Connection with Dynamics: General Introduction

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Abstract

This is a very brief nontechnical introduction to a few theoretical issues related to the density-velocity relation. The aim of this introduction is not an exhaustive analysis of the current theoretical situation but rather setting a stage for the following talks. The selection of topics has been determined by the sequel program

1 Formation of the structure in the Universe: Scenarios

1.1 Gravitational vs non-gravitational

Relating the observations of the spatial distribution and motions of galaxies with the highly isotropic cosmic microwave background radiation (hereafter CMBR) we conclude that the structure in the Universe has risen from some kind of seeds after cosmological decoupling which took place at redshift $z \approx 10^3$. The nature of the seeds must be such that they did not disturb CMBR more than about one thousandth of a percent at the decoupling and on the other hand could be amplified to make galaxies, clusters and superclusters of galaxies by the present time. Following Newton, Jeans, Lifshitz most of us believe that the 'amplifier' is gravitation. Even the explosion scenario, the model primarily stressing the role of the nuclear power released in numerous supernova explosions [23], invokes the gravitational amplification of the large-scale density perturbations [29]. However, the explosion models are probably ruled out as an explanation for structure on scales $\sim 15 Mpc$ by the tight limits imposed by the COBE measurements of the y-distortions of CMBR determined by the Comptonization process [21].

1.2 Primordial fluctuations

Scenarios based on gravitational instability must assume the initial seeds of some kind (see e.g. [26]). Quite understandably two types of seeds have been proposed: Gaussian and non-Gaussian random fluctuations. The assumption of Gaussian primordial perturbations is based on three lines of argument. First, an inflation scenario predicts that the Gaussian fluctuations are a natural outcome of the inflationary stage. In early eighties it also was claimed that the perturbations had the scale invariant (Harrisson-Peebles-Zel'dovich) spectrum, however at present constraints on the form of the spectrum are somewhat weaker (see e.g. [22] and references therein). The second argument is based on the central limit theorem which states that if many processes operate the outcome is a Gaussian distribution independently of nature of the processes. Finally, the hypothesis of the Gaussian primordial perturbations is a completely statistically specified model allowing a fair amount of observational tests. A Gaussian random function is statistically fully specified by its spectrum. The hypothesis of Gaussian primordial fluctuations is model independent in a sense that we do not need to specify a physical process for generating the initial conditions. The initial spectrum can be used as a functional parameter.

On the contrary, for generating the non-Gaussian initial conditions one has to specify the physical process first and calculate the initial density and/or velocity fluctuations generating by the process. Referring to non-Gaussian initial perturbations without specifying the undelying physical model is almost meaningful except, perhaps, 'small' non-Gaussianity, which can be characterized by a few first moments (skewness or kurtosis). [11].

There were several well specified models to generate non-Gaussian initial perturbations known under nicknames domain walls, cosmic strings, textures. Expressing very personal view, I take a risk to say that non-Gaussian models introducing additional parameters have not gained a clear advantage over the Gaussian models in explaining the structure of the universe. The COBE measurements of the large-scale angular fluctuations impose strong constrains on non-Gaussian models.

1.3 Dark matter

At present almost nobody doubts that a substantial amount of dark matter is present in the universe. However, there are two important questions needed to be answered: (1) what is the form (or forms) of the dark matter, and (2) what is the amount of dark matter in the universe. The inflation scenario suggests that the spatial curvature of the universe is zero which means that $\Omega_{tot} = 1$. In the absence of a substantial Λ-term it means that $\Omega_{tot} \approx \Omega_{matter} = 1$, which requires non-baryonic dark matter, because from the cosmological nucleosynthesis we know that the mean density in baryons can not exceed about $\Omega_b \approx 0.12h^{-1}$ (see e.g. [27] and references therein). However, being theoretically very beautiful the inflation scenario has been properly tested neither experimentally nor observationally. Thus, the astronomical estimates of $\Omega_{dyn} \sim 0.2$ must be taken very seriously [25]. If they are correct then we may live in an open universe. Unfortunately, the formation of the structure in the baryon dominated open universe is in a serious conflict with the new COBE limit on the y-distortions of the CMBR $y < 2.5 \times 10^{-5}$ [34].

The Cold Dark Matter (hereafter CDM) model, a favorite of eighties, seems also to be in a serious trouble. The accurate measurements of galaxy clustering on large scales showed that rms fluctuations within spheres of radius $20h^{-1}Mpc$ are 2-3 times stronger than predicted by the CDM model with the COBE normalization [31]. Several modifications of the CDM model have been proposed to deal with this problem: cooperative galaxy formation [5], a tilted CDM scenario [6], an Mixed Dark Matter (MDM) [8] or Cold+Hot Dark Matter (C+HDM) [16], [17], [28] models, mixing $\sim 60\%$ of cold and $\sim 30\%$ of hot dark components. As usually, the new models were claimed to resolve the above problem.

2 Nonlinear dynamics

Beyond the uncertainties mentioned above (Ω_{tot}, the dark matter species composition, the primordial spectrum) a model based on the gravitaional instability has to deal primarily with the problem of the nonlinear evolution. Of course, in complete form the nonlinear evolution must include galaxy formation as well. Unfortunately, at present there is no theory of galaxy formation. Except, perhaps, very few very general principles, we do not understand how galaxies formed and hence we skip its discussion here.

