Generation and Computerized Simulation of Meshing and Contact of Modified Involute Helical Gears

Faydor L. Litvin, Ningxin Chen, and Jian Lu
University of Illinois at Chicago
Chicago, Illinois

Prepared for
Propulsion Directorate
U.S. Army Aviation Systems Command
and
Lewis Research Center
under Grant NAG3–1469

National Aeronautics and Space Administration
Office of Management
Scientific and Technical Information Program
1995
GENERATION AND COMPUTERIZED SIMULATION OF MESHING AND
CONTACT OF MODIFIED INVOlUTE HELICAL GEARS

by
Faydor L. Litvin
Principal Investigator
Ningxin Chen and Jian Lu
Research Associates
University of Illinois at Chicago, Chicago, IL

Abstract

The design and generation of modified involute helical gears that have a localized and stable bearing contact, and reduced noise and vibration characteristics are described. The localization of the bearing contact is achieved by the mismatch of the two generating surfaces that are used for generation of the pinion and the gear. The reduction of noise and vibration will be achieved by application of a parabolic function of transmission errors that is able to absorb the almost linear function of transmission errors caused by gear misalignment. The meshing and contact of misaligned gear drives can be analyzed by application of computer programs that have been developed. The computations confirmed the effectiveness of the proposed modification of the gear geometry. A numerical example that illustrates the developed theory is provided.
Nomenclature

\(a\) \hspace{1cm} \text{Parabola parameter (fig. 2(a))}

\(a_c\) \hspace{1cm} \text{Modification coefficient of pinion rack-cutter surface (fig. 5)}

\(b\) \hspace{1cm} \text{Slope of linear function (fig. 1(b))}

\(E_{pc}\) \hspace{1cm} \text{The shortest distance between the pinion and rack-cutter } \Sigma_c \text{ (fig. 6(b))}

\(E_{pg}\) \hspace{1cm} \text{The shortest distance between the pinion-gear axes (fig. 7)}

\(E_{gt}\) \hspace{1cm} \text{The shortest distance between the gear and rack-cutter } \Sigma_t \text{ (fig. 6(a))}

\(m_{21}\) \hspace{1cm} \text{Gear ratio}

\(M_{ij}\) \hspace{1cm} \text{Coordinate transformation matrix (from } S_j \text{ to } S_i \text{)}

\(n_r\) \hspace{1cm} \text{Unit normal vector to rack-cutter surface } \Sigma_r \text{ (} r = c, t \text{)}

\(n_j^{(i)}\) \hspace{1cm} \text{Unit normal vector to surface } \Sigma_i \text{ represented in coordinate system } S_f \text{ (} i = p, g \text{)}

\(N_i\) \hspace{1cm} \text{Number of teeth of the pinion (} i = 1, p \text{) or the gear (} i = 2, g \text{)}

\(N_r\) \hspace{1cm} \text{Normal vector to rack-cutter surface } \Sigma_r \text{ (} r = c, t \text{)}

\(p_n\) \hspace{1cm} \text{Circular pitch in normal section (fig. 3)}

\(P_n\) \hspace{1cm} \text{Diametral pitch in normal section}

\(r_i\) \hspace{1cm} \text{Radius of the pitch circle of the pinion (or gear) (} i = p, g \text{)}

\(r_i, r_j\) \hspace{1cm} \text{Position vector of surface } \Sigma_i\)

\(r_j^{(i)}\) \hspace{1cm} \text{Position vector of surface } \Sigma_i \text{ represented in coordinate system } S_f\)

\(s, s_r\) \hspace{1cm} \text{Displacement of rack-cutter } \Sigma_r \text{ (} r = c, t \text{) (fig. 6)}

\(S_i\) \hspace{1cm} \text{Coordinate system } i\)

\(u_i, \theta_i\) \hspace{1cm} \text{Surface parameters of } \Sigma_i \text{ (} i = p, g \text{)}

\(u_r, \theta_r\) \hspace{1cm} \text{Surface parameters of } \Sigma_r \text{ (} r = c, t \text{)}

\(v^{(r)}\) \hspace{1cm} \text{Velocity of rack-cutter surface point (} r = c, t \text{)}

\(v^{(ij)}\) \hspace{1cm} \text{Relative velocity of surface } \Sigma_i \text{ point with respect to surface } \Sigma_j \text{ point}

\(\alpha_o\) \hspace{1cm} \text{Normal pressure angle (fig. 3)}

\(\beta_o\) \hspace{1cm} \text{Helix angle on the pinion (gear) pitch cylinder (figs. 3 and 4)}

\(\delta\) \hspace{1cm} \text{Elastic approach of pinion and gear tooth surfaces}
\( \Delta E \)  
Change of center distance

\( \Delta \lambda_p \)  
Change of pinion lead angle on the pitch cylinder

\( \Delta q \)  
Displacement of contact point caused by misalignment

\( \Delta \gamma_z \)  
Misalignment angle formed by crossed gear axes (fig. 8(a))

\( \Delta \gamma_y \)  
Misalignment angle formed by intersected gear axes (fig. 8(b))

\( \Delta \phi_2, \Delta \psi_2 \)  
Transmission error (fig. 2)

\( \Delta \phi_2 \)  
Vector of the angle of compensating turn of gear 2

\( \lambda_p \)  
Lead angle on pinion pitch cylinder

\( \Sigma_i \)  
Pinion \((i = p)\) and gear \((i = g)\) tooth surfaces

\( \Sigma_r \)  
Rack-cutter surfaces \((r = c, t)\)

\( \phi_i, \psi_i \)  
Rotation angle of gear \(i (i = 1, 2, p, g)\) (figs. 2 and 7)

\( \psi_{gt} \)  
Rotation angle of gear being in mesh with the rack-cutter \(\Sigma_i\) (fig. 6(a))

\( \psi_{pc} \)  
Rotation angle of pinion being in mesh with the rack-cutter \(\Sigma_c\) (fig. 6(b))
1. Introduction

Conventional helical involute gears are designed for transformation of rotation between parallel axes. Theoretically, the gear tooth surfaces are in line tangency at every instant, along a straight line that is a tangent to the helix on the gear base cylinder. However, the line contact of gear tooth surfaces can be realized only for an ideal gear drive. In reality, the crossing of axes of rotation (instead of being parallel) and errors of lead angle result in the so-called edge contact, as a specific instantaneous point contact caused by curve-to-surface tangency. Here, the contacting curve is the edge of the tooth surface of one of the mating gears and the contacting surface is the tooth surface of the other one.

Trying to avoid the edge contact, the manufacturers of helical gears use various methods of crowning (deviation) of the theoretical gear tooth surfaces. However, the applied methods of crowning have not been complemented with the analysis of transmission errors caused by misalignment. Our investigation shows that improper crowning may avoid edge contact but cannot avoid the appearance of transmission errors of the shape shown in fig. 1. The function of such transmission errors is piecewise, almost linear, and has the frequency equal to the cycle of meshing of one pair of teeth. The above-mentioned transmission errors cause high vibration and noise and therefore such transmission errors must be avoided. This can be achieved by application of computer numerically controlled (CNC) machines that have opened new perspectives for generation of gear tooth surfaces with improved topology.

The intent of this paper is to describe a modified topology of low-noise involute helical gears that satisfies the following requirements:

1. The noise and vibration of helical gears are reduced substantially by application of a predesigned function of transmission errors of a parabolic type (fig. 2). Such a function can absorb (see below) an almost linear function of transmission errors shown in fig. 1.

2. The bearing contact is localized. Theoretically, the tooth surfaces are in tangency at every instant at a point instead of a line. The contact of gear tooth surfaces at every instant
is spread over an elliptical area due to elastic deformation of gear teeth. The dimension of
the instantaneous contact ellipse can be controlled by choosing proper design parameters.

(3) The proposed gear tooth surfaces can be generated by two rack-cutters designed for
generation of the pinion and gear, respectively. A nonlinear transmission function in the
process for gear generation must be provided and this can be accomplished by application of
the CNC machine. A linear transmission function is provided in the process for the pinion
generation.

2. Interaction of Parabolic and Linear Function of Transmission Errors

The ideal gears transform rotation with constant gear ratio \( m_{21} = \frac{N_1}{N_2} \), and the ideal
transmission function is

\[
\phi_2(\phi_1) = \frac{N_1}{N_2} \phi_1
\]  

where \( N_1 \) and \( N_2 \) are the tooth numbers of the pinion and gear, respectively.

However, the crossing of gear axes (instead of being parallel), intersection of these axes
and errors of lead angle cause a transmission function \( \phi_2(\phi_1) \) that is shown in fig. 1(a).
Our investigation (see sections 4-6) shows that the function of transmission errors caused by
above mentioned errors of misalignment is a piecewise almost linear function of transmission
errors \( \Delta \phi_2(\phi_1) \) with the frequency of a cycle of meshing for one pair of teeth (fig. 1(b)).

Here:

\[
\Delta \phi_2(\phi_1) = \phi_2(\phi_1) - \frac{N_1}{N_2} \phi_1
\]

Transmission errors of this type cause a discontinuity of the transmission function and a
big jump of the angular velocity of the driven gear at transfer points (when one pair of teeth is changed to another one). Therefore, vibration and noise become inevitable.

It was proven [1,2,4] that a predesigned parabolic function of transmission errors interacting with a linear function will become a parabolic function with the same parabola parameter. A parabolic function of transmission errors is much more preferable than a linear function since the transmission function will be a continuous one, the jump of angular velocity of the driven gear and the stroke at the transfer point will be substantially reduced.

Fig. 2(a) shows the sum of two functions of transmission errors

$$\Delta \phi_2(\phi_1) = \Delta \phi_2^{(1)}(\phi_1) + \Delta \phi_2^{(2)}(\phi_1) = b\psi_1 - a\psi_1^2$$  \hspace{1cm} (3)

The first one, $\Delta \phi_2^{(1)}(\phi_1)$, is caused by misalignment. The second one, $\Delta \phi_2^{(2)}(\phi_1)$, is a predesigned parabolic function which exists even if misalignment does not appear. It is easy to verify that equation (3) represents in the new coordinate system ($\Delta \psi_2$, $\psi_1$) the parabolic function (fig. 2(b)) that is designated as

$$\Delta \psi_2 = -a\psi_1^2$$  \hspace{1cm} (4)

The parabola parameter $a$ in equations (3) and (4) is the same. Axes of coordinate system ($\Delta \psi_2$, $\psi_1$) and ($\Delta \phi_2$, $\phi_1$) are parallel but the origins are different. The coordinate transformation from ($\Delta \phi_2$, $\phi_1$) to ($\Delta \psi_2$, $\psi_1$) is represented with the following equations

$$\Delta \psi_2 = \Delta \phi_2 - \frac{b^2}{4a}, \hspace{0.5cm} \psi_1 = \phi_1 - \frac{b}{2a}$$  \hspace{1cm} (5)
The difference between functions $\Delta \phi_2(\phi_1)$ and $\Delta \psi_2(\psi_1)$ is the location of the couple of points $(A, B)$ and the respective points $(A^*, B^*)$ (fig. 2(a)). The symmetrical location of $(A, B)$ is turned into the asymmetrical location of $(A^*, B^*)$. However, the interaction of several functions $\Delta \psi_2(\psi_1)$ determined for several tooth surfaces being in mesh may provide a symmetrical parabolic function of transmission errors as shown in fig. 2(b). This can be achieved if the parabolic function $\Delta \phi_2(\phi_1)$ will be predesigned in the area (fig. 2(a))

$$\phi_1(B) - \phi_1(A) \geq \frac{2\pi}{N_1} + 2c$$

where $c = \frac{b}{2a}$. Requirement (6), if observed, enables to provide a continuous function $\Delta \psi_2(\psi_1)$ for the range of $\frac{2\pi}{N_1}$ where $N_1$ is the pinion tooth number. It will be shown below (see sections 5 and 6) that functions of transmission errors caused by angular errors (such as the crossing and intersection of the axes of rotation, error of the lead angles) are indeed piecewise linear functions, and the coefficient $b$ can be determined knowing the angular error caused by misalignment and the design parameters of the gear drive.

3. Surfaces of Rack-Cutters

The imaginary process of generation of conjugate tooth surfaces is based on application of two rack-cutters that are provided respectively by a plane $\Sigma_t$ and a cylindrical surface $\Sigma_c$ that differs slightly from plane $\Sigma_t$ (see fig. 3). The rack-cutter surfaces $\Sigma_t$ and $\Sigma_c$ are rigidly connected each to other in the process of the imaginary generation, and they are in tangency along a straight line, $O_b z_b$ (fig. 5). This line and the parallel axes of the gears form angle $\beta_8$, that is equal to the helix angle on the pinion (gear) pitch cylinder. The normal sections of the rack-cutters are shown in figs. 3 and 5. Rack-cutter surface $\Sigma_c$ generates the pinion tooth surface $\Sigma_p$, and the rack-cutter surface $\Sigma_t$ generates the gear tooth surface $\Sigma_g$. 
Gear Rack-Cutters $\Sigma_i$

Using figs. 3, 4 and 5, we represent the transformation matrix from system $S_3$ to $S_r$ $(r = c, t)$ and $S_b$ to $S_3$ as follows

$$M_{ra} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \beta_o & \sin \beta_o & 0 \\ 0 & -\sin \beta_o & \cos \beta_o & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(7)

$$M_{ab} = \begin{bmatrix} \cos \alpha_o & -\sin \alpha_o & 0 & -d_p \cos \alpha_o \\ \sin \alpha_o & \cos \alpha_o & 0 & a_m - d_p \sin \alpha_o \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(8)

$$M_{rb} = \begin{bmatrix} \cos \alpha_o & -\sin \alpha_o & 0 & -d_p \cos \alpha_o \\ \sin \alpha_o \cos \beta_o & \cos \alpha_o \cos \beta_o & \sin \beta_o & (a_m - d_p \sin \alpha_o) \cos \beta_o \\ -\sin \alpha_o \sin \beta_o & -\cos \alpha_o \sin \beta_o & \cos \beta_o & -(a_m - d_p \sin \alpha_o) \sin \beta_o \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(9)

Here $\alpha_o$ and $\beta_o$ are the normal pressure angle and the helix angle of the rack-cutter; $a_m$ is the half of the tooth width of the rack-cutter on middle line $m - m$ (fig. 3), where

$$a_m = \frac{\pi}{4P_n}$$

(10)
and \( P_n \) is the normal diametral pitch of the rack-cutter, \( d_p \) is the distance between middle line \( m - m \) of the rack-cutter and the origin \( O_b \) along axis \( x_b \), fig. 5. Parameter \( d_p \) can be controlled to adjust the location of the contact path on the gear tooth surface.

