

1995 11 20 24
NUMERICAL COMPUTATION OF AERODYNAMICS AND HEAT TRANSFER IN A
TURBINE CASCADE AND A TURN-AROUND DUCT USING ADVANCED
TURBULENCE MODELS

B. LAKSHMINARAYANA AND J. LUO

The Pennsylvania State University
Department of Aerospace Engineering
University Park, PA 16802

532-34
43807
p. 34

The objective of this research is to develop turbulence models to predict the flow and heat transfer fields dominated by the curvature effect such as those encountered in turbine cascades and turn-around ducts.

A Navier-Stokes code has been developed using an explicit Runge-Kutta method with a two layer $k-\epsilon$ /ARSM (Algebraic Reynolds Stress Model), Chien's Low Reynolds Number (LRN) $k-\epsilon$ model and Coakley's LRN $q-\omega$ model. The near wall pressure strain correlation term was included in the ARSM. The formulation is applied to Favre-averaged N-S equations and no thin-layer approximations are made in either the mean flow or turbulence transport equations. Anisotropic scaling of artificial dissipation terms was used. Locally variable timestep was also used to improve convergence. Detailed comparisons were made between computations and data measured in a turbine cascade by Arts et al. at Von Karman Institute. The surface pressure distributions and wake profiles were predicted well by all the models. The blade heat transfer is predicted well by $k-\epsilon$ /ARSM model, as well as the $k-\epsilon$ model. It's found that the onset of boundary layer transition on both surfaces is highly dependent upon the level of local freestream turbulence intensity, which is strongly influenced by the streamline curvature.

Detailed computation of the flow in the turn around duct has been carried out and validated against the data by Monson as well as Sandborn. The computed results at various streamwise locations both on the concave and convex sides are compared with flow and turbulence data including the separation zone on the inner well. The $k-\epsilon$ /ARSM model yielded relatively better results than the two-equation turbulence models. A detailed assessment of the turbulence models has been made with regard to their applicability to curved flows.

NUMERICAL COMPUTATION OF AERODYNAMICS AND HEAT TRANSFER IN A TURBINE CASCADE AND A TURN-AROUND DUCT USING ADVANCED TURBULENCE MODELS*

B. Lakshminarayana and J. Luo

**The Pennsylvania State University
Department of Aerospace Engineering
University Park, Pennsylvania**

OBJECTIVE:

**TO DEVELOP TURBULENCE MODELS TO PREDICT
FLOW AND HEAT TRANSFER FIELDS IN
TURBOMACHINERY INCLUDING CURVATURE,
ROTATION AND HIGH TEMPERATURE EFFECTS**

OUTLINE:

- **INTRODUCTION**
- **NUMERICAL TECHNIQUE**
- **TURBULENCE MODELS**
- **FLOW AND HEAT TRANSFER FIELD IN A HIGH
MACH NUMBER TRANSONIC TURBINE CASCADE**
- **FLOW FIELD IN A TURN-AROUND DUCT**
- **CONCLUSIONS**

***SPONSORED BY NASA HUNTSVILLE WITH LISA GRIFFIN AS THE TECHNICAL
MONITOR**

GOVERNING EQUATIONS (Cartesian)

$$\frac{\partial Q}{\partial t} = - \left(\frac{\partial E}{\partial x} + \frac{\partial F}{\partial y} + \frac{\partial G}{\partial z} \right) + \left(\frac{\partial E_v}{\partial x} + \frac{\partial F_v}{\partial y} + \frac{\partial G_v}{\partial z} \right) + S$$

$$Q = \begin{pmatrix} \rho \\ \rho u \\ \rho v \\ \rho w \\ \rho e_0 \\ \rho k \\ \rho e \end{pmatrix}, E = \begin{pmatrix} \rho u \\ \rho u u + p \\ \rho u v \\ \rho u w \\ (\rho e_0 + p)u \\ \rho u k \\ \rho u e \end{pmatrix}, F = \begin{pmatrix} \rho v \\ \rho v u + p \\ \rho v v + p \\ \rho v w \\ (\rho e_0 + p)v \\ \rho v k \\ \rho v e \end{pmatrix}, G = \begin{pmatrix} \rho w \\ \rho w u \\ \rho w v \\ \rho w w + p \\ (\rho e_0 + p)w \\ \rho w k \\ \rho w e \end{pmatrix},$$

$$E_v = \begin{pmatrix} 0 \\ \tau_{xx} \\ \tau_{xy} \\ \tau_{xz} \\ u\tau_{xx} + v\tau_{xy} + w\tau_{xz} - Q_x \\ \left(\mu_1 + \frac{\mu_1}{Pr_k} \right) \frac{\partial k}{\partial x} \\ \left(\mu_1 + \frac{\mu_1}{Pr_\epsilon} \right) \frac{\partial e}{\partial x} \end{pmatrix}, F_v = \begin{pmatrix} 0 \\ \tau_{yx} \\ \tau_{yy} \\ \tau_{yz} \\ u\tau_{yx} + v\tau_{yy} + w\tau_{yz} - Q_y \\ \left(\mu_1 + \frac{\mu_1}{Pr_k} \right) \frac{\partial k}{\partial y} \\ \left(\mu_1 + \frac{\mu_1}{Pr_\epsilon} \right) \frac{\partial e}{\partial y} \end{pmatrix}, G_v = \begin{pmatrix} 0 \\ \tau_{zx} \\ \tau_{zy} \\ \tau_{zz} \\ u\tau_{zx} + v\tau_{zy} + w\tau_{zz} - Q_z \\ \left(\mu_1 + \frac{\mu_1}{Pr_k} \right) \frac{\partial k}{\partial z} \\ \left(\mu_1 + \frac{\mu_1}{Pr_\epsilon} \right) \frac{\partial e}{\partial z} \end{pmatrix}, S = \begin{pmatrix} 0 \\ 0 \\ 0 \\ \rho(\omega^2 y + 2\omega w) \\ \rho(\omega^2 z - 2\omega v) \\ 0 \\ P - \rho e + D \\ (C_1 P - C_2 \rho e) \frac{e}{k} + E \end{pmatrix}$$

• Relative velocities, constant rotation rate about x axis, ω . Averaged quantities.

• Energy, $e_0 = \epsilon + \frac{q^2}{2} - \frac{\omega^2 r^2}{2}$, rothalpy constant along streamlines for inviscid steady state.

TECHNIQUES

1. RK2D code :

- * 2-D Navier-Stokes code,
Conservative, compressible formulation
- * Favre-Averaged Mean and Turbulence equations
- * 4-stage explicit Runge-Kutta scheme
- * 2nd and 4th order artificial dissipation
(with eigenvalue and local velocity scaling)
- * Coupled with compressible Low-Reynolds number K- ϵ model , q- ω model , ARSM , NLSM (Nonlinear -stress model) , AHFM(Algebraic Heat Flux model)
- * Characteristic boundary conditions, H grids (generated by a combined algebraic and elliptic method to keep smoothness and orthogonality near the wall)

2. TEXSTAN code

- * 2-D boundary layer code developed by Crawford

- * Extension of STAN 5, Patankar-Spalding numerical scheme

- * Include 7 differential two-equation turbulence models (Jones-Launder, Chien, Lam- Bremhorst , etc.) and mixing length model

RSM (Reynolds Stress Model)
(Gibson & Launder 1978)

Reynolds stresss transport equation :

$$U_k \frac{\partial \overline{u_i u_j}}{\partial x_k} = -\overline{u_i u_k} U_{j,k} - \overline{u_j u_k} U_{i,k} + \overline{\frac{p}{\rho} (u_{i,j} + u_{j,i})}$$

$$- \frac{\partial}{\partial x_k} \left[\overline{u_i u_j u_k} + \frac{\overline{p u_j}}{\rho} \delta_{ik} + \frac{\overline{p u_i}}{\rho} \delta_{jk} - \nu \frac{\partial \overline{u_i u_j}}{\partial x_k} \right] - 2\nu \overline{\frac{\partial u_i}{\partial x_k} \frac{\partial u_j}{\partial x_k}}$$

i.e., $C_{ij} - D_{ij} = P_{ij} + \phi_{ij} - \epsilon_{ij}$

where $\epsilon_{ij} = \frac{2}{3} \epsilon$ (Dissipation)

$\phi_{ij} = \phi_{ij1} + \phi_{ij2} + \phi_{ij1,w} + \phi_{ij2,w}$ (Pressure-strain correlation)

$\phi_{ij1} = -C_1 \frac{\epsilon}{k} (\overline{u_i u_j} - \frac{2}{3} k \delta_{ij})$ (Return-to-isotropy part)

$\phi_{ij2} = -C_2 (P_{ij} - \frac{2}{3} P_k \delta_{ij})$ (Rapid part)

$$\phi_{ijw,1} = c'_1 \frac{\varepsilon}{k} (\overline{u_k u_m} n_k n_m \delta_{ij} - \frac{3}{2} \overline{u_i u_k} n_k n_j - \frac{3}{2} \overline{u_j u_k} n_k n_i) f_n$$

(Near-wall term)

$$\phi_{ijw,2} = c'_2 (\phi_{km,2} n_k n_m \delta_{ij} - \frac{3}{2} \phi_{ik,2} n_k n_j - \frac{3}{2} \phi_{jk,2} n_k n_i) f_n$$

(Near-wall term)

$f_n = k^{3/2} / (2.55 x_n \varepsilon)$ (x_n is the distance normal to the wall)

Constants : $c_1=1.8$, $c_2=0.6$, $c'_1=0.5$, $c'_2=0.3$

ARSM (Algebraic Reynolds Stress Model)

ARSM assumption:

$$C_{ij} - D_{ij} = \frac{\overline{u_i u_j}}{k} (C_k - D_k) = \frac{\overline{u_i u_j}}{k} (P_k - \varepsilon)$$

$$\Rightarrow C_{ij} - D_{ij} = \frac{\overline{u_i u_j}}{k} (C_k - D_k) = \frac{\overline{u_i u_j}}{k} (P_k - \varepsilon)$$

$$\Rightarrow C_{ij} - D_{ij} = \frac{\overline{u_i u_j}}{k} (C_k - D_k) = \frac{\overline{u_i u_j}}{k} (P_k - \varepsilon)$$

where

$$P_k = -\overline{u_i u_j} U_{i,j}$$

$$P_{ij} = -\overline{u_i u_k} U_{j,k} - \overline{u_j u_k} U_{i,k}$$

$\phi_{ij1,w}$ and $\phi_{ij2,w}$ as in RSM

NLSM (Nonlinear Stress Model)
(Shih , Zhu & Lumley 1992)

