DIMENSIONAL SCALING FOR IMPACT CRATERING AND PERFORATION

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SUMMARY

POD Associates have revisited the issue of generic scaling laws able to adequately predict (within better than 20%) cratering in semi-infinite targets and perforations through finite thickness targets. The approach used was to apply physical logic for hydrodynamics in a consistent manner able to account for chunky-body impacts such that the only variables needed are those directly related to known material properties for both the impactor and target. The analyses were compared and verified versus CTH hydrodynamic code calculations and existing experimental data. Comparisons with previous scaling laws were also performed to identify which (if any) were good for generic purposes. This paper is a short synopsis of the full report (ref. 1) available through the NASA Langley Research Center, LDEF Science Office.

INTRODUCTION

The need for scaling laws exists because the options (i.e. experiments and/or computer simulations) are very expensive and time-consuming. Interpretation of the many LDEF/SOLAR MAX/other impacts cannot be done directly via either experiments or simulations. To derive the new scaling laws, POD's approach was: (1) use physical logic to determine expectations. (While this does not guarantee the final answer, it should indicate the correct form of the relationships); (2) Use the CTH impact hydrodynamics code (from Sandia National Laboratory, Albuquerque) to map out specific responses and determine sensitivities to parameter changes. (The CTH code was chosen because it is "the" code presently supported by DOD, and has been "proven" against many different...
impact experiments); (3) Compare (1) and (2) with *existing* scaling laws and experiments to determine which (*if any*) are good fits to data, and are credible for impact conditions not readily accessible to experiments.

A "good" fit is one which obeys physics, has credibility, and requires *only* changes due to *known* material parameters. Accuracy need only be in the range of 10 - 20% for many cases. This allows an experimenter/code analyst to "home-in" on specific cases, as required.

POD's approach is primarily based on consideration of *momentum* and *stresses*. This approach assumes that, immediately following the short-lived pseudo one-dimensional (1-D) shock stress, the Bernoulli stress can be used as the "initial stress driver" for the remainder of the analyses. The logic is based on the concept of Bernoulli stress generated after a pulse reverberation through the projectile giving rise to an expanding (diverging) pulse in the target. The pulse contains fixed total *momentum*, but decreasing areal momentum, which induces hoop strains and stresses, which themselves decrease with radial distance. When the hoop stress drops to the local yield value, cratering stops. *Energy* is not directly invoked in the analysis. All energy solutions have the problem of needing to determine the correct *partition* between projectile, target, melting or vaporization, plastic flow and elastic waves.

POD's investigations mostly concentrated on impacts of aluminum into aluminum (both 6061-T6) and aluminum into Teflon, since these cases are representative of many of the LDEF cratering events.

**CRATERING IN INFINITE TARGETS**

For crater *diameters* \(d_0\), the *reverberation* pulse is assumed to be limited to one reverberation in the projectile (diameter \(d_p\)), since "later" momentum no longer contributes to the lateral push. Using this logic, coupled with dispersion of the pulse, we obtain

\[
d_p/d_o = 1.0857 (\rho_p/\rho)^{0.2857} (\rho/Y)^{0.2857} (c/c_p)^{0.2857} u_0^{0.5714}/(1+(\rho_p/\rho)^{1/2})^{0.5714} \tag{1}
\]

where \(\rho\) is density, \(Y\) is yield strength, \(u_0\) is normal impact speed, \(c\) is sound speed, and the subscripts apply for the projectile (p) or target (t). Note that the yield value is the static one, since we are describing the *terminal* phase of cratering.

