Summary of Research
For the period November 18, 1993 through November 17, 1994

Prepared for
National Aeronautics and Space Administration
Langley Research Center
Hampton, VA 23681-0001

Under
Research Grant NAG-1-1552
Dr. Gregory Jones, Technical Monitor
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by
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Abstract

A different approach for calibrating hot-wires, which simplifies the calibration procedure and reduces the tunnel run-time by an order of magnitude was sought. In general, it is accepted that the directly measurable quantities in any flow are velocity, density, and total temperature. Very few facilities have the capability of varying the total temperature over an adequate range. However, if overheat temperature parameter, \( a_w \), is used to calibrate the hot-wire then the directly measurable quantity, voltage, will be a function of the flow variables and the overheat parameter i.e., \( E = f(u, \rho, a_w, T_w) \) where \( a_w \) will contain the needed total temperature information. In this report, various methods of evaluating sensitivities with different dependent and independent variables to calibrate a 3-wire hot-wire probe using a constant temperature anemometer (CTA) in subsonic/transonic flow regimes is presented. The advantage of using \( a_w \) as the independent variable instead of total temperature, \( T_o \), or overheat temperature parameter, \( \tau \), is that while running a calibration test it is not necessary to know the recovery factor, the coefficients in a wire resistance to temperature relationship for a given probe. It was deduced that the method employing the relationship \( E = f(u, \rho, a_w) \) should result in the most accurate calibration of hot wire probes. Any other method would require additional measurements. Also this method will allow calibration and determination of accurate temperature fluctuation information even in atmospheric wind tunnels where there is no ability to obtain any temperature sensitivity information at present. This technique greatly simplifies the calibration process for hot-wires, provides the required calibration information needed in obtaining temperature fluctuations, and reduces both the tunnel run-time and the test matrix required to calibrate hot-wires. Some of the results using the above techniques are presented in an appendix.
Nomenclature

\[ a_w = \frac{R_w - R_{adw}}{R_{adw}}, \] Overheat temperature parameter

\[ d_w \] Wire diameter

\[ E \] Mean voltage across wire

\[ Kn \] Knudsen number

\[ k_i \] Thermal conductivity of air at \( T_o \)

\[ \ell \] Wire length

\[ M \] Mach number

\[ m \] Mass of fluid flow

\[ Nu_i \] Nusselt number

\[ P_w \] Power to hot-wire

\[ Q_w \] Heat transfer rate

\[ R_{ref} \] Resistance of wire at reference temperature \( T_{ref} \)

\[ Re_i \] Reynolds number based on viscosity evaluated at \( T_o \) and wire diameter

\[ R_w \] Resistance of wire

\[ S_u \] Velocity sensitivity

\[ S_p \] Density sensitivity

\[ S_{Te} \] Total temperature sensitivity

\[ T_{adw} \] Recovery temperature of wire

\[ T_o \] Total temperature

\[ T_{ref} \] Reference temperature

\[ T_w \] Wire temperature

\[ u \] Velocity

\[ \alpha \ & \beta \] Coefficients in wire resistance to temperature relationship
Introduction

The heat transfer from heated wires in fluid flows has been investigated for many years for various reasons. One of the main reasons in fluid mechanics is the application of the results to hot wire anemometry. The early work in this field was performed by Boussinesq (1905) and King (1914). Even though "King's law" was derived over 75 years ago, it is still being used, in various forms, to predict the heat transfer from heated wires in subsonic incompressible flows. From King's law it can be shown that the heat transfer parameter, Nusselt number, is a function only of Reynolds and Prandtl numbers.

In the 1950's, it was shown that the Nusselt number can be a function of Mach number for Mach numbers as low as 0.10, indicating that King's law does not apply. The reason for this disagreement was determined by Spangenberg to be due to gas rarefaction effects. Spangenberg found that $Nu = f(Re, M, Kn)$, where the apparent Mach number or compressibility effect was a slip flow or Knudsen number effect. Since the Knudsen number is related to the Reynolds and Mach numbers, only two of the three quantities in the relationship for the Nusselt number are required. Experimental results obtained by King and others indicate that the Nusselt number is also a function of a temperature parameter. The functional relationships expressed for Nusselt number are $Nu = f(M, Re, \theta)$ or $Nu = f(M, Kn, \tau)$.
above functional relationship, it has been shown that the mean voltage measured across the heated wire is \( E = f(u, \rho, T_o, T_w) \).

### Theoretical Consideration

To obtain the sensitivity relationships for use in hot wire anemometry, the following heat transfer equation is usually used to describe the heat loss from a heated wire mounted normal to the flow:

\[
Q_w = 2\pi \ell k_i (T_w - \eta T_o) Nu_i
\]  

(1)

For steady state conditions the electrical power supplied to the wire is equal to the heat loss from the wire \( P_w = Q_w \). Differentiating this equation in terms of its logarithmic derivatives yields:

\[
d \log Q_w = d \log Nu_i - \frac{\eta}{\tau} d \log T_o + \frac{\theta}{\tau} d \log T_w - \frac{\eta}{\tau} d \log \eta + d \log k_i
\]  

(2)

where \( d \log T_w = 0 \) for a constant temperature anemometer (CTA).

**For sensitivities depending on \( Nu_i = f(M, Kn, \tau) \):**

Equations were re-derived for the sensitivities of heated wires operated in compressible flow and powered by a constant temperature anemometer using the heat transfer relationship for the variables \( Nu_i = f(M, Kn, \tau) \). The sensitivity equations derived by Baldwin\(^7\) and Sandborn\(^8\) are incomplete for several reasons. In these references the velocity sensitivity includes a \( \partial T_o/\partial u \) term under the constraint that \( T_o = \text{constant} \). There are other terms missing
in the equations due to an incomplete partial differentiation process. Following the assumptions\(^7,8\) that \(Nu_i = f(M, Kn, \tau)\), the change in Nusselt number is:

\[
d\log Nu_i = \frac{\partial \log Nu_i}{\partial \log M} d\log M + \frac{\partial \log Nu_i}{\partial \log Kn} d\log Kn + \frac{\partial \log Nu_i}{\partial \log \tau} d\log \tau
\] (3)

The Mach number is a function of \(u\) and \(T_o\). Thus:

\[
d\log M = \frac{\partial \log M}{\partial \log u} d\log u + \frac{\partial \log M}{\partial \log T_o} d\log T_o
\] (4)

\[
d\log M = \left(1 + \frac{\gamma - 1}{2} M^2\right) d\log u - \frac{1}{2} \left(1 + \frac{\gamma - 1}{2} M^2\right) d\log T_o
\] (5)

The Knudsen number is a function of only density for a given wire diameter, and can be expressed as\(^7\):

\[
Kn = \frac{1.587 \times 10^{-4}}{\rho d_w}
\] or \(d\log Kn = -d\log \rho\) (6)

The recovery temperature ratio, \(\eta\), is assumed to be a function of \(M\) and \(Kn\). Then:

\[
d\log \eta = \frac{\partial \log \eta}{\partial \log M} d\log M + \frac{\partial \log \eta}{\partial \log Kn} d\log Kn
\] (7)

The thermal conductivity for air is evaluated at \(T_o\), giving:

\[
d\log k_i = \frac{\partial \log k_i}{\partial \log T_o} d\log T_o
\] (8)
The power to the heated wire can be written as:

\[ d\log P_w = d\log E_w - d\log R_w \]  \hspace{1cm} (9)

For a constant temperature anemometer:

\[ d\log R_w = 0 \]  \hspace{1cm} (10)

The quantity \( \tau = \frac{T_w - \eta T_o}{T_o} \), therefore, the change in \( \tau \) is:

\[ d\log \tau = -\frac{\theta}{\tau} \frac{d\log T_o}{\tau} - \frac{\eta}{\tau} d\log \eta \]  \hspace{1cm} (11)

Equations (5-11) can be substituted into (2) to give:

\[ d\log E_w = S_u d\log u + S_\rho d\log \rho + S_{\tau} d\log T_o \]  \hspace{1cm} (12)

where

\[ S_u = \frac{1}{2} \left( 1 + \frac{\tau - 1}{2} \right) \left[ \frac{\partial \log Nu}{\partial \log M} - \frac{\eta}{\tau} \frac{\partial \log \eta}{\partial \log \tau} \left( \frac{\partial \log Nu}{\partial \log \tau} + 1 \right) \right] \]  \hspace{1cm} (13)

\[ S_\rho = \frac{1}{2} \left[ -\frac{\partial \log Nu}{\partial \log Kn} + \frac{\eta}{\tau} \frac{\partial \log \eta}{\partial \log \tau} \left( \frac{\partial \log Nu}{\partial \log \tau} + 1 \right) \right] \]  \hspace{1cm} (14)

\[ S_{\tau} = \frac{1}{2} \left( -\frac{1}{2} S_u - \left( 1 + \frac{\eta}{\tau} \right) \frac{\partial \log Nu}{\partial \log \tau} - \frac{\eta}{\tau} + \frac{\partial \log k}{\partial \log T_o} \right) \]  \hspace{1cm} (15)
The sensitivity equations as derived by Baldwin in reference 7 using functional relationship $Nu_t = f(M, Kn, \tau)$ are as follows:

$$\frac{dI}{I} = \frac{1}{2} \left\{ \left[ \left( 1 + \frac{\gamma - 1}{2} M^2 \right) \left( \frac{\partial \log Nu_t}{\partial \log M} - \frac{M}{\tau} \frac{\partial \eta}{\partial M} \right) - \frac{\eta (\gamma - 1) M^2}{\tau} \right] \frac{dU}{U} \right\}$$

These equations (13-15) for velocity, density and total temperature sensitivities are different from equation (16) for several reasons. In equation (16) the velocity was differentiated with respect to $T_0$ under the assumption that $T_0 = constant$. Also, the differentiation processes in was incomplete.

The re-derived equations were used to obtain the velocity, density and total temperature sensitivities for hot-wires and are compared with the sensitivities computed using the functional relationship $E = f(u, \rho, T_o, T_w)$. The constraints required by calculus and the functional form of the equation dictated by the data were met by both methods for computing sensitivities. These results are presented in the enclosed appendix.

For sensitivities depending on $Nu_t = f(M, Kn, a_w)$:

Following the assumptions that $Nu_t = f(M, Kn, a_w)$, the change in Nusselt number is:

$$d \log Nu_t = \frac{\partial \log Nu_t}{\partial \log M} d \log M + \frac{\partial \log Nu_t}{\partial \log Kn} d \log Kn + \frac{\partial \log Nu_t}{\partial \log a_w} d \log a_w$$

The quantity $a_w = \frac{R_w - R_{adv}}{R_{adv}}$ and $R_{adv} = f(M, Kn, T_o)$. Therefore, the change in $a_w$ is:
Substituting equations (5-10), (17) and (18) in equation (2) leads to equation (12) where

\[ S_v = \frac{1}{2} \left\{ \left( 1 + \frac{\gamma}{2} M^2 \right) \left[ \frac{\partial \log (1 + \frac{\gamma}{2} M^2)}{\partial \log M} \left( \frac{1}{2} d \log u - \frac{1}{2} d \log T_o \right) \right] \right\} \]  

(19)

\[ \frac{S}{\rho} = \frac{1}{2} \left\{ \frac{\partial \log T_i}{\partial \log a_w} - \frac{\partial \log k_i}{\partial \log a_w} \right\} \]  

(20)

\[ S_{r_v} = \frac{1}{2} \left\{ -2S_v - \frac{\partial \log k_i}{\partial \log a_w} \right\} \]  

(21)

For sensitivities depending on \( E = f(M, Kn, a_w) \):

If we use the relationship \( E = f(M, Kn, a_w) \) then the change in voltage is:

\[ d \log E = \frac{\partial \log E}{\partial \log M} d \log M + \frac{\partial \log E}{\partial \log Kn} d \log Kn + \frac{\partial \log E}{\partial \log a_w} d \log a_w \]  

(22)

Substituting equations (5) through (10) and (18) in equation (22) leads to equation (12) where

\[ S_v = \left( 1 + \frac{\gamma}{2} M^2 \right) \left[ \frac{\partial \log E}{\partial \log M} - \frac{\partial \log E}{\partial \log a_w} \frac{\partial \log k_i}{\partial \log a_w} \right] \]  

(23)
\[ S = \left( \frac{\partial \log E}{\partial \log u} \right)_{\rho, T_w, T_o} \]  
(24)

\[ S_{T_o} = \left( \frac{\partial \log E}{\partial \log T_o} \right)_{\rho, u, T_w} \]  
(25)

Resistance to temperature relationship can be written as:

\[ R_w = R_{ref} \left\{ 1 + \alpha(T_w - T_{ref}) + \beta(T_w - T_{ref})^2 \right\} \]  
(26)

Using equation (26) the quantity \( \frac{\partial \log R_{adv}}{\partial \log T_o} \) can be obtained as:

\[ \frac{\partial \log R_{adv}}{\partial \log T_o} = \frac{T_{adv} \left[ \alpha + 2\beta(T_{adv} - T_{ref}) \right]}{1 + \alpha(T_{adv} - T_{ref}) + (T_{adv} - T_{ref})^2} \]  
(27)

For sensitivities depending on \( E = f(u, \rho, T_o) \):

For a constant temperature anemometer (CTA), the change in voltage across the wire can be expressed as \( E = f(u, \rho, T_o) \). Once the wire is calibrated and functional relationships are defined, the sensitivities can be evaluated by taking the partial change in \( E \) with respect to \( u, \rho \) and \( T_o \) in turn. The equations for sensitivities can be written as:

\[ S_u = \left( \frac{\partial \log E}{\partial \log u} \right)_{\rho, T_w, T_o} \]  
(28)

\[ S_{\rho} = \left( \frac{\partial \log E}{\partial \log \rho} \right)_{u, T_w, T_o} \]  
(29)
This method of obtaining sensitivities is the direct method since the output voltage, which is a directly measurable quantity, is a function of velocity, density and total temperature which are in turn directly measurable quantities.

