Sensitivity of Combustion-Acoustic Instabilities to Boundary Conditions for Premixed Gas Turbine Combustors

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ABSTRACT
Premixed combustors, which are being considered for low NOx engines, are susceptible to instabilities due to feedback between pressure perturbations and combustion. This feedback can cause damaging mechanical vibrations of the system as well as degrade the emissions characteristics and combustion efficiency. In a lean combustor instabilities can also lead to blowout. A model was developed to perform linear combustion-acoustic stability analysis using detailed chemical kinetic mechanisms. The Lewis Kinetics and Sensitivity Analysis Code, LSENS, was used to calculate the sensitivities of the heat release rate to perturbations in density and temperature. In the present work, an assumption was made that the mean flow velocity was small relative to the speed of sound. Results of this model showed the regions of growth of perturbations to be most sensitive to the reflectivity of the boundary when reflectivities were close to unity.

INTRODUCTION
Premixed flames are being considered for future low NOx combustors. They are susceptible to instabilities due to feedback between pressure perturbations and combustion. This coupling can cause damaging mechanical vibrations of the system as well as degradation of the emissions characteristics and combustion efficiency. In a lean flame, fluctuations can also lead to blowout. Models have been developed to predict linear stability of interactions between combustion and pressure oscillations [e.g., 1-4]. Typically, either a single-step reaction mechanism or an empirical expression is used to describe the combustion chemistry. In the present work we have included detailed chemical reaction mechanisms in order to provide a more realistic description of the chemical kinetics.

Boundary conditions are obviously important since an acoustic-combustion instability occurs when more acoustic energy is generated in the combustor than is dissipated or emitted out through the boundaries. The boundary conditions are often given as an acoustic impedance of the boundary, perhaps as pure reflection or a choked nozzle. Although some work has been done on transmission of combustion noise out of an engine [5], the boundary conditions for real combustors, operating at high pressures and temperatures, are difficult to measure. The purpose of the present work is to determine the effects of uncertainties in boundary conditions on the prediction of combustion-acoustic instabilities.
MODEL

A low Mach number model, which assumes that the mean flow velocity is much less than the speed of sound, was developed to study combustion-acoustic interactions. The model was constructed by linearly perturbing the compressible, one-dimensional, inviscid flow equations with heat release. Solution variables were decomposed into mean (steady) and fluctuating components,

\[ p = \overline{p} + \hat{p}, \quad T = \overline{T} + \hat{T}, \quad Q = \overline{Q} + \hat{Q}, \quad \text{etc.} \]

where the overbar and hat denote mean and fluctuating quantities, respectively, \( p \) is the pressure, \( T \) is the temperature and \( Q \) is the heat release rate per unit length. The fluctuating components were further separated into (1) a function of time \( (e^{ivt}) \) to represent the frequency and growth/decay rate of oscillations and (2) a function of position, \( x \). The latter function was itself decomposed into real and imaginary parts to represent the phase and magnitude of the oscillations at each spatial location, e.g.,

\[ \hat{p} = [P_1(x) + iP_2(x)]e^{ivt}, \]

where the functions \( P_1(x) \) and \( P_2(x) \) are determined by integrating the perturbed equations (equations 4-7, described below). The perturbation in heat release rate, \( \hat{Q} \), is the energy source for amplifying pressure waves as they propagate through the combustion zone.

The mean-flow quantities were obtained by solving the first-order, nonlinear ordinary differential equations (ODEs) describing steady, one-dimensional, inviscid chemically reacting flow, which can be written as:

\[ \frac{d\mathbf{y}}{d\xi} = \mathbf{f}(\mathbf{y}), \]

where \( \mathbf{y} \) is the vector of dependent variables and \( \xi \) the independent variable—time or distance. The dependent variables are the species specific mole numbers \( \{\sigma_i\} \) (i.e., moles of species \( i \) per unit mass of mixture), temperature, density, \( \rho \), and velocity, \( u \). The chemistry was modeled with a modified version of LSENS, the Lewis Kinetics and Sensitivity Analysis Code [6,7], which uses the backward differentiation formula (BDF) method as implemented in LSODE [8,9].

Once the mean flow equations are solved, the perturbed quantities are found by integrating equations 4-7:

\[ \frac{d\hat{u}}{dx} = -\frac{i\omega \hat{p}}{\rho} - \frac{\hat{u}}{\rho} \frac{d\hat{p}}{dx} \]

\[ \frac{d\hat{p}}{dx} = -i\omega \rho \hat{u} \]

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Here \( c_p \) is the constant-pressure mixture mass-specific heat, \( A \) the flow cross-sectional area and \( R \) the universal gas constant. Equations 4-7 were derived by perturbing the continuity, momentum and energy conservation equations and the ideal gas equation of state.