For crater *depths* \(P\), the total projectile momentum is involved, but the effective shock speed is updated and a "cut-off" speed is defined, below which Bernoulli flow no longer occurs. From this we obtain

\[
P/d_p = (1/4)(4/3)^{1/3} (\rho_p/\rho)^{1/3} (\rho/Y)^{1/3} ((c_{0,t} + s(u_0-u_{cem}))/((1+(\rho_p/\rho)^{1/2}))(u_0-u_{cem}))^{1/3} \tag{2}
\]

where \(c_{0,t}\) is the low stress target sound speed, \(s\) is the Hugoniot term from the shock-speed versus
particle speed relationship, and \( u_{\text{crit}} \) is the limit velocity needed to ensure Bernoulli flow, given by

\[
u_{\text{crit}} = \left( \frac{2Y}{\rho} \right)^{1/2} \left( 1 + \left( \frac{\rho}{\rho_T} \right)^{1/2} \right)
\]

Figures 1 and 2 indicate the results of equations 1 and 2 versus CTH data for the cases of Al into Al and Al into Teflon, respectively. For the Al/Al case the fit is within a few percent, while for the Al/Teflon case POD's predictions are about 18% low versus CTH across the entire impact velocity range considered. Figure 3 shows the comparison between POD's predictions and those of Cour-Palais (ref. 2) for an Al/Al impact. The two sets of predictions are seen to be quite close, indicating that the previous NASA use of the Cour-Palais predictions should be giving credible results.

PERFORATIONS IN FINITE TARGETS

The Ballistic Limit

We define the ballistic limit as the condition where a through-hole is just produced. The logic is based on both the creation of a front surface crater and the reflection of the diverging shock off the target rear surface. When the reflected shock (tensile) exceeds the tensile strength of the target at the "normal" depth of crater, perforation occurs. This logic gives a result which has two parts: one relating to crater depth and one relating to tensile spall. The result is one term dependent on yield strength and 2/3 power of impact speed, and one term dependent on tensile strength and approximately unit power of impact speed. The net result is a speed index \( 2/3 < n < 1.0 \), as experimentally observed, and a need for both material strength terms. We obtain

\[
T/d_p = \left( \frac{1}{8} \right) \left( \frac{4}{3} \right)^{1/3} \left( \frac{\rho_T}{\rho} \right)^{1/3} \left( \frac{Y}{\rho} \right)^{1/3} \left\{ (c_0+\tau u_0-u_{\text{crit}}) \left( 1 + \left( \frac{\rho}{\rho_T} \right)^{1/2} \right) (u_0-u_{\text{crit}}) \right\}^{1/3} \\
+ \left( \frac{1}{4} \right) \left( \rho_p u_0^2 / (2\sigma_s (1 + \left( \frac{\rho}{\rho_T} \right)^{1/2}) \right) \right)^{1/n} 
\]

where \( T \) is the target thickness, \( n \) is an index describing the rate of decrease in compressive shock strength versus propagation distance. For aluminum \( n \approx 2.0 \), while for Teflon \( n \approx 2.4 \), based on data from the CTH calculations. The term \( \sigma_s \) is the tensile strength of the target. Thus the perforation ballistic limit requires both yield and tensile strengths. Figures 4 and 5 show comparisons between the new POD predictions and those of others for the cases of Al/Al and Al/Teflon impacts, respectively. It is seen that the most recent equation of McDonnell (ref. 3) gives very similar data to those of POD for the Al/Al case, but a somewhat larger variation for the Al/Teflon case.
A foil implies $T << d_p$, and for this case the pulse-time is limited by the transit across the foil and becomes $t = 2T/c_t$. Additionally, the index for stress decrease, $n$, becomes very large owing to the two free surfaces, the jetting, and the lip development. We obtain

$$
\frac{d_c}{d_p} = (\rho_r/\rho_p)^{1(n+1)}(\rho_r/Y_r)^{1(n+1)}u_0^{2(n+1)}(2T/d_p)^{1(n+1)}/(1+(\rho_r/\rho_p)^{1/2})^{2(n+1)}
$$

[5]

for $n >> 2$ this implies $d_c/d_p \rightarrow 1.0$, regardless of material properties or impact speed. This accords with experiment.

Intermediate Thickness Targets

To describe the intermediate case we assumed that the index $n$ is itself a function of $T/d_p$. The simplest possibility chosen was

$$
n = n_0(1+md_p/T)
$$

[6]

where $n_0$ is the "infinite target" index. To equate $d_c/d_p$ in both [1] and [5] we need $n = 2.5$ when $T = 2/3d_p$ which implies $m = 0.166$.