For sensitivities depending on $Nu_i = f(M, Re_i, \theta)$:

For the sake of completeness, the sensitivity equations for constant current anemometers (CCA) using the functional relationship of $Nu_i = f(M, Re_i, \theta)$, as derived by Morkovin\textsuperscript{5}, are repeated here. They are:

\begin{equation}
S_u = \frac{1}{2} \left\{ \left( 1 + \frac{\gamma - 1}{2} M^2 \right) \left[ \frac{\partial \log Nu_i}{\partial \log M} \bigg|_{Re_i, \theta} - \frac{\eta}{\tau_w} \frac{\partial \log \eta}{\partial \log M} \bigg|_{M, \theta} \right] + S_p \right\} \tag{31}
\end{equation}

\begin{equation}
S_p = \frac{1}{2} \left\{ \frac{\partial \log Nu_i}{\partial \log Re_i} \bigg|_{M, \theta} - \frac{\eta}{\tau_w} \frac{\partial \log \eta}{\partial \log Re_i} \bigg|_{M, \theta} \right\} \tag{32}
\end{equation}

\begin{equation}
S_{r_1} = \frac{1}{2} \left\{ S_u + S_p (2 m_i - 1) - \frac{\partial \log Nu_i}{\partial \log \theta} \bigg|_{M, Re_i} - \frac{\eta}{\tau_w} + \frac{\partial \log k_i}{\partial \log T_o} \right\} \tag{33}
\end{equation}

\textbf{Experimental Description:}

The data were collected using the "Probe Calibration Tunnel" (PCT) at NASA Langley research center which is an open jet tunnel in which velocity, density and total temperature can be controlled independently. The combination of a staged pressure system, low mass flow rates through the facility and the availability of large vacuum systems enables this facility to run
continuously. It has two interchangable, subsonic and transonic nozzles, having diameters of 1.50 and 2.25 inches respectively, that yields a continuous flow capability over the Mach number range of 0.05 to 1.0. Tunnel stagnation pressure and temperature can be varied from a minimum of 0.20 atmospheres to a maximum of 10 atmospheres and from 500°R to 600°R, respectively. This corresponds to a Reynolds number range of $1 \times 10^6 < Re/ft < 51 \times 10^6$ for a Mach number of 1. The 3-wire hot-wire probe utilized three different diameters of Platinum coated Tungsten wires (diameters of 0.0001, 0.0002 and 0.0003 inch) and was used to obtain most of the calibration data. The test matrix consisted of 12 velocities, 7 densities and 5 total temperatures for the cases where $E = f(u, \rho, a_w, T_w)$. For $E = f(M, Kn, a_w)$, the test matrix consisted of 12 Mach numbers, 7 Knudsen numbers and 8 overheats. The lower and upper bounds of the test matrix for the two methods are given in Tables I and II. In this study all Knudsen numbers were greater than 0.01, indicating that all the data were in the slip flow regime based on the definition of the slip flow.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total pressure, psi</td>
<td>3.40306</td>
<td>46.2992</td>
</tr>
<tr>
<td>Static pressure, psi</td>
<td>2.83727</td>
<td>31.5238</td>
</tr>
<tr>
<td>Velocity, ft/sec</td>
<td>113.785</td>
<td>950.953</td>
</tr>
<tr>
<td>Mach number</td>
<td>0.0982</td>
<td>0.941</td>
</tr>
<tr>
<td>Total temperature, °R</td>
<td>500</td>
<td>600</td>
</tr>
<tr>
<td>Static temperature, °R</td>
<td>417.33</td>
<td>579.73</td>
</tr>
<tr>
<td>Reynolds number/foot</td>
<td>0.3 x $10^6$</td>
<td>$12.3 x 10^6$</td>
</tr>
<tr>
<td>Density</td>
<td>0.01875</td>
<td>0.14693</td>
</tr>
<tr>
<td>Mass flow, lb/sec</td>
<td>0.012832</td>
<td>1.71468</td>
</tr>
</tbody>
</table>

Table I. Minimum and Maximum Test Conditions of Test Matrix for Functional Relationship $E = f(u, \rho, a_w, T_w)$. 
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total pressure, psi</td>
<td>3.675</td>
<td>29.4</td>
</tr>
<tr>
<td>Static pressure, psi</td>
<td>3.44859</td>
<td>26.2216</td>
</tr>
<tr>
<td>Mach number</td>
<td>0.1</td>
<td>0.95</td>
</tr>
<tr>
<td>Total temperature, °R</td>
<td>540</td>
<td>540</td>
</tr>
<tr>
<td>Static temperature, °R</td>
<td>457.433</td>
<td>539.73</td>
</tr>
<tr>
<td>Reynolds number/foot</td>
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<td>13.3 x 10^6</td>
</tr>
<tr>
<td>Knudsen number</td>
<td>0.00494</td>
<td>0.10369</td>
</tr>
<tr>
<td>Overheat ratio</td>
<td>0.3</td>
<td>1.0</td>
</tr>
<tr>
<td>Density</td>
<td>0.01875</td>
<td>0.13333</td>
</tr>
<tr>
<td>Mass flow, lb/sec</td>
<td>0.013103</td>
<td>1.7193</td>
</tr>
</tbody>
</table>

Table II. Minimum and Maximum Test Conditions of Test Matrix for Functional Relationship $E = f(M, Kn, a_w)$.

**Discussion:**

To illustrate the complexity of the sensitivity equations, consider the relationship between voltage across a wire mounted normal to the flow and the quantities $u, \rho$ and $T_o$ for the constant temperature anemometer. Utilizing a small perturbation assumption we can write:

$$\frac{e'}{E} = S_u \frac{u'}{u} + S_\rho \frac{\rho'}{\rho} + S_{T_o} \frac{T_o'}{T_o}$$  \hspace{1cm} (34)
Evaluating the partial derivatives in the above sensitivity equations requires care when carrying out the mean flow calibration. For example, consider equations (31-33). The evaluation of \( \frac{\partial \log \textit{Nu}_t}{\partial \log \textit{Re}_t} \bigg|_{M, \theta} \) must be obtained by varying \( P_o \) only and the Mach number and the total temperature must be held constant. Also, the evaluation of \( \frac{\partial \log \textit{Nu}_t}{\partial \log \textit{M}} \bigg|_{\textit{Re}_t, \theta} \) requires that the total pressure be changed when the Mach number is varied in order to maintain Reynolds number constant. Similar constraints must be observed when \( \frac{\partial \log \eta}{\partial \log \textit{Re}_t} \bigg|_{M} \) and \( \frac{\partial \log \eta}{\partial \log \textit{M}} \bigg|_{\textit{Re}_t} \) are evaluated. Care must also be taken in evaluating the partial derivatives when using other dependent and independent variables. This method of obtaining sensitivities is time consuming when conducting experiments in large wind tunnels since the Mach number and the total pressures must be varied in order to maintain the prescribed non-varying independent variable at a constant level when the variation of the remaining dependent variables being sought.

If \( \textit{Nu}_t = f(M, \textit{Re}_t, \theta) \) and the above described constraint is applied, the operational envelope for the facility must be considered, since there is a skewing of the \( \textit{Nu}_t \) vs. \( \textit{Re}_t \) curves for constant Mach numbers because \( \textit{Re}_t = f(u, \rho)^9 \). Due to this skewing, the region over which the partial derivatives can be evaluated will reduce rapidly. The advantage of using the suggested functional relationship i.e., \( E_b = f(m, T_o, T_w) \) or \( E_b = f(m, a_w, T_w) \) can be seen by plots of \( \textit{Nu}_t \) vs. \( \text{Kn} \) for constant Mach number where the data are not as skewed and a more complete set of derivatives can be evaluated from a given number of data points\(^5 \). This efficient use of data with the wire voltage correlated in terms of these primitive variables cannot be ignored and that will reduce the amount of data needed for calibration by an order of magnitude. Eventually, that attribute leads to better accuracy in the calibration process.

The effect of wire temperature is complicated due to its nonlinear variation and because it is not fully understood. In the literature several methods based on various heat transfer
relationships, using different dependent and independent variables for a given type of anemometer, are available to compute hot-wire sensitivities. Among these, the most extensively used correlations are $Nu = f(M_a, Kn, T_w)$ and $Nu = f(M_a, Re, \theta)$. All these methods should give identical results for the sensitivity of a given hot-wire at given test conditions. Hot-wire calibrations have been performed as part of the PCT facility flow quality investigation, in various flow regimes. At higher speeds, where compressible flow effects occur, it has been found that King's Law is not valid$^{1,2,10}$. Hence, those flow regimes have been investigated.

In the 1950's, Kovasznay$^{11,12}$ extended hot-wire anemometry to compressible flows where it was found experimentally that in supersonic flow the heated wire was sensitive only to mass flow and total temperature. Kovasznay developed a graphical technique to obtain these fluctuations, which is used primarily in supersonic and hypersonic flows. Kovasznay's$^{12}$ compressible flow results showed that there was a significant difference between the heat transfer in compressible and incompressible flows. In subsonic compressible, transonic and low supersonic flows, effects due to compressibility influence the heat transfer from a wire. In high supersonic and hypersonic flows a strong shock occurs ahead of the wire and the heat transfer from the wire is influenced by subsonic flow downstream of the shock front. Because of this, it was found experimentally that in supersonic/hypersonic flows $Nu = f(Re, \theta)$ only, and the heat transfer from the wire was again a function of mass flow, total temperature, and wire temperature. For this reason some understanding of subsonic flow while studying supersonic or hypersonic flow, becomes essential.

In continuum flow the mean free path of the particles is less than the diameter of the wire and conventional heat transfer theories are applicable. When the diameter of the wire approaches a few mean free paths between the particles, the flow does not behave as a continuum, but exhibits some effects of the finite spacing between the particles. These effects have been studied$^{13,14}$ by assuming finite velocity and temperature jump boundary conditions.
This gas rarefaction regime was noted as slip flow. In free molecular flow the fluid is assumed to be composed of individual particles and the distance between the particles is sufficiently large that their impact with and reflection from a body is assumed to occur without interaction between the particles.

Conclusions

The following conclusions have been made:

1. The equation obtained for a constant temperature anemometer (CTA) based on the assumption that \( E = f(u, \rho, T_0) \) resulted in an equation in which neither wire length nor the coefficients in the wire resistance vs. temperature relationship were required. This should result in a more accurate method for calibrating hot-wire probes in facilities where total temperature can be varied.

2. The equation obtained for a constant current anemometer (CCA) based on the assumption that \( E = f(u, \rho, T_0, T_w) \) results in an equation that requires the variation of four independent variables and is believed to be too complicated for routine use. The equation also requires evaluation of \( \beta \) who's elimination was desired.

3. The equation obtained for a CCA and CTA under the assumption that \( Nu_t = f(M, Kn, a_w) \) and \( T_c = \text{constant} \) resulted in an equation where \( \alpha \) and \( \beta \) occurred in only one term in the equation for the total temperature sensitivity. Consequently, this calibration of wires should be more accurate than those obtained using conventional methods.
The sensitivities computed based on the methods presented using the data obtained in the "Probe Calibration Tunnel" are comparable. However, differences in $S_T$ are more significant than for $S_u$ and $S_p$.

In general, the velocity sensitivity ranges from 0.05 to 0.40, density sensitivity ranges from 0.10 to 0.40 and total temperature sensitivity ranges from -0.2 to -1.0.

The equations (13-15) for the sensitivities of a constant temperature anemometer are different from those in references 7 and 8. In those references, the velocity sensitivity has a term for $\frac{\partial \log T_o}{\partial \log u}$ under the constraints that $T_o = \text{constant}$. There are other terms missing in the equations due to the incomplete partial differentiation process.
References


Appendix
A Rational Technique for Calibrating Hot-Wire Probes in Subsonic to Supersonic Speeds

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June 20-23, 1994 / Colorado Spring, CO
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Abstract

Hot-wire anemometry is a well developed technique for measuring fluctuations in flowing fluids where its application depends on the mean flow calibration of the heated wire. The conventional technique usually used to calibrate hot-wire probes is based on the determination of Nusselt number and wire recovery temperature ratio as functions of Reynolds number and Mach number. This method requires the measurement of the wire length and the coefficients in the temperature - resistance equation for the wire material. The accurate measurement of these quantities are difficult to make, particularly the wire length and the coefficient of the second degree term in the temperature - resistance equation. The lack of accurate values for these two quantities are possible sources of error in the determination of the sensitivities for the heated wire to heat transfer from the wire. A technique is proposed that reduces the dependency of the sensitivities of wires to these quantities.

Nomenclature

\[ a'_w = \frac{(R_e - R_{	ext{wire}})}{R_{	ext{wire}}} \]
\[ A = \left[ 1 - \frac{\theta}{\alpha'_w (1 + \theta)} \frac{\partial \ln E}{\partial \ln a'_w} \right]^{-1} \]
\[ c_p \] specific heat at constant pressure
\[ c_v \] specific heat at constant volume
\[ d_w \] diameter of the hot-wire
\[ dA_w \] elemental surface area of the wire
\[ e' \] fluctuating voltage across the sensor
\[ E \] mean voltage across the sensor
\[ h \] heat transfer coefficient
\[ I \] current across the sensor
\[ k \] thermal conductivity of air evaluated at subscript temperature
\[ K \] \( \frac{\partial \ln T_e}{\partial \ln R_e} \)
\[ Kn \] Knudsen number
\[ m \] \( \frac{\partial \mu}{\partial \ln T_e} \)
\[ M \] Mach number
\[ Nu \] Nusselt number evaluated at subscript temperature
\[ P \] electrical power to the hot-wire
\[ q \] dynamic pressure

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Introduction

Historically, much hot-wire data were analyzed by using equations derived by Morkovin to obtain the sensitivities of the heated wire to changes in velocity, density, and total temperature. This is particularly true for wires operating in the compressible flow regime. The use of these equations requires a knowledge of the wire length, \( a \), and \( \beta \), in the temperature - resistance relationship for the wire material. The operation of wires at high temperatures requires a knowledge of \( \beta \) values. The calibration of the wire at elevated temperature places a severe demand on the construction of the probe to withstand these temperatures. Because of this, values for \( \beta \) are often not obtained from calibration but from handbooks.

The length of the wire must also be known and for wires having slack it is difficult to accurately measure their lengths. Since the wire is heated, the length measured when cold will differ from the value when the wire is hot. The approximate values of \( \beta \) and errors in measuring the length of the wire can lead to error in the calibrated values of the mean flow sensitivities. The mean flow calibration is also constrained by the average values used and the approximations made for the heat transfer equation. The use of general functional relationships for heat transfer from the heated wire should lead to more accurate mean flow calibrations.

Two methods are proposed for calibrating the sensitivities of heated wires to flow variables. One method eliminates the need to measure the wire length or \( a \), and \( \beta \). In the second method, the quantities \( a \) and \( \beta \) affect only the total temperature sensitivity of the wire to a minor degree. Equations, based on these techniques, will be presented for constant temperature and constant current anemometers (CTA and CCA). Calibration data for a constant temperature anemometer will be presented based on these two calibration techniques.

Theoretical Consideration

Rational for using Functional Relationships

Consider the heat transfer from a heated wire with slack as shown in figure 1. For this problem the general heat transfer equation due to forced convection can be written in integral form as:

\[
Q = \int h(T_\infty - T_w) dA_w
\]

where

\[
dA_w = dx ds = r dx d\varphi
\]
Using equation (2) we can write equation (1) as:

\[ Q = 2\pi \int_{\phi_0}^{\phi} \left[ h(T_w - T_{\text{adv}}) \right] dx \, d\phi \]  

(3)

Here \( h \), \( T_w \) and \( T_{\text{adv}} \) are local values. Using stagnation values of these quantities to normalize the above equation along with the stagnation values at zero sweep, the above equation can be rewritten as:

\[ Q = 2\pi h_{\Lambda,0} (T_w - T_{\text{adv},0}) \int_{\phi_0}^{\phi} \frac{(T_w - T_{\text{adv}})}{h_{\Lambda,0} (T_w - T_{\text{adv}})} \, dx \, d\phi \]  

(4)

where \( \Lambda \), \( T_w \) and \( T_{\text{adv},0} \) are \( f(x) \). \( T_{\text{adv}} \) and \( T_{\text{adv},0} \) are \( f(M, Kn, T_w, \Lambda) \). Mach number and Reynolds number are \( f(\phi) \). \( h_{\Lambda,0} = f(\phi) = f(M, Kn) \) and \( h_{\Lambda,0} = f(M_w, Kn_w, \Lambda) = f(x) \). The quantities that are \( f(x) \) due to conduction to the supports and to the local sweep. This is a more complex equation than the usual heat transfer equation used to derive hot-wire equations.