Reynolds stress :

$$\begin{aligned}\overline{u_i u_j} &= \frac{2}{3} k \delta_{ij} - \nu_t (U_{i,j} + U_{j,i}) \\ &+ \frac{C_{\tau 1}}{A_2 + \eta^3} \frac{k^3}{\varepsilon^2} (U_{i,k} U_{k,j} + U_{j,k} U_{k,i} - \frac{2}{3} \pi \delta_{ij}) \\ &+ \frac{C_{\tau 2}}{A_2 + \eta^3} \frac{k^3}{\varepsilon^2} (U_{i,k} U_{j,k} - \frac{1}{3} \pi \delta_{ij}) \\ &+ \frac{C_{\tau 3}}{A_2 + \eta^3} \frac{k^3}{\varepsilon^2} (U_{k,i} U_{k,j} - \frac{1}{3} \pi \delta_{ij})\end{aligned}$$

where

$$\pi = U_{i,j} U_{j,i}$$

$$\tilde{\pi} = U_{i,j} U_{i,j}$$

$$\nu_t = C_\mu \frac{k^2}{\varepsilon}$$

$$C_\mu = \frac{2/3}{A_1 + \eta + \alpha \xi}$$

$$\xi = \frac{k}{\varepsilon} \Omega$$

$$\Omega = (2\Omega_{ij}^* \Omega_{ij}^*)^{\frac{1}{2}}$$

$$\Omega_{ij}^* = (U_{i,j} - U_{j,i}) / 2$$

$$\eta = \frac{k}{\varepsilon} S$$

$$S = (2S_{ij} S_{ij})^{\frac{1}{2}}$$

$$S_{ij} = (U_{i,j} + U_{j,i}) / 2$$

Constants:

$C_{\tau 1}$	$C_{\tau 2}$	$C_{\tau 2}$	A_1	α	A_2
-4	13	-2	1.25	0.9	1000

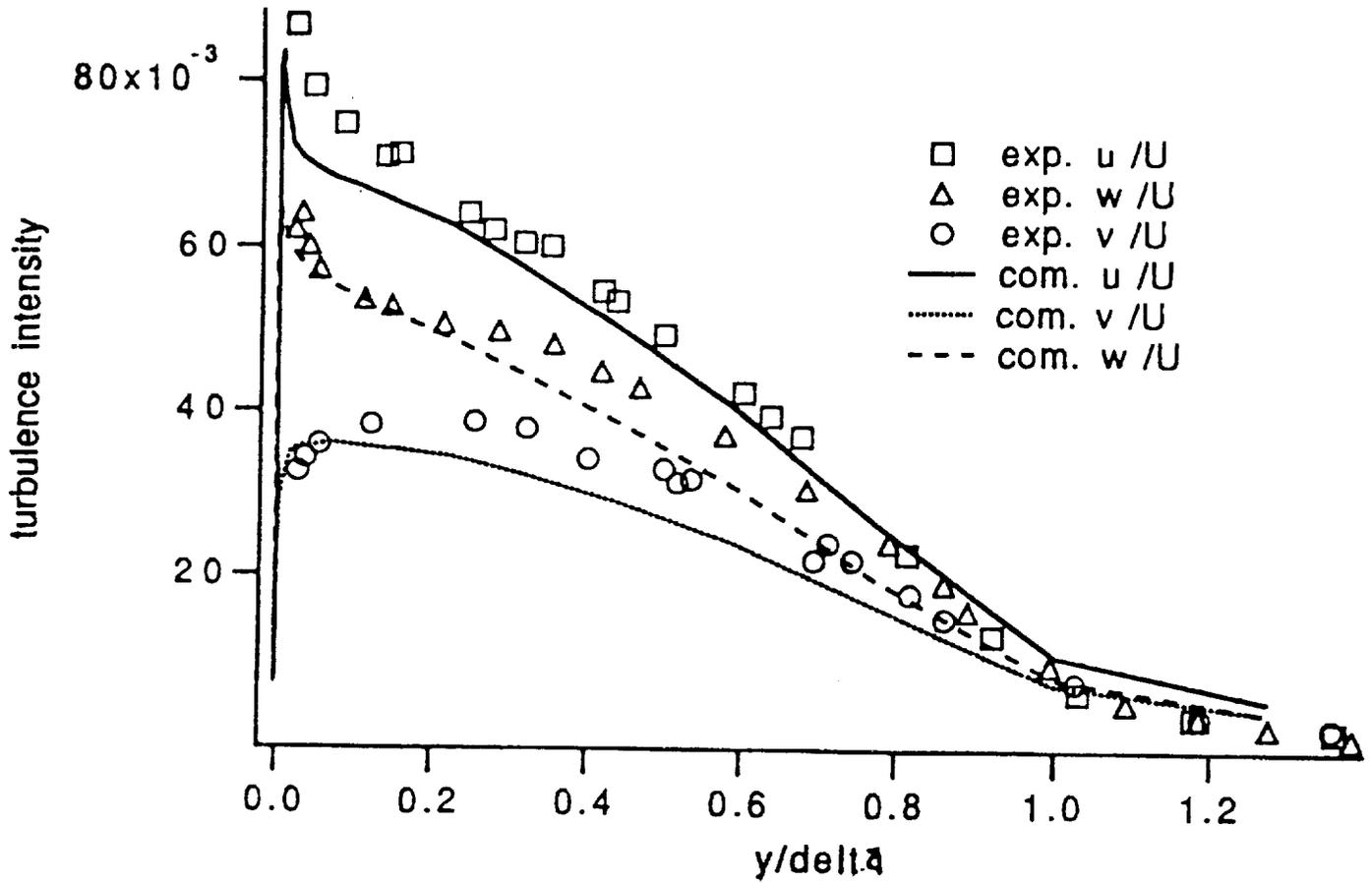
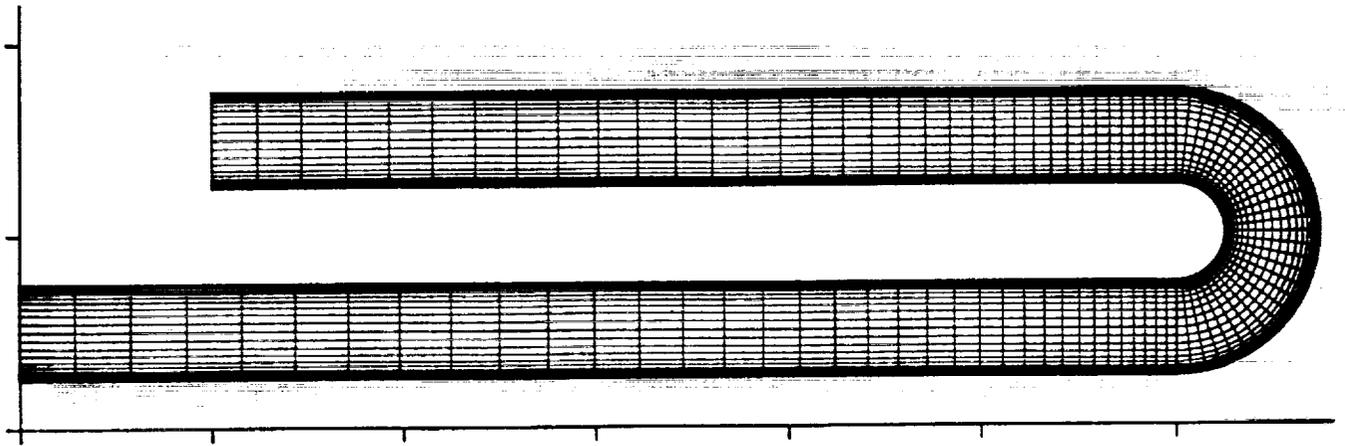


Fig. Validation of the ARSM model: Turbulence intensity profile in the flat-plate turbulent boundary layer : experiment by Klebanoff; computation by ARSM

180-degree TURN AROUND DUCT(TAD)

Geometry & Grid :



EARLIER RESEARCH ON TAD FLOW

- Measurements:

*Sandborn (1988), Sandborn and Shin (1989) (Water flow, $Re=7 \times 10^4 - 5 \times 10^5$ (Re based on duct height and bulk velocity))

*Monson, Seegmiller, McConnaughey & Chen (1989,1990) (Air flow, $Re=10^5, 10^6$)

*Sharma et al (1987) (Axisymmetric TAD air flow, $Re=10^5$)

- Earlier Computations:

- *Chen and Sandborn (1986) (K- ϵ and curvature-corrected K- ϵ)

- *Monson, Seegmiller & McConnaughey (1989,1990) (mixing-length & K- ϵ with curvature-correction)

- *Avva etal (1990) (High Re and Low-Re K- ϵ)

- *Gallardo & Lakshminarayana (1993) (curvature-modified K- ϵ)

- Agreement in above computations are not satisfactory.

