Development of an Algebraic Stress/Two-Layer Model for Calculating Thrust Chamber Flow Fields

By
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ABSTRACT

Following the consensus of a workshop in Turbulence Modeling for Liquid Rocket Thrust Chambers, the current effort was undertaken to study the effects of second-order closure on the predictions of thermochemical flow fields. To reduce the instability and computational intensity of the full second-order Reynolds Stress Model, an Algebraic Stress Model (ASM) coupled with a two-layer near wall treatment was developed. Various test problems, including the compressible boundary layer with adiabatic and cooled walls, recirculating flows, swirling flows and the entire SSME nozzle flow were studied to assess the performance of the current model. Detailed calculations for the SSME exit wall flow around the nozzle manifold were executed. As to the overall flow predictions, the ASM removes another assumption for appropriate comparison with experimental data, to account for the non-isotropic turbulence effects.
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11th Workshop for CFD Applications in Rocket Propulsion
April 20-22, 1993
NASA-Marshall Space Flight Center
• Improve Predictive Capabilities of Turbulent Transport in Thrust Chamber

• Non-Isotropic and Compressibility Effects are the Focus of the Study

• Simplified Reynolds Stress Modeling

• Further Modeling in Turbulent Transport of Thermal Energy and Chemical Species - $u'_iC'$ and $u'_iT'$ etc.
Motivation and Objective

- Higher Order Models Are Desirable For Calculating Thrust Chamber Flow Fields
  --- 1991 Thrust Chamber Turbulence Modeling Workshop

- To Develop a Simplified 2nd-Order Turbulence Model For Thrust Chamber Flow Calculation
  --- Near wall treatment
  --- Efficiency and stability
APPRAOCH

- PDE's for Reynolds stress $\overline{u_i u_j}$ can be derived. Modeling any unknown in terms of Reynolds stress, the mean strain rate etc.

- Simplifications of the Differential Reynolds stresses Equations
  - Algebraic Stress Model (ASM)

- Non-linear constitutive relations (Spezial)
**APPROACH (DRS Equation)**

- Differential Reynolds Stress Equation

\[
\frac{D}{Dt} \rho \bar{u}_i \bar{u}_j = P_{ij} + D_{ij} + \pi_{ij} + C_{ij} - \epsilon_{ij}
\]

\[
P_{ij} = -\rho \left( \bar{u}_i \bar{u}_k \frac{\partial \bar{u}_j}{\partial x_k} + \bar{u}_j \bar{u}_k \frac{\partial \bar{u}_i}{\partial x_k} \right) \quad \text{production}
\]

\[
D_{ij} = \frac{\partial}{\partial x_k} \left( \rho \bar{u}_i \bar{u}_j \bar{u}_k + \delta_{ik} \bar{u}_j \bar{p} + \delta_{jk} \bar{u}_i \bar{p} \right) - \left( \mu S_{ik} \bar{u}_j + \mu S_{jk} \bar{u}_i \right) \quad \text{diffusion}
\]

\[
\pi_{ij} = p' \left( \frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} \right) \quad \text{pressure-strain correlation}
\]

\[
C_{ij} = -\left[ \bar{u}_i \frac{\partial \bar{p}}{\partial x_j} + \bar{u}_j \frac{\partial \bar{p}}{\partial x_i} \right] \quad \text{compressibility}
\]

\[
\epsilon_{ij} = \mu \left[ S_{ik} \frac{\partial \bar{u}_j}{\partial x_k} + S_{jk} \frac{\partial \bar{u}_i}{\partial x_k} \right] \quad \text{dissipation}
\]


APPROACH --- ASM

- Similitude Principle (Mellor and Yamada)

\[ P_{ij} - \frac{2}{3} \delta_{ij} P_k + \phi_{ij} + C_{ij} \equiv 0 \]

- Algebraic Reynolds Stress Model of Rodi

\[ \frac{D}{Dt} \rho u_i u_j - D_{ij} \equiv \frac{u_i u_j}{k} \left[ \frac{Dk}{Dt} - D_k \right] \]

\[ \Rightarrow \{ P_{ij} + \phi_{ij} + C_{ij} - \varepsilon_{ij} \} = \frac{u_i u_j}{k} \left[ P_k - \varepsilon \right] \]
APPRAOCHE --Pressure-Strain Term

\[ \pi_{ij} = p' \left( \frac{\partial u_i''}{\partial x_j} + \frac{\partial u_j''}{\partial x_i} \right) \]

return to isotropy

\[ = -C_1 \frac{\varepsilon}{k} \left( \rho u_i'' u_j'' \delta_{ij} \right) \]