Surface \( \Sigma_t \) of the gear rack-cutter is a plane that is represented in \( S_b \) as

\[
\mathbf{r}_t^{(b)} = \begin{bmatrix} u_t & 0 & l_t \end{bmatrix}^T
\]  \hspace{1cm} (11)

where \( (u_t, l_t) \) are the surface parameters.

Rack-cutter surface \( \Sigma_t \) is represented in coordinate system \( S_t \) by the matrix equation

\[
\mathbf{r}_t(u_t, l_t) = \mathbf{M}_{tb} \mathbf{r}_t^{(b)}
\]  \hspace{1cm} (12)

Equations (9), (11) and (12) yield

\[
\mathbf{r}_t = \begin{bmatrix}
(u_t - d_p) \cos \alpha_o \\
((u_t - d_p) \sin \alpha_o + a_m) \cos \beta_o + l_t \sin \beta_o \\
-[(u_t - d_p) \sin \alpha_o + a_m] \sin \beta_o + l_t \cos \beta_o
\end{bmatrix}
\] \hspace{1cm} (13)

The unit normal to \( \Sigma_t \) is represented in \( S_t \) by equations

\[
\mathbf{n}_t = \frac{\mathbf{N}_t}{|\mathbf{N}_t|}, \hspace{0.5cm} \mathbf{N}_t = \frac{\partial \mathbf{r}_t}{\partial u_t} \times \frac{\partial \mathbf{r}_t}{\partial l_t}
\]  \hspace{1cm} (14)

that yield
\[ n_t = \begin{bmatrix} -\sin \alpha_o & \cos \alpha_o \cos \beta_o & -\cos \alpha_o \sin \beta_o \end{bmatrix}^T \] (15)

**Pinion Rack-Cutter Surface \( \Sigma_c \)**

Rack-cutter \( \Sigma_c \) generates the pinion. The normal section of rack-cutter surface \( \Sigma_c \) (fig. 5) is a parabolic curve. We remind that the normal section of rack-cutter surface \( \Sigma_t \) is a straight line directed along axis \( x_b \) in fig. 5. The parabolic curve is in tangency with the \( x_b \)-axis at point \( N(O_b) \). Rack-cutter surfaces \( \Sigma_c \) and \( \Sigma_t \) are in tangency along a straight line that is parallel to axes \( z_a \) and \( z_b \) and passes through point \( O_b \) that coincides with point \( N \). The deviation of the parabolic curve from the \( x_b \)-axis affects the dimensions of the instantaneous contact ellipse.

Rack-cutter surface \( \Sigma_c \) is represented in \( S_b \) as follows

\[ r_c^{(t)} = \begin{bmatrix} u_c & -a_c u_c^2 & l_c \end{bmatrix}^T \] (16)

where \( a_c \) is coefficient of the parabolic normal section, and \((u_c, l_c)\) are the surface parameters of \( \Sigma_c \).

Rack-cutter surface \( \Sigma_c \) is represented in coordinate system \( S_c \) by the matrix equation

\[ r_c(u_c, l_c) = M_{cb} r_c^{(t)} \] (17)

Equations (9), (16) and (17) yield
The unit normal of $\Sigma_c$ is represented as

$$n_c = \frac{\frac{\partial r_c}{\partial u_c} \times \frac{\partial r_c}{\partial l_c}}{|\frac{\partial r_c}{\partial u_c} \times \frac{\partial r_c}{\partial l_c}|}$$

Equations (18) and (19) yield

$$n_c = \frac{1}{(1 + 4a_c^2 u_c^2)^{0.5}} \begin{bmatrix} 
\sin \alpha_o - 2a_c u_c \cos \alpha_o \\
-(\cos \alpha_o + 2a_c u_c \sin \alpha_o) \cos \beta_o \\
(\cos \alpha_o + 2a_c u_c \sin \alpha_o) \sin \beta_o 
\end{bmatrix}$$

Using equations (13), (15), (18) and (20), it is easy to verify that surfaces $\Sigma_c$ and $\Sigma_t$ are in tangency along the $z_b$ axis when $u_c = u_t = 0$.

4. Pinion and Gear Surfaces Generated by Rack-Cutters

In the process for generation the two rigidly connected rack-cutters perform translational motion while the pinion and the gear perform rotational motions as shown in fig. 6. To provide a predesigned parabolic function of transmission errors for each cycle of meshing, it is necessary to observe certain relations between the motions of the rack-cutters and gears, respectively.
The angle $\psi_{pc}$ of pinion rotation and the displacement $s_c$ of rack-cutter $\Sigma_c$ are related by the following linear function

$$\psi_{pc} = \frac{s_c}{r_p}$$ (21)

Here: $r_p$ is the radius of the pinion pitch cylinder.

The angle $\psi_{gt}$ of gear rotation and the displacement $s_t$ of rack-cutter $\Sigma_t$ are related as follows

$$\psi_{gt} = \frac{N_p}{N_g} \left( \frac{s_t}{r_p} - a \left( \frac{s_t}{r_p} - \psi^{(0)} \right)^2 \right)$$ (22)

Here: $N_p$ and $N_g$ are the tooth numbers of the pinion and gear, respectively, and $\psi^{(0)}$ is the initial position angle of the gear for the modification gear rotation.

Equation of Meshing between Rack-Cutter $\Sigma_c$ and Pinion $\Sigma_p$

The equation of meshing between rack-cutter $\Sigma_c$ and the pinion tooth surface $\Sigma_p$ is represented as

$$f(u_c, l_c, \psi_{pc}) = N_c^{(c)} \cdot v_c^{(sp)} = 0$$ (23)

where $\psi_{pc}$ is the angle of rotation of the pinion in the process for generation. The normal $N_c^{(c)}$ to $\Sigma_c$ in $S_c$ can be obtained by equation (20), and the relative velocity of the pinion with respect to $\Sigma_c$ may be represented as
\( v_c^{(pc)} = \omega_c^{(p)} \times (R_p + r_c) - (0 \quad r_p \quad 0)^T \frac{d\psi_{pc}}{dt} \)  \hspace{1cm} (24)

Here: \( R_p = (O_pO_c)_c = (r_p \quad r_p \psi_{pc} \quad 0)^T \), \( r_p = E_{pc} \) is the radius of the pitch cylinder of the pinion (fig. 6), \( \omega_c^{(p)} = \omega_c^{(p)}(0 \quad 0 \quad 1)^T \).

Substitution of equations (19) and (24) into (23), yields the following equation of meshing between \( \Sigma_p \) and \( \Sigma_c \)

\[
f(u_c, l_c, \psi_{pc}) = l_c \sin \beta_o + r_p \psi_{pc} + a_m \cos \beta_o + \frac{[(u_c - d_p) + 2a_c^2u_c^2] \cos \beta_o}{\sin \alpha_o - 2a_cu_c \cos \alpha_o} = 0 \]  \hspace{1cm} (25)

**Surface of Pinion \( \Sigma_p \)**

In the process of generation of pinion surface, rack-cutter \( \Sigma_c \) performs uniform translation and the workpiece performs uniform rotation (fig. 6(b)). The transformation matrix from system \( S_c \) to \( S_p \) can be represented as

\[
M_{pc} = \begin{bmatrix}
\cos \psi_{pc} & \sin \psi_{pc} & 0 & r_p \cos \psi_{pc} + r_p \psi_{pc} \sin \psi_{pc} \\
-\sin \psi_{pc} & \cos \psi_{pc} & 0 & -r_p \sin \psi_{pc} + r_p \psi_{pc} \cos \psi_{pc} \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix} \]  \hspace{1cm} (26)

Pinion surface \( \Sigma_p \) in system \( S_p \) is represented as

\[
r_p(u_c, l_c, \psi_{pc}) = M_{pc}r_c \]

\[
l_c = -\left\{ \frac{(u_c - d_p) + 2a_c^2u_c^2}{\sin \alpha_o - 2a_cu_c \cos \alpha_o} \cos \beta_o + r_p \psi_{pc} + a_m \cos \beta_o \right\} / \sin \beta_o \]  \hspace{1cm} (27)

13
Substituting equation (26) into (27), we obtain equation of $\Sigma_p$ as

$$\tau_p(u_c, \psi_{pc}) = \tau_p(u_r, \theta_r)$$

(28)

Equation of Meshing between Rack-Cutter $\Sigma_t$ and Gear $\Sigma_g$

The equation of meshing between rack-cutter $\Sigma_t$ and the gear tooth surface $\Sigma_g$ is represented as

$$f(u_t, l_t, \psi_{gt}) = n_t^{(t)} \cdot v_t^{(gt)} = 0$$

(29)

where $\psi_{gt}$ is the angle of rotation of the gear in the process for generation. The unit normal $n_t^{(t)}$ to $\Sigma_t$ in $S_t$ is represented by equation (15), and $v_t^{(gt)}$ is the relative velocity of the gear with respect to rack-cutter $\Sigma_t$.

We recall that the rack-cutter $\Sigma_t$ performs translation with constant velocity, but the gear performs rotation with variable angular velocity that is represented as (see equation (22))

$$\omega_t^{(g)} = \left[ \begin{array}{ccc} 0 & 0 & -\left( \frac{N_p}{N_g} - 2a(\psi_{pc} - \psi^{(0)}) \right) \end{array} \right]^T \frac{d\psi_{pc}}{dt}$$

(30)

The relative velocity $v_t^{(gt)}$ is represented as

$$v_t^{(gt)} = \omega_t^{(g)} \times (R_g + r_t) - \left( 0 \quad \frac{N_p}{N_g} r_g \quad 0 \right)^T \frac{d\psi_{pc}}{dt}$$

(31)
where

\[ R_g = \begin{pmatrix} -r_g & \frac{N_p}{N_g} r_g \psi_{pc} & 0 \end{pmatrix}^T \]  

(32)

and \( r_g = E_{gt} \) is the radius of pitch cylinder of the gear (fig. 6).

Substitution of equations (15), (30), (31) and (32) into (29) yields the following equation of meshing between \( \Sigma_g \) and \( \Sigma_t \)

\[ f(u_t, l_t, \psi_{gt}) = (u_t - d_p) \cos \beta_o + \sin \alpha_o (l_t \sin \beta_o + \frac{N_p}{N_g} r_g \psi_{pc} + a_m \cos \beta_o) \]

\[ + \frac{2aN_g r_g (\psi_{pc} - \psi^{(0)})}{N_p - 2aN_g (\psi_{pc} - \psi^{(0)})} \cos \alpha_o \cos \beta_o \]  

(33)

where

\[ \psi_{gt} = \frac{N_p}{N_g} \psi_{pc} - a(\psi_{pc} - \psi^{(0)})^2 \]  

(34)

**Surface of Gear \( \Sigma_g \)**

It must be remembered that the gear with the tooth surface \( \Sigma_g \) performs rotation about its axis with varied angular velocity while rack-cutter \( \Sigma_t \) performs uniform translation (fig. 6(a)). The transformation matrix from system \( S_t \) to \( S_g \) can be represented as
\[
M_{gt} = \begin{bmatrix}
-\cos \psi_{gt} & \sin \psi_{gt} & 0 & r_g \cos \psi_{gt} + \frac{N_p}{N_g} \psi_{pa} \sin \psi_{gt} \\
-\sin \psi_{gt} & -\cos \psi_{gt} & 0 & r_g \sin \psi_{gt} - \frac{N_p}{N_g} \psi_{pa} \cos \psi_{gt} \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]  

(35)

Gear tooth surface \( \Sigma_g \) in system \( S_g \) can be represented as

\[
r_g(u_t, l_t, \psi_{gt}) = M_{gt} r_t
\]

\[
l_t = -\{\left(u_t - d_p\right) \cot \beta_o + \left(\frac{N_p}{N_g} r_{g,pa} + a_m \cos \beta_o\right) \frac{1}{\sin \beta_o} \}
\]

\[
+ \frac{2aN_g \gamma_g(\psi_{pa} - \psi^{(0)})}{N_p - 2aN_g(\psi_{pa} - \psi^{(0)})} \cot \alpha_o \cot \beta_o \}
\]

(36)

The derivation of equation (36) is based on transformation of equations (33) and (34). Equations (35) and (36) enable to represent the gear tooth surface in two-parameter form as follows

\[
r_g(u_t, \psi_{gt}) = r_g(u_g, \theta_g)
\]

(37)

5. Computerized Simulation of Meshing and Contact of Pinion-Gear Tooth Surfaces

We consider that the surfaces of the pinion and the gear generated by worms \( \Sigma_w \) and \( \Sigma_h \) are represented in coordinate systems \( S_p \) and \( S_g \), respectively. The fixed coordinate system \( S_f \)
is rigidly connected to the housing of the gear drive (figs. 7 and 8). The movable coordinate systems \(S_p\) and \(S_g\) are rigidly connected to the pinion and the gear, respectively. An auxiliary coordinate system \(S_h\) is applied for simulation of meshing when the gear axis is crossed or intersected with the pinion axis instead of being parallel, and when the shortest distance between the pinion and gear axes is changed. The errors of misalignment are referred to the gear. The misalignment angle \(\Delta \gamma\) is decomposed into two components, \(\Delta \gamma_x\) and \(\Delta \gamma_y\), that represent the crossing angle and the intersection angle, respectively. The pinion performs rotational motion about the \(z_f\)-axis. The axis of gear rotation is \(z_h\). The shortest distance between the axes of rotation is designated as \(E_{pg}\).