The conventional heat transfer equation due to forced convection can be written as:

\[ Q = \pi \varepsilon d m (T_{w,m} - T_{\text{adv,m}}) \]  

(5)

Nondimensionalizing equation (5) similar to that done in equation (4) gives:

\[ Q = \pi n \varepsilon h_{\Lambda,0} (T_w - T_{\text{adv}}) \frac{h_{\text{adv}} (T_{w,m} - T_{\text{adv,m}})}{h_{\Lambda,0} (T_w - T_{\text{adv},0})} \]  

(6)

Here equation (4) and (6) represent the same quantity but approached in a different way. Evaluating the terms in equations (4) and (6) are difficult. However, note that both equations can be represented in terms of functional relationship to basic flow quantities as:

\[ Q = f(u, \rho, T_w, T_v) \]  

(7)

which can be used to evaluate the sensitivities of the wire to heat transfer more accurately than equation (4) or (5) or (6) as far as the hot-wire application is concerned.

**Derivation of Sensitivity Equations**

The functional relationship given by Morkovin that governs the voltage measured across a heated wire was obtained from equation (5) and results in:

\[ E = f(u, \rho, T_v, T_w) \]  

(8)

The change in voltage with respect to the independent variables is:

\[ \frac{dE}{du} = \frac{\partial E}{\partial u} d\log u + \frac{\partial E}{\partial \rho} d\log \rho + \frac{\partial E}{\partial T_v} d\log T_v + \frac{\partial E}{\partial T_w} d\log T_w \]  

(9)

For a constant current anemometer the change in wire temperature must be considered. The temperature of the wire should be obtained from the wire resistance using the following equation:

\[ \frac{R_w}{R_{\text{ref}}} = 1 + \alpha (T_w - T_{\text{ref}}) + \beta (T_w - T_{\text{ref}})^2 \]  

(10)

Equation (10) is required to relate \( T_w \) to \( R_w \). For \( T_w = f(R_w) \) the following equation can be written:

\[ d\log T_w = K d\log R_w \]  

(11)

Ohm's law gives:

\[ d\log R_w = d\log u - d\log I \]  

(12)

Using equation (11) and (12), equation (9) can be written as:

\[ \left[ 1 - K \frac{\partial E}{\partial \log T_w} (1 + \alpha) \right] d\log E = \left[ \frac{\partial E}{\partial \log u} + \frac{\partial E}{\partial \log \rho} + \frac{\partial E}{\partial \log T_w} \right] d\log u + \frac{\partial E}{\partial \log T_v} d\log T_v \]  

(13)

The final equation for the change in \( E \) becomes:

\[ d\log E = S_u d\log u + S_p d\log \rho + S_T d\log T_w \]  

(14)

where

\[ S_u = \frac{\partial E}{\partial \log u} \left[ 1 - K \frac{\partial E}{\partial \log T_w} (1 + \alpha) \right] ; \quad S_p = \frac{\partial E}{\partial \log \rho} \left[ 1 - K \frac{\partial E}{\partial \log T_w} (1 + \alpha) \right] \]  

(15)

\[ S_T = \frac{\partial E}{\partial \log T_w} \left[ 1 - K \frac{\partial E}{\partial \log T_w} (1 + \alpha) \right] \]
For a CTA, $dT_v = 0$ and the final equation for the change in $E$ is same as equation (14) where:

$$S_v = \frac{\partial E}{\partial \log u} \quad S_p = \frac{\partial E}{\partial \log \rho} \quad S_T = \frac{\partial E}{\partial \log T_v} \quad (16)$$

Equation (15) is not recommended for calibrating a CCA since the flow variables of $u$, $\rho$, $T_v$ and $T_c$ must be varied. This makes the calibration process too lengthy. The term $K$, in equation (11) also requires values of $\alpha$ and $\beta$. However, equation (14) and (16) for a CTA does not require the knowledge of wire length or $\alpha$ and $\beta$.

It is not often possible to vary $T_c$ in wind tunnels. To overcome this problem, the following functional relationship is suggested:

$$E = f(M, K, a'_v) \quad (17)$$

where $T_c$ is constant. Using equation (17), the change in the voltage across the heated wire is:

$$d \log E = \frac{\partial E}{\partial \log M} d \log M + \frac{\partial E}{\partial \log K} d \log K + \frac{\partial E}{\partial \log a'_v} d \log a'_v \quad (18)$$

From the definition of $a'_v$, the change in $a'_v$ is:

$$d \log a'_v = \frac{\theta}{a'_v} (d \log R_w - d \log R_{ref}) \quad (19)$$

One possible functional relationship for $R_{ref}$ is:

$$R_{ref} = f(M, K, T_c) \quad (20)$$

From which the change in $R_{ref}$ is:

$$d \log R_{ref} = \frac{\partial R_{ref}}{\partial \log M} d \log M + \frac{\partial R_{ref}}{\partial \log K} d \log K + \frac{\partial R_{ref}}{\partial \log a'_v} d \log a'_v \quad (21)$$

Using equation (21) in (19), substituting $d \log a'_v$ into equation (18), and rearranging leads to:

$$d \log E = S_v d \log u + S_p d \log \rho + S_T d \log T_v + \frac{\theta}{a'_v} \frac{\partial E}{\partial \log R_w} d \log R_w \quad (22)$$

Finally, Ohm's law can be used to express the wire resistance in terms of the voltage across the wire as follows:

$$d \log R_w = (1+\epsilon) d \log E \quad (23)$$

Using equation (23) in (22), the final equation for the change in $E$ is same as equation (14) where:

$$S_v = AS'_v = A \left[ \frac{\partial E}{\partial \log M} - \frac{\theta}{a'_v} \frac{\partial E}{\partial \log a'_v} \right] \quad (24)$$

$$S_p = AS'_p = A \left[ \frac{\theta}{a'_v} \frac{\partial E}{\partial \log a'_v} - \frac{\partial E}{\partial \log M} \right] \quad (25)$$

$$S_T = AS'_T = A \left[ 1 + \frac{\theta}{a'_v} \frac{\partial E}{\partial \log a'_v} \right] \quad (26)$$

$$A = \left[ 1 - \frac{\theta}{a'_v} (1+\epsilon) \frac{\partial E}{\partial \log a'_v} \right] \quad (27)$$

Using equation (10) the quantity $\frac{\partial E}{\partial \log R_{ref}}$ can be obtained as:

$$\frac{\partial E}{\partial \log R_{ref}} = \frac{T_{ref} [\alpha_1 + 2\beta_1 (T_{ref} - T_v)]}{1 + \alpha_1 (T_{ref} - T_v) + \beta_1 (T_{ref} - T_v)^2} \quad (28)$$

The evaluation of $\frac{\partial E}{\partial \log R_{ref}}$, requires values for $\alpha_1$ and $\beta_1$, and the latter should be obtained from the temperature-resistance calibration of the wire. The values for $\alpha_1$ can be easily obtained, however, it is often necessary to use handbook values for $\beta_1$. In any case, the values of $\alpha_1$ and $\beta_1$ occur in only one term in the equation for the sensitivity of the wire to total temperature and any error in $\beta_1$ would be limited to this sensitivity. The possible error in evaluating $\frac{\partial E}{\partial \log R_{ref}}$ by neglecting the value for $\beta_1$ can be obtained from:

$$\left[ \frac{\partial E}{\partial \log R_{ref}} \right]_{\beta_1} = \left[ \frac{T_{ref} [\alpha_1 + 2\beta_1 (T_{ref} - T_v)]}{(\alpha_1 T_{ref})_{\beta_1}} \right] \quad (29)$$
where
\[
[T_{\text{ref}}]_{1,4} = \frac{-(a_1 - 2 \beta T_{\text{ref}})}{2 \beta} \pm \sqrt{\frac{\alpha_1}{4 \beta^2} + \frac{1}{\alpha_1} \left( \frac{R_{\text{ref}}}{R_{\text{ref}} - 1} \right)}
\] (30)

and
\[
[T_{\text{ref}}]_{1,4} = \frac{1}{\alpha_1} \left( \frac{R_{\text{ref}}}{R_{\text{ref}} - 1} \right) + T_{\text{ref}}
\] (31)

For a CTA \( d \log R \), \( = 0 \), and equations (24) - (26) are valid with \( A = 1 \).

Facility and Description of Present Work

The "Probe Calibration Tunnel" (PCT)\(^2\) is an open jet tunnel with the ability to independently control velocity, density and total temperature. The primary purpose of this facility is to economically calibrate probes for NASA's major test facilities at the Langley Research Center. Typical operational cost for the PCT is 1% of the cost of operating a major facility\(^2\). The combination of the staged pressure system and low mass flow requirement of the facility and the large volume vacuum systems enables economic and accurate calibration of probes. Due to these feature, the flow in this facility is continuous. Schematic diagram of the facility is presented in figure 2.

The PCT has two interchangeable, subsonic and transonic, nozzles having diameter of 1.50 and 2.25 inches, that gives a continuous flow capability over Mach number range of 0.05 to 1.0. Tunnel stagnation pressure and temperature can be varied from a minimum of 0.20 atmosphere to a maximum of 10 atmospheres and 500°R to 600°R, respectively. This corresponds to a Reynolds number range of \( 1 \times 10^6 < Re/ft < 51 \times 10^6 \) for a Mach number of 1.

In this study, all the Knudsen number were greater than 0.01 indicating that all the data were in the slip flow regime based on the definition of the slip flow boundary in reference 7.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Minimum</th>
<th>Maximum</th>
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<tr>
<td>Total Pressure, psi</td>
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<tr>
<td>Static Pressure, psi</td>
<td>2.83727</td>
<td>31.5238</td>
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<tr>
<td>Velocity, ft/sec</td>
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<td>Mach Number</td>
<td>0.0982</td>
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<tr>
<td>( q ), psi</td>
<td>0.0064</td>
<td>14.328</td>
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<tr>
<td>Total Temperature, °R</td>
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<tr>
<td>Static Temperature, °R</td>
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<td>579.73</td>
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<td>Reynolds number/foot</td>
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<td>( 12.3 \times 10^6 )</td>
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<tr>
<td>Heater Power, Watts</td>
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<td>4545</td>
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<tr>
<td>Density</td>
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<td>0.14693</td>
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<tr>
<td>Mass flow, lb/sec</td>
<td>0.012832</td>
<td>1.71468</td>
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Table I. Minimum and Maximum Test Conditions of Test Matrix for Functional Relationship \( E = f(u, \rho, T_c, T_e) \); Method I.

<table>
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<th>Parameter</th>
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<td>Mach Number</td>
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<td>( q ), psi</td>
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<td>Total Temperature, °R</td>
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<td>540</td>
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<td>Static Temperature, °R</td>
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<td>539.73</td>
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<td>Reynolds number/foot</td>
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<td>( 13.3 \times 10^6 )</td>
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<tr>
<td>Knudsen number</td>
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<tr>
<td>Heater Power, Watts</td>
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<td>2985</td>
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<tr>
<td>Density</td>
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<td>0.13333</td>
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<tr>
<td>Mass flow, lb/sec</td>
<td>0.013103</td>
<td>1.1793</td>
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<tr>
<td>Overheat Ratio</td>
<td>0.3</td>
<td>1.0</td>
</tr>
</tbody>
</table>

Table II. Minimum and Maximum Test Conditions of Test Matrix for Functional Relationship \( E = f(M, Kn, \alpha'') \); Method II.
Results and Discussion

Data obtained in the "Probe Calibration Tunnel" was used to calculate the sensitivities of several hot-wires using the two methods developed for CTA. The data were obtained such that the partial derivatives of the functional relationship such as \( E = f(u, \rho, T) \) and \( E = f(M, K_n, a') \) were satisfied as dictated by the calculus, i.e., while obtaining data by varying one independent variable, the other independent variables were held constant. Constant Temperature Anemometers (CTA) were used in obtaining the hot-wire data. Thus, the restriction of the calculus with respect to \( T \) was satisfied automatically and the first functional relationship reduces to \( E = f(u, \rho, T) \). Polynomials of appropriate degree were chosen to give the *best curve fit* to the data to evaluate the partial derivatives required in the sensitivities equations.

The test conditions in this study for \( E = f(u, \rho, T) \) case are presented in Table I. Examples of curve fits to the data are shown in figures 6 (a-b) where the dependent variable, measured voltage, is presented against each of the independent variable, while the other independent variables were held constant. Computing sensitivities in this case was simple and direct. The logarithmic derivative of the measured voltage to each independent variable gives the sensitivity of the hot-wire to corresponding variable as shown in equation (16).

In the second set of data, \( a' \) was chosen over \( T_e \) to demonstrate that even though many wind tunnels do not have the capability of varying total temperature, the limitations of the tunnel can be overcome by using \( a' \). In obtaining the second set of data with respect to overheat ratio, all the hot-wires were operated at assigned overheat values. The test conditions for this case are given in Table II. The curve fit of measured voltage vs. various independent variables are presented in figure 7 (a-b). Here the sensitivities were computed using equations (24)-(26).

To obtain fluctuation quantities each hot-wire on the probe was assigned different overheat value in an attempt to make \( S_p \neq S_p \), a necessary condition for obtaining fluctuations. However, no fluctuation quantities are presented in the present study.

Velocity Sensitivity:

As seen in figures 6-a and 7-a, the voltage increases rapidly at low velocities or Mach numbers and at transonic speed the slope of the curves tend to decrease. For the case \( E = f(M, K_n, a') \) there was a tendency for \( \frac{dE}{dM} \to 0 \) at intermediate Mach numbers before increase as \( M \to 1 \). Thus, in correlating \( E \) vs. \( M \) at transonic speeds might result in zero or negative slope. This variation was also observed in the data presented by other researchers. In order to correlate these data higher order polynomials were required. This could lead to large difference in the sensitivities at higher Mach number due to poor curve fitting at the end points. Precautions need to be taken in using the proper degree of polynomials because they could lead to unreasonable values for the sensitivities. One such difficulty in using higher order polynomial was presented in reference 9 where it lead to a hairpin like variation of \( S_p \) with \( S_p \).

Examples of the velocity sensitivity, \( S_p \), for both sets of data are presented in figures 8 (a-b) and generally, \( S_p \) ranged from 0.05 to 0.40. In both cases \( S_p \) increased with increased \( u \) or \( M \) at low subsonic speeds and decreased at intermediate subsonic speeds. As \( S_p \) approached sonic velocity, the sensitivities increased rapidly.

A comparison of \( S_p \) for both the methods is presented in figure 9. It is expected that both methods, at identical test conditions, should give similar results. Most of the values for \( S_p \) agreed within about 25 percent, however, some of the differences shown may be due to different degree of polynomial curve fits used in each technique. Also, it may be due to the differences in the operation of the anemometer for each method. In the first method, \( T_e \) is held constant and \( T_s \) is varied. In the second method, \( T_e \) is held constant and \( T_s \) is varied. Due to this difference in the operation of the hot-wire the end-loss to the support could be different. This difference in end loss could contribute to differences in all the three sensitivities.

