( \hat{D}\phi_\star/\partial t = 0; (\nabla \phi_\star + v_R) \cdot n = 0 \) on \( B_m \) & \( W_m \)

- Entropy & rotational velocity: \( X = \Delta \eta + \Psi e_n \rightarrow x \) as \( \xi \to -\infty \)
  \[ s(x) = s_{-\infty} \exp(i\kappa_{-\infty} \cdot X) \]
  \[ v_R(x) = [\nabla(X \cdot A_{-\infty} + s_{-\infty} \nabla \Phi/2) \times \exp(i\kappa_{-\infty} \cdot X)] \]

- Unsteady velocity potential
  \[ \hat{D}(A^{-2} \hat{D}\phi/\partial t)/\partial t - \hat{\rho}^{-1} \nabla \cdot (\hat{\rho} \nabla \phi) = \hat{\rho}^{-1} \nabla \cdot [\hat{\rho} \nabla \phi_\star] \]
  where \( \phi_\star = F(A_{-\infty}, \Psi) \exp(i\kappa_{-\infty} \cdot X) \)

- Surface conditions:
  - Blades: \( \nabla \phi \cdot n = f(r_b) \)
  - Wakes: \( [\hat{D}\phi/\partial t] = 0 \) and \( [\nabla \phi] \cdot n = 0 \)
  - Shocks: \( [\hat{\rho} \nabla \phi + \hat{\rho} \nabla \Phi] \cdot n = f(r_{sh} \cdot n, \nabla \Phi); r_{sh} \cdot n = -[\phi]/[\Phi_\star] \)

- Far field conditions:
  - \( \phi_{1,\infty} \) prescribed; \( \phi_{R,\infty} \) must be determined
  - Analytic far-field solutions for \( \phi = \phi_1 + \phi_R \)
NUMERICAL SOLUTION DOMAIN

- Extended blade-passage region of finite extent in axial-flow direction

NUMERICAL APPROXIMATION

- Implicit, least-squares, finite-difference model

\[
(L\phi)_o \approx (L\phi)_o = q^o \phi_o + \sum_{m=1}^{m} \beta_m (\phi_m - \phi_o)
\]

- Transonic differencing strategies
- Cascade, local and composite mesh solutions
- Direct solution procedure
  - Block tridiagonal system of algebraic equations for subsonic flow
  - Block pentadiagonal system for transonic flow with fitted shocks
AERODYNAMIC RESPONSE AT A BLADE SURFACE

- Surface pressure (transonic flow):
  \[ \tilde{P}(\tau_B, t) = P(\tau_B) + \text{Re}\{p_B(\tau_B) \exp(i\omega t)\} + \sum_n \tilde{P}_{Sn}(\tau_B, t) + \ldots \]

- Blade motion: \( r_B(x) = \sum_{i=1}^I \delta_i R_i(x) \)

- Unsteady airloads:
  \[ q_i = \tilde{Q} q_{ir} d\tau = -\tilde{Q} \left( P \frac{\partial r_B}{\partial \tau} \cdot e + p_B n - \sum_n r_{Sn} [P] n \right) R_i d\tau \]

- Work per cycle/pressure-displacement function
  \[ W_C = \tilde{Q} \frac{dW}{dt} d\tau = \tilde{Q} \frac{w(\tau)}{d\tau} d\tau = \pi \tilde{Q} \text{Im}\{\epsilon'_i q_i\} d\tau = \pi \text{Im}\{\sum_{i=1}^I \epsilon'_i q_i\} \]

EXAMPLE RESPONSE PREDICTIONS

- Compressor exit guide vane (EGV): \( \Theta = 15\,\text{deg}, \ G = 0.6 \)
  - Thick, highly-cambered NACA 0012 airfoils
  - Subsonic flow: \( M_{\infty} = 0.3, \ \Omega_{\infty} = 40\,\text{deg} \)
  - Vortical excitation: \( \omega = 10, \ \sigma = -2\pi \)
  - Acoustic excitation from downstream:
    \( \omega = 10, \ \sigma = 0 \)