The rotation matrices from system \(S_h\) to \(S_f\) for crossed and intersecting angles are represented in the followings (fig. 8)

\[
L_{fh} = \begin{bmatrix}
-1 & 0 & 0 \\
0 & -\cos \Delta \gamma_x & \sin \Delta \gamma_x \\
0 & \sin \Delta \gamma_x & \cos \Delta \gamma_x
\end{bmatrix}
\]

(38)

\[
L_{fh} = \begin{bmatrix}
-\cos \Delta \gamma_y & 0 & -\sin \Delta \gamma_y \\
0 & -1 & 0 \\
-\sin \Delta \gamma_y & 0 & \cos \Delta \gamma_y
\end{bmatrix}
\]

(39)

We represent the pinion and gear tooth surfaces, \(\Sigma_p\) and \(\Sigma_g\), and their unit normals in coordinate system \(S_f\). The conditions of continuous tangency of surfaces \(\Sigma_p\) and \(\Sigma_g\) are represented by the following equations \([1,2]\).

\[
r^{(p)}_f(u_p, \theta_p, \phi_p) = r^{(g)}_f(u_g, \theta_g, \phi_g)
\]

(40)
Vector equation (41) provides only two independent equations since $|n_f^{(p)}| = |n_f^{(g)}| = 1$. The total number of independent equations provided by (40) and (41) is five that relate six parameters

$$f_i(u_p, \theta_p, \phi_p, u_g, \theta_g, \phi_g) = 0 \quad (i = 1, 2, \ldots 5)$$

(42)

The continuous solution of the system of nonlinear equations (42) is based on the following procedure:

(1) Using an initial guess, we determine a set of parameters that satisfy equation system (42). Thus

$$P^{(0)} = (u_p^{(0)}, \theta_p^{(0)}, u_g^{(0)}, \theta_g^{(0)}, \phi_g^{(0)})$$

(43)

(2) One of the variable parameters, say $\phi_p$, is chosen as the input one, and is supposed that the Jacobian

$$\left. \frac{D(f_1, f_2, f_3, f_4, f_5)}{D(u_p, \theta_p, u_g, \theta_g, \phi_g)} \right|_{P^{(0)}}$$

(44)

differs from zero. The derivatives in the Jacobian are taken at point $P^{(0)}$.

(3) Then, equation system (42) can be solved in the neighborhood of $P^{(0)}$ by functions
\[ \phi_g(\phi_p), u_p(\phi_p), \theta_p(\phi_p), u_g(\phi_p), \theta_g(\phi_p) \]  

(4) Vector function \( r_p(u_p, \theta_p) \) that determines the pinion surface \( \Sigma_p \) and functions \( u_p(\phi_p), \theta_p(\phi_p) \) enable to determine the path of contact on \( \Sigma_p \).

(5) Similarly, we can obtain the path of contact on the gear surface \( \Sigma_g \) using vector function \( r_g(u_g, \theta_g) \) and functions \( u_g(\phi_p), \theta_g(\phi_g) \).

(6) The paths of contact on pinion and gear tooth surfaces slightly deviate from helices in the case of an aligned gear drive. The line of action for an aligned gear drive (the set of contact points in \( S_f \)) slightly deviates from a straight line that is parallel to the gear axes.

(7) The transmission function \( \phi_g(\phi_p) \) deviates from the ideal transmission function, and the function of transmission errors coincides with the predesigned parabolic function.

(8) The determination of dimensions and orientation of the instantaneous contact ellipse needs the knowledge of the principal curvatures and directions of contacting surfaces and the elastic approach of surfaces. This problem can be substantially simplified if the pinion-gear principal curvatures and directions are expressed in terms of the principal curvatures and directions of the generating surfaces and parameters of motion [1,2].

6. Numerical Example

The method developed in this report is illustrated with the example discussed below. The design parameters of the pinion and gear are listed in Table 1. The numerical simulation of meshing is performed for an aligned and misaligned gear drives with various errors of alignment for the pinion and gear.

Case 1. Aligned gear drive

Figure 9 shows the transmission errors for the aligned gear drive. The TCA performed
Table 1: Design parameters of pinion and gear

<table>
<thead>
<tr>
<th></th>
<th>pinion</th>
<th>gear</th>
</tr>
</thead>
<tbody>
<tr>
<td>tooth number</td>
<td>$N_p = 20$</td>
<td>$N_g = 100$</td>
</tr>
<tr>
<td>normal diametral pitch</td>
<td>$P_n = 5 \frac{1}{\text{in}}$</td>
<td>$P_n = 5 \frac{1}{\text{in}}$</td>
</tr>
<tr>
<td>normal pressure angle</td>
<td>$\alpha_o = 20^\circ$</td>
<td>$\alpha_o = 20^\circ$</td>
</tr>
<tr>
<td>helix angle on pitch cylinder</td>
<td>$\beta_o = 30^\circ$</td>
<td>$\beta_o = 30^\circ$</td>
</tr>
<tr>
<td>tooth length</td>
<td>$L = 1.6 \text{ in.}$</td>
<td>$L = 1.6 \text{ in.}$</td>
</tr>
<tr>
<td>modification coefficient</td>
<td>$a_c = 0.0008$</td>
<td>$a = 0.0014$</td>
</tr>
<tr>
<td>elastic approach</td>
<td>$\delta = 0.007 \text{ mm}$</td>
<td>$\delta = 0.007 \text{ mm}$</td>
</tr>
</tbody>
</table>

confirms that the predesigned parabolic function of transmission errors exists. The maximum transmission error is approximately 8 arc seconds. Figure 10 shows the contact pattern and the contact path. The path of contact on the tooth surface is in the longitudinal direction. The major axis of the instantaneous contact ellipse is 6 mm for the assumed elastic approach of the surfaces equal to 0.007 mm.

Case 2. The pinion-gear rotation axes are crossed

Figures 11 and 12 show the transmission errors and the contact pattern for the case when the crossing angle $\Delta \gamma_{cross}$ is 4 arc minutes. The maximum transmission error is 8 arc seconds and contact paths are shifted up and down on the gear and pinion surfaces, respectively. Figures 13 and 14 show the transmission errors and contact pattern for $\Delta \gamma_{cross} = -4$ arc minutes.

Case 3. The pinion-gear rotation axes are intersected

Figures 15 and 16 show the transmission errors for the misalignment $\Delta \gamma_{intersect} = 4$ arc minutes. Figures 17 and 18 show the transmission errors and contact pattern for the mentioned above error of alignment.
Case 4. Influence of error of lead angle

Figures 19 and 20 show the transmission errors and contact pattern when the error of the lead angle is 4 arc minutes.

Figures 21 and 22 show the transmission errors and contact pattern when the error of the lead angle is -4 arc minutes.

For all of above cases, the maximum transmission error does not exceed 8 arc second (with very small deviations of this value).

7. Conclusion

From the analytical study presented in this report the following conclusions can be drawn:

(1) The interaction of a parabolic and a linear functions of transmission errors has been discussed to prove the possibility to absorb almost, linear functions of transmission errors caused by misalignment.

(2) Mismatched surfaces of two rack-cutters for generation of modified involute gears have been proposed.

(3) Generation and geometry of pinion-gear modified tooth surfaces have been determined.

(4) Computerized simulation of meshing and contact of pinion-gear tooth surfaces has been developed.

(5) An algorithm for determination of relations between the curvatures of the generating and the generated surfaces has been developed.

(6) An algorithm for determination of the contact ellipse has been developed.

(7) Directions for users of application of developed computer programs for the design of gears with the modified geometry and computerized simulation of their meshing and contact have been developed (Appendix C).
References


5. Reishauer CNC Gear Grinding Machines, Catalogs, Switzerland.
Fig. 1 Transmission function and transmission errors for a misaligned gear drive
Fig. 2 Interaction of parabolic and linear functions
Fig. 3 Normal sections of rack-cutters
Parallel to gear axes

Fig. 4 Orientation of rack-cutters with respect to gear axes
Fig. 5 Normal section of pinion rack-cutter surface
Fig. 6 Generation of pinion and gear by rack-cutters
Fig. 7 Coordinate system applied for tooth contact analysis (TCA)
Fig. 8 Coordinate systems $S_h$ and $S_f$
Fig. 9 Transmission errors for the aligned gear drive
Fig. 10 Contact paths and pattern for the aligned gear drive
CROSS ING ANGLE BETWEEN AXES: 4 MINUTES

Fig. 11 Transmission errors for the alignment error
\(\Delta \gamma_{\text{cross}} = 4\) arc minutes
Fig. 12 Contact pattern for the alignment error

\[ \Delta \gamma_{\text{cross}} = 4 \text{ arc minutes} \]
Fig. 13 Transmission errors for the alignment error

\[ \Delta \gamma_{\text{cross}} = -4 \text{ arc minutes} \]
CROSSING ANGLE BETWEEN AXES: -4 MINUTES

Fig. 14 Contact pattern for the alignment error

$$\Delta \gamma_{cross} = -4 \text{ arc minutes}$$
Fig. 15 Transmission errors for misaligned gear drive with \( \Delta \gamma_{\text{intersecting}} = 4 \) arc minutes
Fig. 16 Contact pattern and paths for the misaligned gear drive with $\Delta \gamma_{\text{intersecting}} = 4$ arc minutes
Fig. 17 Transmission errors for the misaligned gear drive with $\Delta \gamma_{intersecting} = -4$ arc minutes
Fig. 18 Contact pattern for the misaligned gear drive with $\Delta \gamma_{\text{intersecting}} = -4$ arc minutes
Fig. 19 Transmission errors for the misaligned gear drive with $\Delta \lambda_0 = 4$ arc minutes
Fig. 20 Contact pattern and contact path for misaligned gear drive with $\Delta \lambda_0 = 4$ arc minutes
Fig. 21 Transmission errors for the misaligned gear drive with $\Delta \lambda_0 = -4$ arc minutes
Fig. 22 Contact pattern and contact paths for the misaligned gear drive with $\Delta \lambda_0 = -4$ arc minutes
Appendix A. Relations between the Curvatures of the Generating and Generated Surfaces

Direct Relations between Principal Curvatures and Directions of Mating Surfaces

The main advantage of this approach (proposed by Litvin) is the possibility to determine the principal curvatures and directions of the generated surface in terms of principal curvatures and directions of the generating tool surface, and the parameters of motion. In this case, the tool surfaces for the generation of the gear and the pinion tooth surfaces are rack cutters. The equations developed permit a simplified computational procedure.

The system of equations that relate the principal curvatures and directions of the generating and generated surfaces is as follows. Consider that unit vectors $e_f$ and $e_h$ represent the principal directions on the tool surface $\Sigma_1$ at point $P$ of tangency of surfaces $\Sigma_1$ and $\Sigma_2$ (fig. A1). The principal curvatures on the mating surfaces $\kappa_f$ and $\kappa_h$ of the tool are given; the parameters of motion (see below) are also given.

The goal is to determine angle $\sigma$ that is formed by unit vectors $e_f$ and $e_q$, and principal curvatures $\kappa$, and $\kappa_q$. (The unit vectors $e_s$ and $e_q$ represent the principal directions on surface $\Sigma_2$). The system of equations for determination of $\sigma$, $\kappa_s$, and $\kappa_q$ is as follows.

$$\tan 2\sigma = \frac{2b_{15}b_{25}}{b_{25}^2 - b_{15}^2 - (\kappa_f - \kappa_h)t_{33}} \quad (A1)$$

$$\kappa_q - \kappa_s = \frac{2b_{15}b_{25}}{t_{33}\sin 2\sigma} \frac{b_{25}^2 - b_{15}^2 - (\kappa_f - \kappa_h)t_{33}}{t_{33}\cos 2\sigma} \quad (A2)$$

$$\kappa_q + \kappa_s = \kappa_f + \kappa_h + \frac{b_{15}^2 + b_{25}^2}{t_{33}} \quad (A3)$$

Here:
The nomenclature for equations (A4) to (A6) is described as follows:

\( \omega^{(1)} \) angular velocity of the generating tool

\( \omega^{(2)} \) angular velocity of the generated gear

\( \omega^{(12)} \) defined as \( \omega^{(1)} - \omega^{(2)} \)

\( v_{tr}^{(1)} \) transfer motion velocity of the generating tool

\( v_{tr}^{(2)} \) transfer motion velocity of the generated gear

\( v^{(12)} \) defined as \( v_{tr}^{(1)} - v_{tr}^{(2)} \)

\( n \) surface unit normal vector

The equations discussed above are used in the TCA program for determination of the contact ellipse at the points of contact path of the modified helical gear drive.