Rotta Model

rapid term

\[ -C_2 (P_{ij} \frac{2}{3} \delta_{ij} P_k) \]

IP Model

wall damping term

\[ +\pi_{ijw} \]

Lumped with the two-layer model

in which

\[ P_k = \frac{1}{2} P_{ii} \]
\[ \frac{\rho u_i u_j}{k} = \frac{(1-C_2)(P_{ij} - \frac{2}{3}\delta_{ij}P_k) - \frac{2}{3}\delta_{ij}C_k + C_{ij}}{P_k + \varepsilon(C_1-1) + C_k} + \frac{2}{3}\delta_{ij} \]

where

\[ P_{ij} = -\rho \left[ u_i u_k \frac{\partial \bar{u}_j}{\partial x_k} + u_j u_k \frac{\partial \bar{u}_i}{\partial x_k} \right] \]

\[ P_k = \frac{1}{2} P_{ii} \]
k-ε Equations

\[ \frac{\partial k}{\partial t} = \frac{\partial}{\partial x_j} \left( \frac{\partial k}{\partial x_j} \right) + \mu_G \nabla^2 k \]

\[ \frac{\partial \varepsilon}{\partial t} = \frac{\partial}{\partial x_j} \left( \frac{\partial \varepsilon}{\partial x_j} \right) + \frac{C_1}{C_2} \frac{\partial k}{\partial x_j} \frac{\partial k}{\partial x_j} \]

where the production term \( \mu_G \) takes the form

\[ \mu_G = \rho \frac{\partial u_i}{\partial x_j} \frac{\partial u_j}{\partial x_i} \]
<table>
<thead>
<tr>
<th>Turbulence model costants</th>
<th>$C_1$</th>
<th>$C_{e2}$</th>
<th>$C_k$</th>
<th>$C_e$</th>
<th>$C_1$</th>
<th>$C_2$</th>
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</thead>
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<tr>
<td></td>
<td>1.45</td>
<td>1.92</td>
<td>0.22</td>
<td>0.15</td>
<td>2.5</td>
<td>0.5</td>
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</table>
Two-Layer Wall Treatment

Outer Layer --- resolved by ASM

Inner Layer --- Patched with a One-Equation Model

Matching at

\[ R_k = \frac{k^{1/2}y}{\mu} = 200 \]
\[ V_1 = u' \gamma' = C_\mu \frac{\kappa^{3/2}}{\mu} l_{\mu} = C_\mu \frac{\kappa^{3/2}}{\mu} l_{\mu} = C_\mu \frac{\kappa^{3/2}}{\mu} \frac{l_\varepsilon}{\kappa^{3/2}} \]

within Intertia Sublayer \( \varepsilon = C_\mu \frac{\kappa^{3/2}}{\mu} \frac{l_\varepsilon}{\kappa^{3/2}} \) → to be used in Eddy Viscosity

<table>
<thead>
<tr>
<th>( l_{\mu} = C_1 y\left[1 - \exp\left(-\frac{R_\kappa}{A_{\mu}}\right)\right] )</th>
<th>( l_{\varepsilon} = C_1 y\left[1 - \exp\left(-\frac{R_\kappa}{A_{\varepsilon}}\right)\right] )</th>
<th>( C_1 = \kappa C_\mu^{3/4} )</th>
</tr>
</thead>
</table>

Matching at \( R_\kappa = \frac{\nu}{\kappa^{3/2}} y \)
IMPLEMENTATIONS

- Implemented into MAST-2D
- Non-Staggered Grids, Sequential Solver
- Chakravarthy-Osher TVD Scheme
- PISO-C Algorithm
- Conjugate Gradient Matrix Solver
- Time Marching
Validations

- Incompressible & Compressible Flat Plate
  Cooled & Heated Wall, Up To Mach 10

- Incompressible & Compressible Recirculating Flows

- Incompressible Swirling Flows

- Thrust Chamber Flows
Compressible Flat Plate Flow

Fig. 1 Variation of $C_f/C_{f0}$ with $M_\infty$ for adiabatic wall boundary condition.
Compressible Flat Plate Flow

Fig. 2 Variation of $C_f/C_{f0}$ with $T_w/T_{aw}$ for $M_\infty = 5.0$. 