Density Sensitivity

The measured voltages across the hot wires monotonically increased or decreased with increasing density or Knudsen number, respectively. At higher Knudsen number the slope changed sign which was due to large change in Knudsen number and
insufficient data. A second degree polynomial was used to correlate the data. The density sensitivity, $S_p$, varies from 0.1 to 0.4 and an example of $S_p$ for both the cases are presented in figures 10 (a-b). Except for the data where Knudsen number had a large change, the density sensitivities in both the cases were comparable with each other and a case is presented in figure 11.

A comparison of velocity sensitivity, $S_v$, with density sensitivity, $S_p$, is presented in figures 12 (a-b) and shows that $S_v > S_p$ for most of the data. However, few data points do approach $S_v = S_p$ and $S_v > S_p$ in some cases for $E = f(M,Kn,a')$. The cases where $S_v > S_p$ indicates that the wire was in the slip flow regime. This observation is consistent with Spangenberg's data as noted in reference 9. It was also noted that at higher Mach numbers and lower overheat ratio, $a'$, $S_v$ approaches $S_p$. As $a'$ was increased $S_v$ became greater than $S_p$.

**Total Temperature Sensitivity**

For method II, the manufacturer's suggested values of $a_t$ and Morkovin\(^1\) suggested values of $\beta_t$ for tungsten wire were used to compute $S_{\zeta}$. $a_t$ was also computed using the data obtained during the test and this value was comparable with the manufacturer's provided value of $a_t$. The data obtained in the present study was not adequate to evaluate $\beta_t$.

An example of the data used to obtain the total temperature sensitivity, $S_{\zeta}$, is presented in figures 6(b) and 7(b) and $S_{\zeta}$ ranges from 0.2 to 1.0. However, the total temperature sensitivity obtained using the first method was always larger than the values computed using the second method. A few examples of these variations are presented in figures 13 (a-b). This range of temperature sensitivity is consistent with the earlier studies\(^6,9\) from the authors. The temperature sensitivities using the overheat parameter may be more accurate because better curve fits were possible since more data points were available when using the $E = f(u,p,T_0)$ method. This shows the possible advantage of using this method versus the other.

A comparison of total temperature sensitivity, $S_{\zeta}$, between the two methods is presented in figure 14. There is a significant difference between the values of $S_{\zeta}$ for the two methods. This difference might be explained if the end losses are taken into account since they might be different due to the difference in the operation of the anemometer for each method.

In general, the range of data available from the present study is greater than those obtained from previous studies\(^3-6\), particularly in regards to temperature information. In some of the earlier studies logarithmic functions were used to correlate the data which gave no variation of the sensitivity to its corresponding independent variable. Therefore, polynomial functions seem to be a better technique for correlating the data compared to logarithmic functions. However, careful analysis of the present data is required for a better understanding of the differences between the sensitivities, particularly the total temperature sensitivity, so that they could be applied for computing the fluctuation quantities.

**Conclusion**

Conventional methods for calibrating hot-wire probes use equations which include quantities that are difficult to evaluate. These quantities are wire length and $\beta_t$ in the temperature - resistance equation for the wire material. This could lead to possible errors in obtaining the sensitivities of the heated wire to changes in velocity, density and total temperature. Methods were developed that either eliminated the need for the measurement of $t$ and $\beta_t$, which are difficult to make accurately or reduced the effect of $\beta_t$. From a study of these methods the following conclusions can be made:

1. The equation obtained for a CTA based on the assumption that $E = f(u,p,T_0)$ resulted in an equation in which neither wire length or $a_t$ and $\beta_t$ are required. This should result in a more accurate method for calibrating hot-wire probes in facilities where $T_0$ can be varied.

2. The equation obtained for a CCA based on the assumption that $E = f(u,p,T_0,T_{\zeta})$ results in an equation that requires the variation of four independent variables and is believed to be too complicated for routine use. The equation also requires the evaluation of $\beta_t$ who's elimination was desired.

3. The equation obtained for a CCA and CTA under the assumption that $E = f(M,Kn,a'_t)$ and
$T_0$ = constant resulted in an equation where $a_i$ and $b_i$ occurred in only one term in the equation for the total temperature sensitivity. Because of this the calibration of wires should be more accurate than those obtained using conventional methods.

4. The sensitivities computed based on the methods presented using the data obtained in "Probe Calibration Tunnel" gives comparable results for $S_m$ and $S_p$. However, there were significant differences in the values for $S_T$. The differences in the sensitivities could be due to different degree of the polynomial curve fits used in each technique or due to the differences in the operation of the anemometer in each method.

5. In general, the velocity sensitivity ranges from 0.05 to 0.40, density sensitivity ranges from 0.10 to 0.40 and total temperature sensitivity ranges from -0.2 to -1.0.

6. Polynomial functions seem to be better for correlating hot-wire data compared to logarithmic functions.

Acknowledgment

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References


Figure 1. Schematic Diagram of Heated Wire.
Figure 2. Schematic Diagram of "Probe Calibration Tunnel".

Figure 3. Schematic Diagram of Data Acquisition System.

Figure 4. Operational Envelope of the "Probe Calibration Tunnel".

Figure 5. Comparison of Present Test Range with Earlier Calibration Tests.

Figure 6. Data Correlated as $E = f(u, \rho, T_e)$; $d_e = 0.0001$ inch.
Figure 6. Concluded.

Figure 7. Data Correlated as $E = f(M, Kn, a'_w)$; $T_e = 540^\circ R$; $d_w = 0.0001$ inch.

Figure 8. Velocity Sensitivity, $S_a$; $T_e = 540^\circ R$; $d_w = 0.0001$ inch.

Figure 8. Concluded.
Figure 9. Comparison of $S_s$ for both methods; $T_s = 540^\circ R$; $d_w = 0.0001$ inch.

Figure 10. Density Sensitivity, $S_p$; $T_s = 540^\circ R$; $d_w = 0.0001$ inch.

(a) For the case of $E = f(u, \rho T_s)$

(b) For the case of $E = f(M, Kn, \alpha^*)$; $\alpha^* = 0.6$.

Figure 10. Concluded.

Figure 11. Comparison of $S_p$ for both methods; $T_s = 540^\circ R$; $d_w = 0.0001$ inch.

(a) For the case of $E = f(u, \rho T_s)$

Figure 12. Comparison of $S_s$ vs. $S_p$; $T_s = 540^\circ R$; $d_w = 0.0001$ inch.
(b) For the case of $E = f(M, Kn, a')$; $Kn = 0.025923$

Figure 12. Concluded.

(a) For the case of $E = f(u, \rho, T_e)$; $\rho = 0.073465$ lb/ft$^3$

Figure 13. Total Temperature Sensitivity, $S_T$; $d_w = 0.0001$ inch.

(b) For the case of $E = f(M, Kn, a')$; $T_e = 540^\circ R$; $Kn = 0.025923$

Figure 13. Concluded.
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Hot-Wire Anemometry in Subsonic Slip and Transonic Flow Regimes

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Abstract

Much has been written about the improbability and impossibility of obtaining solutions to the mean square equation for constant current anemometry in subsonic slip and transonic flows. For these flow conditions, the fluctuating voltage across a wire mounted normal to the flow is a function of velocity, density, and total temperature. In principal, the fluctuations of these quantities can be measured, however, to date there are no known acceptable solutions to the mean square equation in these flow regimes. In this study, data presented in the 1950's by Spangenberg were used to show that, for a large region in the Nusselt number - Reynolds number or Nusselt number - Knudsen number regimes, there exists the possibility of obtaining solutions to the mean square hot-wire equation. These data were used to compute the sensitivities of the heated wire to changes in velocity, density, and total temperature; indicate regions where the velocity and density sensitivities were different (a condition required for a solution to the mean square equation); and show a second, necessary condition for a solution. Examples of fluctuation and mode diagrams for subsonic compressible and subsonic slip flows are also presented under the assumption that there are solutions to the hot-wire anemometry equation.

Nomenclature

\( a_0, a_k \) coefficients in equations (22) - (27)

\( b_0, b_k \) coefficients in equations (24) - (26)

\( c_s, c_v \) coefficients in equation (26)

\( a, b, c, d \) constants in equation (A-4)

\( A \) quantity given by equation (20)

\( c_s \) specific heat at constant pressure

\( c_v \) specific heat at constant volume

\( d \) wire diameter

\( e' \) fluctuating voltage across sensor

\( E_o \) mean voltage across sensor

\( f, g, h \) constants in equation (A-4)

\( h \) coefficient of heat transfer

\( I_w \) current through the wire

\( k_s \) thermal conductivity of air based on \( T_0 = 0^\circ C \)

\( K \) \( \log T_0 / \log R_w \)

\( K_n \) Knudsen number

\( L \) wire length

\( L \) characteristic length

\( m \) mean mass flow

\( M \) Mach number

\( Nu \) Nusselt number

\( n \) number of data points

\( P_e \) electrical power to wire

\( Q \) heat transfer from wire

\( q \) sensitivity ratio \( = S_v / S_T \)

\( R_w \) resistance of wire

\( R_{ref} \) wire resistance at reference temperature

\( r \) sensitivity ratio \( = S_u / S_T \)

\( s \) sensitivity ratio \( = S_p / S_T \)

\( S_m \) mass sensitivity

\( S_v \) velocity sensitivity

\( S_p \) density sensitivity

\( S_T \) total temperature sensitivity

\( T \) temperature

\( T_0 \) total temperature

\( u \) velocity

\( V \) general dependent variable

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Very few attempts have been made to measure fluctuations in subsonic compressible flow using CCA because of the complexity that exists when the heat transfer from the wire is a function of three independent variables.

Some efforts have been made to measure fluctuations in subsonic compressible flow using a constant temperature anemometer (CTA). For these investigations a three-wire probe was used and the voltages across the wires were digitized and a system of three equations solved to obtain the instantaneous changes in velocity, density, and total temperature as a function of time. Statistical techniques were then used to obtain fluctuations and correlations of interest.7

For compressible flow; where the wire is sensitive to velocity, density, and total temperature; equations for CCA are usually derived using mean square values. This results in a single equation with six unknowns. Much has been written about the improbability4 and impossibility8 of obtaining solutions to this mean square equation. Solutions to this equation requires that the velocity and density sensitivities of the heated wire be sufficiently different. In this report, data published in the 1950's by Spangenberg9 was used to show that there exist a large region in the Nusselt number-Reynolds number or Nusselt number-Knudsen number regimes where the velocity and density sensitivities appeared to be sufficiently different to permit suitable solutions to the mean square equation. It was further shown that solutions were possible only when the velocity and density sensitivities were non-linearly related in specific manners. Again Spangenberg's data were used to show that this second condition can often be satisfied, further indicating that solutions to the mean square equation are possible. Examples of fluctuation and mode diagrams for subsonic compressible and subsonic slip flows are also presented under the assumption that there are solutions to the hot-wire anemometry equation.

Spangenberg's Data

Spangenberg's data were presented in terms of $Nu = f(M, \rho, r_\infty)$ for $T_s = constant$. Data were presented for several wire diameters and wire lengths. The data to be considered herein were obtained using a Pt-10% Rh wire having a diameter of 0.00015 inch and a length of 2.80 mm. Data were limited to the following conditions:

Introduction

Constant current anemometry (CCA) has been extensively used to measure fluctuations in incompressible and compressible flows1-3. In compressible flow the heated wire is, in general, sensitive to changes in velocity, density, and total temperature and fluctuations in these quantities can, in principal, be measured4. In supersonic flow it has been found experimentally that the mean voltage measured across a heated wire is only a function of changes in mass flow and total temperature5. Using this result for supersonic flow, Kovasznay developed a graphical technique for determining fluctuations of mass flow, total temperature, and their correlation6.
The Nusselt numbers presented were corrected for heat loss to the supports of the wire. This corrected Nusselt number, \( N_u_c \), is suitable for comparing heat transfer results from different experiments using different wire materials and different \( l/d \)’s. For computing fluctuations, the sensitivities of the wire must be obtained using the uncorrected Nusselt number, \( N_u \). For the results presented herein, \( N_u_c \) values were converted to \( N_u \) values using the method presented by Spangenberg.

Examples of Spangenberg’s data are presented in figure 1. In figure 1a the uncorrected Nusselt number is presented as a function of Mach number for various values of density for a given value of \( r_v \). The Nusselt number increases rapidly with Mach number at very low values. The slopes of the curves decrease somewhat as the Mach number increases. At the higher Mach numbers (0.6 < \( M < 0.95 \)) the Nusselt number often reaches a peak prior to decreasing. This variation of \( N_u \) with \( M \) at the higher Mach numbers often results in zero or negative slopes for \( N_u \) vs \( M \). The Mach numbers at which the slopes of the curves reach zero increase with increasing density. This variation of \( N_u \) vs \( M \) was also observed in data presented in references 10 and 11. Polynomials were used to fit curves to the data from which the required derivatives were obtained, and a fourth degree equation was required to obtain “good” fits. The variation of \( N_u \) with \( \rho \), shown in figure 1b, is monotonically increasing with increasing density. A second degree equation was used to fit curves to the \( N_u \) vs \( \rho \) data.

The variation of Nusselt number with respect to \( r_v \) is presented in figure 1c and this variation of \( N_u \) is rather complex and nonlinear. The Nusselt number can often increase then decrease with increasing values of \( r_v \). A closer look at \( N_u \) vs \( r_v \) data revealed that, for most of the data, the values of \( N_u \) decreased from 0.629 ≤ \( r_v \) ≤ 1.629 but most often increased or tended to increase from 1.629 ≤ \( r_v \) ≤ 2.192. This variation of \( N_u \) over this later interval of \( r_v \) was present in about 80 percent of the data and had a profound affect on the relationship between the velocity and density sensitivities. It is not known if this variation can be supported by the limited amount of data which is available. If third degree curves were fitted to the data, there was a significant change in the derivatives for most of the data at the higher values of \( r_v \). Because of these changes, the values of \( N_u \) were fitted to values of \( r_v \) using both second and third degree equations. The results obtained using these two curve fits on the variation of the velocity and density sensitivities will be presented subsequently.

Data presented by Baldwin in Ref. 10 also showed a similar variation of the Nusselt number with \( r_v \). This complex and nonlinear variation of Nusselt number with \( r_v \) or overheat is probably the reason that solutions to the mean square equation for CCA might exist.

In general, the scatter in the values of \( N_u \) with respect to \( M \) and \( \rho \) are acceptably small; the apparent scatter of \( N_u \) with respect to \( r_v \) is somewhat larger. This is probably due to the relatively small change in \( N_u \) with respect to \( r_v \) or the limited number of values of \( r_v \) that are available.