**Numerical Example**

The input and output for the determination of the principal curvatures of the pinion tooth surface are shown in Tables A1 and A2. The input and output for the determination of the principal curvatures of the gear tooth surface are shown in Tables A3 and A4.
Table A1 Input Data

<table>
<thead>
<tr>
<th>Description</th>
<th>Symbol</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>first principal curvature of tool</td>
<td>$\kappa_f$</td>
<td>-0.0016 (1/mm)</td>
</tr>
<tr>
<td>second principal curvature of tool</td>
<td>$\kappa_h$</td>
<td>0.0 (1/mm)</td>
</tr>
<tr>
<td>first principal direction of tool</td>
<td>$e_f$</td>
<td>$[0.0016 - 0.4999 0.8660]^T$</td>
</tr>
<tr>
<td>second principal direction of tool</td>
<td>$e_h$</td>
<td>$[0.9387 0.2992 0.1710]^T$</td>
</tr>
<tr>
<td>angular velocity of tool (1/sec)</td>
<td>$\omega^{(1)}$</td>
<td>$[0.0 0.0 0.0]^T$</td>
</tr>
<tr>
<td>angular velocity of pinion (1/sec)</td>
<td>$\omega^{(2)}$</td>
<td>$[0.0 0.0 1.0]^T$</td>
</tr>
<tr>
<td>transfer velocity of tool (mm/sec)</td>
<td>$v_{tr}^{(1)}$</td>
<td>$[0.0 58.6588 0.0]^T$</td>
</tr>
<tr>
<td>transfer velocity of pinion (mm/sec)</td>
<td>$v_{tr}^{(2)}$</td>
<td>$[-0.1514 58.5951 0.0]^T$</td>
</tr>
<tr>
<td>surface normal of tangent point</td>
<td>$n$</td>
<td>$[-0.3420 0.8138 0.4698]^T$</td>
</tr>
</tbody>
</table>

Table A2 Output Data

<table>
<thead>
<tr>
<th>Description (for pinion)</th>
<th>Symbol</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>first principal curvature</td>
<td>$\kappa_s$</td>
<td>-0.001543 (1/mm)</td>
</tr>
<tr>
<td>second principal curvature</td>
<td>$\kappa_q$</td>
<td>0.03907 (1/mm)</td>
</tr>
<tr>
<td>first principal direction</td>
<td>$e_s$</td>
<td>$[0.1751 - 0.4361 0.8827]^T$</td>
</tr>
<tr>
<td>second principal direction</td>
<td>$e_q$</td>
<td>$[0.9222 0.3865 0.0080]^T$</td>
</tr>
</tbody>
</table>
### Table A3 Input Data

<table>
<thead>
<tr>
<th>Description</th>
<th>Symbol</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>first principal curvature of tool</td>
<td>( \kappa_f )</td>
<td>0.0 (1/mm)</td>
</tr>
<tr>
<td>second principal curvature of tool</td>
<td>( \kappa_h )</td>
<td>0.0 (1/mm)</td>
</tr>
<tr>
<td>first principal direction of tool</td>
<td>( \mathbf{e}_f )</td>
<td>([0.9397 \ 0.2962 \ 0.1710]^T)</td>
</tr>
<tr>
<td>second principal direction of tool</td>
<td>( \mathbf{e}_h )</td>
<td>([0.0 \ -0.5 \ 0.8660]^T)</td>
</tr>
<tr>
<td>angular velocity of tool (1/sec)</td>
<td>( \omega^{(1)} )</td>
<td>([0.0 \ 0.0 \ 0.0]^T)</td>
</tr>
<tr>
<td>angular velocity of gear (1/sec)</td>
<td>( \omega^{(2)} )</td>
<td>([0.0 \ 0.0 \ -0.2003]^T)</td>
</tr>
<tr>
<td>transfer velocity of tool (mm/sec)</td>
<td>( \mathbf{v}_{tr}^{(1)} )</td>
<td>([0.0 \ 58.6588 \ 0.0]^T)</td>
</tr>
<tr>
<td>transfer velocity of gear (mm/sec)</td>
<td>( \mathbf{v}_{tr}^{(2)} )</td>
<td>([-0.2096 \ 58.7218 \ 0.0]^T)</td>
</tr>
<tr>
<td>surface normal of tangent point</td>
<td>( \mathbf{n} )</td>
<td>([-0.3420 \ 0.8138 \ 0.4698]^T)</td>
</tr>
</tbody>
</table>

### Table A4 Output Data

<table>
<thead>
<tr>
<th>Description (for gear)</th>
<th>Symbol</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>first principal curvature</td>
<td>( \kappa_s )</td>
<td>-0.007836 (1/mm)</td>
</tr>
<tr>
<td>second principal curvature</td>
<td>( \kappa_q )</td>
<td>0.0 (1/mm)</td>
</tr>
<tr>
<td>first principal direction</td>
<td>( \mathbf{e}_s )</td>
<td>([0.9219 \ 0.3874 \ 0.0]^T)</td>
</tr>
<tr>
<td>second principal direction</td>
<td>( \mathbf{e}_q )</td>
<td>([0.1820 \ -0.4331 \ 0.8827]^T)</td>
</tr>
</tbody>
</table>
Fig. A1 Principal Directions
Appendix B. Contact Ellipse

Determination of Dimensions and Orientation of Instantaneous Contact Ellipse

The gear tooth surfaces are in point contact at every instant. Due to elastic deformation of gear tooth surfaces the contact is spread over an elliptical area and the center of the ellipse coincides with the instantaneous contact point. The bearing contact is formed as the set of instantaneous contact ellipses.

The dimensions and orientation of the instantaneous contact ellipse can be determined using the data about the principal curvatures and directions of the contacting surfaces, and the elastic approach of the surfaces. The elastic approach depends on the applied load but we will consider it as a given value that is known from experimental data.

The determination of the instantaneous contact ellipse is based on the following equations (proposed by Litvin):

\[
\cos 2\alpha^{(1)} = \frac{g_1 - g_2 \cos 2\sigma}{(g_1^2 - 2g_1g_2 \cos 2\sigma + g_2^2)^{1/2}} \tag{B1}
\]

\[
\sin 2\alpha^{(1)} = \frac{g_2 \sin 2\sigma}{(g_1^2 - 2g_1g_2 \cos 2\sigma + g_2^2)^{1/2}} \tag{B2}
\]

\[
2a = 2 \left| \frac{\delta}{A} \right|^{1/2}, \quad 2b = 2 \left| \frac{\delta}{B} \right|^{1/2} \tag{B3}
\]

where

\[
A = \frac{1}{4} [\kappa^{(1)} - \kappa^{(2)} - (g_1^2 - 2g_1g_2 \cos 2\sigma + g_2^2)^{1/2}] \tag{B4}
\]

\[
B = \frac{1}{4} [\kappa^{(1)} - \kappa^{(2)} + (g_1^2 - 2g_1g_2 \cos 2\sigma + g_2^2)^{1/2}] \tag{B5}
\]

\[
\kappa^{(i)} = \kappa^{(i)}_I + \kappa^{(i)}_H, \quad g_i = \kappa^{(i)}_I - \kappa^{(i)}_H \tag{B6}
\]
Here (fig. B1) \( \alpha^{(1)} \) is the angle that is formed by axis \( \eta \) of the contact ellipse with the unit vector \( e_1^{(1)} \) of the principal direction on surface \( \Sigma_1 \); \( \sigma \) is the angle formed by unit vectors \( e_1^{(1)} \) and \( e_{1/2}^{(1)} \) of the principal directions of the contacting surfaces; \( 2a \) and \( 2b \) are the axes of the contact ellipse; \( \delta \) is the elastic approach; and \( \kappa_i^{(i)} \) and \( \kappa_i^{(j)} \) are two principal directions of tooth surface \( i \).

**Numerical Example**

The input and output for the determination of the contact ellipse are shown in Tables B1 and B2.

**Table B1 Input Data**

<table>
<thead>
<tr>
<th>Description</th>
<th>Symbol</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>pinion first principal curvature</td>
<td>( \kappa_s )</td>
<td>-0.001543 (1/mm)</td>
</tr>
<tr>
<td>pinion second principal curvature</td>
<td>( \kappa_q )</td>
<td>0.03907 (1/mm)</td>
</tr>
<tr>
<td>pinion first principal direction</td>
<td>( e_s )</td>
<td>([0.1751 - 0.4361 0.8827]^T)</td>
</tr>
<tr>
<td>pinion second principal direction</td>
<td>( e_q )</td>
<td>([0.9222 0.3865 0.0080]^T)</td>
</tr>
<tr>
<td>gear first principal curvature</td>
<td>( \kappa_f )</td>
<td>-0.007836 (1/mm)</td>
</tr>
<tr>
<td>gear second principal curvature</td>
<td>( \kappa_h )</td>
<td>0.0 (1/mm)</td>
</tr>
<tr>
<td>gear first principal direction</td>
<td>( e_f )</td>
<td>([-0.9219 - 0.3874 0.0]^T)</td>
</tr>
<tr>
<td>gear second principal direction</td>
<td>( e_h )</td>
<td>([-0.1820 0.4331 0.8827]^T)</td>
</tr>
<tr>
<td>elastic approach</td>
<td>( \delta )</td>
<td>0.007 (mm)</td>
</tr>
</tbody>
</table>

**Table B2 Output Data**

<table>
<thead>
<tr>
<th>Description</th>
<th>Symbol</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>long axis of contact ellipse</td>
<td>( 2a )</td>
<td>6.026 (mm)</td>
</tr>
<tr>
<td>short axis of contact ellipse</td>
<td>( 2b )</td>
<td>1.092 (mm)</td>
</tr>
<tr>
<td>angle between long axis and the first principal direction of pinion</td>
<td>( \alpha^{(1)} )</td>
<td>89.87 (deg)</td>
</tr>
</tbody>
</table>
Fig. B1 Orientation and dimensions of contact ellipse
Appendix C. Directions for Users of Application of Computer Program

C.1 Introduction

The name of the computer program is HELTCA.FOR. It is written in FORTRAN77 language. The operating system is CMS-9.0. A subroutine DNEQNF to solve a system of nonlinear equations should be available in the Math-Library or working environment. The subroutine is not included in the program. The program will call the subroutine DNEQNF several times.

C.2 Input Block

The input block consists of three parts: (1) design parameters of pinion and gear; (2) the controlled modification parameters; and (3) parameters for TCA.

Part 1. Design parameters of pinion and gear

In the beginning of the computer program, you can read the following lines:

C... A11.....COEFFICIENT FOR TRANSFORMATION OF DEGREE TO RADIANS
   A11=DACOS(-1.D0)/180.D0
C
C... KHD=1 FOR RIGHT-HAND PINION AND LEFT-HAND GEAR
C... KHD=2 FOR LEFT-HAND PINION AND RIGHT-HAND GEAR
KHD=2

If you write “KHD=1”, the computer will use the necessary equations for the case of right-hand pinion and left-hand gear. The computations will be accomplished for a left-hand pinion and right-hand gear if you use “KHD=2”.

Then, the variable definition for the pinion follows:

C
C... INPUT THE DESIGN PARAMETERS OF PINION
C...
C... TN1..........GEAR NUMBER OF TEETH
C... PN1..........NORMAL DIAMETRICAL PITCH (1/MM)
C... PSIN1.......NORMAL PRESSURE ANGLE (RAD.)
C... BETAP1......HEICAL ANGLE ON PITCH CYLINDER (RAD.)
C... ADG1........ADDENDUM (MM)
In accordance with our numerical example (see Table 1), the following data would be used:

\[
\begin{align*}
T\text{N}1 &= 20.0D0 \\
P\text{N}1 &= 5.0D0/25.4D0 \\
P\text{S}\text{I}1 &= 11*20.0D0 \\
L\text{A}M\text{D}P1 &= 11*60.0D0 \\
B\text{E}T\text{A}P1 &= 11*30.0D0 \\
F\text{W}1 &= 25.4D0*1.6D0 \\
A\text{D}G1 &= 1.0D0/PN1 \\
D\text{E}G1 &= 1.25D0/PN1
\end{align*}
\]

The following variables are used for the gear:

\[
\begin{align*}
C\ldots \ T\text{N}2 &\ldots \ \text{GEAR NUMBER OF TEETH} \\
C\ldots \ P\text{N}2 &\ldots \ \text{NORMAL DIAMETRAL PITCH} \ (1/\text{MM}) \\
C\ldots \ P\text{S}\text{I}2 &\ldots \ \text{NORMAL PRESSURE ANGLE} \ (\text{RAD.}) \\
C\ldots \ B\text{E}T\text{A}P2 &\ldots \ \text{HELICAL ANGLE} \ \text{ON PITCH CYLINDER} \ (\text{RAD.}) \\
C\ldots \ A\text{D}G2 &\ldots \ \text{ADDENDUM} \ (\text{MM}) \\
C\ldots \ D\text{E}G2 &\ldots \ \text{DEDENDUM} \ (\text{MM}) \\
C\ldots \ L\text{A}M\text{D}P2 &\ldots \ \text{LEAD ANGLE} \ \text{ON PITCH CYLINDER} \ (\text{RAD.}) \\
C\ldots \ F\text{W}2 &\ldots \ \text{FACE WIDTH} \ (\text{MM}) \\
C\ldots \ R\text{P}T2 &\ldots \ \text{RADIUS} \ \text{OF PITCH CYLINDER} \ (\text{MM}) \\
C\ldots \ R\text{B}T2 &\ldots \ \text{RADIUS} \ \text{OF BASE CYLINDER} \ (\text{MM}) \\
C\ldots \ R\text{A}T2 &\ldots \ \text{RADIUS} \ \text{OF ADDENDUM CYLINDER} \ (\text{MM}) \\
C\ldots \ R\text{D}T2 &\ldots \ \text{RADIUS} \ \text{OF DEDENDUM CYLINDER} \ (\text{MM}) \\
C\ldots \ P\text{S}I2 &\ldots \ \text{TRANSVERSE PRESSURE ANGLE} \ (\text{RAD.}) \\
C\ldots \ L\text{A}M\text{D}B2 &\ldots \ \text{LEAD ANGLE} \ \text{ON BASE CYLINDER} \ (\text{RAD.}) \\
C
\end{align*}
\]

From the design parameters listed in Table 1, we have
TN2=100.D0
PN2=5.D0/25.4D0
A11=DACOS(-1.D0)/180.D0
PSIN2=A11*20.D0
LAMDP2=A11*60.D0
BETAP2=A11*30.D0
FW2=25.4D0*1.6D0
ADG2=1.D0/PN2
DEG2=1.25D0/PN2

If KHD=2, the computer program will change the values of some design parameters as follows

IF(KHD.EQ.2) THEN
  LAMDP1=-LAMDP1
  BETAP1=-BETAP1
  LAMDP2=-LAMDP2
  BETAP2=-BETAP2
ENDIF

The computer program call the following subroutines "DATAT1" and "DATAT2" to calculate other tooth element proportions and output the whole data in file 55 (see below).