The nonlinear dynamics of density perturbations is more than complex even in the simplest possible model of the dustlike collisionless medium (see e.g. [33]). This model perfectly suits for the description of the evolution of density perturbations in dark matter comprised of weakly interacting particles. Avoiding the complexity of galaxy formation, we simply label density peaks as 'galaxies' assuming that galaxies must form in the regions of high density. This simple trick is usually used when the results of N-body simulations are compared with observations. In case the resulting sample of simulated 'galaxies' does not have the desired statistical properties one can adjust the selection criterion by introducing the so called bias. Bias in most cases is simply a numerical parameter to adjust the initial amplitude of the density fluctuations, but sometimes it can label a potential physical process which either suppress or enhance the galaxy formation in some regions [5].

Probably the most popular approach to the nonlinear stage consists in running N-body simulations. Its advantage is related to the capability of N-body simulations to deal with generic and random initial conditions. One can also relatively easy simulate various observational effects like the distributions in the redshift space instead of the coordinate space, the modulation of the density of galaxies due to a selection function, the influence of the boundaries of a sample, etc.

The major disadvantage of N-body simulations is the lack of good tests of what we simulate. The accuracy of the results of N-body simulations is almost always overestimated. By itself the method of N-body simulations does not bring much understanding of the nonlinear dynamics.

2.1 Approximations

Constructing approximations is another technique to study the nonlinear stage of the density perturbations. A good approximation brings a deep insight into the physics of the process, but rarely can compete with N-body simulations when a theoretical model is compared with observations. The most successful approximation describing the nonlinear stage of the gravitational instability was suggested by Zel'dovich in 1970 [35] (for review see [33]). Formally it is an extrapolation of a simple linear solution in the Lagrangian form (for more details see [32]).

Before describing the Lagrangian formalism we note in passing, that a simple modification of the Zel'dovich approximation [7] has made its application to hierarchical clustering quite sensible though not as quite good as in the pancake scenario. Another quite good modification of the Zel'dovich approximation is the adhesion approximation [13], [12], [30].

2.2 Eulerian formalism vs Lagrangian formalism

Usually the density and velocity distributions are given in the Eulerian form which, of course, is quite natural. We are used to the Eulerian space since we live in it; the structure we observe and study is in Eulerian space as well. The Lagrangian space is an auxiliary theoretical construction often used in hydrodynamics. In the Lagrangian description the density and velocity are assigned to particles (instead of spatial coordinates) which often are labeled by their coordinates in the unperturbed state. Thus the densities and velocities are functions of the unperturbed coordinates and (time or a parameter equivalent to time). The Lagrangian formalism was a clue to the success of the Zel'dovich approximation and at present is explicitly used for the analysis of the density-velocity relation [9], [32], the velocity and mass distribution functions [18], and also in attempts to construct the exact solution of the gravitational instability in a dust-like medium for arbitrary initial conditions [2], [20].

The disadvantage of the Lagrangian formalism is related to the fact that even we knew the exact spatial density and velocity distributions in Lagrangian coordinates we still would not be able to compare them with the observational galaxy (or rather mass) distributions unless we knew explicitely the relation between the Lagrangian and Eulerian coordinates. In the Zel'dovich approximation this relation was found to the linear order. The relation between the Lagrangian and Eulerian coordinates can be written to higher orders, but it already becomes nonlocal in the second order. Nonlocality is only one problem of the higher order approximations. It is probably even worse that the higher approximations do not converge homogeneously to the solution. This means that improving the lower order approximation in some places it makes it worse in others. However, the higher order approximations are necessary for calculating the higher order moments of the density and velocity distribution functions [1], [3], [15].

Finally, it is worth mentioning that the Gaussian initial density perturbations are Gaussian only in the Lagrangian space. In the Eulerian space they are non-Gaussian already to the linear order. Thus the Gaussian initial perturbations can be confirmed only when a very specific non-Gaussian density fluctuation field is found.

3 Density, velocity and gravitational potential

Studying the large-scale structure of the universe we usually analyzed the density and peculiar velocity fields and almost always ignored the gravitational potential. Perhaps, there were reasons for that. First, the gravitational potential could not be measured directly until recent time, and second, knowing the Poisson equation we always can relate the density and potential perturbations. If the density of galaxies was not thought to be proportional to the mass density then a magic biasing mechanism could be invoked.

Trying to predict the mass and velocity distributions theoretically, we specified the power spectrum of the initial density fluctuations. Since in the growing mode the initial velocity and potential fields are not independent from the density fluctuation field the statistical information stored in the power spectrum is complete and we need not to care about the velocity and potential spectra. Formally being quite right this concept in practice caused a few wrong corollaries. First, it was often wrongly implied that the typical scales of density, velocity and potential fields are about the same. A search for a great attractor was one consequence of that assumption. It may happen that the large scale streaming motion is not caused by a single concentration of mass [10], but a rather smooth and very large trough in the initial gravitaional potential.

Theoretically the importance of the initial gravitational potential for the large-scale galaxy distribution and motion was manifested in the adhesion approximation [13], [19]. Now it may become an observable quantity [14].

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