A GENERALIZED EULERIAN-LAGRANGIAN ANALYSIS, WITH APPLICATION TO LIQUID FLOWS WITH VAPOR BUBBLES†

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ABSTRACT

Under a NASA MSFC SBIR Phase II effort an analysis has been developed for liquid flows with vapor bubbles such as those in liquid rocket engine components. The analysis is based on a combined Eulerian-Lagrangian technique, in which Eulerian conservation equations are solved for the liquid phase, while Lagrangian equations of motion are integrated in computational coordinates for the vapor phase. The novel aspect of the Lagrangian analysis developed under this effort is that it combines features of the so-called particle distribution approach with those of the so-called particle trajectory approach and can, in fact, be considered as a generalization of both of those traditional methods. The result of this generalization is a reduction in CPU time and memory requirements. Particle time step (stability) limitations have been eliminated by semi-implicit integration of the particle equations of motion (and, for certain applications, the particle temperature equation), although practical limitations remain in effect for reasons of accuracy. The analysis has been applied to the simulation of cavitating flow through a single-bladed section of a labyrinth seal. Models for the simulation of bubble formation and growth have been included, as well as models for bubble drag and heat transfer. The results indicate that bubble formation is more or less "explosive": for a given flow field, the number density of bubble nucleation sites is very sensitive to the vapor properties and the surface tension. The bubble motion, on the other hand, is much less sensitive to these properties, but is affected strongly by the local pressure gradients in the flow field. In situations where either the material properties or the flow field are not known with sufficient accuracy, parametric studies can be carried out rapidly to assess the effect of the important variables. Future work will include application of the analysis to cavitation in inducer flow fields.

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OBJECTIVE:

DEVELOPMENT OF AN ANALYSIS FOR LIQUID FLOWS WITH VAPOR BUBBLES (SUCH AS THOSE IN BEARINGS, SEALS, AND PUMPS) FOR USE IN DESIGN OF LIQUID ROCKET ENGINE COMPONENTS

APPROACH:

USE A COMBINED EULERIAN-LAGRANGIAN ANALYSIS

- CONTINUOUS (LIQUID) PHASE TREATED BY SOLVING EULERIAN CONSERVATION EQUATIONS (N-S EQUATIONS) WITH DISCRETE PHASE SOURCE TERMS
- DISCRETE (VAPOR BUBBLE) PHASE TREATED BY INTEGRATING LAGRANGIAN EQUATIONS OF MOTION IN COMPUTATIONAL COORDINATES

LAGRANGIAN ANALYSIS - METHODOLOGY

- GENERALIZATION OF PARTICLE TRAJECTORY AND PARTICLE DISTRIBUTION MODELS
  ⇒ REDUCED CPU TIME AND STORAGE REQUIREMENTS
- STABLE INTEGRATION OF LAGRANGIAN EQUATIONS OF MOTION IN COMPUTATIONAL COORDINATES
  ⇒ NO PARTICLE TIME STEP LIMITATION OTHER THAN FOR ACCURACY
LAGRANGIAN ANALYSIS - FEATURES

- LAGRANGIAN DISCRETE PHASE ANALYSIS INTERFACES WITH EULERIAN CONTINUOUS PHASE ANALYSIS VIA ONE SINGLE SUBROUTINE

- GENERALIZED ARRAY ADDRESSING AND INDEX / VARIABLE NUMBERING

⇒ LAGRANGIAN ANALYSIS CAN BE HOOKED UP TO A VARIETY OF EULERIAN CODES ("FLOW SOLVERS")

- CODE IS MODULAR W.R.T. PHYSICAL MODELS

APPLICATION TO VAPOR BUBBLES

- MODELS NEEDED FOR BUBBLE FORMATION, GROWTH / COLLAPSE AND MOTION

- CRITICAL PHYSICAL PARAMETERS:
  - SURFACE TENSION
  - VAPOR PRESSURE - TEMPERATURE RELATION (CLAUSIUS-CLAPEYRON EQUATION)
**BUBBLE FORMATION**

- NUCLEATION DUE TO LOCAL P DROP;
  USE HOMOGENEOUS NUCLEATION MODEL
  (KATZ AND BLANDER)

\[
J \left( \frac{\text{NUCLEI}}{\text{M}^3\text{S}} \right) = n \left( \frac{2\sigma}{\pi M} \right)^{\frac{1}{2}} \exp\left[ \frac{-16\pi \sigma^3}{3kT(P_e - P_L)^2} \right]
\]