CALL DATAT1
CALL DATAT2

Part 2. Control of modification parameter for application in the TCA program

At this stage we can read

THE FOLLOWING DATA IS FOR THE TO-BE CONTROLLED MODIFICATION PARAMETERS

AA... MODIFICATION PARAMETER OF GEAR
AP... MODIFICATION PARAMETER OF PINION RACK-CUTTER
DP... TANGENT POINT N OF PROFILES OF PINION & GEAR RACK-CUTTERS
THET2P..INITIAL ANGLE FOR MODIFICATION OF GEAR (RAD.)

You must input the four parameters for modification of pinion and gear surfaces, for example:

AA=-0.0014D0
THET2P=-0.08D0
AP=-0.0008
DP=-DSIN(PSIN1)*DCOS(-1.D0)*RPT1/TN1/8.D0
The above four controlling parameters should be tried several times in order to obtain better contact pattern and transmission errors optimal for a given design.

**Part 3. Parameters for TCA**

In this part, the alignment errors expected should be input:

```
C
C PARAMETERS FOR TCA
C
C.. KM.. SWITCH 1 FOR CROSSING ANGLE $\Delta \gamma_x$ & 2 FOR INTERSECTION ANGLE $\Delta \gamma_y$
C.. DGAM..ANGLE OF MISALIGNMENT(CROSSING OR INTERSECTION) (ARC MINUTE)
C.. DEE... CHANGE OF CENTER DISTANCE (MM)
C
```

If a crossing angle of misalignment is considered, should input "KM=1". Input "KM=2" if an intersection angle of misalignment is considered. For an aligned gear drive, input "DGAM=0.0". For instance, if $\Delta \gamma_x = 4'$, $\Delta E = 0$, input the following lines:

```
KM=1
KM=2
DGAM=4.0*A11/60.D0
DEE=0.000D0
```

Then you will read the following sentence:

```
C
C.... THE INPUT BLOCK IS READY
```

Usually, you cannot make changes anything after this step.

**C.3 Output Block**

After the input block is filled out, you can read the following explanation for the output files:

```
C
C... OUTPUT DATA FILES ARE THE FOLLOWINGS
C
C... FILE 55... TOOTH PROPORTIONS OF PINION AND GEAR
C... FILE 85... CONTACT PATH ON PINION SURFACE (2D)
C... FILE 86... CONTACT PATH ON GEAR SURFACE (2D)
C... FILE 87... LENGTH AND DIRECTIONS OF CONTACT ELLIPSE ON PINION AND GEAR SURFACES (2D)
```
1) File 55

In File 55 the information about the pinion and gear are listed.

2) Files 85 and 86

There are two coordinates in File 85 for each contact point of the pinion: Radial \((x_p^2 + y_p^2)^{0.5}\) and axial \(z_p\). There are two coordinates in File 86 for each contact point of the gear: Radial \((x_g^2 + y_g^2)^{0.5}\) and axial \(z_g\).

3) File 87

There are 5 values in File 87 for each pair of contact points of the pinion and gear. The first one is the value of the major semi-axis of the contact ellipse. The second and third values are the cosine directions of the major axis of the contact ellipse on the pinion tooth surface. The last two values are the cosine direction of the major axis of the contact ellipse on the gear tooth surface.

C.4 Computer Program
C................. TCA FOR MODIFIED HELICAL GEARS.................

PROGRAM HELTCA
IMPLICIT REAL*8(A-H, O-Z)
REAL*8 XI(9), X(9), F(9)
REAL*8 LF1(3,3), LH2(3,3), L1F(3,3), L2H(3,3)
REAL*8 R1F(3), R2F(3), N1F(3), N2F(3)
REAL*8 DPHI(2,180), DDPHI(2,180,4)
REAL*8 ELAL(180), EL1(2,180), EL2(2,180)
REAL*8 LFH(3,3), LHF(3,3)
REAL*8 V1(3), V2(3), V3(3), V4(3), V5(3)
REAL*8 RG2(3), NG2(3)
REAL*8 RG1(3), NG1(3)
REAL*8 UI(3), UJ(3), UK(3)
REAL*8 LAMDP2, LAMDB2
REAL*8 LAMDP1, LAMDB1
REAL*8 UT2, KT21, KT22
REAL*8 AVCL(3), VTR1(3), AVC2(3), VTR2(3)
REAL*8 KSIG1, KSIG2, KFF, KH
REAL*8 EFN(3), EHNS(3), W1VT2(3), W12(3), W2VT1(3)
REAL*8 KM2, KT2
REAL*8 KFP, KHP, KFG, KHG
COMMON /A300/ ES(3), EQ(3)
COMMON /A310/ KFF, KH
COMMON /A340/ EFF(3), EHH(3)
COMMON /A360/ A, B, SI(3), FI(3)
COMMON /A200/ W1(3), W2(3), WI2(3), VT1(3), VT2(3), V12(3)
COMMON /A210/ EX(3), EF(3), EH(3)
COMMON /A212/ EF2(3), EH2(3), KF2, KH2
COMMON /A220/ KP, KH
COMMON /A230/ ET(3), EM(3)
COMMON /A380/ KS, KQ
COMMON /A400/ VT11(3), VT12(3), VT21(3), VT22(3)
COMMON /A401/ KHP, KFP, KHG, KFG
EXTERNAL FCNG, FCNC, FCNT
COMMON /AXIS/ UI, UJ, UK
COMMON /NET/ RR, DD
COMMON /DD/ DF, KPRI
COMMON /DATT2/ TN2, PN2, PSIN2, BETAPI2, ADG2, DEG2, LAMDP2,
& UP2, FW2, RPT2, RBT2, RDT2, PSIT2, LAMDB2
COMMON /DATT/ TN1, PN1, PSIN1, BETAPI1, ADG1, DEG1, LAMDP1,
& FW1, RPT1, RBT1, RDT1, PSIT1, LAMDB1
COMMON /B2/ XNP1, YNP1, ZNP1, XNP2, YNP2, ZNP2
COMMON /B4/ X1, Y1, Z1, XN1, YN1, ZN1
COMMON /B5/ X2, Y2, Z2, XN2, YN2, ZN2
COMMON /B6/ THET2P, DGPHI2
COMMON /W1/ ETAW1, UPP1, SPP1, ETAW2, UPP2, SPP2
COMMON /SG1/ RG1, NG1, AP, DP
COMMON /SG2/ RG2, NG2, AA
COMMON /ATT/ PH11, PH12, R1F, R2F, N1F, N2F, LFH, CC, D GAM, DPHI2
COMMON /MVT/ LF1, LH2
COMMON /AST/ ICONT
COMMON /ATS/ DPHI1

C...... ALL coefficient for transformation of degree to radian
AII=DCOS(-1.0)/180.0
C...... KHD=1 for right-hand pinion and left-hand gear
C...... KHD=2 for left-hand pinion and right-hand gear
KHD=2
C...... input the design parameters of pinion
C...... TN1........... gear number of teeth
C...... PN1........... normal diametral pitch (mm)
C. ... PSIN1......NORMAL PRESSURE ANGLE (RAD.)
C. ... BETAP1......LEADING ANGLE OF THE HELIX ON PITCH CYLINDER (RAD.)
C. ... ADG1...... ADDENDUM (MM)
C. ... DEG1...... DEEDENDUM (MM)
C. ... LAMDP1......HELIX ANGLE ON PITCH CYLINDER (RAD.)
C. ... FW1......FACE WIDTH (MM)
C. ... RPT1......RADIUS OF PITCH CYLINDER (MM)
C. ... RBT1......RADIUS OF BASE CYLINDER (MM)
C. ... RAT1......RADIUS OF ADDENDUM CYLINDER (MM)
C. ... RDT1......RADIUS OF DEEDENDUM CYLINDER (MM)
C. ... PSIT1......TRANSVERSE PRESSURE ANGLE (RAD.)
C. ... LAMDB1......HELIX ANGLE ON BASE CYLINDER (RAD.)

TN1=20.0
PN1=5.0/25.4D0
PSIN1=A11*20.0
LAMDP1=A11*60.0D0
BETAP1=A11*30.0D0
FW1=25.4D0*1.6D0
ADGI=1.0D0/PN1
DEGI=1.25D0/PN1

C. ... INPUT THE DESIGN PARAMETERS OF GEAR
C. ... TN2......GEAR NUMBER OF TEETH
C. ... PN2......NORMAL DIAMETRAL PITCH (1/MM)
C. ... PSIN2......NORMAL PRESSURE ANGLE (RAD.)
C. ... BETAP2......LEADING ANGLE OF THE HELIX ON PITCH CYLINDER (RAD.)
C. ... ADG2...... ADDENDUM (MM)
C. ... DEG2...... DEEDENDUM (MM)
C. ... LAMDP2......HELIX ANGLE ON PITCH CYLINDER (RAD.)
C. ... FW2......FACE WIDTH (MM)
C. ... RPT2......RADIUS OF PITCH CYLINDER (MM)
C. ... RBT2......RADIUS OF BASE CYLINDER (MM)
C. ... RAT2......RADIUS OF ADDENDUM CYLINDER (MM)
C. ... RDT2......RADIUS OF DEEDENDUM CYLINDER (MM)
C. ... PSIT2......TRANSVERSE PRESSURE ANGLE (RAD.)
C. ... LAMDB2......HELIX ANGLE ON BASE CYLINDER (RAD.)

TN2=100.0
PN2=5.0/25.4D0
A11=DACOS (-1.0D0)/180.0D0
PSIN2=A11*20.0D0
LAMDP2=A11*60.0D0
BETAP2=A11*30.0D0
FW2=25.4D0*1.6D0
ADG2=1.0D0/PN2
DEG2=1.25D0/PN2
IF (KHD.EQ.2) THEN
LAMDP1=-LAMDP1
BETAP1=-BETAP1
LAMDP2=-LAMDP2
BETAP2=-BETAP2
ENDIF
CALL DATAT1
CALL DATAT2

C. ... THE FOLLOWING IS FOR CONTROLLING PARAMETERS
C. ... AA...... MODIFICATION PARAMETER OF GEAR
C. ... AP...... MODIFICATION PARAMETER OF PINION RACK-CUTTER
C. ... DP...... TANGENT POINT N OF PROFILES OF PINION & GEAR RACK-CUTTERS
C. ... THET2P......INITIAL ANGLE FOR MODIFICATION OF GEAR (RAD.)
C
C
PARAMETERS FOR TCA
C.. KM .. SWITCH 1 FOR CROSSING & 2 FOR INTERSECTING MISALIGNMENT
C.. DGAM.. ANGLE OF DISALIGNMENT(CROSSING OR INTERSECTING) (ARC MINUTE)
C.. DEE... CHANGE OF CENTER DISTANCE (MM)

KM=1
KM=2
DGAM=0.D0
DEE=0.0000D0
AA=-0.0014D0
THET2P=-0.08D0
AP=-0.0008
DP=-DSIN(Psin1)*DCOS(-1.D0)*RPT1/TN1/8.D0

C... THE INPUT BLOCK IS OVER HERE

C
C... OUTPUT DATA FILES ARE THE FOLLOWINGS
C
C... FILE 55... TOOTH PROPORTIONS OF PINION AND GEAR
C... FILE 85... CONTACT PATH ON PINION SURFACE (2D)
C... FILE 86... CONTACT PATH ON GEAR SURFACE (2D)
C... FILE 87... DIRECTIONS OF LONG AXIS OF CONTACT ELLIPS (2D)
C... FILE 90 ...TRANSMISSION ERRORS

DO 901 I=1,3
  UI(I)=0.D0
  UJ(I)=0.D0
  UK(I)=0.D0
901 CONTINUE
UI(1)=1.D0
UJ(2)=1.D0
UK(3)=1.D0

C .. EE2.... GEAR RATIO
  EE2=TN1/TN2
C .. CC....... CENTER DISTANCE OF GEAR DRIVE
  CC=RPT1+RPT2+DEE

C .. CALCULATE CONTACT POINT ON MEAN SECTION WITHOUT MISALIGNMENT
ICONT=1
N=6
ERRREL=0.1D-6
ITMAX=1000

C
CALL MIAL(KM,0.D0,LFH)
C
XX(1)=0.D0
XX(2)=0.D0
XX(3)=0.D0
XX(4)=0.D0
XX(5)=0.D0
XX(6)=0.D0
DD=0.D0

C
CALL DNEQNF(FCNG,ERRREL,N,ITMAX,XX,X,FNORM)
C
PHISS1=X(6)
PHIMEA=X(6)
C .. CALCULATE CONTACT POINT ON EDGE SECTION WITHOUT MISALIGNMENT
DD=-0.5D0*FWI
ERRREL=0.1D-8
C
CALL DNEQNF(FCNG,ERRREL,N,ITMAX,XX,X,FNORM)
C
PHISS2=X(6)
C .. CALCULATE CONTACT POINT ON EDGE SECTION WITH MISALIGNMENT