- High speed compressor cascade: \( \Theta = 45\,\text{deg}, \ G = 1 \)
  - Cambered NACA 0006 airfoils
  - Subsonic flow: \( M_{\infty} = 0.7, \ \Omega_{\infty} = 58\,\text{deg} \)
  - Transonic flow: \( M_{\infty} = 0.8, \ \Omega_{\infty} = 55\,\text{deg} \)
  - SDOF blade motions: \( \delta_i = (1, 0), \ \omega = 1 \)

- Linear/nonlinear result comparisons
  - NGUST analysis (Navier-Stokes)
  - NPHASE analysis (Euler)
COMPRESSOR EXIT GUIDE VANE

NGUST Computational Grid

Steady surface pressure coefficient

---

CASPOF (Full potential)
--- NGUST (Euler)
--- NGUST (Navier-Stokes)
VORTICITY WAVE IN AN EGV CASCADE

Unsteady vorticity, \( \vec{v}_{R,-\infty} = (0.05\vec{q}, 0) \), \( \sigma = -2\pi \), \( \omega = 10.0 \)

Unsteady pressure, \( \vec{u}_{R,-\infty} = (0.05\vec{q}, 0) \), \( \sigma = -2\pi \), \( \omega = 10.0 \)
VORTICITY WAVE IN AN EGV CASCADE

FIRST HARMONIC UNSTEADY PRESSURE DIFFERENCE
$\vec{v}_{R,-\infty} = (0.05\bar{q}, 0)$, $\sigma = -2\pi$, $\omega = 10.0$

![Graph showing real and imaginary components of pressure difference for various cases.](image)
COMPRESSOR EXIT GUIDE VANE

Unsteady Pressure Response

\[ p_{+\infty} = (0.04, 0), \ \omega = 10.0, \ \sigma = 0.0 \]

Linearized Inviscid (LINFLO)  Navier-Stokes (NGUST)

EXIT ACOUSTIC WAVE IN AN EGV CASCADE

FIRST HARMONIC UNSTEADY PRESSURE DIFFERENCE

\[ p_{I,+\infty} = (0.04P, 0), \ \sigma = 0, \ \omega = 10.0 \]

REAL  IMAGINARY
EXIT ACOUSTIC WAVE IN AN EGV CASCADE

FIRST HARMONIC UNSTEADY PRESSURE DIFFERENCE
\[ p_{I,+\infty} = (0.04P, 0), \quad \sigma = 0, \quad \omega = 10.0 \]

- \[ p_{I,+\infty} = 0.04P \]
- \[ p_{I,+\infty} = 0.12P \]
- \[ p_{I,+\infty} = 0.20P \]

NACA 0006 CASCADE
NPHASE Computational Grid
HIGH SPEED COMPRESSOR CASCADE

Surface Mach Number Distributions
— — Potential, — — — Euler

--- Subsonic flow

Subsonic flow

<table>
<thead>
<tr>
<th>M</th>
<th>0.0</th>
<th>0.2</th>
<th>0.4</th>
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HIGH SPEED COMPRESSOR CASCADE

Pressure Displacement Function Distributions for Torsional Blade Vibrations at $\alpha = 2 \, \text{deg}, \omega = 1$

— — Linearized Analysis (LINFLO)
--- Nonlinear Euler Analysis (NPHASE)

--- Subsonic Flow

Subsonic Flow

<table>
<thead>
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<th>$x$</th>
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HIGH SPEED COMPRESSOR CASCADE

Work per Cycle versus Interblade Phase Angle for Torsional Blade Vibrations at $\alpha = 2 \text{ deg}$, $\omega = 1$

- Linearized Analysis (LINFLO)
- Nonlinear Euler Analysis (NPHASE)