**Theoretical Considerations**

Because of the manner in which Spangenberg presented his data, equations must be derived for the change in voltage across the wire in terms of the sensitivity of the wire to changes in velocity, density, and total temperature. The equation for the heat transfer from the heated wire is generally given as:

\[
Q = \pi d\theta(T_v - T\infty)
\]

For steady state conditions, power supplied to the heated wire equals the aerodynamic heat transfer from the wire. Therefore:

\[
P_v = l^2 R_v = \pi\theta_k(T_v - \eta T_v)N_u
\]

Making the small perturbation assumption, the change in \( R_v \) can be expressed as:

\[
(1 - 2\varepsilon)d\log R_v = d\log N_u + d\log r_v + d\log T_v
\]

where:

\[
\varepsilon = -\frac{\partial \log l_v}{\partial \log R_v}
\]

For Spangenberg’s data:

\[
N_u = f(M, \rho, r_v)
\]
Note that there is a direct relationship between density and Knudsen number. Therefore, the Knudsen number could be used as independent variable. The change in $Nu_c$ is:

$$d \log Nu_c = \frac{\partial \log Nu_c}{\partial \log \rho} d \log \rho + \frac{\partial \log Nu_c}{\partial \log M} d \log M + \frac{\partial \log Nu_c}{\partial \log \tau_w} d \log \tau_w$$

(6)

It is desired to obtain the final equation in the following form:

$$E_\nu = f(u, \rho, T_w)$$

(7)

The change in $M$ is:

$$d \log M = \frac{1}{\alpha} \left( d \log u - \frac{1}{2} d \log T_w \right)$$

(8)

and

$$d \log \tau_w = \frac{\theta}{\tau_w} d \log T_w + \frac{\eta}{\tau_w} d \log \eta - \frac{\eta}{\tau_w} d \log T_0$$

(9)

The recovery temperature ratio can be written as:

$$\eta = f(\rho, M)$$

(10)

and the change in $\eta$ is:

$$d \log \eta = \frac{\partial \log \eta}{\partial \log \rho} d \log \rho + \frac{1}{\alpha} \frac{\partial \log M}{\partial \log \rho} \left( d \log u - \frac{1}{2} d \log T_w \right)$$

(11)

The temperature of the wire must be obtained from the measured resistance using:

$$\frac{R_w}{R_{cm}} = 1 + \alpha \left( T_w - T_{cm} \right) + \beta \left( T_w - T_{cm} \right)^2$$

(12)

Therefore:

$$T_w = f(R_w)$$

(13)

and

$$d \log T_w = K d \log R_w$$

(14)

Finally, Ohm's law can be used to express the wire resistance in terms of the voltage across the wire as follows:

$$d \log R_w = \frac{d \log E_\nu}{(1 + \epsilon)}$$

(15)

Morkovin\textsuperscript{4} gives the following equation for the changes in $E_\nu$ due to changes in $u$, $\rho$, and $T_w$ for a CCA:

$$d \log E_\nu = -S_u d \log u - S_\rho d \log \rho + S_{T_w} d \log T_w$$

(16)

For the present case the sensitivities can be obtained using equations (3) - (15) and are given by:

$$S_u = \frac{\partial \log Nu_c}{\partial \log M} \frac{\partial \log \eta}{\partial \log \rho} \left( \frac{\partial \log Nu_c}{\partial \log \tau_w} + 1 \right)$$

(17)

$$S_\rho = \frac{\partial \log Nu_c}{\partial \log \rho} \frac{\partial \log \eta}{\partial \log M} \left( \frac{\partial \log Nu_c}{\partial \log \tau_w} + 1 \right)$$

(18)

$$S_{T_w} = \frac{1}{2a} \frac{\partial \log Nu_c}{\partial \log M} \frac{\partial \log \eta}{\partial \log \rho} \left( \frac{\partial \log Nu_c}{\partial \log \tau_w} + 1 \right)$$

(19)

where

$$A = \frac{\left( 1 + \epsilon \right)}{\left( 1 - 2 \epsilon \right) - \frac{\theta K}{\tau_w} \left( \frac{\partial \log Nu_c}{\partial \log \tau_w} + 1 \right)}$$

(20)

For Spangenberg's data $\eta$ varied with $M$ and plots of $\eta$ vs $M$ were presented in his report. The values for $\alpha$, and $\beta$, for the wire material were also presented. Therefore, after the partial derivatives in equation (17) - (20) were determined the sensitivities to changes in $u$, $\rho$, and $T_w$ could be calculated.

The value of $\epsilon$ cannot be evaluated from Spangenberg's data because the voltage of the battery used is unknown and there are several variable resistors, having unknown resistances, in the electrical circuit. Because of these problems the absolute values of the sensitivities could not be obtained. The values of $\epsilon$ does not, however, influence the ratio of the sensitivities, the important parameters, since the quantity $A$ cancels out when the ratios are formed.

**The Partial Derivatives**

Spangenberg's data were measured and tabulated in a form that is necessary to obtain the partial derivatives of the dependent variable with respect to one independent variable while holding the remaining independent variables constant. These are the only data known to the authors where measurements were obtained in this necessary manner. For the present report, polynomials were
used to fit the data to obtain the desired derivatives of $N_\nu$ with respect to the independent variables. The degree of the polynomial was chosen to give the "best fit" to the data. In general, this method gives:

$$V = f(x,y,z)$$

where the function was assumed to be:

$$V(x,y,z) = a_0 + a_1x + a_2x^2 + \ldots a_nx^n$$

and:

$$a_n = f(y,z) \quad m = 0,1,2,\ldots,n$$

Using polynomials, the values for the $a$'s can be expressed as:

$$a_n = b_0 + b_1y + b_2y^2 + \ldots b_ny^n$$

where

$$b_n = f(x); \quad m = 0,1,2,\ldots,n$$

In a similar manner:

$$b_n = c_0 + c_1x + c_2x^2 + \ldots c_nx^n$$

From equation (22), the derivatives becomes:

$$\frac{\partial \log V}{\partial \log x} = \frac{a_1x + 2a_2x^2 + \ldots na_nx^n}{a_0 + a_1x + a_2x^2 + \ldots a_nx^n}$$

**The Mean Square Equation**

In the past hot wire data, using a CCA, were obtained from mean square values. Following Kovasznay, squaring equation (16), dividing by $S_x^2$, and taking the mean gives:

$$\bar{\phi}^2 = q^2\left(\frac{u^2}{\bar{u}}\right)^2 + \beta^2\left(\frac{\rho}{\bar{\rho}}\right)^2 + \left(\frac{\bar{T}_e}{T_0}\right)^2 + 2q\frac{u^\prime \bar{u}^\prime}{\bar{u}} - 2q\frac{\bar{T}_e}{\bar{T}_0} - 2\frac{\rho T^\prime_0}{\rho \bar{T}_0}$$

This is a single equation in six unknowns, \(i.e., \frac{u}{\bar{u}}, \frac{\rho}{\bar{\rho}}, \frac{\bar{T}_e}{\bar{T}_0}, \frac{\bar{T}_e}{T_0}\). Since $\frac{\phi}{\phi}^2$, $q$ and $s$ are functions of the overheat of the wire, one can, in principal, operate a single wire at six overheats and solve a system of six equations for the fluctuations and their correlations. The unsuccessful attempts that were made in the past to obtain solutions in this manner were thought to be due to the mean flow calibrations not being accurate enough or the sensitivities not being sufficiently different to permit accurate solutions. Demetriades stated that no solutions to equation (29) are possible unless the independent variable occurs to at least the fifth power. It is shown in the Appendix that if $s$ is assumed to be a function of $q$, solutions to equation (29) exist if, after substituting the functional relationship for $s$ into the equation, the resulting equation has six terms. This result requires that values for $s$ have very restrictive, non-linear variations with respect to $q$ as the overheat of the wire is changed. Table I presents conditions where solutions to equation (29) exist based on $s$ being a simple, finite, power series of $q$.

Figure 2 is presented to show to what extent $q \equiv s$ for $r_*=2.192$. From the figure it can be seen that $s$ is, in general, greater than $q$ and the difference can be large since $q$ can often approach zero at the higher values of $M$. At the higher values of $M$ the sign of $q$ can change and negative values are not presented in the figure.

For Spangenberg's data, which is considered herein, the Mach number ranged from 0.05 to 0.95. The equation given by Morkovin for the Knudsen number is:

$$Kn = \frac{7.75x10^{-9}}{\mu t}$$ (30)
where \( \rho \) is in \( \text{gm/cm}^3 \) and \( d \) is in \( \text{cm} \). Using this equation, the value for the Knudsen number for these data ranged from 0.017 to 0.051. Reference 12 suggests that continuum flow occurs for \( Kn < 0.001 \) and slip flow occurs for \( 0.001 \leq Kn \leq 2 \). Other references suggest that continuum flow occurs for \( Kn < 0.01 \). These quoted boundaries are not sharply defined, but if it is assumed that slip flow occurs for \( Kn > 0.01 \), all of Spangenberg's data considered herein is in the slip flow regime. This result is in agreement with the data presented in figure 2 where \( S_p/S_z \) vs \( S_p/S_z \), however, a few data points do approach \( S_p/S_z \).

**Solution to Mean Square Equation**

Solutions to equation (29) can be obtained by using the method of least squares. Applying this method results in the following matrix equation:

\[
\begin{align*}
\Sigma q^2 & \Sigma q^2 \Sigma q^2 \Sigma q^2 \\
\Sigma s^2 & \Sigma s^2 \Sigma s^2 \Sigma s^2 \\
\Sigma q^2 & \Sigma q^2 \Sigma q^2 \Sigma q^2 \\
\Sigma s^2 & \Sigma s^2 \Sigma s^2 \Sigma s^2 \\
\Sigma q^2 & \Sigma q^2 \Sigma q^2 \Sigma q^2 \\
\Sigma s^2 & \Sigma s^2 \Sigma s^2 \Sigma s^2 \\
\text{(31)}
\end{align*}
\]

In equation (31) \( q \), \( s \) and \( \phi \) are functions of the overheat parameter, \( \tau_w \). Solutions to equation (31) can be obtained as follows. Obtain a set of values for \( q \), \( s \) and \( \phi \) for a suitable number of values for \( \tau_w \), say 10 or 12. Fit suitable curves to \( q \) and \( \phi \) vs \( \tau_w \). Also obtain curve fits for \( s = f(q) \) that satisfies the requirements noted in the Appendix to determine if a solution to equation (31) is possible. The "best curve fit" to these data using the acceptable coefficients can be used to relate \( s \) to \( q \) and ultimately \( s \) and \( \tau_w \).

Using the equations obtained for \( q \), \( s \) and \( \phi \) vs \( \tau_w \), compute a set of values, say 20, to be used in equation (31) to obtain the fluctuations and their correlations.

An additional datum point can be obtained for \( s \) and \( q \) by using the limiting values for these variables. These equations are:

\[
\begin{align*}
q &= -\frac{1}{a} \left( \frac{1}{2a} \frac{\Delta \log \eta}{\Delta \log M} \right) - \left( \frac{1}{2a} \frac{\Delta \log \eta}{\Delta \log M} - 1 \right) ; \quad \tau_w = 0 \\
q &= -\frac{1}{a} \left( \frac{1}{2a} \frac{\Delta \log \eta}{\Delta \log M} \right) ; \quad \tau_w = 0
\end{align*}
\]

An indication of the accuracy and suitability of these results can be obtained by computing the condition number for the 6x6 matrix in equation (31). The smaller the condition number the more accuracy can be expected from the calculated results.

No fluctuating voltages were presented by Spangenberg, therefore, no fluctuating quantities could be calculated. His data were used to obtain relationships between \( q \) and \( s \), to calculate the elements of the matrix in equation (31), and to obtain the condition numbers for these matrices. This procedure serves to illustrate that solutions to equation (31) are possible and give some indication of the accuracy of possible solutions.

Examples of the variation of \( s \) with \( q \) obtained from Spangenberg's data is presented in figure 3. The case where the Nusselt number was assumed to be a cubic relationship with respect to \( \tau_w \), most often resulted in a hairpin like variation of \( s \) with \( q \). This type of variation does not appear reasonable, therefore, a curve is also presented in the figure where \( Nu_w \) was assumed to be given by a second degree equation. This assumption resulted in what appears to be a more reasonable variation of \( s \) with \( q \). It must be again noted that the behavior obtained with the cubic curve fit might be due to the limited amount of data available.

There were several sets of data where \( Nu_w \) could be fitted with a cubic equation with respect to \( \tau_w \), without the derivative of \( Nu_w \) increasing at the higher values of \( \tau_w \). Examples of \( s \) vs \( q \) for these cases are presented in figure 4. The powers of \( q \) presented in Table I were fitted to the \( s-q \) data, and the powers that gave the "best curve fits" to the data are presented in the Table II. Most of the curves fitted
to the data appear to be good based on the \( R^2 \) values except for the case \( M = 0.60 \). Curves fitted to the data using the values of the powers presented in Table II are given in figure 4. The equations with these powers were used to relate \( s \) to \( q \) and in turn to \( r \). Twenty values for \( q \) were selected, the values for \( s \) computed, and the elements for the 6x6 matrix in equation (31) calculated. Using the elements of the matrix, the condition numbers for the cases presented in figure 4 were determined and these results are presented in the Table II. These large numbers indicate that the 6x6 matrix in equation (31) is very ill conditioned. It is not known if these results are general or limited to the cases investigated which has a limited number of overheats. There are techniques that can be used to improve solutions for equations having ill conditioned matrices. It is not known if these techniques can be used to assure suitable solutions to the hot wire equation. There are cases where suitable solutions can be obtained to sets of equations where the matrix results in large condition numbers. It must be demonstrated that satisfactory results can be obtained for the present equation. Thus, the necessary condition for a solution to the mean square equation has been established; it remains to be demonstrated that sufficient conditions can be established for suitable solutions.

There is an additional problem that could have an adverse effect on solutions to equation (29). Equations (32) and (33) show that \( s \) and \( q \), in general, cannot be zero at the same time. This means that conditions required to obtain the total temperature fluctuations do not lie on the curve defined by \( s = f(q) \). Therefore, the total temperature fluctuations must be obtained by extrapolation to the condition where \( q = s = 0 \). It must be noted that this condition is not determined by the accuracy of the mean flow calibration data but by the relationships used to obtain equation (29). This problem is not limited to the subsonic compressible flow case, but could also exist in supersonic flow if \( d \log \eta /d \log Re \neq 0 \). In supersonic flow the extrapolation is in two dimensional space, whereas in subsonic flow the extrapolation must be in three dimensional space (see Fluctuation and Mode diagram section). Extrapolation, in three dimensional space, of a nonlinear function could result in highly inaccurate solutions. At the present time it is not known whether other relationship could be used to obtain values for \( s \) and \( q \) such that this problem can be rectified. Possible remedy might be found in the techniques presented in reference 11.

Fluctuation and Mode Diagrams

Fluctuation Diagrams

It is recognized that, in general, if solutions to equation (29) can be obtained using the method of least squares then the fluctuations and their correlations can be obtained without considering the fluctuation diagram. However, the fluctuation diagram can often be used to advantage in determining the dominant mode in a flow.