60
N=6  
ERRREL=0.1D-6  
ITMAX=1000  

C CALL MIAL(KM,DGAM,LFH)  

C XI(1)=X(1)  
XI(2)=X(2)  
XI(3)=X(3)  
XI(4)=X(4)  
XI(5)=X(5)  
XI(6)=X(6)  

C CALL DNEQNF(FCNG,ERRREL,N,ITMAX,XI,X,FNORM)  

1330 CONTINUE  
PHII=X(6)  
PHIISTA=X(6)  

C .. THE FOLLOWING IS FOR TCA  

ICONT=2  
N=5  
ERRREL=0.1D-5  
ITMAX=400  
STP=(PHISS2-PHISSI)/36.D0  
NN=72  
DO 1010 I=1,NN  
PHII=PHIIISTA-(I-1)*STP  
XI(1)=X(1)  
XI(2)=X(2)  
XI(3)=X(3)  
XI(4)=X(4)  
XI(5)=X(5)  

C CALL DNEQNF(FCNG,ERRREL,N,ITMAX,XI,X,FNORM)  

IF(RGI(3).GT.(0.5D0*FW1)) GO TO 1011  
C... ROTATING VELOCITY OF CUTTER IN CUTTER SYSTEM  
W1(1)=0.D0  
W1(2)=0.D0  
W1(3)=0.D0  
C... ROTATING VELOCITY OF PINION IN CUTTER SYSTEM  
W2(1)=0.D0  
W2(2)=0.D0  
W2(3)=1.D0  
C... NORMAL OF CUTTER IN CUTTER SYSTEM  
EX(1)=XNP1  
EX(2)=YNP1  
EX(3)=ZNP1  
C... TRANSFER VELOCITIES OF CUTTER AND PINION IN CUTTER SYSTEM  
CALL EQVEC(VT1,VT11)  
CALL EQVEC(VT2,VT12)  
C... RELATIVE VELOCITIES OF CUTTER WRT. PINION IN CUTTER SYSTEM  
CALL ADDVEC(W12,W1,W2,-1.D0)  
CALL ADDVEC(V12,VT1,VT2,-1.D0)  
KF=KFP  
KH=KHP  
C... PRINCIPAL CURVATURES AND DIRECTIONS OF PINION  
CALL CURVT(1)  
C... PRINCIPAL DIRECTIONS OF PINION IN PINION SYSTEM  
CALL EQVEC(EFF,ES)

61
CALL EQVEC(EHH, EQ)

C... ROTATING VELOCITY OF CUTTER IN CUTTER SYSTEM
W1(1) = 0.D0
W1(2) = 0.D0
W1(3) = 0.D0

C... ROTATING VELOCITY OF GEAR IN CUTTER SYSTEM
W2(1) = 0.D0
W2(2) = 0.D0
W2(3) = -DGPHI2

C... NORMAL OF CUTTER IN CUTTER SYSTEM
EX(1) = XNP2
EX(2) = YNP2
EX(3) = ZNP2

C... TRANSFER VELOCITIES OF CUTTER AND PINION
CALL EQVEC(VT1, VT21)
CALL EQVEC(VT2, VT22)
CALL EQVEC(EF, EF2)
CALL EQVEC(EH, EH2)
KF = KF2
KH = KH2

C... RELATIVE VELOCITY OF CUTTER WRT. GEAR IN CUTTER SYSTEM
CALL ADDVEC(W12, W1, W2, -1.D0)
CALL ADDVEC(V12, VT1, VT2, -1.D0)
KF = KFG
KH = KHG

C... PRINCIPAL CURVATURES AND DIRECTIONS OF PINION
CALL CURVT(2)

C... PRINCIPAL DIRECTIONS OF PINION IN PINION SYSTEM
CALL TRANSM(L1F, LF1)
CALL MAVEC(V1, LH2, ES)
CALL MAVEC(V2, LFH, V1)
CALL MAVEC(ES, L1F, V2)
CALL MAVEC(V1, LH2, EQ)
CALL MAVEC(V2, LFH, V1)
CALL MAVEC(EQ, L1F, V2)

C... CONTACT ELLIPSES
CALL ELLIP
ELAL(I) = A

C... AXIS OF PINION ELLIPSE ON TANGENT PLANE
CALL EQVEC(V1, SI)
CALL DOTVEC(V1N, V1, V1)
EL1(1, I) = DSQRT(V1(1)**2 + V1(2)**2 / V1N)
EL1(2, I) = V1(3) / DSQRT(V1N)

C... AXIS OF GEAR ELLIPSE ON TANGENT PLANE
CALL TRANSM(LHF, LFH)
CALL TRANSM(L2H, LH2)
CALL MAVEC(V2, LF1, V1)
CALL MAVEC(V1, LH2, V2)
CALL MAVEC(V2, L2H, V1)
CALL EQVEC(V1, V2)
CALL DOTVEC(V1N, V1, V1)
EL2(1, I) = -DSQRT(V1(1)**2 + V1(2)**2 / V1N)
EL2(2, I) = V1(3) / DSQRT(V1N)
AG1 = DATAN(EL1(2, I) / EL1(1, I)) / A11
AG2 = DATAN(EL2(2, I) / EL2(1, I)) / A11
DPHI(1, I) = PHI1
DPHI(2, I) = PHI2 - TN1/TN2*PHI1
DDR1 = DSQRT(RG1(1)**2 + RG1(2)**2)
DDR2 = DSQRT(RG2(1)**2 + RG2(2)**2) - 210.D0
KM = 4
C
OUI=FLOAT(I-1)/FLOAT(KM)
OUP=AINT(OUI)
IF(OUI.EQ.OUP) THEN
  WRITE(85,*) DDR1,RG1(3)
  WRITE(86,*) DDR2,RG2(3)
  WRITE(87,*) ELAL(I),EL1(I,I),EL1(2,I),EL2(I,I),EL2(2,I)
ENDIF
1010 CONTINUE
1011 CONTINUE
AD=-10000.D0
DO 1090 I=I,NN,KM
  IF(DPHI(2,I).NE.0.D0) THEN
    IF(DPHI(2,I).GT.AD) THEN
      AD=DPHI(2,I)
    ENDIF
  ENDIF
1090 CONTINUE
SS=360.D0/TN1
DO 1020 J=1,4
  DO 1030 I=I,NN,KM
    BD=(DPHI(2,I)-AD)/AII*3600.D0
    DDPHI(I,I,J)=DPHI(1,I)/AII+SS*(J-1)
    DDPHI(2,I,J)=BD
  1030 CONTINUE
1020 CONTINUE
KM=4
DO 1040 J=1,4
  DO 1050 I=I,NN,KM
    IF(DDPHI(I,I,J).NE.0.D0) THEN
      WRITE(90,*) DDPHI(I,I,J),DDPHI(2,I,J)
    ENDIF
  1050 CONTINUE
1040 CONTINUE
KM=4
WRITE(87,*), ELAL(I),EL1(I,I),EL1(2,I),EL2(I,I),EL2(2,I)
1060 CONTINUE
WRITE(6,*), '***** PROGRAM FINISHED *****'
STOP
END
C
C ...THE SUBROUTINE IS FOR TCA
C...

SUBROUTINE FCNG(X,F,N)
IMPLICIT REAL*8(A-H,O-Z)
REAL*8 X(N),F(N)
REAL*8 LFI(3,3),LH2(3,3),LFH(3,3)
REAL*8 R1F(3),R2F(3),N1F(3),N2F(3),CO(3)
REAL*8 AVCl(3),VTR1(3),AVC2(3),VTR2(3)
REAL*8 UP, LAMDP, LAMDB
REAL*8 KF2,KH2
REAL*8 KFP,KHP,KFG,KHG
COMMON /AXIS/, UI,UJ,UK
COMMON /SG1/, RG1,NG1,AP,DP
COMMON /SG2/, RG2,NG2,AA
COMMON /ATT/, PHI1,PHI2,R1F,R2F,N1F,N2F,LFH,CC,DGAM,DPH12
COMMON /AST/, ICONT
COMMON /MVT/, LF1,LH2
REAL*8 KSIG1,KSIG2,KFF,KHH

63
REAL*8  EFN(3), EHN(3), W1VT2(3), WV12(3), W2VT1(3), KF, KH, KS, KQ
REAL*8  KM2, KT2
COMMON /A300/  ES(3), EQ(3)
COMMON /A310/  KFF, KHH
COMMON /A340/  EFF(3), EHH(3)
COMMON /A340/  EF1(3), EH1(3)
COMMON /A360/  A, B, SI(3), FI(3)
COMMON /A200/  W1(3), W2(3), W12(3), VT1(3), VT2(3), V12(3)
COMMON /A210/  EX(3), EF(3), EH(3)
COMMON /A212/  EF2(3), EH2(3), KF2, KH2
COMMON /A220/  KF, KH
COMMON /A230/  ET(3), EM(3)
COMMON /A380/  KS, KQ
COMMON /A401/  KHP, KFP, KHG, KFG
REAL*8  RG1(3), NG1(3)
REAL*8  RG2(3), NG2(3)
REAL*8  LAMDPI, LAMDB2
REAL*8  LAMDP1, LAMDB1

& UP2, FW2, RPT2, RBT2, RDT2, PSIT2, LAMDB2
COMMON /B2/XNP1, YNP1, ZNP1, XNP2, YNP2, ZNP2
COMMON /B4/X1, Y1, Z1, XN1, YN1, ZN1
COMMON /B5/X2, Y2, Z2, XN2, YN2, ZN2
COMMON /B6/THTET2P, DGPHI2
COMMON /W1/  ETAW1, UPP1, SPP1, ETAW2, UPP2, SPP2
COMMON /NET/  RR, DD
COMMON /DATT/  TN1, PN1, PSIN1, BETAP1, ADG2, DEG2, LAMDP1,
& FW1, RPT1, RBT1, RAT1, RDT1, PSIT1, LAMDB1
UPP1=X(1)
ETAW1=X(2)
UPP2=X(3)
ETAW2=X(4)
PHI2=X(5)
IF(ICONT.EQ.1) THEN
   PHI1=X(6)
ENDIF

RPT=RPT1
CNST=DARCOS(-1.0D00)/180.0
PI=DARCOS(-1.0D00)
PI2=2.0D0*PI
PSIT= PSIT1
PSIN= PSIN1
BETAP= BETAP1
DSIN1=DSIN(ETAW1)
DCOS1=DCOS(ETAW1)
ANF1=DARCOS(RBT1/RPT1)
CINV1=DTAN(ANF1)-ANF1
ANG1=PI/2.0D0/TN1
AM=PI/PN1/4.0D0
SA=AM*DCOS(BETAP)
SS1= RPT1*(ETAW1)
DSS1= RPT1

C... EQUATION OF MESHING OF PINION
FF1=SPPI*DSIN(BETAP)
FF2=(-(UPP1+DP)-2.0D0*AP**2*UPP1**3)
FF3=DSIN(PSIN)+2.0D0*AP*UPP1*DCOS(PSIN)
F5=FF2/FF3*DCOS(BETAP)-SA-RPT1*ETAW1
SPPI= F5/DSIN(BETAP)

C... SURFACE OF PINION RACK CUTTER
XP = (UPP1+DP) *DCOS(PSIN)
YP = (UPP1+DP) *DSIN(PSIN)+AM)*DCOS(BETAP) +
& AP*UPP1**2*DCOS(PSIN)*DCOS(BETAP)+SSP1*DSIN(BETAP)
ZP = -(UPP1+DP)*DSIN(PSIN)+AM)*DSIN(BETAP) -
& AP*UPP1**2*DCOS(PSIN)*DSIN(BETAP)+SSP1*DCOS(BETAP)
X1 = DCOS(ETA1)*XP+DSIN(ETA1)*YP+RPT*DCOS(ETA1)
&+SS1*DSIN(ETA1)
Y1 = -DSIN(ETA1)*XP+DCOS(ETA1)*YP+RPT*DSIN(ETA1)
&+SS1*DCOS(ETA1)
Z1 = ZP

C... NORMAL OF RACK CUTTER
PNN = DSQRT(1.0+(2.0*AP*UPP1)**2)
XNPP = -2.0*AP*UPP1/PNN
YNPP = 1.0/PNN
ZNPP = 0.0

XNP1 = (DCOS(PSIN)*XNPP-DSIN(PSIN)*YNPP)
YNP1 = (DSIN(PSIN)*DCOS(BETAP)*XNPP+DCOS(PSIN)*DCOS(BETAP)*YNPP)
ZNPP = (-DSIN(PSIN)*DSIN(BETAP)*XNPP+DCOS(PSIN)*DSIN(BETAP)*YNPP)

C... NORMAL OF PINION IN S1
XN1 = DCOS(ETA1)*XNP1+DSIN(ETA1)*YNP1
YN1 = -DSIN(ETA1)*XNP1+DCOS(ETA1)*YNP1
ZN1 = ZNP1

DX1 = -DSIN(ETA1)*XP+DCOS(ETA1)*YP+SS1*DCOS(ETA1)
DY1 = -DCOS(ETA1)*XP-DSIN(ETA1)*YP-SS1*DSIN(ETA1)
DZ1 = ZN1

RG1(1) = XI
RG1(2) = YI
RG1(3) = ZI
NG1(1) = XN1
NG1(2) = YN1
NG1(3) = ZN1

KHP = 0.0

XFP = 2.0*AP/(DSQRT((1.0+(2.0*AP*UPP1)**2))**3)