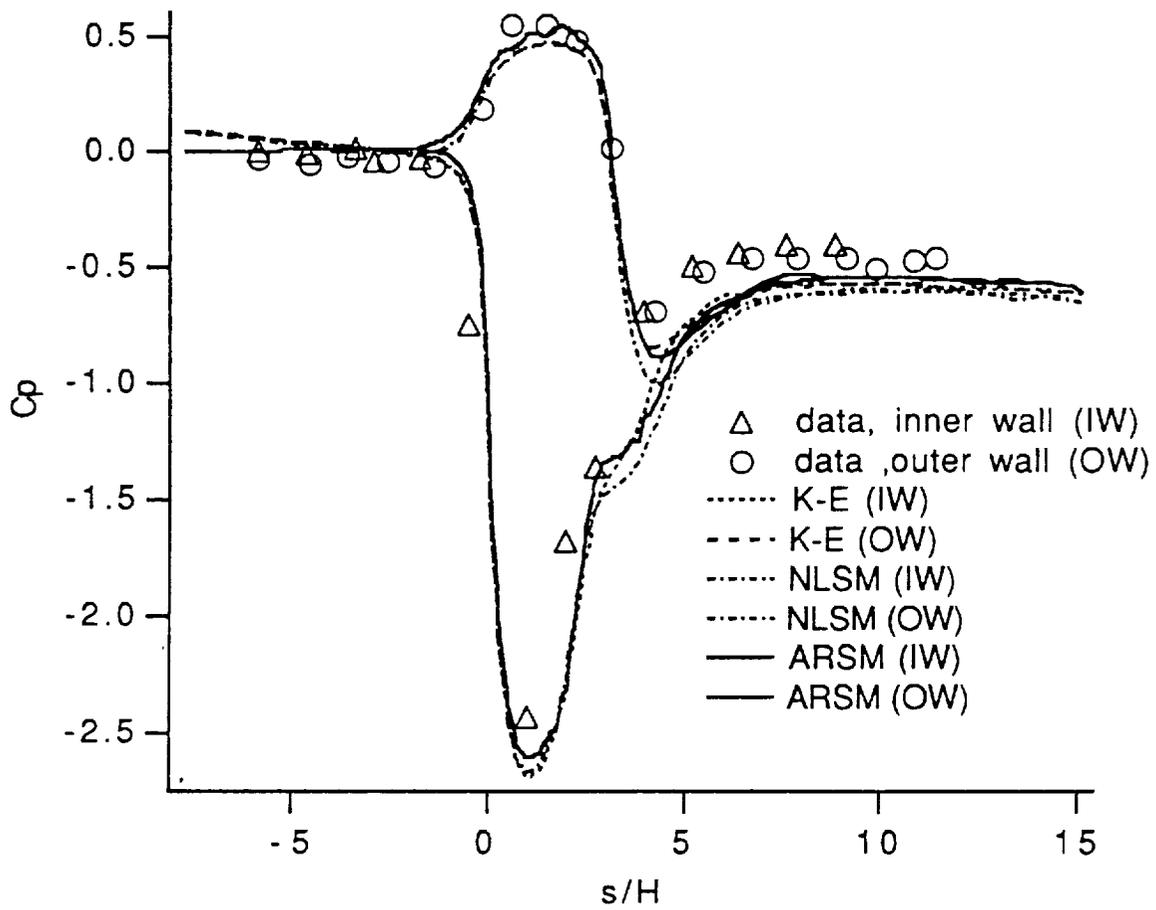


Fig.1 Static pressure coefficient on turnaround duct inner and outer walls

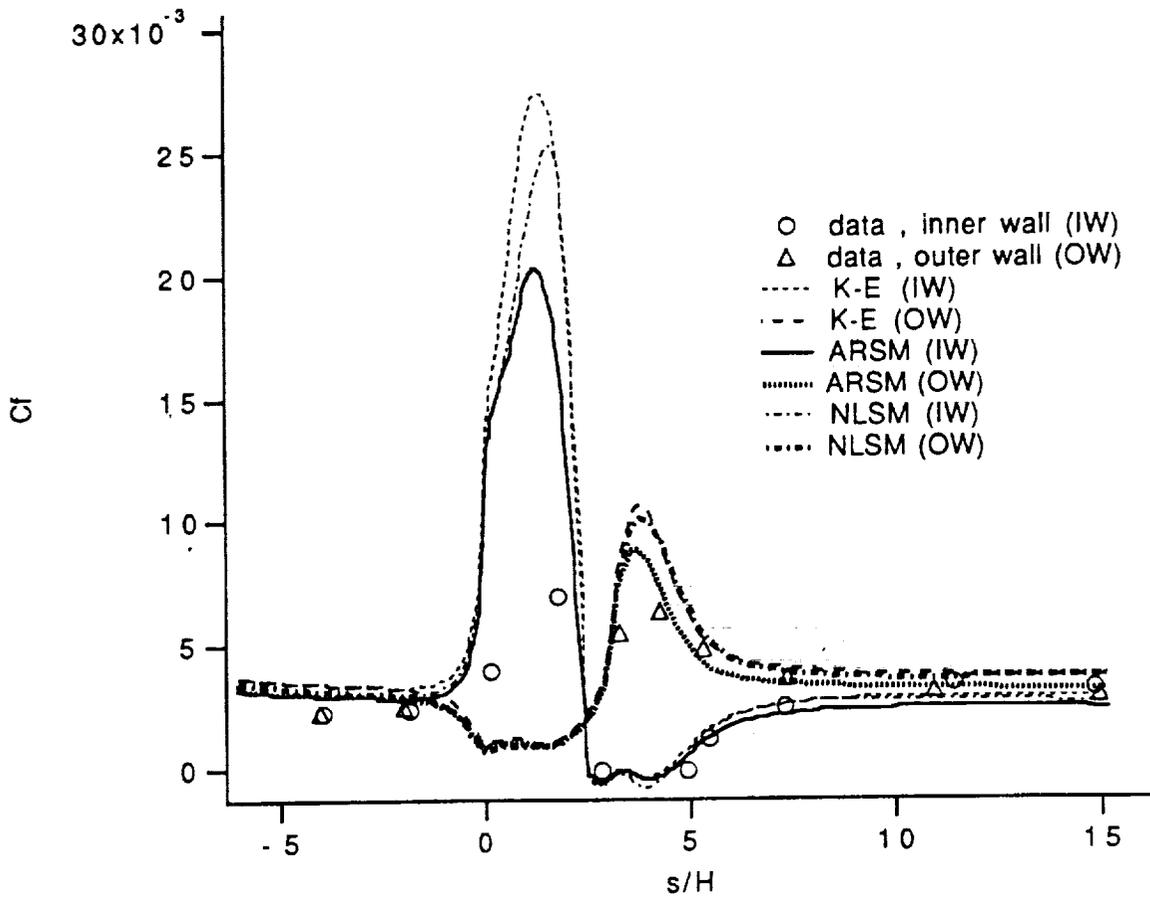


Fig. 2 Skin friction coefficient on turnaround duct inner and outer walls