This approach gives a response very similar to that observed by Hörz (ref. 4). The asymptotic response for aluminum becomes

$$
(d_c - d_p)/d_p = (3.0 T/d_p)(\ln(A) - \ln(T/d_p))
$$

[7]

where

$$
A = 2 (\rho_r/\rho_p) (\rho_r/Y_r) u_0^2 / (1 + (\rho_r/\rho_p)^{1/2})^2
$$

[8]

This implies an almost linear response as $T \rightarrow 0$, but the $\log(T/d_p)$ term gives a variable rate as $T/d_p$ increases. The $\log(A)$ term also gives a very weak dependence on material properties and impact speed. This should be compared with results of Sawle (ref. 5), Maiden (ref. 6), and Brown (ref. 7) who suggest

$$
(d_c - d_p)/d_p \propto (T/d_p)^n
$$

[9]

where $n = 2/3$ (Sawle), or $n = 2/3$ (Maiden), or $n = 0.646$ (Brown). However, as noted by Herrmann (ref. 8), these various equations also contain impact velocity indexes which imply very large holes at high speeds, in contrast to experimental data. Figure 6 indicates the predictions of POD’s equation 5, using the variable value of $n$ from equation 6, for crater diameter versus the value of $T/d_p$ for an Al target foil. These predictions are compared with the data from Hörz for the
perforation hole size. The two groups of data track each other well.

SUPRALINEARITY

Experimentally, craters increase in size faster than the projectile does, all other factors constant. This phenomenon has been "explained" (by others) by either (a) shape changes during impact, or (b) strain-rate effects which increase the effective yield strength of the target. POD believes neither of these explanations works because:

(a) shape changes invalidate the concept of "chunky bodies" behaving like spheres, and POD's analysis also indicates that exact shape is not important.

(b) if strain-rate were important then we have a velocity dependence which is not included in the logic. Further, the strain-rate increase in yield occurs during the compressive shock only. The logic ignores stress-relaxation and the fact that the hysteretic reversal of stress does not suffer the same increase in yield since the release waves give much lower strain-rates. Only if cratering is a direct function of the shock front should strain-rate have a direct effect.

(c) POD's CTH calculations compared a large projectile versus small ones. No supralinearity was observed, despite different strain-rates. Further, there is nothing in either hydrodynamics, shape, or simple strain-rate that implies a "scale-length", which is necessary to explain supralinearity. To obtain a "length" we require the combination velocity/strain-rate, which thus gives a velocity dependence.

POD invoked the "Petch law" (ref. 9) which does invoke a "scale-length" and gives yield strength as a function of material grain size. Using this approach, POD demonstrated that the supralinear index is not constant, but merely appears to be over the projectile sizes commonly used for experiments.

The Petch law states

$$Y_t = Y_0 \left[1 + (\delta /d)^{1/2}\right]$$  \hspace{1cm} [10]

where $Y_t$ is the observed strength, $Y_0$ is an intrinsic strength, $d$ is the mean grain size and $\delta$ is a material-specific "size" parameter, given by

$$\delta = \pi G \gamma /Y_0^2$$  \hspace{1cm} [11]

where $G$ is the shear modulus and $\gamma$ is the surface energy per area for opening cracks.

The result is to downgrade predictions by the factor

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\[ f = \frac{1}{1 + (\delta / r_c)^{1/2}} \] \[ \text{[12]} \]

and this analysis implies that supralinearity is really a *small projectile down-scaling*, which essentially vanishes for projectiles larger than about 1.0 cm. Since *none* of the hydrocode simulations (done by anybody) include the Petch logic for material strength, it is not surprising that the codes *never* predict supralinearity. Figure 7 illustrates the behavior of equation 12, and indicates that for very small projectiles the supralinear exponent (slope of the line) approaches 1/6, while for large projectiles the exponent approaches zero. For projectiles in the range microns to millimeters, the mean exponent is very close to the Cour-Palais quote of 0.056, *i.e.*, \( d_i / d_p \propto d_p^{0.056} \).