EF1(1) = DCOS(PSIN)
EF1(2) = DSIN(PSIN)*DCOS(BETAP)
EF1(3) = DSIN(PSIN)*DSIN(BETAP)

EH1(1) = 0.0
EH1(2) = DSIN(BETAP)
EH1(3) = DCOS(BETAP)

EH(1) = DCOS(ETA1)*EH1(1)+DSIN(ETA1)*EH1(2)
EH(2) = -DSIN(ETA1)*EH1(1)+DCOS(ETA1)*EH1(2)
EH(3) = EH1(3)

EH(1) = DCOS(ETA1)*EF1(1)+DSIN(ETA1)*EF1(2)
EH(2) = -DSIN(ETA1)*EF1(1)+DCOS(ETA1)*EF1(2)
EH(3) = EF1(3)

C VELOCITY OF RACK-CUTTER IN SP
VT11(1) = 0.0
VT11(2) = RPT1
VT11(3) = 0.0

C VELOCITY OF PINION IN SP
VT12(1) = -(YP+SS1)
VT12(2) = XP+RPT1
VT12(3) = 0.0
RPT = RPT2
PSIT = PSIT2
PSIN = PSIN2
BETAP = BETAP2
SS2 = RPT2*(ETA1/TN2*TN1)
DSS2 = RPT2/TN2*TN1
TTD = AA*(ETA1+THET2)*+2
\[ DTP = 2 \cdot D0 \cdot AA \cdot (ETAW2 - THET2P) \]
\[ GPHI2 = ETAW2 / TN2 \cdot TN1 - TTD \]
\[ DGPHI2 = 1 \cdot D0 / TN2 \cdot TN1 - DTP \]

**C... EQUATION OF MESHING OF GEAR**

\[ GG1 = SPP2 \cdot DSIN(PSIN2) \cdot DSIN(BETAP2) \]
\[ GG1 = DSIN(PSIN2) \cdot DSIN(BETAP2) \]
\[ GG2 = (UPP2 + DP) \cdot DCOS(BETAP2) \]
\[ GG3 = RPT2 \cdot TN2 / ETAW2 \cdot DSIN(PSIN2) \]
\[ GG4 = SA \cdot DSIN(PSIN2) \]
\[ GG5 = 2 \cdot D0 \cdot AA \cdot TN2 \cdot ETAW2 \cdot THET2P \]
\[ GG6 = TN1 \cdot 2 \cdot D0 \cdot AA / (ETAW2 - THET2P) \]
\[ SPP2 = -(GG2 + GG3 + GG4 + GG5 / GG6) / GG1 \]

**C... SURFACE OF GEAR RACK CUTTER**

\[ XP = (UPP2 + DP) \cdot DCOS(PSIN) \]
\[ YP = ((UPP2 + DP) \cdot DSIN(PSIN) + AM) \cdot DCOS(BETAP) + SPP2 \cdot DSIN(BETAP) \]
\[ ZP = -(UPP2 + DP) \cdot DSIN(PSIN) + AM \cdot DCOS(BETAP) + SPP2 \cdot DCOS(BETAP) \]
\[ X2 = -DCOS(GPHI2) \cdot XP + DSIN(GPHI2) \cdot YP + RPT2 \cdot DCOS(GPHI2) \]
\[ &+ SS2 \cdot DSIN(GPHI2) \]
\[ Y2 = -DSIN(GPHI2) \cdot XP - DCOS(GPHI2) \cdot YP + RPT2 \cdot DSIN(GPHI2) \]
\[ &- SS2 \cdot DCOS(GPHI2) \]
\[ Z2 = ZP \]

**C... NORMAL OF RACK CUTTER IN SW**

\[ XNP2 = DSIN(PSIN) \]
\[ YNP2 = -DCOS(PSIN) \cdot DCOS(BETAP) \]
\[ ZNP2 = DCOS(PSIN) \cdot DSIN(BETAP) \]
\[ XNP2 = DSIN(PSIN) \cdot DSIN(BETAP) \]
\[ YNP2 = DCOS(PSIN) \cdot DCOS(BETAP) \]
\[ ZNP2 = -DCOS(PSIN) \cdot DSIN(BETAP) \]

**C... NORMAL OF GEAR IN S2**

\[ XN2 = -DCOS(GPHI2) \cdot XNP2 + DSIN(GPHI2) \cdot YNP2 \]
\[ YN2 = DSIN(GPHI2) \cdot XNP2 - DCOS(GPHI2) \cdot YNP2 \]
\[ ZN2 = ZNP2 \]
\[ DX2 = (DSIN(GPHI2) \cdot XP + DCOS(GPHI2) \cdot YP - RPT2 \cdot DSIN(GPHI2) + \]
\[ &SS2 \cdot DSIN(GPHI2) \cdot DGPHI2 + DSS2 \cdot DSIN(GPHI2) \]
\[ DY2 = (-DCOS(GPHI2) \cdot XP + DSIN(GPHI2) \cdot YP + RPT2 \cdot DCOS(GPHI2) + \]
\[ &SS2 \cdot DSIN(GPHI2) \cdot DGPHI2 - DSS2 \cdot DCOS(GPHI2) \]
\[ DZ2 = 0. D0 \]
\[ RG2(1) = X2 \]
\[ RG2(2) = Y2 \]
\[ RG2(3) = Z2 \]
\[ NG2(1) = XN2 \]
\[ NG2(2) = YN2 \]
\[ NG2(3) = ZN2 \]
\[ KFG = 0. D0 \]
\[ KHG = 0. D0 \]
\[ EF2(1) = DCOS(PSIN) \]
\[ EF2(2) = DSIN(PSIN) \cdot DCOS(BETAP) \]
\[ EF2(3) = -DSIN(PSIN) \cdot DSIN(BETAP) \]
\[ EH2(1) = 0. D0 \]
\[ EH2(2) = DSIN(BETAP) \]
\[ EH2(3) = DCOS(BETAP) \]

**C VELOCITY OF RACK-CUTTER IN S2**

\[ VT21(1) = 0. D0 \]
\[ VT21(2) = RPT1 \]
\[ VT21(3) = 0. D0 \]

**C VELOCITY OF GEAR**

\[ VT22(1) = -(YP + SS2) \cdot DGPHI2 \]
\[ VT22(2) = (XP + RPT2) \cdot DGPHI2 \]
\[ VT22(3) = 0. D0 \]
\[ S1 = DSIN(PHI1) \]
C1=DCOS(PHI1)
S2=DSIN(PHI2)
C2=DCOS(PHI2)
LF1(1,1)=C1
LF1(1,2)=S1
LF1(2,1)=-S1
LF1(2,2)=C1
LF1(1,3)=0.D0
LF1(2,3)=0.D0
LF1(3,1)=0.D0
LF1(3,2)=0.D0
LF1(3,3)=1.D0
LH2(1,1)=C2
LH2(1,2)=-S2
LH2(2,1)=S2
LH2(2,2)=C2
LH2(1,3)=0.D0
LH2(2,3)=0.D0
LH2(3,1)=0.D0
LH2(3,2)=0.D0
LH2(3,3)=1.D0
CALL MAVEC(V1,LH2,NG2)
CALL MAVEC(N2F,LFH,V1)
CALL MAVEC(V2,LH2,RG2)
CALL MAVEC(V3,LFH,V2)
CALL ADDVEC(R2F,V3,UI,CC)
CALL MAVEC(N1F,LF1,NG1)
CALL MAVEC(R1F,LF1,RG1)
F(1)=(R1F(1)-R2F(1))
F(2)=(R1F(2)-R2F(2))
F(3)=(R1F(3)-R2F(3))
F(4)=N1F(1)-N2F(1)
F(5)=N1F(2)-N2F(2)
IF(INTERM.EQ.1) THEN
  F(6)=ZI-DD
ENDIF
RETURN
END

C... FOR PINION DATA **
C...

SUBROUTINE DATAT1
IMPLICIT REAL*8(A-H,O-Z)
REAL*8 UP,LAMDPI,LAMDBI
COMMON /DATT/ TN1,PNI,PSINI,BETAPI,ADGI,DEGI,LAMDPI,
& FW1,RPT1,RBT1,RAT1,RDT1,PSIT1,LAMDBI
RPT1=TN1/(2.D0*PNI*DCOS(BETAPI))
PSIT1=DATAN(DTAN(PSINI)/DCOS(BETAPI))
RBT1=RPT1*DCOS(PSIT1)
RAT1=RPT1+ADGI
RDT1=RPT1-DEGI
LAMDBI=DATAN(DTAN(LAMDPI)/DCOS(PSIT1))
BETAPI=DATAN(DTAN(BETAPI)*DCOS(PSIT1))
WRITE(55,110)
110 FORMAT(/2X,'*****************************************',/
& 2X,'* DATA OF PINION *',/
& 2X,'********************************************',/)
WRITE(55,120) TN1,PNI,PSINI,BETAPI,ADGI,DEGI,LAMDPI,FW1,
& RPT1,RBT1,RAT1,RDT1,PSIT1,LAMDBI
120 FORMAT(2X,'GEARNUMBER OF TEETH TN1=',F14.7/
&2X,'NORMAL DIAMETRAL PITCH (1/MM) PN1='F14.7/
&2X,'NORMAL PRESSURE ANGLE (RAD.) PSIN1='F14.7/
&2X,'LEADING ANGLE OF HELIX' /
&4X,'ON PITCH CYLINDER (RAD.) BETAP1='F14.7/
&2X,'ADDENDUM (MM) ADG1='F14.7/
&2X,'DEDENDUM (MM) DEG1='F14.7/
&2X,'HELIX ANGLE ON PITCH CYLINDER(RAD) LAMDP1='F14.7/
&2X,'FACE WIDTH (MM) FW1='F14.7/
&2X,'RADIUS OF PITCH CYLINDER (MM) RPT1='F14.7/
&2X,'RADIUS OF BASE CYLINDER (MM) RBT1='F14.7/
&2X,'RADIUS OF ADDENDUM CYLINDER (MM) RAT1='F14.7/
&2X,'RADIUS OF DEEDENDUM CYLINDER (MM) RDT1='F14.7/
&2X,'TRANSVERSE PRESSURE ANGLE (RAD) PSIT1='F14.7/
&2X,'HELIX ANGLE ON BASE CYLINDER (RAD) LAMDB1='F14.7/

RETURN
END

C...

THE SUBROUTINE IS FOR DATA OF THEORITICAL GEAR SURFACE

SUBROUTINE DATAT2
IMPLICIT REAL*8(A-H,O-Z)
REAL*8 UP2,LAMDP2,LAMDB2
COMMON /DAT2/ TN2,PN2,PSIN2,BETAP2,ADG2,DEG2,LAMDP2,
& UP2,FW2,RPT2,RBT2,RAT2,RDT2,PSIT2,LAMDB2
RPT2=TN2/(2.D0*PN2*DCOS(BETAP2)
PSIT2=DATAN(DTAN(PSIN2)/DCOS(BETAP2))
RBT2=RPT2*DCOS (PSIT2)
RAT2=RPT2+ADG2
RDT2=RPT2-DEG2
LAMDB2=DATAN(DTAN(LAMDP2)/DCOS(PSIT2))
BETAB2=DATAN(DTAN(BETAP2)*DCOS(PSIT2))
DEL2=0.007
WRITE(6,110)
WRITE(55,110)
110 FORMAT(/2X,'*******************************************/
& 2X,'* DATA OF GEAR 2 *',/
& 2X,'*******************************************/)
WRITE(55,120) TN2,PN2,PSIN2,BETAP2,ADG2,DEG2,LAMDP2,
& FW2,RPT2,RBT2,RAT2,RDT2,PSIT2,LAMDB2,DEL2
120 FORMAT (2X,'GEAR NUMBER OF TEETH TN2='F14.7/
&2X,'NORMAL DIAMETRAL PITCH (1/MM) PN2='F14.7/
&2X,'NORMAL PRESSURE ANGLE (RAD.) PSIN2='F14.7/
&2X,'LEADING ANGLE OF HELIX' /
&4X,'ON PITCH CYLINDER (RAD) BETAP2='F14.7/
&2X,'ADDENDUM (MM) ADG2='F14.7/
&2X,'DEDENDUM (MM) DEG2='F14.7/
&2X,'HELIX ANGLE ON PITCH CYLINDER(RAD) LAMDP2='F14.7/
&2X,'FACE WIDTH (MM) FW2='F14.7/
&2X,'RADIUS OF PITCH CYLINDER (MM) RPT2='F14.7/
&2X,'RADIUS OF BASE CYLINDER (MM) RBT2='F14.7/
&2X,'RADIUS OF ADDENDUM CYLINDER (MM) RAT2='F14.7/
&2X,'RADIUS OF DEEDENDUM CYLINDER (MM) RDT2='F14.7/
&2X,'TRANSVERSE PRESSURE ANGLE (RAD) PSIT2='F14.7/
&2X,'HELIX ANGLE ON BASE CYLINDER (RAD) LAMDB2='F14.7/
&2X,'ELASTIC APPROACH (MM) DEL='F14.7)

RETURN
END

C...
ADDISON OF TWO VECTORS

SUBROUTINE ADDVEC(VA, VB, VC, DD)
IMPLICIT REAL*8 (A-H, O-Z)
REAL*8 VA(3), VB(3), VC(3)
DO 101 I=1,3
   VA(I) = VB(I) + DD*VC(I)
101 CONTINUE
RETURN
END