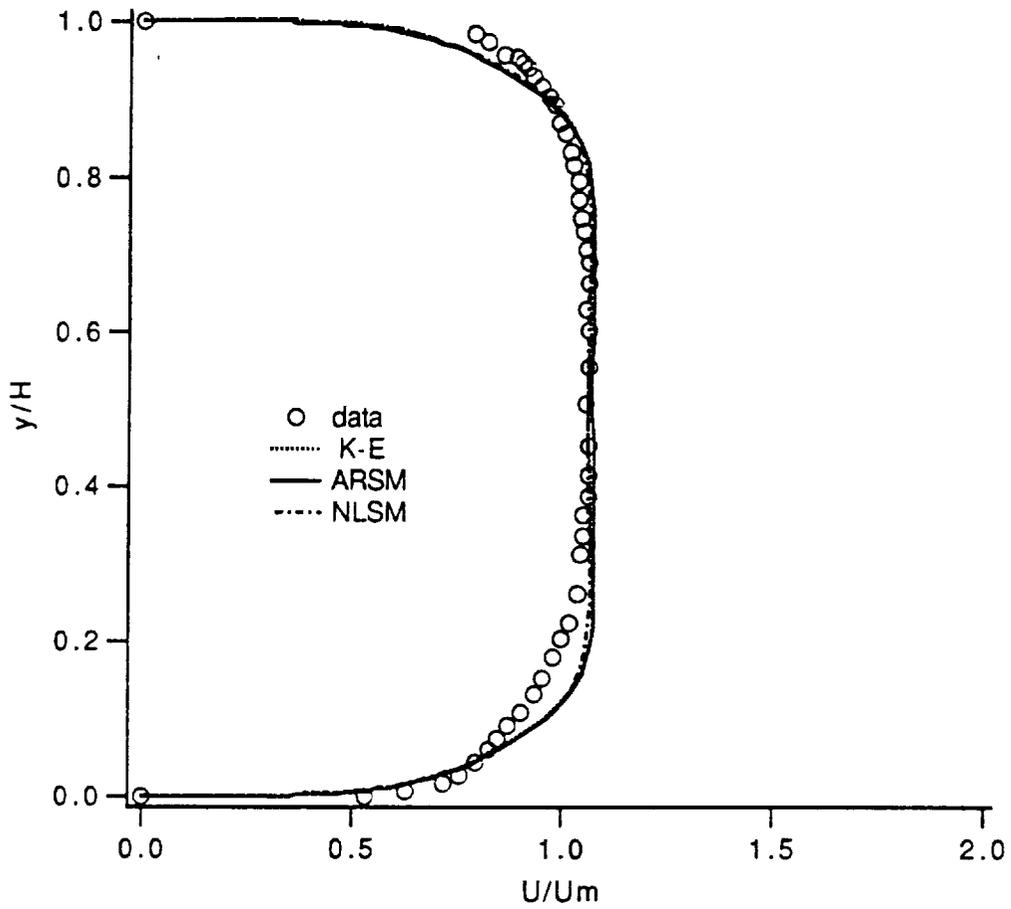


Fig.3 (a) Longitudinal velocity in turnaround duct, $x/H=-4$

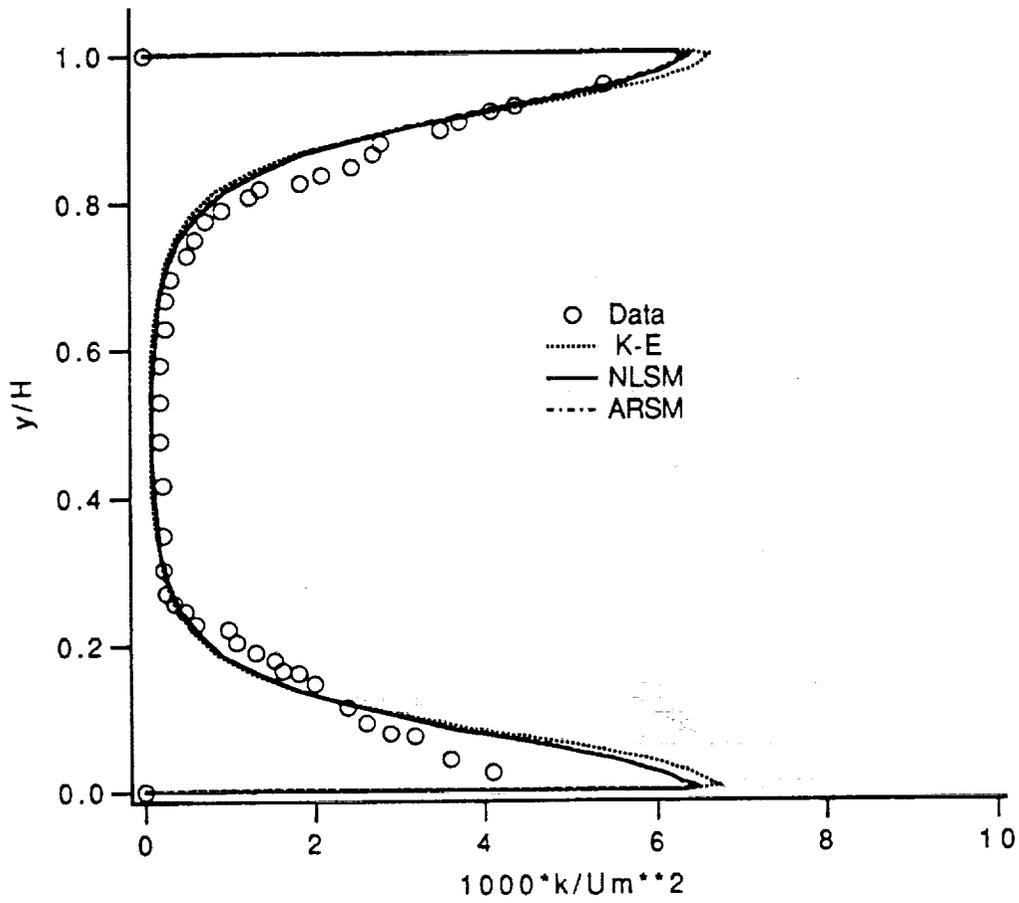
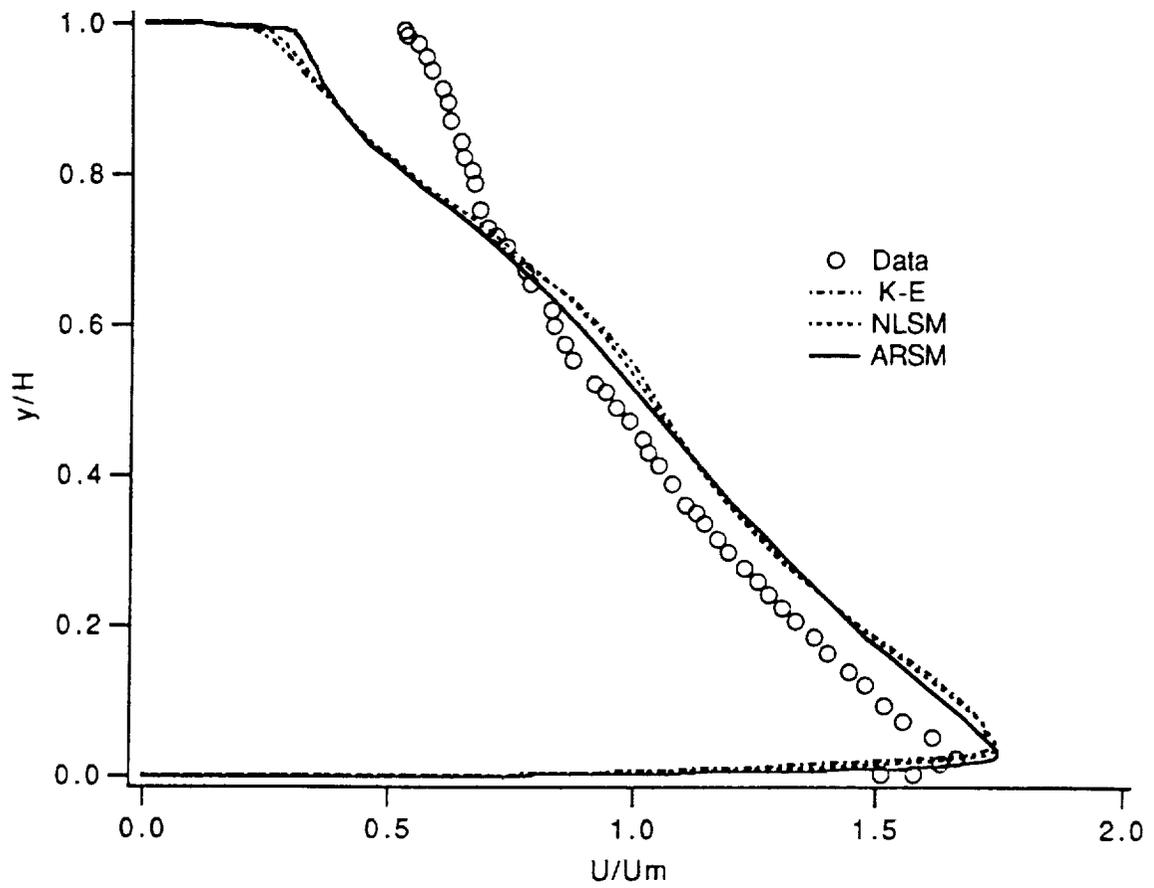