- J : NUMBER OF NUCLEATION SITES PER UNIT VOLUME AND TIME
- \(P_e\) : EQUILIBRIUM VAPOR PRESSURE
- \(P_L\) : PRESSURE IN LIQUID
- \(T\) : TEMPERATURE OF THE LIQUID
- \(\sigma\) : SURFACE TENSION
- \(k\) : BOLTZMANN CONSTANT
- \(M\) : MOLECULAR WEIGHT
- \(n\) : MOLECULAR NUMBER DENSITY IN LIQUID

**BUBBLE GROWTH**

- AFTER THE INITIAL GROWTH PERIOD (WHICH IS INERTIA CONTROLLED) THE PRESSURE INSIDE THE BUBBLE IS DETERMINED BY THE YOUNG-LAPLACE EQUATION

\[
P_e - P_L = \frac{2\sigma}{R}
\]

- THE BUBBLE TEMPERATURE IS ASSUMED TO BE EQUAL TO THE SATURATION TEMPERATURE AT THE PRESSURE \(P_e\)

- BUBBLE GROWTH IS DETERMINED BY THE ENERGY EQUATION
ENERGY EQUATION FOR BUBBLE

\[ q = m c_p \dot{\psi}_p + \dot{m} h_{fg} \]

- \( c_p \): VAPOR-PHASE SPECIFIC HEAT
- \( h_{fg} \): LATENT HEAT OF EVAPORATION
- \( \dot{m} \): BUBBLE MASS GROWTH RATE ("EVAPORATION RATE")
- \( \dot{\psi}_p \): RATE OF CHANGE OF BUBBLE TEMPERATURE

\[ \dot{\psi}_p = \left( \frac{dT}{dp} \right)_{sat} \vec{u} \cdot \nabla T \]

HEAT TRANSFER RATE TO BUBBLE

\[ q = Nu \pi D \chi (T - T_p) \]

- \( Nu \): NUSSELT NUMBER
- \( D \): BUBBLE DIAMETER
- \( \chi \): THERMAL CONDUCTIVITY OF THE LIQUID
- \( T \): LIQUID TEMPERATURE
- \( T_p \): BUBBLE TEMPERATURE
BUBBLE EQUATION OF MOTION

\[ \ddot{\mathbf{r}} = m \left( 1 + \frac{\Delta_A \rho}{2 \rho_p} \right) \frac{d^2 \mathbf{x}}{dt^2} \]

- \( \rho \): LIQUID DENSITY
- \( \rho_p \): VAPOR DENSITY
- \( \Delta_A \): ADDED MASS COEFFICIENT (=1 FOR STOKES FLOW)
- \( m \): BUBBLE MASS
- \( \mathbf{x} \): BUBBLE POSITION VECTOR
- \( \mathbf{F} \): FORCE ON BUBBLE
  - DRAG FORCE
  - BUOYANCY
  - PRESSURE GRADIENT

DRAG COEFFICIENT

- FOR BUBBLES
  \[ C_D \left( = \frac{2F_d}{\pi \rho a^2 u^2} \right) = \begin{cases} 14.9 / Re^{0.78} & Re > 2 \\ 16 / Re & Re < 2 \end{cases} \]
  GOOD FIT TO NUMERICAL SOLUTION
  COMPARES WELL WITH EXPERIMENTS, WIDELY USED

- FOR SOLID SPHERES
  \[ C_D = \frac{24}{Re} \left[ 1 + 0.15 Re^{0.687} \right] \]
  \[ = 0.438, \quad Re > 1000 \]

- NOTE: FOR SAME \( a \) AND \( u \), DRAG ON A BUBBLE IS SMALLER (FACTOR 6 FOR \( Re = 900 \))
COMPUTATIONAL TEST CASE

SINGLE BLADE LABYRINTH SEAL
APPROXIMATES ONE BLADE OF THE LOX PREBURNER
BOOST PUMP IMPELLER SEAL IN THE SSME

FLOW CONDITIONS:

- INLET PRESSURE: 3.67 MPa (532 psi)
- INLET TEMPERATURE: 130 K (234 °R)
- DENSITY: 1088 kg/m$^3$ (167.9 lb/ft$^3$)
- PRESSURE DROP: 1.47 MPa (213 psi)

GEOMETRY

SEAL GAP: $1.524 \times 10^{-4}$ m (0.006 inch)
NEAR-WALL RESOLUTION IN THE GAP = 1.0 \times 10^{-4} (\approx 0.015 \, \mu m)

80 \times 160 GRID
AXIAL VELOCITY

PRESSURE
NUCLEATION SITE DENSITY