Equations 4-7 can be solved (for real and imaginary components of perturbed quantities), given an upstream value of the perturbed quantities and a model of the perturbed heat release rate as a function of other perturbed quantities. The heat release rate was modeled as follows. The heat release rate per unit length at a given location is a function of the temperature, density, and composition at that point:

\[
Q = \overline{Q} + \hat{Q} = f(T, \rho, \sigma, i = 1, 2, 3, \ldots, NS)
\]

where \( NS \) is the total number of chemical species (reacting and inert). The perturbation in heat release rate is given by

\[
\hat{Q} = \left( \frac{\partial Q}{\partial T} \right) \hat{T} + \left( \frac{\partial Q}{\partial \rho} \right) \hat{\rho}
\]

A benefit of this approach is that for linear perturbations \( \partial Q/\partial T \) and \( \partial Q/\partial \rho \) are functions of position only, and not time. Therefore, they only need to be calculated once at each \( x \) location for a given mean flow. These partial derivatives are determined using a modified version of LSENS. In addition to solving the ODE’s for the mean reacting flow, LSENS generates the linear sensitivity coefficients \( \{ \partial y/\partial \eta_i \} \) and \( \{ \partial y/\partial \eta_j \} \), where \( \eta_i \) is a variable of interest (e.g., a rate coefficient parameter or an initial condition value). This information can be used to compute the sensitivity coefficients \( \{ \partial q/\partial \eta_i \} \), where \( q \) is the heat release rate per unit mass:

\[
\hat{q} = -\sum_{i=1}^{NRS} \hat{\alpha}_i h_i.
\]

Here \( NRS \) is the number of reacting species and \( h_i \) is the molar-specific enthalpy of species \( i \). In the present model \( \eta_i = T \) and \( \eta_j = \rho \). If perturbations in fuel injection was added to the model, \( \eta \)'s would need to be added to represent sensitivities to upstream fuel concentrations. The partial derivative \( \partial \hat{q}/\partial \eta_i \) is
\[
\frac{\partial q_i}{\partial n_j} = -\sum_{i=1}^{NRS} \frac{\partial s_i}{\partial n_j} - \sum_{i=1}^{NRS} \sigma_i c_{p,i} \frac{\partial T}{\partial n_j},
\]

where \(c_{p,i}\) is the constant-pressure molar-specific heat of species \(i\). This is used to calculate the partial derivatives in equation 9, which is, itself, used to model the perturbation in heat release rate in equation 6.

RESULTS AND DISCUSSION

The model was used to predict regions of growth and decay of disturbances in a constant area, premixed combustor. Various boundary conditions and flow conditions were considered. A lower reflectivity resulted in a more stable system, because more acoustic energy was lost from the boundaries of the combustor. The level of the reflectivity where the system became unstable is a function of frequency, geometry, and flow conditions.

Many different situations were modeled in the present study. The specific frequencies and sizes of the regions of disturbance growth varied with flow conditions, however the general trend was consistent. The following is an example to illustrate the results. Calculations were performed for a 0.8 meter long, premixed combustor with propane as the fuel. The chemical kinetic mechanism of Jachimowski [10] was used. The equivalence ratio was 0.8 and the temperature of the incoming fuel/air mixture was 1000 K.

In this example, waves propagating both in the flow direction and reverse direction were considered. These were related at the boundaries (upstream and downstream) by a reflectivity. Various reflectivities were considered. In general, a phase shift upon reflection was modeled in some cases as well, but for this example, no phase shift was included. In addition to reflectivity, the phase difference between the velocity and pressure perturbations at the upstream boundary was varied. For a simple plane wave, the pressure and density would be in phase, but in the presence of combustion they could be out of phase. In fact, this phase difference between velocity and pressure has a significant effect on whether a disturbance grows or decays. In this case, there were no frequencies for which disturbances grew for reflectivities below 0.96. However, some other situations resulted in disturbance growth for reflectivities as low as 0.85.

Results are given as regions of growth of disturbances as a function of frequency and the phase difference between velocity and pressure at the upstream boundary. Figures 1 and 2 show the regions of growth of disturbances for reflectivities of 0.97 and 0.98 respectively. There is a noticeable increase in the size of the growth regions for an increase in reflectivity of 0.01; for a reflectivity of 0.97 the area of the regions of growth was 4.7% of the total area of the enclosed rectangle. When the reflectivity was 0.98, the regions of growth represented 10.4% of the total area of the rectangle. However, the difference in areas of growth are more pronounced when increasing the reflectivity from 0.99 to 1.0 (from 20.4% to 55.2% of the total area). This trend was seen for all cases we considered; the regions of growth/decay are more sensitive to the reflectivity as the reflectivity approaches unity.
Figure 1 Regions of growth of disturbances for a reflectivity of 0.970

Figure 2 Regions of growth of disturbances for a reflectivity of 0.980

Figure 3 Regions of growth of disturbances for reflectivity of 0.990

Figure 4 Regions of growth of disturbances for a reflectivity of 1.000
REFERENCES


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