**OBLIQUE IMPACTS**

Because oblique impacts are 3-D, hydrodynamic code calculations are time-consuming. Thus POD performed limited numbers of such computations. Previous arguments have suggested that oblique impacts behave as if only the impact velocity component normal to the target surface were involved. This is supported by experiments.

Data show that until obliquities greater than about 60° are involved, the craters remain almost hemispherical. For larger obliquities the craters become obviously elongated in the downstream direction. Ricocheting of the projectile is observed at these higher obliquities.

POD's CTH calculations *confirm* the "cosine law", indicating that the crater depth and "lateral" crater diameter (i.e. perpendicular to the projectile plane of impact/ricochet) develop as if for the impact speed \( u_0 \cos \theta \), where \( \theta \) is the angle between the impact velocity and the normal to the target surface.

POD's analysis indicates no contrary behavior. All of POD's equations remain valid provided the term \( u_0 \) is replaced everywhere by the term \( u_0 \cos \theta \).

POD's analysis also accounts for projectile ricochet. Such ricochets depend either on the ability of the projectile component of speed parallel to the target surface "out-running" the induced disturbance (this is the "stone bounce on water" logic), or on simple geometry arguments for the upper portions of the projectile to "pass over" the induced crater lips. The CTH results appear to be consistent with these arguments.

Because POD's analyses do *not* invoke energy for cratering there is no reason to expect a deviation from the cosine law even at very high impact speeds.
OTHER DATA COMPARISONS

POD has recently received details of work done by Wingate et al. (Los Alamos National Laboratory) (ref. 10), presented at the 1992 HVIS Symposium. The work involves Cu/Cu impacts, and compares four hydrodynamic code predictions. The codes are: EPIC, MESA, SPH and CALE, and experimental data is also compared. The following table lists the codes results and the POD predictions.

The calculations are for an impact at 6 km/s. The projectile diameter was \( d_p = 0.4747 \) cm (0.5 g). Properties for copper were:

\[
\rho_p = \rho_t = 8.93 \, \text{g/cm}^3, \quad c_{0,t} = 3.94 \, \text{Km/s}, \quad s = 1.49, \quad Y_t = 2.4 \, \text{Kbars}
\]

and to compute our values POD used equation [1] for \( d_c \), and [2] for \( P \).

Results

<table>
<thead>
<tr>
<th>quantity</th>
<th>experiment</th>
<th>EPIC</th>
<th>MESA</th>
<th>SPH</th>
<th>CALE</th>
<th>POD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P ) (cm)</td>
<td>1.4</td>
<td>1.8</td>
<td>1.59</td>
<td>1.73</td>
<td>1.51</td>
<td>1.55</td>
</tr>
<tr>
<td>( d_c ) (cm)</td>
<td>2.54</td>
<td>2.4</td>
<td>2.8</td>
<td>2.6</td>
<td>2.44</td>
<td>2.71</td>
</tr>
<tr>
<td>( P/d_c )</td>
<td>0.55</td>
<td>0.75</td>
<td>0.57</td>
<td>0.67</td>
<td>0.62</td>
<td>0.572</td>
</tr>
</tbody>
</table>

We observe that POD's predictions are close to the experimental data. Also note that the variations in the code answers are themselves about 19% (for \( P \)), 17% (for \( d_c \)) and 32% (for \( P/d_c \)). The ratios for the POD values versus experiment are:

1.107 (for \( P \)), 1.07 (for \( d_c \)) and 1.04 (for \( P/d_c \)).