DOT PRODUCT OF TWO VECTOR

SUBROUTINE DOTVEC(AA, VA, VB)
IMPLICIT REAL*8 (A-H, O-Z)
REAL*8 VA(3), VB(3)
AA = 0.0
DO 102 I=1,3
   AA = AA + VA(I) * VB(I)
102 CONTINUE
RETURN
END

CROSS PRODUCT OF TWO VECTOR

SUBROUTINE CROVEC(VA, VB, VC)
IMPLICIT REAL*8 (A-H, O-Z)
REAL*8 VA(3), VB(3), VC(3)
VA(1) = VB(2) * VC(3) - VB(3) * VC(2)
VA(2) = VB(3) * VC(1) - VB(1) * VC(3)
VA(3) = VB(1) * VC(2) - VB(2) * VC(1)
RETURN
END

PRODUCT OF MATRIX AND A VECTOR

SUBROUTINE MAVEC(VA, MC, VB)
IMPLICIT REAL*8 (A-H, O-Z)
REAL*8 MC(3,3), VA(3), VB(3)
DO 103 I=1,3
   VA(I) = 0.0
   DO 104 J=1,3
      VA(I) = MC(I, J) * VB(J) + VA(I)
104 CONTINUE
103 CONTINUE
RETURN
END

PRODUCT OF A VECTOR AND A SCALAR

SUBROUTINE PDSVEC(VA, VB, T)
IMPLICIT REAL*8 (A-H, O-Z)
REAL*8 VA(3), VB(3)
DO 105 I=1,3
   VA(I) = T * VB(I)
105 CONTINUE
RETURN
END

STANDARDIZATION OF A VECTOR
SUBROUTINE STDVEC(VA, VB)
IMPLICIT REAL*8 (A-H, O-Z)
REAL*8 VA(3), VB(3)
CC=0.
DO 106 I=1,3
    CC=CC+VB(I)**2
106 CONTINUE
CN=DSQRT(CC)
DO 107 I=1,3
    VA(I)=VB(I)/CN
107 CONTINUE
RETURN
END

SUBROUTINE EQVEC(VA, VB)
IMPLICIT REAL*8 (A-H, O-Z)
REAL*8 VA(3), VB(3)
DO 108 I=1,3
    VA(I)=VB(I)
108 CONTINUE
RETURN
END

SUBROUTINE TRIVEC(AA, VA, VB, VC)
IMPLICIT REAL*8 (A-H, O-Z)
REAL*8 VA(3), VB(3), VC(3), V(3)
CALL CROVEC(V, VB, VC)
CALL DOTVEC(AA, VA, V)
RETURN
END

SUBROUTINE TRANSM(AA, BB)
IMPLICIT REAL*8 (A-H, O-Z)
REAL*8 AA(3,3), BB(3,3)
AA(1,1)=BB(1,1)
AA(1,2)=BB(2,1)
AA(1,3)=BB(3,1)
AA(2,1)=BB(1,2)
AA(2,2)=BB(2,2)
AA(2,3)=BB(3,2)
AA(3,1)=BB(1,3)
AA(3,2)=BB(2,3)
AA(3,3)=BB(3,3)
RETURN
END

SUBROUTINE MAPMA(AA, BB, CC)
IMPLICIT REAL*8 (A-H, O-Z)
REAL*8 AA(3,3), BB(3,3), CC(3,3)
AA(1,1)=BB(1,1)*CC(1,1)+BB(1,2)*CC(2,1)+BB(1,3)*CC(3,1)
AA(1,2)=BB(1,1)*CC(1,2)+BB(1,2)*CC(2,2)+BB(1,3)*CC(3,2)

70
\[
AA(1,3) = BB(1,1) * CC(1,3) + BB(1,2) * CC(2,3) + BB(1,3) * CC(3,3)
\]
\[
AA(2,1) = BB(2,1) * CC(1,1) + BB(2,2) * CC(2,1) + BB(2,3) * CC(3,1)
\]
\[
AA(2,2) = BB(2,1) * CC(1,2) + BB(2,2) * CC(2,2) + BB(2,3) * CC(3,2)
\]
\[
AA(2,3) = BB(2,1) * CC(1,3) + BB(2,2) * CC(2,3) + BB(2,3) * CC(3,3)
\]
\[
AA(3,1) = BB(3,1) * CC(1,1) + BB(3,2) * CC(2,1) + BB(3,3) * CC(3,1)
\]
\[
AA(3,2) = BB(3,1) * CC(1,2) + BB(3,2) * CC(2,2) + BB(3,3) * CC(3,2)
\]
\[
AA(3,3) = BB(3,1) * CC(1,3) + BB(3,2) * CC(2,3) + BB(3,3) * CC(3,3)
\]
RETURN
END

C
THE SUBROUTINE IS FOR MISALIGNMENT

SUBROUTINE MIAL (K, DGAMM, LFH)
IMPLICIT REAL*8 (A-H,O-Z)
REAL*8 LFH(3,3)
S3 = DSIN (DGAMM)
C3 = DCOS (DGAMM)

C ** FOR CROSSING ANGLE MISALIGNMENT
IF (K .EQ. 1) THEN
  LFH(1,1) = -1.0D0
  LFH(1,2) = 0.0D0
  LFH(1,3) = 0.0D0
  LFH(2,1) = 0.0D0
  LFH(2,2) = -C3
  LFH(2,3) = -S3
  LFH(3,1) = 0.0D0
  LFH(3,2) = -S3
  LFH(3,3) = C3
ENDIF

C ** FOR INTERSECTING ANGLE MISALIGNMENT
IF (K .EQ. 2) THEN
  LFH(1,1) = -C3
  LFH(1,2) = 0.0D0
  LFH(1,3) = S3
  LFH(2,1) = 0.0D0
  LFH(2,2) = -1.0D0
  LFH(2,3) = 0.0D0
  LFH(3,1) = S3
  LFH(3,2) = 0.0D0
  LFH(3,3) = C3
ENDIF

RETURN
END

C .... COMPUTE THE PRINCIPAL CURVATURES OF MODIFIED cutter SURFACE

SUBROUTINE CURVT (KK)
IMPLICIT REAL*8 (A-H,O-Z)
REAL*8 EFN(3) , EHN(3) , WIVT2(3) , WVT2(3) , W12(3) , W2VT1(3) , KF, KH, KS, KQ
REAL*8 KM2, KT2, KFF, KHH
COMMON/A200/ WI(3), W2(3), WI2(3), VTI(3), VT2(3), VI2(3)
COMMON/A210/ EX(3), EF(3), EH(3)
COMMON/A220/ W1(3), W2(3), W12(3), VT1(3), V2(3), V12(3)
COMMON/A230/ KF, KH
COMMON/A240/ ET(3), EM(3)
COMMON/A300/ ES(3), EQ(3)
COMMON/A310/ KFF, KHH
COMMON/A380/ KS, KQ

EFN(1) = EX(2) * EF(3) - EX(3) * EF(2)
EFN(2) = -(EX(1) * EF(3) - EX(3) * EF(1))
EFN(3) = EX(1) * EF(2) - EX(2) * EF(1)
EHN(1) = EX(2) * EH(3) - EX(3) * EH(2)
EHN(2) = -(EX(1) * EH(3) - EX(3) * EH(1))
EHN(3) = EX(1) * EH(2) - EX(2) * EH(1)
W1VT2(1) = W1(2) * VT2(3) - W1(3) * VT2(2)
W1VT2(2) = -(W1(1) * VT2(3) - W1(3) * VT2(1))
W1VT2(3) = W1(1) * VT2(2) - W1(2) * VT2(1)
W2VT1(1) = W2(2) * VT1(3) - W2(3) * VT1(2)
W2VT1(2) = -(W2(1) * VT1(3) - W2(3) * VT1(1))
W2VT1(3) = W2(1) * VT1(2) - W2(2) * VT1(1)
W12(1) = W12(2) * V12(3) - W12(3) * V12(2)
W12(2) = -(W12(1) * V12(3) - W12(3) * V12(1))
W12(3) = W12(1) * V12(2) - W12(2) * V12(1)
V12F = 0.0
V12H = 0.0
WNEF = 0.0
WNEH = 0.0
VWN = 0.0
W1TN = 0.0
W2TN = 0.0
DO 1 I = 1, 3
   V12F = V12(I) * EF(I) + V12F
   V12H = V12(I) * EH(I) + V12H
   WNEF = W12(I) * EFN(I) + WNEF
   WNEH = W12(I) * EHN(I) + WNEH
   VWN = EX(I) * WVT2(I) + VWN
   W1TN = EX(I) * W1VT2(I) + W1TN
1
W2TN = EX(I) * W2VT1(I) + W2TN
C... COMPUTE THE CURVATURE OF THE GENERATED SURFACE
B13 = -KF * V12F - WNEF
B23 = -KH * V12H - WNEH
B33 = -KF * V12F ** 2 - KH * V12H ** 2 + VWN - W1TN + W2TN
B11 = B13 ** 2 / B33
B12 = B13 * B23 / B33
B22 = B23 ** 2 / B33
T1 = 2.0D00 * B13 * B23
T2 = B23 ** 2 - B13 ** 2 - (KF - KH) * B33
SIG1F = 0.5D00 * DATAN(T1 / T2)
C....PRINCIPAL CURVATURES OF THE GENERATED SURFACE
IF (DABS(SIG1F) .LE. 0.1D-5) THEN
   KQ = 0.5D00 * ((KF + KH) + (B13 ** 2 + B23 ** 2) / B33
   & + (B23 ** 2 - B13 ** 2 - (KF - KH) * B33) / (B33 * DCOS(SIG1F))
   KS = KQ - (B23 ** 2 - B13 ** 2 - (KF - KH) * B33) / (B33 * DCOS(SIG1F))
ELSE
   KQ = 0.5D00 ** (KF + KH) - 0.5D00 ** (B13 ** 2 + B23 ** 2) / B33
   & + B13 * B23 / (B33 * DSIN(2.0D00 ** (+SIG1F)))
   KS = KQ - 2.0D00 ** B13 * B23 / (B33 * DSIN(2.0D00 ** (+SIG1F)))
ENDIF
SIGSF = -SIG1F
C....PRINCIPAL DIRECTIONS OF THE GENERATED SURFACE
DO 2 I = 1, 3
   EQ(I) = DCOS(SIGSF) * EH(I) - DSIN(SIGSF) * EF(I)
2
   ES(I) = DSIN(SIGSF) * EH(I) + DCOS(SIGSF) * EF(I)
   IF (KK .LT. 2) GO TO 100
   Q3 = ES(1) * ET(1) + ES(2) * ET(2) + ES(3) * ET(3)
   Q4 = EQ(1) * ET(1) + EQ(2) * ET(2) + EQ(3) * ET(3)
   IF (Q3, EQ. Q4) THEN
      Q34 = DATAN(Q4)
   ELSE
      Q34 = DATAN2(Q4, Q3)
   ENDIF
ENDIF
COMPUTE THE DIMENSIONS AND DIRECTIONS OF THE CONTACT ELLIPSE

SUBROUTINE ELLIP
IMPLICIT REAL*8(A-H,O-Z)
REAL*8 KSIGI,KSIG2,KFF,KHH,KS,KQ
COMMON /A300/ ES(3),EQ(3)
COMMON /A380/ KS,KQ
COMMON /A310/ KFF,KHH
COMMON /A340/ EFF(3),EHH(3)
COMMON /A360/ A,B,SI(3),FI(3)
DEL=0.007D0
PI=DACOS(-0.1D0)
SI=ES(1)*EFF(1)+ES(2)*EFF(2)+ES(3)*EFF(3)
S2=EQ(1)*EFF(1)+EQ(2)*EFF(2)+EQ(3)*EFF(3)
SIGSF=DATAN2(S2,SI)

KSIGI=KFF+KHH
KSIG2=KS+KQ
GI=KFF-KHH
G2=KS-KQ
A=(KSIG1-KSIG2-(GI**2-2.D0*GI*G2*DCOS(2.D0*SIGSF)+G2**2)**0.5)/4.D0
B=(KSIG1-KSIG2+(GI**2-2.D0*GI*G2*DCOS(2.DO*SIGSF)+G2**2)**0.5)/4.DO
A=(DEL/ABS(A))**0.5
B=(DEL/ABS(B))**0.5

S1=G2*DSIN(2.D0*SIGSF)
S2=G1-G2*DCOS(2.D0*SIGSF)
ALF1=0.5D0*DATAN2(S1,S2)
S3=DSQRT(G1**2-2.D0*G1*G2*DCOS(2.D0*SIGSF)+G2**2)
SS2=S1/S3
SC2=S2/S3
ALF1=DATAN(SS2/(1.D0+SC2))
ALF1=DABS(ALF1)

DO 100 I=1,3
   SI(I)=DSIN(ALF1)*EFF(I)+DCOS(ALF1)*EHH(I)
   FI(I)=DCOS(ALF1)*EFF(I)-DSIN(ALF1)*EHH(I)
100 CONTINUE
RETURN
END
The design and generation of modified involute helical gears that have a localized and stable bearing contact, and reduced noise and vibration characteristics are described. The localization of the bearing contact is achieved by the mismatch of the two generating surfaces that are used for generation of the pinion and the gear. The reduction of noise and vibration will be achieved by application of a parabolic function of transmission errors that is able to absorb the almost linear function of transmission errors caused by gear misalignment. The meshing and contact of misaligned gear drives can be analyzed by application of computer programs that have been developed. The computations confirmed the effectiveness of the proposed modification of the gear geometry. A numerical example that illustrates the developed theory is provided.