Subsonic Flow

Transonic Flow

INVISCID/VISCID INTERACTION ANALYSES

- High Reynolds Number Flow
- Inviscid region: Euler or potential flow equations
  - Surface conditions modified to account for viscous displacement effects
- Viscous region: Prandtl's equations
  - Direct solution: $P \rightarrow \delta$
  - Inverse solution: $\delta \rightarrow P$
- Inviscid viscous interaction law
  - Weak interaction $\Rightarrow$ sequential solution, pressure determined by inviscid flow
  - Strong interaction $\Rightarrow$ simultaneous solution, pressure determined by inviscid and viscous flows
CASCADE FLOW WITH LOCAL REGIONS OF STRONG INTERACTION

SFLOW-IVI: INVISCID REGION

- Field equation
  \[ \hat{\rho} \nabla \Phi = 0 \text{ or } A^2 \nabla^2 \Phi = \nabla \Phi \cdot \nabla(\nabla \Phi)^2/2 \]

- Surface b.c.'s account for viscous displacement effects; i.e.,
  - Blades: \( \nabla \Phi \cdot \hat{n} \big|_{S} = \rho^{-1} d(\rho_u \delta)/ds \)
  - Wakes: \( [\nabla \Phi] \cdot \hat{n} \big|_{W} = \rho^{-1} d(\rho_w \delta_{w})/ds \)

- Inlet flow conditions prescribed
- Exit flow conditions determined by Kutta cond. & global mass conservation
SFLOW-IVI: VISCOSOUS REGION

- Classical Viscous-Layer Eqs. (Boundary layers & Wakes)
  - Weak interaction:
    specify \( du_e/ds \rightarrow \text{calc. } \delta^* \) (direct)
  - Strong interaction:
    specify \( \bar{m} = \rho u_e \delta^* \rightarrow \text{calc. } u_e \) (inverse)

- Turbulence and transition
  - Algebraic eddy-viscosity model
    * Blade: Cebeci-Smith w/separation modification
    * Wake: Chang, et al
  - Instantaneous transition

- Solutions in terms of Levy Lees variables

SFLOW-IVI: COUPLING PROCEDURE

\[
\begin{align*}
  n &= n + 1 \\
  \bar{m}^n & \rightarrow \text{Inviscid Solver} \rightarrow u_{eI} \\
  \bar{m}^n & \rightarrow \text{Viscous Solver} \rightarrow u_{eV} \\
  \text{Yes} & \quad \max_i |u_{eV_i} - u_{eI_i}|/u_{eI_i} < \varepsilon \\
  \text{Stop} & \\
  \overline{m}^{n+1} &= \overline{m}^n [1 + \omega (u_{eV}/u_{eI} - 1)] \\
  \text{No} \\
\end{align*}
\]
NUMERICAL RESULTS

- Two Cascade Configurations
  - Compressor exit guide vane (EGV)
  - High-speed compressor cascade
- Effect of Varying Re
- Comparison with Navier-Stokes solutions
- Incidence Angle Study (EGV)

COMPRESSOR EXIT GUIDE VANE

Effect of Varying Re: --- Re = 10^5
- - - - Re = 10^6
------ Inviscid
COMPRESSOR EXIT GUIDE VANE

Comparison with Navier-Stokes Solution: \( Re = 10^6 \)

--- IVI
- - - N-S

\[ C_p \]

\( \tau_w \)

\( \delta \)

\( \rho \)

\( x \)

\( z \)
COMPRESSOR EXIT GUIDE VANE

Loss Parameter, $\bar{\omega}$, & separation point location, $x_{sep}$, versus Inlet Flow Angle:

$Re = 10^5$, $M_{-\infty} = 0.3$

\begin{itemize}
  \item $Re = 10^5$
  \item $M_{-\infty} = 0.3$
\end{itemize}

COMPRESSOR EXIT GUIDE VANE

Streamlines in Trailing-Edge Region: $Re = 10^6$

\begin{itemize}
  \item $\Omega_{-\infty} = 36^\circ$
  \item $\Omega_{-\infty} = 45^\circ$
  \item $\Omega_{-\infty} = 54^\circ$
\end{itemize}
HIGH SPEED COMPRESSOR CASCADE