Before considering the fluctuation diagram for subsonic compressible flows, consider the case for supersonic flow where the velocity and density sensitivities are equal. Under these conditions equation (16) becomes:

\[
\frac{\dot{E}}{E} = -S_m \frac{\dot{m}'}{m} + \frac{T_r}{T_e} S_T
\]

Dividing the above equation by \( S_T \), squaring, and taking the mean gives:

\[
\frac{\dot{E}^2}{E^2} = 2\left(\frac{\dot{m}'}{m}\right)^2 - 2\pi \eta \left(\frac{\dot{m}'}{m}\right)(\frac{T_r}{T_e}) + \left(\frac{T_r}{T_e}\right)^2
\]

This is a single equation with three unknowns \( \dot{m}', \frac{T_r}{T_e}, \frac{\dot{m}'}{m}, \frac{T_r}{T_e} \). In principle, a single wire can be operated at three overheats and a system of three equations solved for the fluctuations and their correlation. Kovasznay noted that this technique would probably result in inaccurate answers. He suggested that the wire be operated at several overheats and a graphical technique be used to obtain the fluctuations. Equation (35) is an equation of a hyperbola where the asymptote gives the mass flow fluctuation and the intercept of the curve with the \( \phi \)-axis gives the total temperature fluctuation. Note that the method of least squares can also be used to obtain solutions to equation (35). An example of a general fluctuation diagram for supersonic flow is presented in figure 5.

Kovasznay demonstrated that fluctuations in compressible flow are composed of three basic fluctuations which are vorticity, entropy, and sound. He denoted these three basic fluctuations as modes. Vorticity is a velocity fluctuation that is usually described as turbulence and has no pressure or static temperature fluctuation. Entropy consists of static temperature fluctuation or temperature spottiness.
The sound field consists of isentropic fluctuations associated with pressure fluctuations and include density, velocity, and temperature fluctuations. If fluctuations are considered to be composed of a single mode then the fluctuation diagram was noted as a mode diagram.

If it is assumed that the velocity and density sensitivities are sufficiently different and sufficiently non-linear so that solutions to equation (29) are possible, what are the characteristics of the fluctuation diagrams in subsonic compressible flow? In equation (29), \( \phi^2 \) is a function of \( q \) and \( s \), therefore, the fluctuation diagram exists on a three-dimensional surface, a hyperboloid, rather than a plane as in the case when \( S_r = S_s \). The locus of points of the fluctuation diagram on the surface of the hyperboloid will depend on the relative changes in \( q \) and \( s \) as the overheat of the wire is changed. However, the important information exists in the \( \phi - q \) and \( \phi - s \) planes. For example when \( s = 0 \), equation (29) reduces to an equation for a hyperbola in the \( \phi - q \) plane where the asymptote gives the velocity fluctuations. If \( q = 0 \), again equation (29) reduces to an equation for a hyperbola in the \( \phi - s \) plane and the asymptote represents the density fluctuations. When \( q \) and \( s \) are zero, the intercept on the \( \phi \)-axis gives the fluctuations for the total temperature. In planes parallel to the \( q - s \) plane, the locus of points of the fluctuation diagram is governed by the velocity and density fluctuations and their correlation. The cross product term, \( qs \), requires a rotation of the axis before the characteristics of this locus can be identified.

Although the fluctuation diagram exists on the surface of a hyperboloid, the fluctuations can be determined from the intersection of the hyperboloid with the \( \phi - q \) and \( \phi - s \) planes. Because of this, the fluctuation and mode diagrams will be defined as the traces of these intersections in the noted planes. A general schematic representation of the fluctuation diagram for equation (29) is presented in figure 6.

**Mode Diagrams**

First, assume that the only fluctuations are vorticity. Then:

\[
\frac{\nu'}{u} = \left( \frac{\nu'}{u} \right)_e; \quad \frac{\nu'}{\rho} = 0; \quad \frac{T_r}{T_e} = \beta \left( \frac{\nu'}{u} \right)_e; \quad \frac{m'}{m} = \left( \frac{\nu'}{u} \right)_e \tag{36}
\]

Next, assume that the only fluctuations are entropy. This results in:

\[
\frac{\nu'}{u} = 0; \quad \frac{\nu'}{\rho} = -\left( \frac{T_r}{T_e} \right)_e; \quad \frac{T_r}{T_e} = \alpha \left( \frac{T_r}{T_e} \right)_e; \quad \frac{m'}{m} = \left( \frac{T_r}{T_e} \right)_e \tag{37}
\]

Finally, assume that the only fluctuations are far-field sound. Then:

\[
\frac{\nu'}{u} = \frac{1}{\gamma M} \left( \frac{p'}{p} \right)_e \cos \beta; \quad \frac{\nu'}{\rho} = \frac{1}{\gamma} \left( \frac{p'}{p} \right)_e; \quad \frac{T_r}{T_e} = \frac{(\gamma - 1)}{\gamma} \left( \frac{p'}{p} \right)_e \tag{38}
\]

\[
\frac{T_r}{T_e} = \alpha \frac{(\gamma - 1)}{\gamma} \left( \frac{p'}{p} \right)_e + \beta \left( \frac{\nu'}{u} \right)_e; \quad \frac{m'}{m} = (M + \cos \beta) \frac{1}{\gamma M} \left( \frac{p'}{p} \right)_e \tag{39}
\]

A schematic of the mode diagrams for supersonic flow is presented in figure 7 for vorticity, entropy, and far-field sound. In all cases the mode diagrams are degenerate hyperbolas. This figure is presented in order to compare the mode diagram in supersonic flows with those in subsonic flows.

If it is assumed that in subsonic flow the fluctuations are only due to vorticity, equation (29) gives:

\[
\phi^2 = (q - \beta)^2 \left( \frac{\nu'}{u} \right)^2 \tag{40}
\]

Results obtained using this equation for the case where \( (u'/u)_e = 0.01 \) are presented in figure 8. The mode diagram for vorticity is identical to the one for supersonic flows except that \( S_r \) is replaced by \( S_s \) and the important part of the mode diagram lies in the \( \phi - q \) plane since \( \rho/p = 0 \). Equation (40) shows that when \( \phi = 0 \), then \( q = \beta \). Also, when \( q = 0 \) then:

\[
\phi = \beta \left( \frac{\nu'}{u} \right)_e \tag{41}
\]

which represents the total temperature fluctuation due to vorticity.

Next assume that the fluctuations are all entropy. Then equation (29) becomes:

\[
\phi^2 = (s + \alpha)^2 \left( \frac{T_r}{T_e} \right)^2 \tag{42}
\]
This equation is identical to the one for supersonic flow except that \( S_m \) is replaced by \( S_p \) and the mode diagram lies in the \( \dot{\psi} - s \) plane.

An example of the mode diagram for entropy when \( (\dot{T}_e/T_e)_s = 0.01 \) is presented in figure 9. Here again, only the \( \dot{\psi} - s \) plane is presented since \( \ddot{u}/u = 0 \). When \( \dot{\psi} = 0 \) then \( s = -\alpha \), and when \( s = 0 \) then:

\[
\dot{\psi} = \alpha \left( \frac{\dot{T}_e}{T_e} \right)_s
\]

(43)

which represent the total temperature fluctuation due to entropy fluctuations.

Now assume that all the fluctuations are far-field sound (i.e., \( \lambda/L \ll 1 \)). The equation for this case is:

\[
\overline{\dot{\psi}^2} = \frac{1}{\gamma^2 \overline{M}^2} \left[ \left( \beta - q \right) \cos \theta + \left( \alpha(y - 1) - s \right) M \right]^2
\]

(44)

Equation (44) represents a three-dimensional surface in the \( \dot{\psi}, q, s \) coordinate system and the mode diagram, in general, lies on this surface. The intersections of the surface with the \( \dot{\psi} - q \) and \( \dot{\psi} - s \) planes are defined, herein, as the mode diagrams. When \( s = 0 \) the above equation represents the mode diagram in the \( \dot{\psi} - q \) plane and when \( q = 0 \) the mode diagram is in the \( \dot{\psi} - s \) plane. An example of the sound mode is presented in figure 10, where the \( \dot{\psi} - s \) plane has been rotated into the plane of the paper. The sound mode is the only mode that has non-zero values in the two planes, \( \dot{\psi} - q \) and \( \dot{\psi} - s \), since sound has both velocity and density fluctuations. All of the mode diagrams described above are degenerate hyperbolas in the \( \dot{\psi} - q \) and \( \dot{\psi} - s \) planes.

Concluding Remarks

From the present study of hot-wire anemometry for a constant-current anemometer applicable to subsonic compressible and subsonic slip flows, the following concluding remarks can be made:

- It was shown that the mean square equation for CCA has possible solutions provided that the functional relationship which adequately relates \( s \) to \( q \) when substituted into the equation results in only six terms. Again Spangenberg’s data were used to show that this condition can often be satisfied.

- The application of the method of least squares to the mean square equation resulted in a 6x6 matrix that was ill-conditioned for the data used. Methods to circumvent this problem, if it is general, must be found before reliable solution can be obtained.

- If a solution exists, the fluctuation diagram was shown to exists on a three dimensional surface - a hyperboloid.

- The intersection of the hyperboloid with two of the co-ordinate planes are hyperbolas, where in one plane, \( (q - \dot{\psi}) \), the asymptote gives the velocity fluctuation and in the other plane, \( (s - \dot{\psi}) \), the asymptote gives the density fluctuation. The intersection of the hyperboloid with the \( \dot{\psi} \) axis gives the total temperature fluctuation.

- The mode diagrams for vorticity, entropy, and sound are degenerate hyperbolas in two of the co-ordinate planes.

Acknowledgment

The second author acknowledge the support of this research by National Aeronautics and Space Administration under contracts NAG1-1552 and NAS1-18585.

References


Appendix

A Note on the Mean Square Equation for CCA

The necessary condition required to assure a solution to the hot-wire equation for the constant current anemometer can be developed as follows. The equation in terms of mean square values is:

\[ \overline{\phi^2} = q^2 \left( \frac{\overline{c^2}}{\overline{u}} \right) + s^2 \left( \frac{\overline{c^2}}{\overline{\rho}} \right) + \left( \frac{\overline{f^2}}{\overline{1_c}} \right)^2 + 2q \frac{\overline{w^2}}{\overline{\rho u}} - 2q \frac{\overline{w^2}}{\overline{\rho u}} - 2 \frac{\overline{w^2}}{\overline{\rho u}} (A-1) \]

where

\[ q = \frac{S_L}{S_c} \] and \[ s = \frac{S_L}{S_c} \]

In equation (A-1) the quantities \( \overline{\phi^2} \), \( q \), and \( s \) are functions of \( r \). Therefore \( s \) can be considered to be function of \( q \). Assume that solutions are sought using the method of least squares and \( s = f(q) \); substitute this functional relationship into equation (A-1):

\[ \overline{\phi^2} = q^2 \left( \frac{\overline{c^2}}{\overline{u}} \right) + f(q)^2 \left( \frac{\overline{c^2}}{\overline{\rho}} \right) + \left( \frac{\overline{f^2}}{\overline{1_c}} \right)^2 + 2qf(q) \frac{\overline{w^2}}{\overline{\rho u}} - 2qf(q) \frac{\overline{w^2}}{\overline{\rho u}} - 2f(q) \frac{\overline{w^2}}{\overline{\rho u}} (A-3) \]

Any functional relationship which adequately relates \( q \) and \( s \) and results in equation (A-3) having six terms will satisfy equation (A-1) and result in a solution. This can be illustrated by assuming the following equation for \( s \):

\[ s = a + bq + cq^2 + dq^4 \]

Substituting equation (A-4) into (A-1) and collecting terms gives:
A solution to this equation is possible for any values of \( f \), \( g \), and \( h \) provided that the resultant equation, after the substitution of values for \( f \), \( g \), and \( h \) into the equation, has 6 terms. Some examples of values for \( f \), \( g \), and \( h \) which result in an equation which has six terms are presented in Table I. However, when equation (32) is considered, the non-integer values for \( f \) cannot be used to determine \( s \) in the vicinity of \( r_w = 0 \). This is due to the limiting value of \( q \) being negative. This results in the values of \( f \) being restricted to the values presented in the first two rows of Table I. The major problem in obtaining a solution to equation (A-1) is evaluating the powers of \( q \) in equation (A-4) which will give the best fit to the mean flow calibration data relating \( s \) to \( q \). This can be done by using standard statistical techniques.

### Table I

Possible Solutions to Mean Square Equation for CCA:

\[
\bar{\sigma}^2 = q^2 \left( \frac{v}{u} \right)^2 + \left[ f(q) \left( \frac{\rho}{\rho_c} \right)^2 + \frac{\tau_T}{\tau_c} \right] + 2g(q) \frac{w}{u} - 2q \frac{w}{u} - 2f(q) \frac{\rho}{\rho_c}
\]

\[
s = f(q) + b + c q^2 + d q^6
\]

<table>
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<th>( g )</th>
<th>( h )</th>
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### Table II

Possible Solutions To Mean Square Equation For CCA Based On Spangenberg's Data:

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<th>( \rho )</th>
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<th>( g )</th>
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<th>Condition Number</th>
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<tr>
<td>0.60</td>
<td>0.0012</td>
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<td>1</td>
<td>0</td>
<td>0.9718</td>
<td>6.54 x 10^8</td>
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</tbody>
</table>
(a) \( \text{Nu}_v \) vs. \( M \) for \( r_e = 1.696 \)

Figure 1. Examples of Spangenberg's Data (Ref. 9).

(b) \( \text{Nu}_v \) vs. \( \rho \) for \( r_e = 1.696 \)

Figure 1. Continued.

(c) \( \text{Nu}_v \) vs. \( r_e \) for \( M = 0.1 \)

Figure 1. Concluded.

Figure 2. Density sensitivity ratio, \( s \) vs. velocity sensitivity ratio, \( q \) for \( r_e = 2.192 \);
(Spangenberg's Data; Ref. 9).
Figure 3. Example of $s$ vs. $q$ for $M = 0.05$ and $ho = 0.0004\text{ gm/cm}^3$ for variable $r_w$; (Spangenberg's Data; Ref. 9).

Figure 4. $s$ vs. $q$ for various Mach numbers and $ho = 0.0012\text{ gm/cm}^3$ for variable $r_w$; (Spangenberg's Data; Ref. 9).

Figure 5. Fluctuation diagram for supersonic flow; $S_* = S_p$.

Figure 6. General fluctuation diagram for subsonic compressible flow; $S_* 
eq S_p$. 
Figure 7. Vortieity-Entropy-Far-field sound modes for supersonic flow; $S_s = S_p$.

Figure 8. Vorticity mode for various Mach numbers; $S_s 
eq S_p; \left(\frac{\bar{u}}{u}_e\right) = 0.01$

Figure 9. Entropy mode for various Mach numbers; $S_s 
eq S_p; \left(\frac{\bar{T}}{T}_e\right) = 0.01$

Figure 10. Far-field sound mode; $S_s 
eq S_p; M = 0.5; \left(\frac{\bar{p}}{p}_e\right) = 0.01$
ADDENDUM AND ERRATA

AIAA 94-2535
HOT-WIRE ANEMOMETRY IN SUBSONIC SLIP AND TRANSONIC FLOW REGIMES

P.C. Stainback
and
K.A. Nagabushana

ADDENDUM

The present analysis of constant current anemometry in subsonic slip and transonic flows using the least square technique appeared to show that, based on Spangenberg's data, measured fluctuations would probably have substantial errors based on the large condition numbers calculated for the 6x6 sensitivity matrix. This was true even though the necessary conditions required for a solution to the mean square equation appeared to be satisfied. Because of this, additional studies of the problem were undertaken in attempts to determine the possible reason for the large condition numbers and to search for possible methods for reducing their magnitudes.