Re=$10^4$ Ma=5

- Van Driest
- $k-\epsilon$
- ASM
Compressible Flat Plate Flow

Fig. 3 Semi-log plots of $u_e^+$ for adiabatic wall boundary condition.
Backward Facing Step

Attachment length:  
- $k$-$\varepsilon$ 4.67H  
- ASM 5.94H  
- EXP. ~ 6H

--- $k$-$\varepsilon$  
--- $k$-$\varepsilon$/ASM

data: $O \overline{u_1u_1}$ $\triangledown \overline{u_2u_2}$ $\Delta \overline{u_1u_2}$

Fig. 5 Reynolds stress profiles for the backward-facing step turbulent flow (9:1), with data from [31]
Confined Swirling Flows

Fig. 6 Decay of mean axial centerline velocity.
Fig. 8a Radial profiles of turbulent intensity $\sqrt{\langle u'^2 \rangle}$
Confined Swirling Flows

Fig. 8b Radial profiles of Reynolds stress ($u'w'$)
SSME Nozzles

Fig. 9a Contour of Mach number for k-\(\epsilon\) with two-layer model.

Fig. 10a Contour of temperature for k-\(\epsilon\) with two-layer model.
SSME Nozzles

Fig. 9b Contour of Mach number for ASM with two-layer model.

Fig. 10b Contour of temperature for ASM with two-layer model.
8 - STEP REACTIONS

<table>
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<th>Reaction</th>
<th>A</th>
<th>N</th>
<th>E</th>
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<tr>
<td>M + O₂ → O + O</td>
<td>0.72000E+19</td>
<td>-1.0000</td>
<td>117908</td>
</tr>
<tr>
<td>M + H₂ → H + H</td>
<td>0.55000E+19</td>
<td>-1.0000</td>
<td>103298</td>
</tr>
<tr>
<td>M + H₂O → H + OH</td>
<td>0.52000E+22</td>
<td>-1.5000</td>
<td>118000</td>
</tr>
<tr>
<td>O + H → OH</td>
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<td>-1.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>H₂O + OH → H₂O + H</td>
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<td>0.0000</td>
<td>180000</td>
</tr>
<tr>
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<td>0.0000</td>
<td>5166</td>
</tr>
<tr>
<td>O₂ + H → OH + O</td>
<td>0.22000E+15</td>
<td>0.0000</td>
<td>16800</td>
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<tr>
<td>H₂ + O → OH + H</td>
<td>0.75000E+14</td>
<td>0.0000</td>
<td>11099</td>
</tr>
</tbody>
</table>

\[ k = AT_{N} \exp\left(-\frac{E}{RT}\right) \]

with \( k \) in cm³·mole⁻¹·s⁻¹ and \( E \) in cal·mole⁻¹

SSME Nozzles

Figure A.5 Sample SSME Nozzle Flow Inputs and Results —— Turbulent, 8-Step Kinetics.

ISP = 452.78 sec.

Exp. ISP = 453.3 sec.
Fig. 11 Behavior of near wall TKE at nozzle exit.
Fig. 12, SSME wall contour and geometry at exit
Fig. 13(a), Grid configurations for SSME nozzle exit manifold

Fig. 13(b), Close-up grids for Figure 13(a)
Fig. 17(a), Pressure levels along the wall near the nozzle exit using the ASM and $k-\varepsilon$ models

Fig. 17(b), Effects of wall temperature on the wall pressure
Fig. 18, Contour of pressure using 75% of the chamber pressure level

Fig. 19, Laminar flow calculations of the SSME exit flow
Summaries

- The Algebraic Stress Model Removes the Isotropic Turbulence Assumption for the Eddy Viscosity Type Models

- Improved On the Reynolds Stresses Predictions

- The ASM Does Not Improve Too Much On SSME Nozzle & Outlet Flows
  ---RSM
  ---Other Mechanisms
  Shock- Boundary Layer Interactions
  Entrainment Issues

- 3-D Calculations Are Desirable
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