Fig.3(b) Turbulent kinetic energy profile



**Fig.5(b) Longitudinal velocity in
turnaround duct, theta=90 deg.**

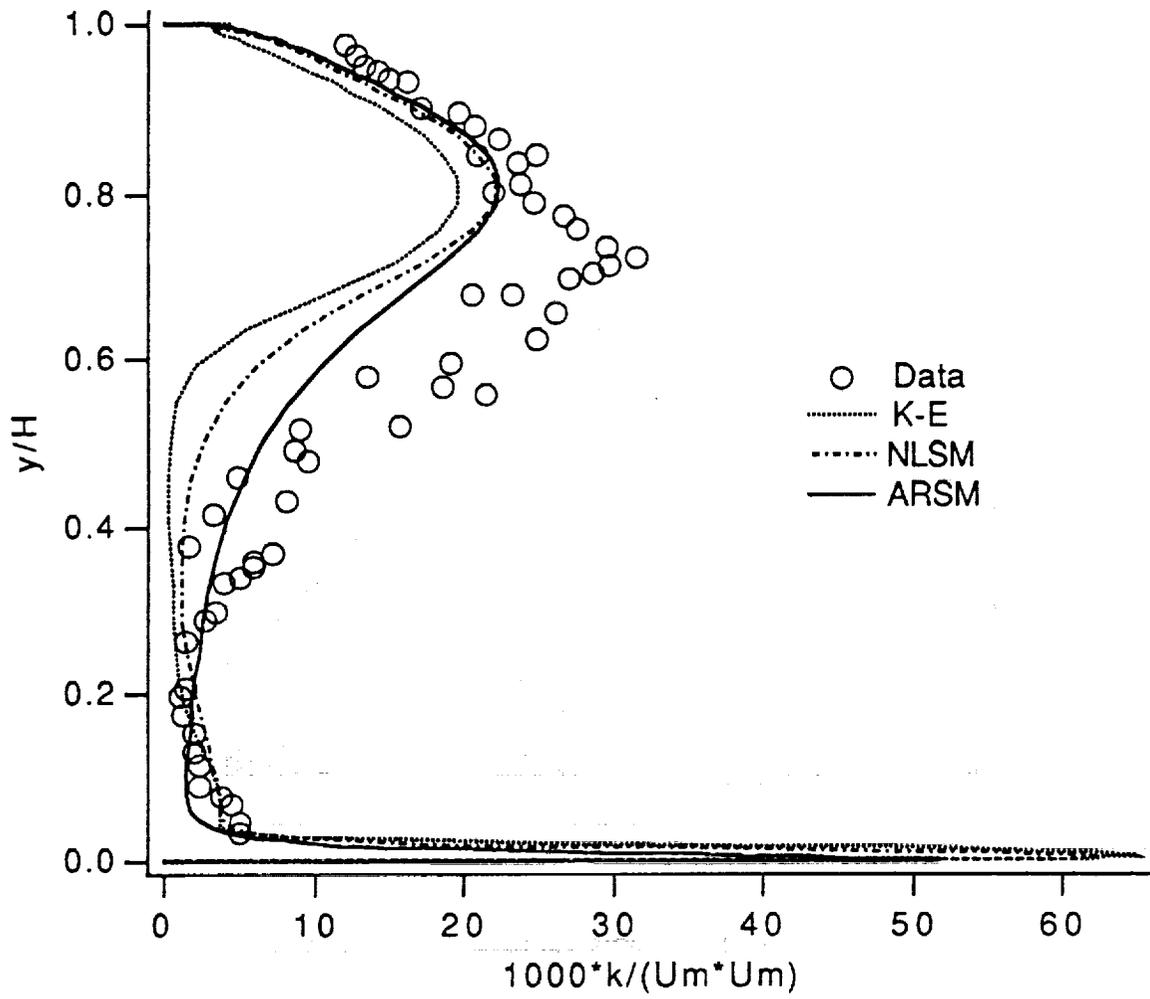


Fig.5(c) Turbulent kinetic energy

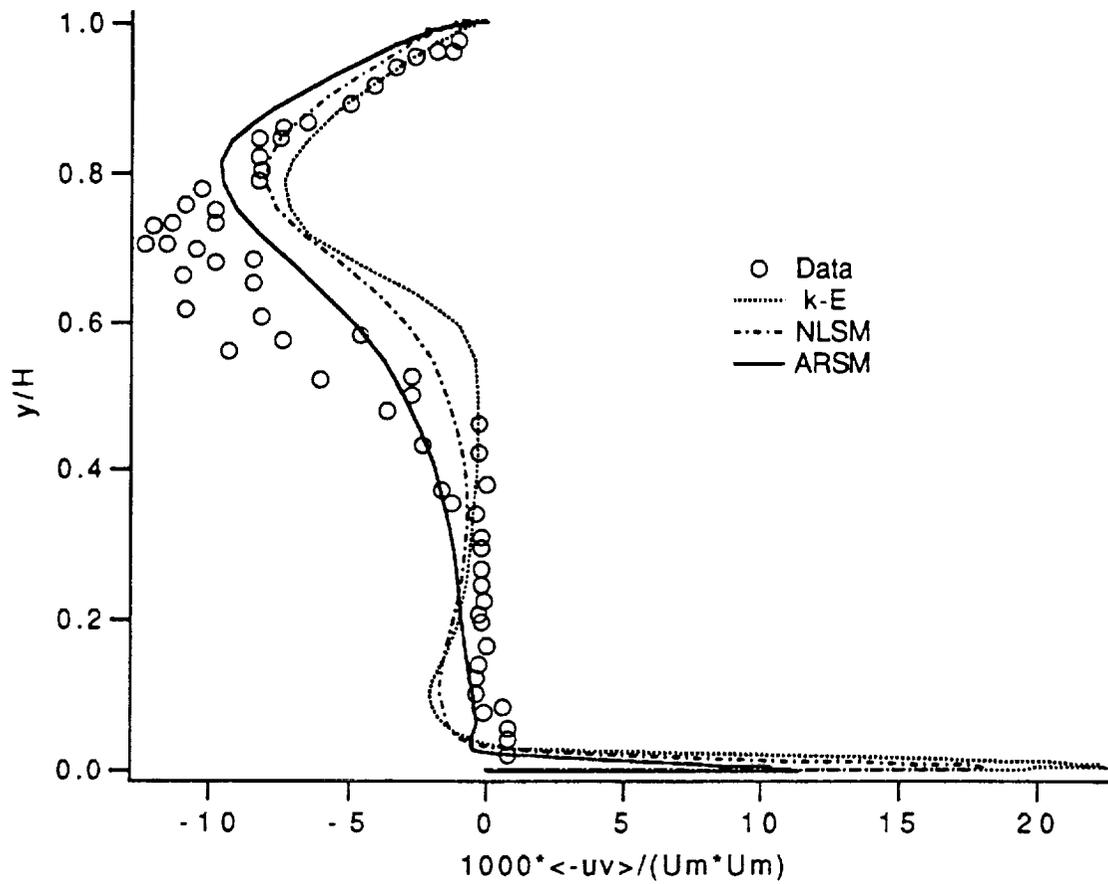


Fig.5(d) Turbulent shear stress

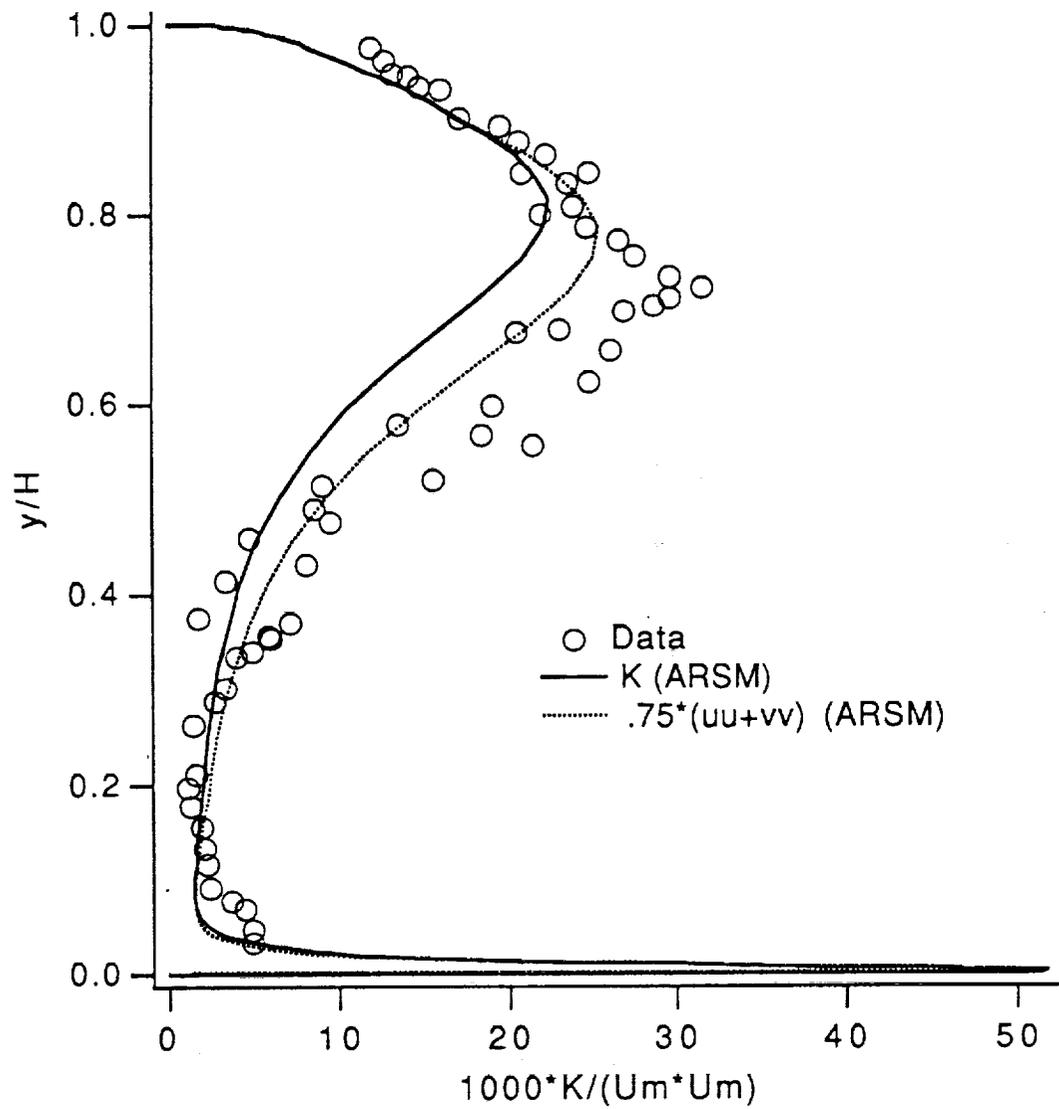


Fig.5 (e) Comparison of K and $0.75 \cdot (uu+vv)$

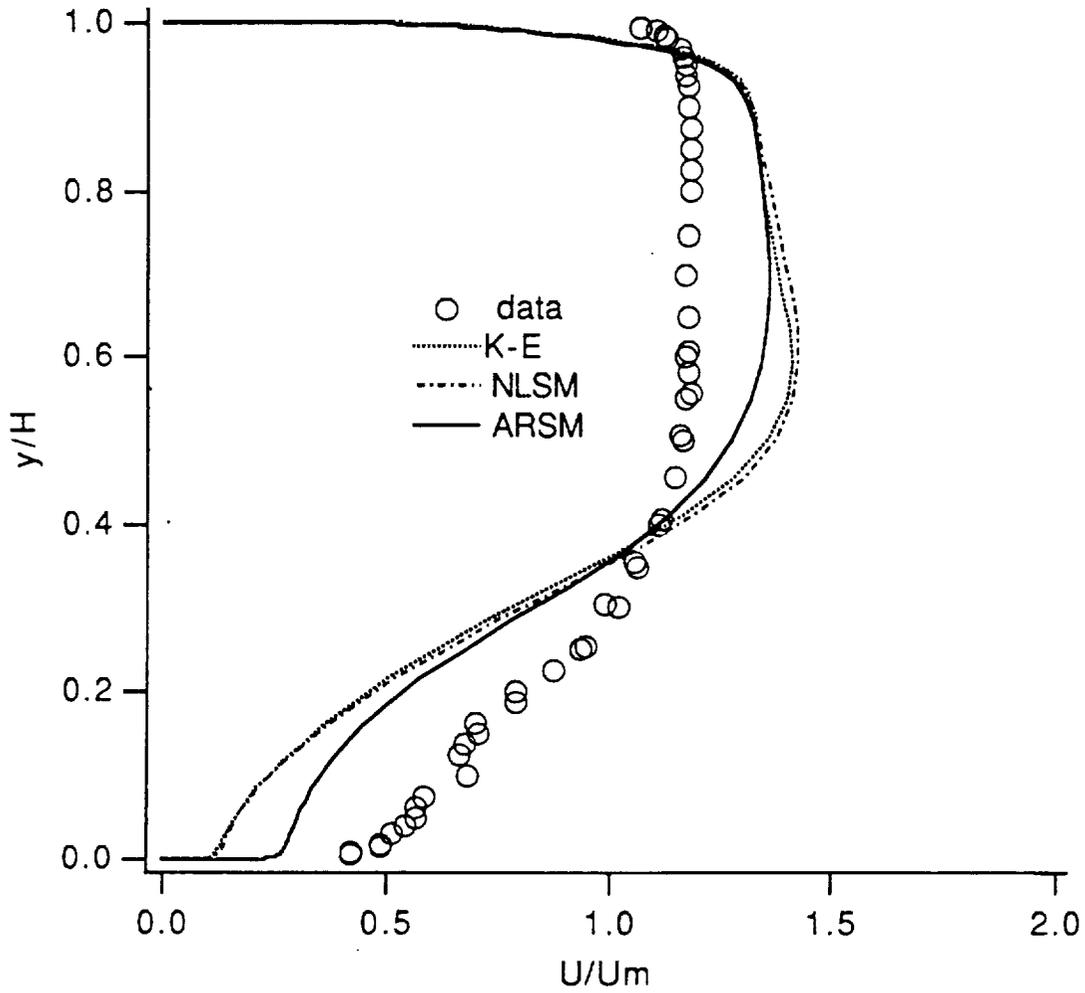


Fig.7(a) Longitudinal velocity in turnaround duct, $x/H = 2$ (downstream of turn)