From an analytical point of view, consider equation (29). One serious problem which could affect the ability to obtain accurate solutions to this equation can be noted by considering the variation of the sensitivity ratios, q and s, with respect to changes in the overheating of the heated wire and their limiting values which are given in equations (32) and (33).

In subsonic slip and transonic flows, the values of s are, in general, larger than those for q. Both of these quantities approach small limiting values as the overheating of the wire approaches zero. However, except for very low densities, the limiting values for s are smaller in magnitude than those for q as the overheating approaches zero. Because of this, there is a range of overheats in the vicinity of zero where q and s are approximately equal and the locus of points for q and s in the q-s plane crosses the q = s line. Because of this, the mean square equation reduces from the case of a single equation with six unknowns to a single equation with three unknowns. In other words as the overheating is reduced, the mean square equation becomes degenerate from a three dimensional problem to a two dimensional one and this could result in an inability of the equation to give accurate results. Therefore, the present method would probably result in answers having large errors and accurate results cannot be expected unless some other suitable technique can be found to solve the mean square equation. Note that this problem in the limit is due to the heat transfer equation used and not due to any errors in the mean flow data. Is the heat transfer equation used adequate? If it is not, can another technique be found to circumvent the problem of obtaining accurate answers to the mean square equation?

After the wire is calibrated and the sensitivities obtained and the RMS and mean voltage measured across the wire, the search for a possible solution to the mean square equation becomes one of geometry. The trace of the values for q and s in the q-s plane and the corresponding values of \( \phi \) must be used to define the hyperboloid from which the fluctuations and their correlations can be accurately determined since the fluctuation diagram lies in its surface. In general, q and s exist in only one quadrant of the q-s plane as the overheating of the wire is changed, and the hyperboloid must be determined from this limited amount of data. Table II shows that attempts to do this resulted in matrices with large condition numbers indicating that any answers obtained would be subject to large errors. These errors might be reduced if additional information could be obtained.

Is there a reasonable method that can be developed to supply additional information to help define more accurately the proper hyperboloid thereby producing more accurate solutions to the
mean square equation? If the functional relationship between \( q \) and \( s \) are valid in one quadrant, values for \( q \) and \( s \) can be obtained in a second quadrant by extending values from the equation into the second quadrant. This technique reduces the condition numbers of the matrix by about two orders of magnitude. There are, however, no corresponding values for \( \phi \) in this second quadrant. A method to circumvent this problem is to assume \( \phi \) is a function of \( q \) in the first quadrant then extend this relationship into the second quadrant. Using this technique the condition numbers of the sensitivity matrix were reduced to the values presented in Table II. These condition numbers indicate that significant improvements in possible solutions can be obtained, however, the condition numbers are still rather large. Because of the symmetry of the hyperboloid, the above technique might be acceptable. However, it should be noted that the coordinates of the hyperboloid is not symmetric with respect to the \( q, s, \phi \) coordinate axes.

Based on the present paper and the Addendum, it appears that the possibility exist for obtaining solutions to the mean square equation for CCA in the subsonic slip and transonic flows. The accuracy of the results are, however, subject to question. In any case, the possibility of a solution suggest that data, including fluctuation measurements, should be obtained in an attempt to further investigate the problem.

**ERRATA**

In general, the values for \( q \) and \( s \) are negative and equation (16) is written so that the terms in equation (29) are all positive. For the present report the negative values for \( q \) and \( s \) were transformed to positive values. However, most of the equations were presented in the non-transformed coordinate system. This fact was not noted in the report and could lead to some confusion. Equations (31) - (33) are written in the non-transformed system.

Page 11 of the Appendix

However, when equation (32) is considered, the non-integer values for \( f \) cannot be used to determine \( s \) in the vicinity of \( r_w \) equal zero. This is due to the limiting value of \( q \) being negative.

The above should read: However, non-integer values cannot be used since the fluctuation diagram exist on the surface of a hyperboloid and this surface must be defined in all quadrants.

The value 3.3 in Table II should be 3.0.

The notations for the coordinate axes in figure 3 should be interchanged.

**New Table II**

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<th>( \rho \text{ gm/cm}^3 )</th>
<th>( f )</th>
<th>( g )</th>
<th>( h )</th>
<th>( R^2 )</th>
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Review of Hot-Wire Anemometry Techniques and the Range of their Applicability for Various Flows

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ABSTRACT

A review of hot-wire anemometry was made to present examples of past work done in the field and to describe some of the recent and important developments in this extensive and ever expanding field. The review considered the flow regimes and flow fields in which measurements were made, including both mean flow and fluctuating measurements. Examples of hot-wire measurements made in the various flow regimes and flow fields are presented. Comments are made concerning the constant current and constant temperature anemometers generally in use and the recently developed constant voltage anemometer. Examples of hot-wire data obtained to substantiate theoretical results are presented. Some results are presented to compare hot-wire data with results obtained using other techniques. The review was limited to wires mounted normal to the flow in non-mixing gases.

NOMENCLATURE

\[ a, - a_t \] \text{ constants in equation (47)}
\[ a \] \text{ speed of sound}
\[ A, B \] \text{ constants in equation (24)}
\[ A_t, A \] \text{ constants in equation (18)}
\[ b, - b_t \] \text{ order of } q \text{ in equation (47)}
\[ B_t, B \] \text{ constants in equation (19)}
\[ A'(T) \] \text{ } L_k
\[ A'_t \] \text{ overheat parameter, } \frac{1}{2} \left( \frac{\partial \log R_w}{\partial \log I} \right)_w
\[ B'(T) \] \text{ } 2L_k \sqrt{\frac{k_c}{c_w}}
\[ c_p \] \text{ specific heat at constant pressure}
\[ c_v \] \text{ specific heat at constant volume}
\[ c_w \] \text{ specific heat of wire}
\[ d \] \text{ wire diameter of mesh}
\[ \frac{d( )}{dt} \] \text{ rate of change of quantity ( ) with respect to time}
\[ d_0 \] \text{ diameter of cylinder}
\[ d_j \] \text{ diameter of jet}
\[ d_w \] \text{ diameter of wire}
\[ E \] \text{ mean voltage across the wire}
\[ E_{\text{out}} \] \text{ anemometer output voltage}
\[ E' \] \text{ finite-circuit parameter, } \frac{(1 - \varepsilon)}{1 + 2A'_t \varepsilon}
\[ f \] \text{ frequency}
\[ F \] \text{ dimensionless frequency}
\[ F_{\text{true}} \] \text{ true one-dimensional spectral density}
\[ F_M \] \text{ measured one-dimensional spectral density}
\[ F_{\text{ref}} \] \text{ turbulence reduction factor}

† Formerly Senior Research Engineer, Analytical Services & Materials, Inc., Hampton, VA 23666
Gr  Grashof Number
h coefficient of heat transfer
h_σ height above wire shock generator to probe
h_w height above shock generator, immediate
postshock value
l current
k thermal conductivity of air evaluated at
subscript temperature
k_1 wave number in the flow direction
Kn Knudsen number
L characteristic length
m mean mass flow
m_σ exponent for mass flow in equations
(16) & (17)

\frac{\Delta \log \mu}{\Delta \log T_e}

Nu Nusselt number evaluated at subscript
temperature
p mean static pressure
p_σ mean total pressure
P electrical power to the hot-wire
Pr Prandtl number
q sensitivity ratio, S_m/S_e
q_σ dynamic pressure
Q forced convective heat transfer
r sensitivity ratio, S_m/S_e
r_σ radial distance in cylindrical polar
coor-ordinate
r_r distance of virtual source of jet from origin
r_w radius of wire
\alpha linear temperature - resistance coefficient
\alpha_1 of wire
\beta second degree temperature - resistance
coefficient of wire
\beta_1 boundary layer thickness
\delta displacement thickness for Blasius flow
\epsilon finite circuit factor, \( -\frac{\Delta \log I_e}{\Delta \log R_e} \)
\eta transformed co-ordinate distance normal to
body
\eta recovery temperature ratio, \( T_{adw}/T_e \)
\theta temperature parameter, \( T_r/T_e \)
\phi angle between plane sound wave and axis of
probe
\lambda mean free path
\mu absolute viscosity
\nu specific heat ratio, \( c_p/c_r \)
\rho density
\tau time lag
\tau_m temperature loading parameter, \( (T_e - T_{adw})/T_{adw} \)
\tau_{adw} temperature parameter, \( (T_e - T_{adw})/T_{adw} \)
\tau_{adw} shear stress at the wall
\delta normalized fluctuation voltage ratio, \( \epsilon'/\epsilon \)/S_e

Subscript

\sigma adiabatic wall condition
\sigma,c adiabatic wall temperature, continuum flow
condition
\sigma,f adiabatic wall temperature, free molecular
flow condition
B due to buoyancy effect
C constant current anemometer
e edge condition
INTRODUCTION

Comte-Bellot noted that the precise origin of hot-wire anemometry cannot be accurately determined. One of the earlier studies of heat transfer from a heated wire was made by Boussinesq in 1905. The results obtained by Boussinesq was extended by King and he attempted to experimentally verify his theoretical results. These earlier investigations of hot-wire anemometry considered only the mean heat transfer characteristics from heated wires. The first quantitative measurements of fluctuations in subsonic incompressible flows were made in 1929 by Dryden and Kuethe using constant current anemometry where the frequency response of the wire was extended by the use of a compensating amplifier. In 1934 Ziegler developed a constant temperature anemometer for measuring fluctuations by using a feedback amplifier to maintain a constant wire temperature up to a given frequency.

In the 1950's, Kovasznay extended hot-wire anemometry to compressible flows where it was found experimentally that in supersonic flow the heated wire was sensitive only to mass flow and total temperature. Kovasznay developed a graphical technique to obtain these fluctuations, which is mostly used in supersonic flow. In subsonic compressible flows the heat transfer from a wire is a function of velocity, density, total temperature, and wire temperature. Because of this complexity, these flow regimes were largely bypassed until the 1970's and 1980's when attempts were made to develop methods applicable for these flows. In recent years there were several new and promising developments in hot-wire anemometry that can be attributed to advances in electronics, data acquisition/reduction methods and new developments in basic anemometry techniques.

Previous reviews, survey reports, and conference proceedings on hot-wire anemometry are included in references 1,9-20. Several books have been published on hot-wire anemometry and chapters have been included in books where the general subject matter was related to anemometry.

This review considers the development of hot-wire anemometry from the earliest consideration of heat transfer from heated wires to the present. Although mean flow measurements are considered, the major portion of the review addresses the measurement of fluctuation quantities. Examples of some of the more important studies are addressed for wires mounted normal to the flow in non-mixing gases. The present review attempts to bring the development of hot-wire anemometry up to date and note some of the important, recent developments in this extensive and ever expanding field.

FLOW REGIMES AND FLOW FIELDS

Based on the applicable heat transfer laws and suitable approximations, hot-wire anemometry can be conveniently divided into the following flow regimes:

1. Subsonic incompressible flow
2. Subsonic compressible, transonic, and low supersonic flows
3. High supersonic and hypersonic flows

Within each of these major flow regimes are the following sub-regimes:

1. Continuum flow
2. Slip flow
3. Free molecular flow

In subsonic incompressible flow the heat transfer from a wire is a function of mass flow, total temperature and wire temperature. Since density variations are assumed to be zero, the mass flow variations reduce to velocity changes only. The non-dimensional heat transfer parameter, the Nusselt number, is usually assumed to be a function of Reynolds and Prandtl numbers and under most flow conditions the Prandtl number is constant. Evidence exist which indicate that \(Nu\) is also a function of a temperature parameter. In subsonic compressible, transonic and low supersonic flows the effects of compressibility influence the heat transfer from a wire. For these conditions the heat transfer from the wire is \(f(u, \rho, T_0, T_w)\) and \(Nu = f(Re, M, \theta)\). In high supersonic and hypersonic flows a strong shock occurs ahead of the wire and the heat transfer from the wire is influenced by subsonic flow downstream of the shock. Because of this, it was found experimentally that \(Nu = f(Re, \theta)\) only, and the heat transfer from the wire is again a function of mass flow, total temperature, and wire temperature.

In continuum flow the mean free path of the particles is very much less than the diameter of the wire and conventional heat transfer theories are applicable. Where the diameter of the wire approaches a few mean free paths between the particles, the flow does not behave as a continuum, but exhibits some effects of the finite spacing between the particles. These effects have been studied by assuming a finite velocity and a temperature jump at the surface of a body. This gas rarefaction regime was noted as slip flow. In free molecular flow the fluid is assumed to be composed of individual particles and the distance between the particles is sufficiently large that their impact with and reflection from a body is assumed to occur without interaction between the particles. Free molecular flow is theoretically studied using the concepts of kinetic theory.

Figure 1 presents a plot of Mach number vs. Reynolds number for lines of constant Knudsen number where \(d_* = 0.00015\) inch and for flow conditions where \(1.5 \leq \rho_0, \text{ psia} \leq 150\). Baldwin's results indicated that the continuum flow regime existed for \(Kn < 0.001\) and slip flow conditions existed for \(0.001 \leq Kn \leq 2.0\). Other references suggest that slip flow conditions were attained only for \(Kn > 0.01\). Even using the larger value of \(Kn\) for the slip flow boundary, (i.e., \(Kn > 0.01\)) operating a 0.00015 inch wire at low Mach numbers and at atmospheric conditions is near the slip flow boundary. If the total pressure is decreased or the wire diameter reduced, the value of \(Kn\) would be shifted farther into the slip flow regime. Free molecular flow conditions are generally assumed to exist for \(Kn > 2.0\).

Types of Flows

<table>
<thead>
<tr>
<th>Approximate (\bar{u}/u)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Freestream of wind tunnels(^{36,37})</td>
</tr>
<tr>
<td>2. Down stream of screens and grids(^{25,38})</td>
</tr>
<tr>
<td>3. Boundary layers(^{39-42})</td>
</tr>
<tr>
<td>4. Wakes(^{43,44})</td>
</tr>
<tr>
<td>5. Jets(^{45-47})</td>
</tr>
<tr>
<td>6. Flow downstream of shocks(^{48})</td>
</tr>
<tr>
<td>7. Flight in Atmosphere(^{49})</td>
</tr>
<tr>
<td>8. Rotating Machinery(^{50,51})</td>
</tr>
<tr>
<td>9. Miscellaneous(^{52-56})</td>
</tr>
</tbody>
</table>

**TYPES OF ANEMOMETERS**

The two types of anemometers primarily used are the constant current anemometer (CCA) and the constant temperature anemometer (CTA). A constant voltage anemometer (CVA) is presently under development. Even though these three anemometers are described as maintaining a given variable "constant", none of these strictly accomplish this. The degree of non-constancy for the CCA is determined by the finite impedance of its circuit. The constancy of the mean wire temperature for a CTA at high frequencies is limited by the rate at which the feedback amplifier can detect and respond to rapid fluctuations in the flow. The CVA maintains the voltage across the wire and leads constant rather than across the wire. The non-constancy effects in the CCA and the CVA can be

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Stainback, P.C. and Nagabushana, K.A.
accounted for by calibration of the CCA and by
knowing the lead resistance in the CVA.