Part of Wingate's work was to explain supralinearity for small (micron size) projectiles. To do so he invoked strain-rate hardening and proposed that the effective yield strength of copper acted as if 5 times larger than normal, thus was set at 12 kbars. This increased yield value reduced the code predictions for crater volume by a factor of 4.1 (EPIC), 4.4 (MESA) and 3.3 (SPH). The POD prediction is 3.973 (Eqn [1]) for the same higher yield. Note that LANL did not actually use a strain-rate model, they merely increased the yield value in the "normal" model.
OTHER SCALING LAWS

POD has compared many existing scaling laws for cratering, which describe either crater depths and/or diameters. Essentially none of these scaling laws can be considered generic, since those few that fit data for aluminum do not fit the data for Teflon, or vice versa. As an example, although the Cour-Palais prediction is very good for an Al/Al impact (figure 3) it is less good for an Al/Teflon impact, as seen in figure 8.

POD has also compared existing equations for the ballistic limit condition. Of these, those recent ones by McDonnell give the best overall consistent fits for both aluminum and Teflon targets. The differences between McDonnell's recent versions are too small to justify a "best choice", since they all give good fits to CTH data and to POD's analysis, and to existing experimental data. However, McDonnell does not describe the condition of intermediate thickness targets.

POD's analyses and CTH calculations agree well with the experimental data of Hörz for cratering in infinite targets and the ballistic limit in finite targets. The analysis also indicates that the intermediate thickness case can be described, although POD has not yet finalized the results.

CONCLUSIONS

POD believes it has a "respectable handle" on scaling laws for cratering and perforations. Our equations fit results (computer and experimental) for targets of aluminum, Teflon and copper. We have indicated that the rules for crater diameter are not the same as for crater depth, and that truly hemispherical craters are a rarity rather than the rule.

POD's equations also adequately describe the ballistic limit condition for both aluminum and Teflon FEP targets. We have demonstrated that the condition really involves two terms, one describing crater depth and one describing the rear-surface generated spall. The response involves both yield strength and tensile strength, and the velocity index is between 2/3 and 1.0.

POD has demonstrated that for oblique impacts the "cosine rule" does apply. We have also indicated the rules for projectile ricochets.

POD has explained supralinearity by using the Petch law, and has concluded that this gives a small-size downscaling, and vanishes for projectiles larger than about 1 cm.

POD believes it would be worthwhile to study other metals, plastics and ceramics. Specifically, POD believes it is possible to formulate the responses for intermediate-thickness target perforations. This would strongly augment the experimental work of Hörz. The latter is the only obvious manner in which perforations can be used to decipher impact projectile details.
REFERENCES

1. "Dimensional scaling for impact cratering and perforations", Watts et al., POD IRAD-93-001, 1993. Work performed under subcontract to Lockheed Engineering and Sciences Company under purchase order number 02N0171219, supported by the LDEF project office, and the LDEF M&D SIG (NASA Langley Research Center, and NASA Johnson Space Center).


Comparison of Watts' Equations with CTH Predictions for Aluminum on Aluminum (Al 6061-T6)

Figure 1

Comparison of Watts' Equations with CTH Predictions for Aluminum on TFE Teflon

Figure 2
Comparison of Watts’ and Cour-Palais’ Equations for Aluminum on Aluminum (Al 6061-T6)

Figure 3

Comparison of Watts’ Equation 4 versus Other Investigators’ Equations for Al into Al

Figure 4
Comparison of Watts' and McDonnell & Sullivan's Equations for Al into TFE Teflon

Figure 5

Comparison of Watts' Equations with Horz Data for Aluminum 6061-T6 Impacts at 6 km/s

Figure 6
Plot of the Petch Law's Supralinearity Downgrading Factor vs. Crater Radius (for Aluminum)

Figure 7

Comparison of Watts' and Cour-Palais' Equations for Aluminum on TFE Teflon

Figure 8
This volume is a compilation of papers presented at the Third Long Duration Exposure Facility (LDEF) Post-Retrieval Symposium. The papers represent the data analysis of the 57 experiments flown on the LDEF. The experiments include materials, coatings, thermal systems, power and propulsion, science (cosmic ray, interstellar gas, heavy ions, micrometeoroid, etc.), electronics, optics, and life science. In addition, papers on preliminary data analysis of EURECA, EOIM-3, and other spacecraft are included.