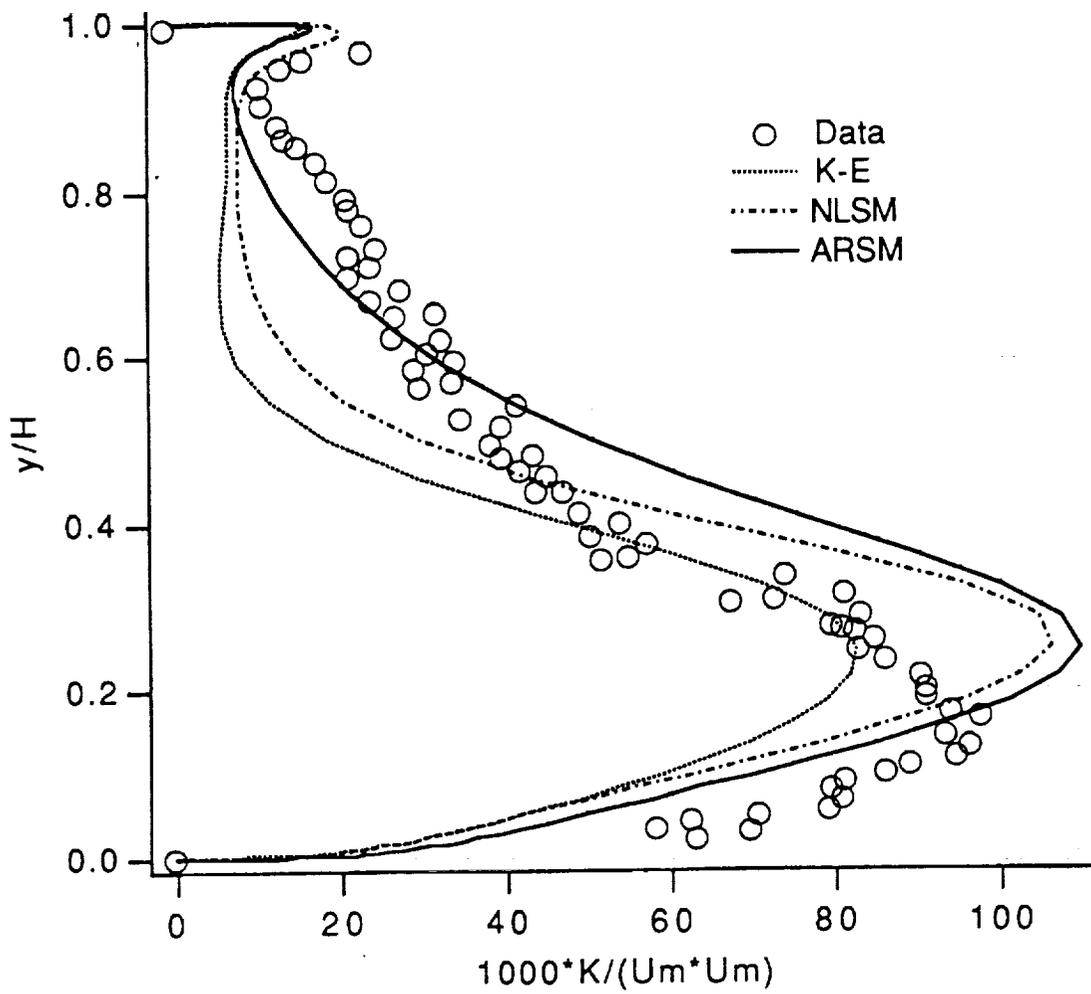


Fig.7(b) Turbulent kinetic energy

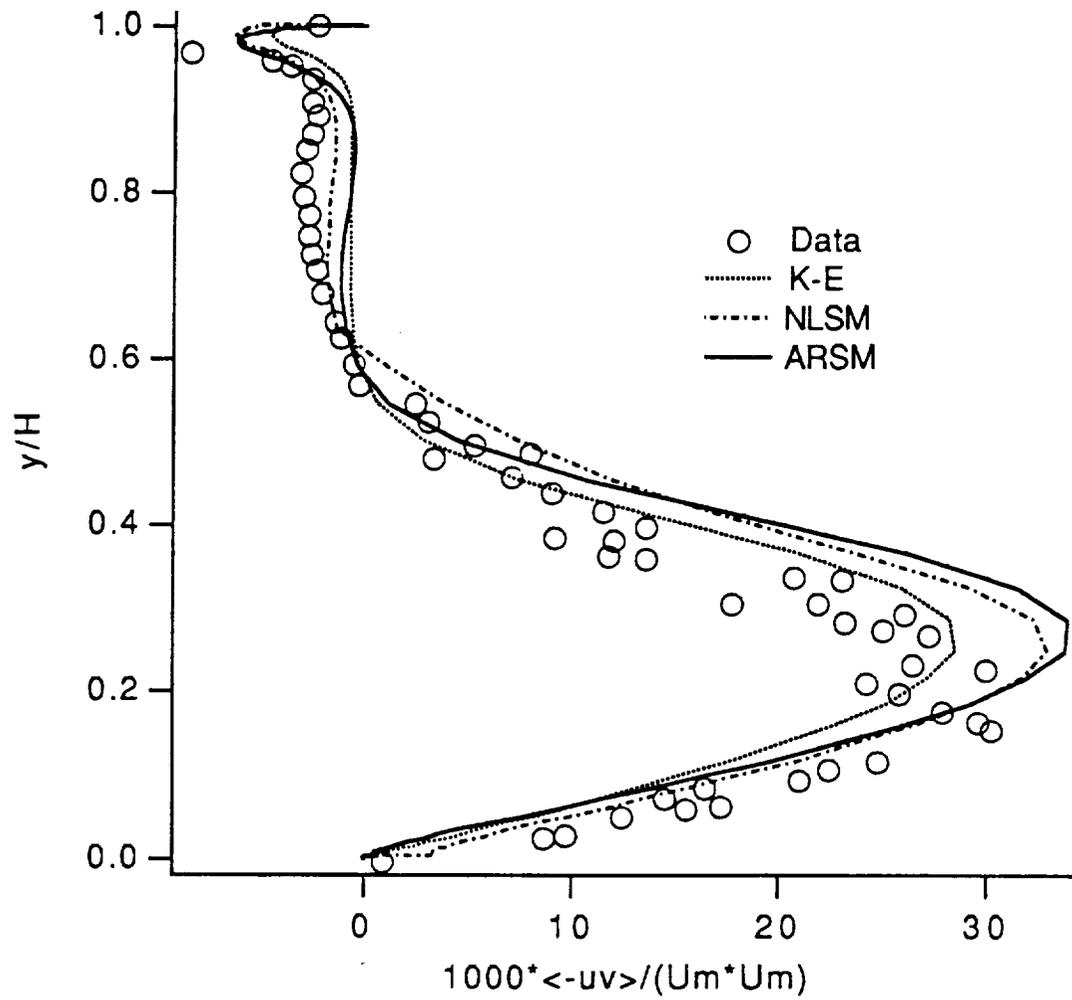


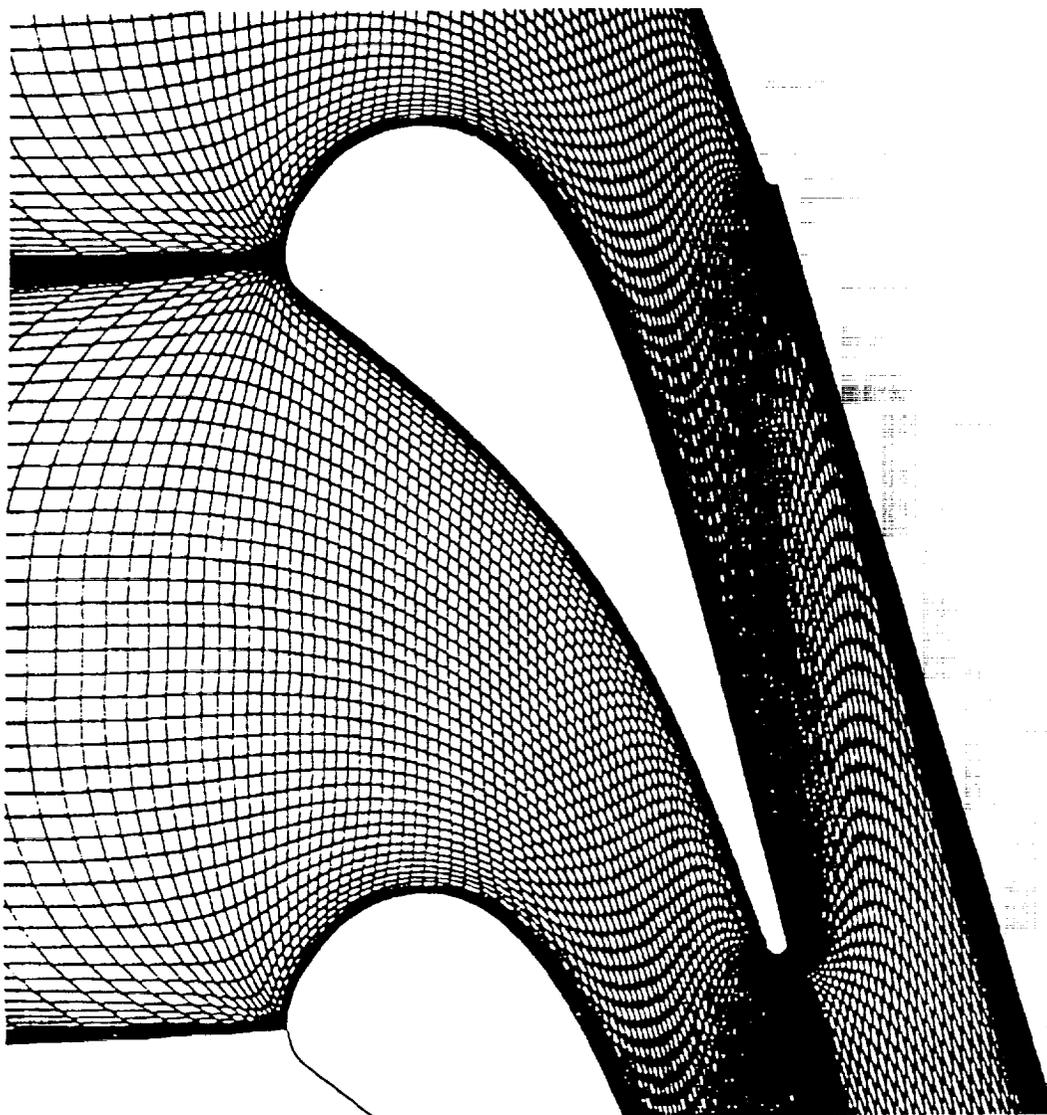
Fig. 7(c) Turbulent shear stress

VKI Turbine Nozzle Guide Vane Cascade

* Measurement by Arts et al (1990) at Von Karman Institute

* $M(\text{inlet})=0.15$, $M(\text{outlet})=0.7$ to 1.11, $Re=0.5 - 2 \times 10^6$, $T_o=415\text{k}$, $T(\text{wall}) = 300\text{k}$

* Geometry and grid



3. Smoothing: Typical values for 2nd and 4th order dissipation taken as 2%-3% and 3%-4% for the turbine cascade computations.