Comparison with Navier-Stokes Solution: $Re = 10^6$

--- IVI
--- --- N-S

--- IVI
--- --- N-S

--- IVI
--- --- N-S
GOAL: UNSTEADY IVI ANALYSIS FOR AEROELASTIC APPLICATIONS

- High Re unsteady cascade flows
- Inviscid region
  - Nonlinear steady (SFLOW) ⇒ \( \Phi \)
  - Linearized unsteady (LINFLO) ⇒ \( s, v_R, \phi \)
    note: \( \tilde{V} = \nabla \Phi + Re\{[\nabla(\phi + \phi') + v_R] \exp(i\omega t)\} \)
  - Surface conditions
    * Blades: \( (\tilde{V} - \tilde{R}) \cdot n = f_B(\tilde{\delta}) \)
    * Wakes: \( [\tilde{V}] \cdot n = f_W(\tilde{\delta}) \)
- Viscous region
  - Unsteady viscous layer analysis UNSVIS
  - UNSVIS is a direct, time marching solution procedure
- Inviscid/viscid coupling
  - Procedure must be developed for unsteady flows
- Issues
  - Must modify UNSVIS to deal with moving blades
  - Matching of inviscid and viscid solutions for \( s \) and \( v_R \) excitations
  - Need inverse unsteady viscous layer calculation
  - Inviscid/viscid coupling ⇒ long computer run times; unless
    * \( \tilde{\delta} \approx \delta + \delta \exp(i\omega t) \), i.e., linearization, or
    * Integral boundary layer calculation
INTERMEDIATE STEP: COUPLED SFLOW-IVI/LINFLO

• Effects of strong steady interactions on unsteady pressure response

• Assumptions
  - $\tilde{\delta}(x, t) = \delta(x) + \tilde{\delta}(x, t)$
  - Strong steady inviscid/viscid interaction
  - Weak unsteady interaction

• SFLOW-IVI will provide steady background flow information for LINFLO calculation
  - Unsteady surface pressure determined by linearized inviscid calculation
  - Unsteady viscous layer determined by direct solution procedure
Figure 1: LINFLO results for EGV cascade undergoing torsional vibration ($\alpha = (1,0)$, $\sigma = 0$ deg, $\omega = 1$); $\Omega_{-\infty} = 40$ deg, $M_{-\infty} = 0.30$: (---) inviscid; (----) viscous, $Re = 10^6$.

Figure 2: LINFLO results for EGV cascade undergoing torsional vibration ($\alpha = (1,0)$, $\sigma = 0$ deg, $\omega = 1$); $\Omega_{-\infty} = 54$ deg, $M_{-\infty} = 0.30$: (---) inviscid; (----) viscous, $Re = 10^6$.
Figure 1: LINFLO results for HSC cascade undergoing torsional vibration
[α = (1,0), σ = 0 deg, ω = 1]; Ω_∞ = 55 deg, M_∞ = 0.70: (---) inviscid; (- - - - -) viscous, Re = 10^6.

Figure 2: LINFLO results for HSC cascade undergoing torsional vibration
[α = (1,0), σ = 180 deg, ω = 1]; Ω_∞ = 55 deg, M_∞ = 0.70: (---) inviscid; (- - - - -) viscous, Re = 10^6.
CONCLUDING REMARKS

• Linearized unsteady aerodynamic analysis: LINFLO
  - Realistic 2D flow configurations
  - Arbitrary modes and frequencies of excitation
  - Efficient prediction of unsteady pressure response

• Steady inviscid/viscid interaction analysis: SFLOW-IVI
  - 2D cascade flows
  - Local strong inviscid/viscid interactions
  - Efficient: CPU < 5 min
  - Robust: wide range of operating conditions

• Future work
  - Transonic/supersonic gust response analysis
  - SFLOW-IVI/LINFLO coupling
  - Unsteady IVI analysis