4. Table: Computed cases :

cases	Mur228	Mur224	Mur239
P_{o1}(bar)	0.915	0.909	3.387
T_{o1} (K)	403	403	412
Re,₂	0.6E+6	0.6E+6	2.1 E+6
Tu_{in}(%)	1	6	6

T(wall) = 300k for all these cases.

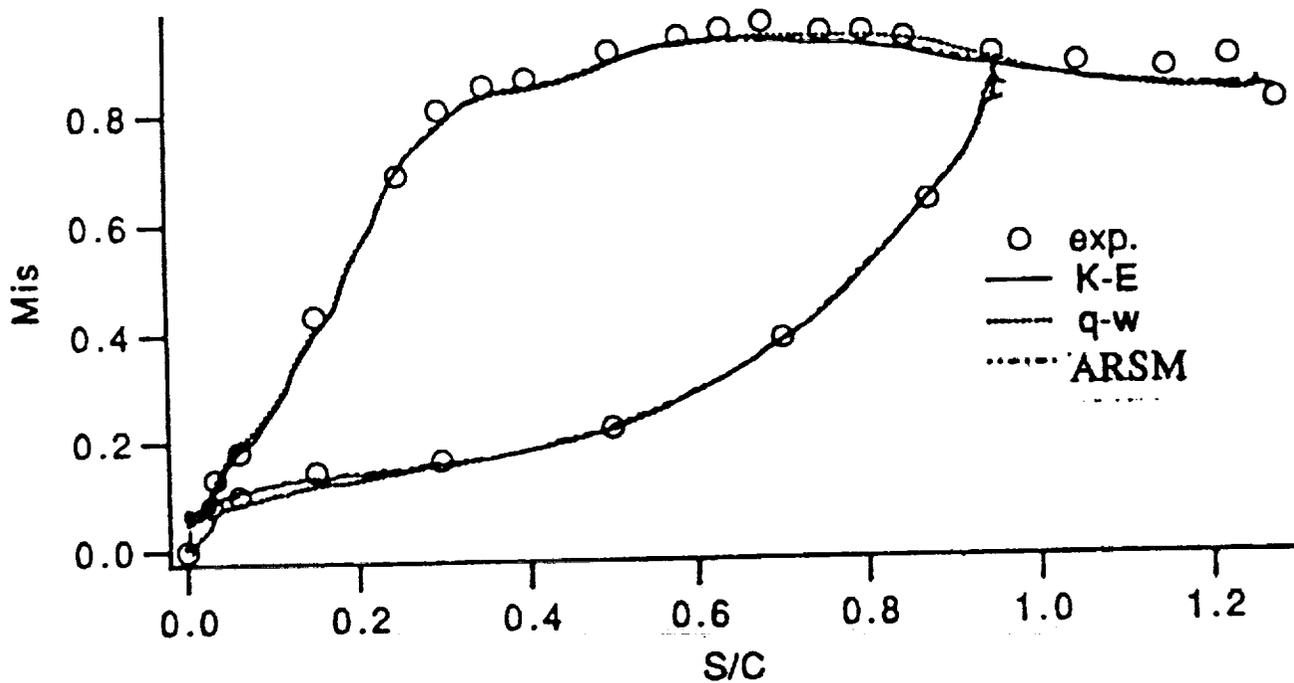


Fig. 9 Blade isentropic Mach number distribution for Mur043 ($P_{o1} = 1.435$ bar, $M_2^{is} = 0.93$, Tu (inlet) = 1%)

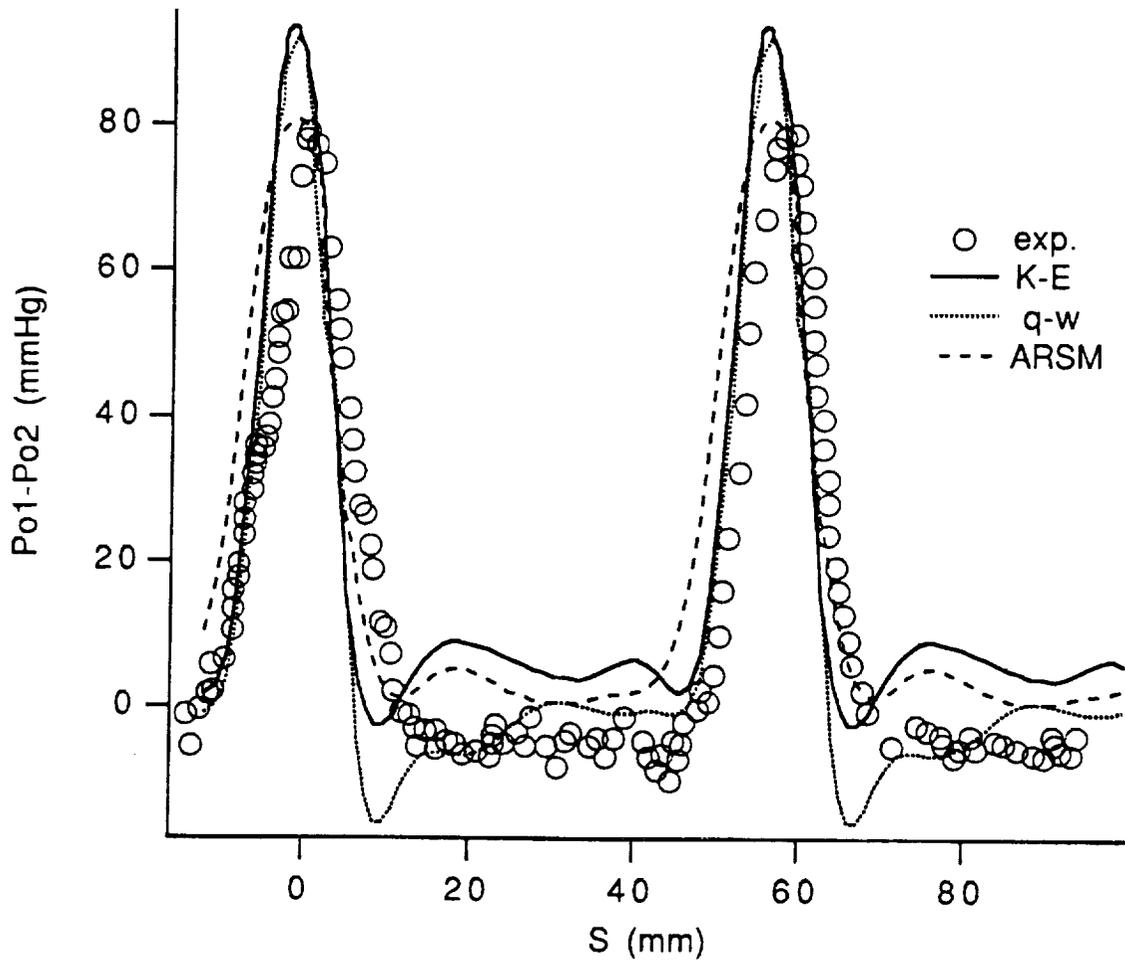


Fig.10 Computed and measured wakes for mur043

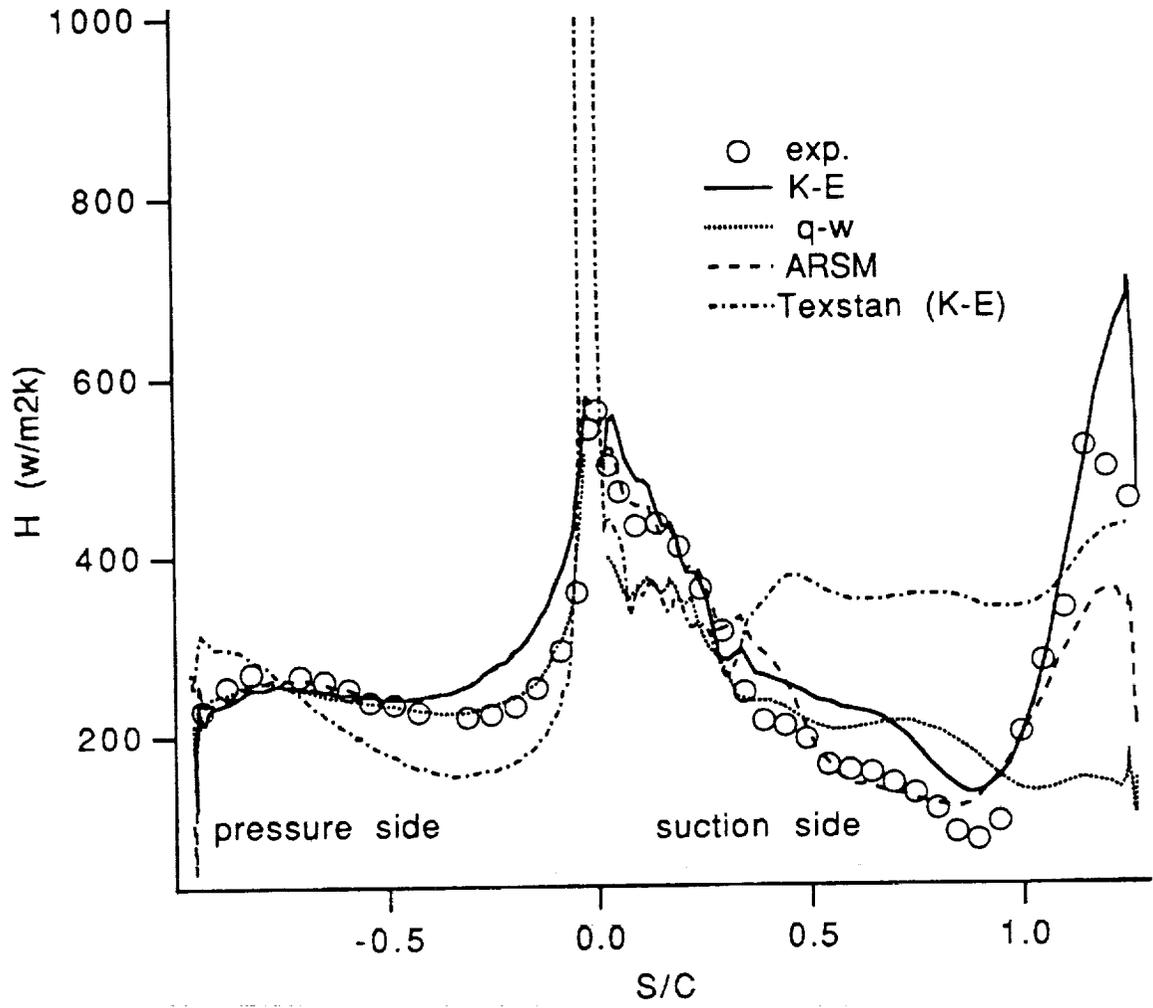


Fig. 11 Heat Transfer Prediction for Mur224

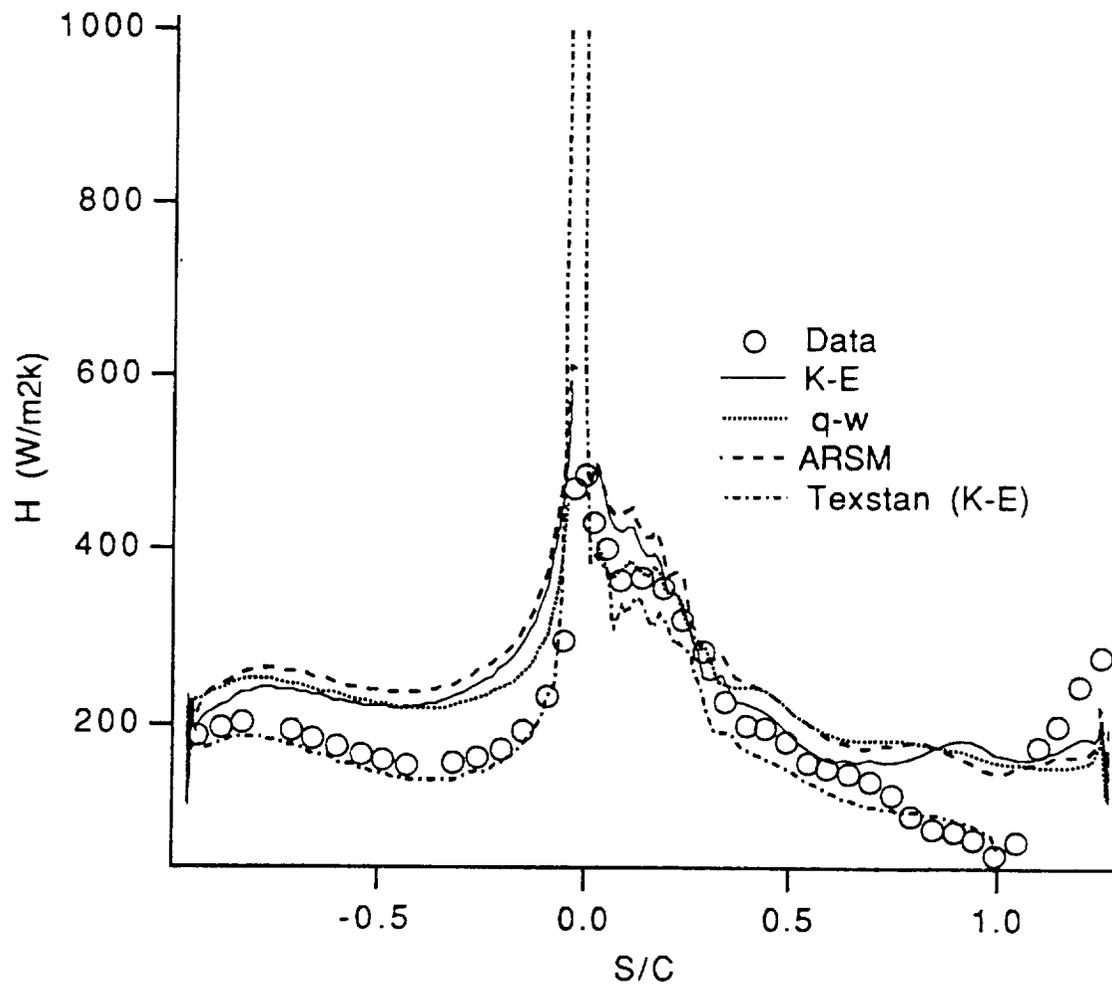


Fig. 12 Heat transfer prediction for Mur228

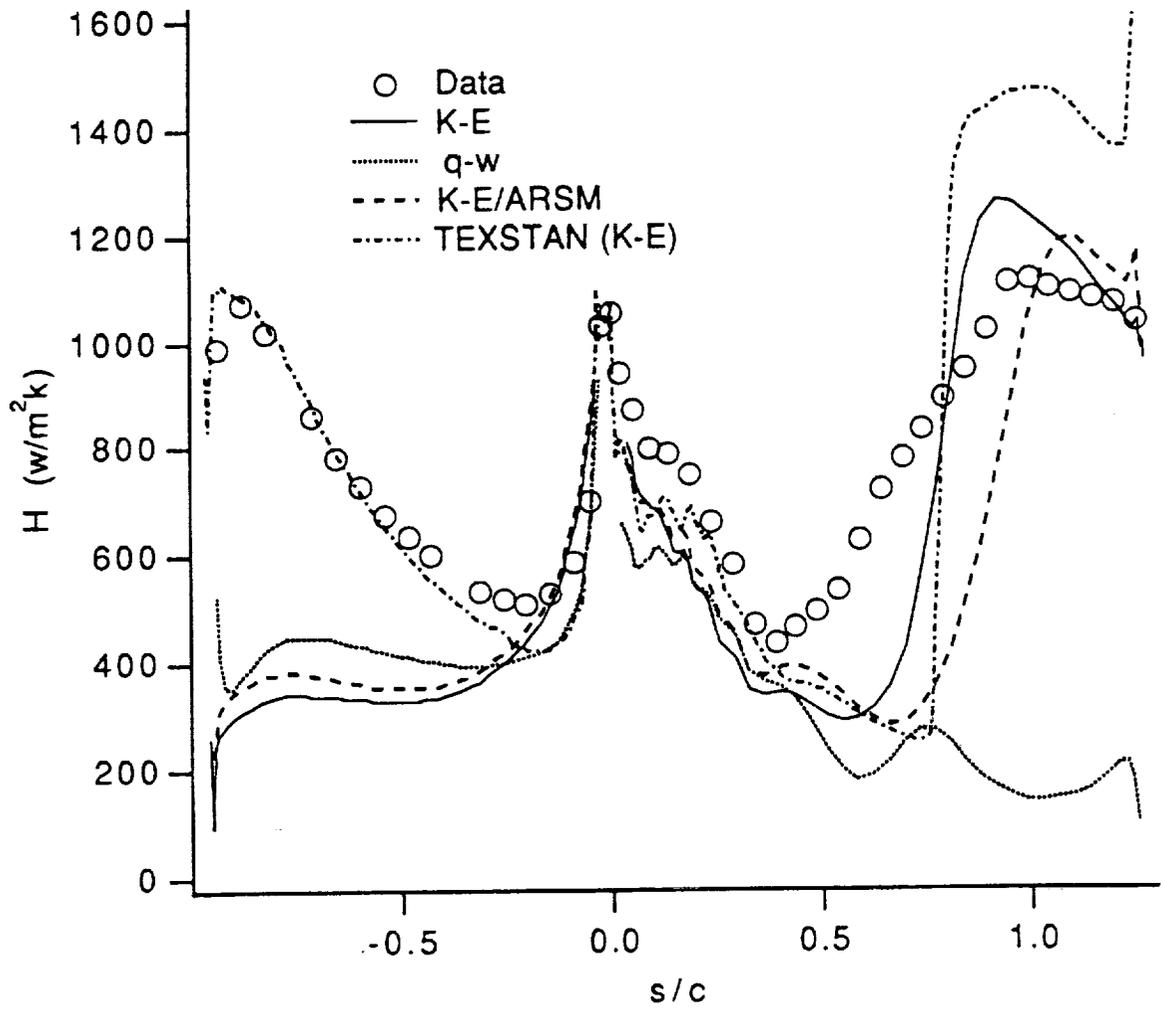


Fig.13 Heat Transfer for Mur239

CONCLUSIONS

*A two-dimensional Navier-Stokes code has been developed using an explicit Runge-Kutta method incorporating the ARSM model, NLSM model, Chien's LRN k - ϵ model and Coakley's LRN q - ω model.

*The surface pressure distributions and wake profiles of a transonic turbine cascade were predicted well by the k - ϵ , ARSM and q - ω models. The heat transfer on suction surfaces were predicted well by the k - ϵ and ARSM models.

*The heat transfer on pressure side for one case (MUR239) was underpredicted. This was caused by the underprediction of mainstream turbulence level, which strongly influences the transition location.

*The boundary layer code predicts the heat transfer on pressure surface well, but it

does not capture the transition on suction surfaces for all the 3 cases.

*The wall damping function (f_{μ}) in Chien's model was modified to yield improved prediction for flow under adverse pressure gradient.

*For TAD flow, good predictions have been obtained for the surface pressure distribution and skin friction coefficients. The ARSM model yields better prediction than NLSM and k - ϵ models, for both the mean and the turbulence quantities.

*The near wall "echo" term ϕ_{ijw} is not correctly modelled for strongly curved flows, especially near the concave surface.

*For more accurate prediction of strongly curved TAD flows, it may be necessary to use existing RSM, including the modeling for the wall region.