The heat balance for an electrically heated
wire, neglecting conduction and radiation is:

\[
\text{Heat Stored} = \text{Electrical Power In} - \text{Aerodynamic Heat Transfer Out}
\]

\[
\frac{dc_w}{dt} T_w = P - Q
\]

\[
\frac{dc_w}{dt} T_w = J^2 R_w - \pi L d J (T_w - T_{\text{ref}})
\]

If the heat storage term is properly compensated, then equation (2) becomes:

\[
J^2 R_w = \pi L d J (T_w - T_{\text{ref}})
\]

The measurement of fluctuations in a flow requires a sensor, in this case a wire, with a time response up to a sufficiently high frequency. The time constant of even small wires are limited and the amplitude response of these wires at higher frequencies decreases with frequency. Therefore, some type of compensation must be made for the wire output. There are two methods for accomplishing this. Earlier approaches utilized a constant current anemometer with a compensating amplifier that had an increase in gain as the frequency increased. An example of the roll off in the frequency of the wire, the gain of the amplifier and the resulting signal is shown schematically in figure 2. In principle, the output from the wire can be compensated to infinite frequencies. However, as the frequency increases, the noise output from the compensating amplifier will equal and ultimately exceed the wire output, which limits the gain that can be obtained. A schematic diagram of a CCA is presented in figure 3.

The constant temperature anemometer uses a feedback amplifier to maintain the average wire temperature and wire resistance constant \([i.e., \frac{dT_w}{dt} = 0 \text{ in equation (2)}]\), within the capability of the amplifier. The practical upper frequency limit for a CTA is the frequency at which the feedback amplifier becomes unstable. A schematic diagram of a CTA is presented in figure 4. A third anemometer, presently under development, is the constant voltage anemometer. This anemometer is based on the alterations of an operational amplifier circuit and does not have a bridge circuit. A schematic diagram of a CVA is presented in figure 5.

The upper frequency response of a CCA is generally accepted to be higher than that of a CTA. There is some evidence that the frequency response of the CVA might equal or exceed that of the CCA. The fluctuation diagram technique described by Kovasznay is usually used with a CCA to obtain data at supersonic speeds. This technique depends on the sensitivity of the wire being a function of wire temperature or overheat and the frequency response of the wire being assessable to compensation to almost zero overheat. This technique has limited application for a CTA, since at low overheats, the frequency response of the anemometer approaches the frequency response of the wire.

An example of the difference between the fluctuation diagrams obtained using a CTA and a CCA is presented in figure 6. The intersection of the diagram with the vertical axis at \(S_{\text{m}}/S_{\text{c}} = 0\) represents the total temperature fluctuation and the data show that the CTA cannot be used to measure these fluctuations. The reason for this is illustrated in figure 7a where the total temperature spectra at low overheats for the two anemometers are presented. In these cases the spectrum obtained with the CTA was attenuated at a frequency that was about two orders of magnitude less than for the CCA. At high overheats the two mass flow spectra were more nearly equivalent (figure 7b). However, in reference 60 the output of a laser was modulated and used to heat a wire to check the frequency response of a CTA. It was shown that the frequency response was essentially unchanged down to an overheat of 0.07.

The CTA can be used to make measurements in supersonic flows by using two wires. For these flows the CTA is operated with two wires having different but high overheats, digitizing the voltages and using two equations to obtain \(m', T',\) and \(m'T',\) as a function of time. Then statistical techniques are used to obtain quantities of interest. In general, the CTA is more suitable for measuring higher levels of fluctuations than a CCA. It remains to be determined how the CVA will compare with the CCA and CTA. At present it appears that the CVA has a higher signal to noise...
ratio than either CCA or CTA. Additional advantages and disadvantages of the CCA versus the CTA are described in references 1, 29, 57, 62 and 63.

At low speeds a linearizer is often used to convert the non-linear relationship between wire voltage and velocity to a linear relationship. There are two types of linearizers in use; the logarithmic and the polynomial. A linearizer makes it possible to directly relate the measured voltage to the velocity. However, the linearization process does not result in better measured quantities.

LIMITATIONS OF HOT-WIRE ANEMOMETRY

Most of the data obtained using hot-wire anemometry is limited to small perturbations. There are cases, however, where this linearization of the anemometry equation is not accurate and non-linear effects can influence both the mean and fluctuating voltages. Since high level fluctuations can influence the mean voltage measured across the heated wire, it is important to calibrate probes in flows with low levels of fluctuations.

Because of the mass associated with the wire supports, there can be a significant amount of heat loss from the wire due to conduction to the relatively cold supports. This heat loss results in a spanwise temperature distribution along the wire that, in turn, causes a variation of heat transfer from the wire along its length. In order to compare the heat transfer results from one wire or probe with another, the heat transfer rates must be corrected for these losses. However, computation of fluctuation quantities requires that the uncorrected values of the heat transfer rates be used. An example of the temperature distribution along a wire and its mean temperature is shown in figure 8. The finite length of the wire and its attendant temperature and heat transfer distribution influences the level of the spectra (especially at higher frequencies), correlations, and phase relationships between sensors.

The spacial resolution of a wire is limited by the length of the wire and the size of the smallest scale of fluctuations in the flow. If the length of the wire is larger than the smallest scale, the resultant magnitude of the spectra will be attenuated at the higher frequencies. The length of the wire with respect to the size of turbulence can have an effect on the measurements of fluctuation intensity, space and time correlations, and the turbulence scales and micro scales. Additional spacial resolution problems encountered near walls was discussed in references 69 and 70. Proximity to walls of wind tunnels or to surfaces of models can introduce errors in measurements due to increased heat transfer from the wire due to conduction to the relatively cold walls. An example of the effect of wire length on normalized spectra is presented in figure 9. The spacial resolution of multi-wire probes is further limited by the distance between the wires. The hot-wire probe intrusion into the flow can cause severe disturbance in certain flows. Examples are flows with large gradient such as boundary layers and vortices. Because of the above, hot-wire anemometry has limited resolution in space, time, and amplitude.

A severe problem is encountered in hypersonic flows when the gas is air. At higher Mach numbers the total temperature must be high enough to prevent liquefaction of air in the test section. There is a maximum recommended operating temperature for each wire material. These two facts places severe limitations on the maximum overheat at which wires can be operated. For example, the maximum recommended operating temperature for Platinum-10% Rhodium wire is 1842°R. For a M = 8 wind tunnel, the total temperature required can be as high as 1360°R. Using a recovery temperature ratio of 0.96, the maximum values for \( r_w \) are 0.394 and \( \theta_{\text{am}} = 1.354 \). If gas rarefaction effects are experienced and \( \eta \) is greater than one, then the problem is even more severe. For \( \eta = 1.1 \) the maximum value for \( r_w \) under the above conditions is 0.254. The above values for \( r_w \) are based on the average temperature for the wire. For small \( L/d_w \) wires the limitation on \( r_w \) would be smaller due to higher temperatures at the mid-portion of the wire. The total temperature at low pressures where \( \eta \) could be larger need not be as high as those at higher pressures, however, the constraint of constant total temperature during the calibration process limits the amount that \( T_0 \) can be reduced. (Also see ref. 71-76).
PROBE PRE-CALIBRATION PROCEDURE

Once a probe is constructed, the following procedure should ensure accurate and reliable measurements. First, the probe should be operated at the maximum q and T that will be used during the proposed test. This is done to pre-stress and pre-heat the wire to ensure that no additional strain will be imposed on the wire during the test that could alter its resistance. For supersonic and high q subsonic flows, the wires should also be checked for strain gaging, that is, stresses generated in the wire due to its vibration. Note, for testing in flows having high values of q, the wires should have slack to reduce the stress in the wires and to help eliminate strain gaging. If strain gaging is significant the wire should be replaced. During this pre-testing many wires might fail due to faulty wires or manufacturing techniques, but it is better that the wires fail in pre-testing rather than during an actual test.

A temperature-resistance relationship for wires is usually requires to compute the heat transfer rate from the heated wires. It is generally recommended that the following equation, which is a second degree equation in AT, be used:

\[ \frac{R}{R_w} = 1 + a_1(T_w - T_{mw}) + \beta_1(T_w - T_{mw})^2 \]  \hspace{1cm} (4)

After the wires have been pre-stressed and pre-heated, they should be placed in an "oven" and the wires calibrated to determine the values for \( a_1 \) and \( \beta_1 \). Once this calibration has been completed, the probes can be placed in a facility for mean flow calibration over the appropriate ranges of velocity, density, total temperature and wire temperature.

STATISTICAL QUANTITIES

Data obtained using hot-wire anemometry are typically reduced to statistical quantities. Over the past few years the analysis of random data has been developed to a very high degree. The rapid developments in electronics (i.e., the A/D converters and high speed computers), have made it possible to obtain almost any statistical quantity of interest within the error constraints of the heated wire. Much of this is due to the fact that the digital processing of data can be used to obtain many quantities that are difficult or impossible to obtain using analog data reduction techniques.

Many types of single point and multi-point statistical quantities can be obtained using hot-wire anemometry. It is routine to measure mean flow and RMS values, histograms and the higher order moments of skewness and kurtosis, auto correlation, and one dimensional spectra. Measurements of multi-point statistical quantities include cross correlations, two-point histograms and higher order two-point moments, cross spectra, and coherence functions. Attempts were made to measure higher moments up to eighth order.

These measurements can be used in various ways to evaluate many characteristics of the flow such as scales, decay rate, energy content etc. The coherence function is a useful statistical quantity that can be used to evaluate various properties of a flow. It can often be used to determine the predominant sound propagating angle and to determine the dominant mode present in a fluctuating flow field.

A few examples of statistical quantities that were measured using hot-wire anemometry are presented in figures 10-13. Integral and micro time and length scales of a flow can be determined from autocorrelation functions such as the one presented in figure 10. The higher moments of skewness and kurtosis (figure 11a-b) can be used to determine if the fluctuations are Gaussian. For a Gaussian distribution the value of the skewness parameter is zero and for the kurtosis the value is 3. Figure 11 shows that both of these moments indicate that the mass flow and total temperature fluctuations are Gaussian over most of the thickness of the boundary layer. The value of third order auto-correlation function, such as the one shown in figure 12, can be used to support turbulent flow theories. An example of space-time correlations measured in a turbulent boundary layer is presented in figure 13. The peak of these correlations at \( t \neq 0 \) indicate the presence of convection. The calculation of the convection velocity, obtained by dividing the separation distance by the time at which the individual curves peaks, indicates that there was no significant variation of the convective velocity over the spacings.
used. An example of normalized spectra measured downstream of a grid is presented in figure 14 and show the increased attenuation of high frequency disturbances with increased distance downstream from the grid. (Also see ref. 89-92).

**GENERAL HEAT TRANSFER RELATIONSHIPS**

The heat transfer from a wire under the limits of the present report (i.e., the wires mounted normal to flow in non-mixing gases) is 29:

\[ Q = f(u, \mu, \rho, c_p, T_e, T_w) \]

(5)

if the fluid properties of \( \mu, c_p, \) and \( k \) are based on \( T_e \), then the above equation becomes:

\[ Q = f(u, \rho, T_e, T_w) \]

(6)

Since \( T_w = \eta T_e \) and \( \eta = f(Kn, M) = f(u, \rho, T_e) \). For incompressible continuum flows equation (6) reduces to:

\[ Q = f(m, T_e, T_w) \]

(7)

Unless noted, the total temperature will be used throughout this report to evaluate \( \mu, c_p, \) and \( k \), where as \( \rho \) will be based on \( T_e \).

For a wire with a given \( L/d_e \), the Nusselt number can be expressed in terms of other dimensionless parameters as:

\[ Nu = f \left( Re, Pr, Gr, \frac{T_e - T_w}{T_e}, \frac{u^2}{c_p(T_e - T_w)} \right) \]

(8)

and can be written as follows to show the effects of compressibility:

\[ Nu = f \left( Re, Pr, Gr, M_a, \frac{T_e - T_w}{T_e} \right) \]

(9)

For relatively constant temperatures, \( Pr = \) constant and if \( Gr < Re^3 \), buoyancy effects will be small and \( Gr \) can be neglected. These approximations lead to:

\[ Nu = f(Re, M_a, T_e) \]

(10)

**MEAN FLOW MEASUREMENTS**

**SUBSONIC INCOMPRESSIBLE - CONTINUUM FLOW**

**Theoretical Considerations**

The functional relationship between the power to the wire or the heat transfer from the wire and the mean flow variables are required to determine the so called "static" calibration of the wire from which the sensitivities to the various flow variables can be obtained in order to calculate the fluctuations. Because of this the mean flow results and probe mean flow calibration procedure are considered together.

The first attempt to obtain a theoretical solution for the heat transfer from a heated wire mounted normal to the flow was carried out by Boussinesq. The equation that he obtained is:

\[ Q = I \left( 2 \sqrt{\pi k_c \rho u w} \right) (T_e - T_w) \]

(11)

Equation (11) can be expressed in terms of non-dimensional quantities as follows:

\[ Nu = \frac{2}{\pi} \sqrt{PrRe} \]

(12)

King re-analyzed the problem of heat transfer from a heated wire and obtained the following relationship:

\[ Q = I \left( k_c + 2 \sqrt{\pi k_c \rho u w} \right) (T_e - T_w) \]

(13)

or in terms of non-dimensional quantities:

\[ Nu = \frac{1}{\pi} \left( 2 \sqrt{PrRe} \right) \]

(14)

From equation (11) and (13) it can be seen that the only difference between Boussinesq's and King's results is the inclusion of the additional term \( k_c \) in King's result that attempts to account for the effects of natural convection. At "high" values of Reynolds number the two results are essentially equal.

Using equation (3), equation (13) can be expressed as:

**Stainback, P.C. and Nagabushana, K.A.**
In this equation George et. al.,\textsuperscript{98} noted that $A_i - A_4$ are functions of $T_r$. They proposed the following equation for the calibration of wires that is independent of $T_c$ for a limited range:

$$Re_r = B_1 + B_2 Nu_1^4 + B_3 Nu_1 + B_4 Nu_1^3 + B_5 Nu_1^2$$  \hspace{1cm} (19)

where $\mu$ is evaluated at $T_c$ and $k$ evaluated at $T_f$.

**Examples of Data**

A summary of heat transfer data from cylinders in terms of $Nu$, vs. $Re$, taken in the subsonic continuum flow regime was presented in reference 21. The results from these experiments are compared with the theoretical results of Boussinesq and King in figure 16. This figure shows that there is a relatively good agreement between the measured results and King's theory over a wide range of Reynolds numbers. There is a substantial difference between Boussinesq’s theory and King’s theory and the measured results for Reynolds numbers less than about 100.

A large amount of heat transfer data was also presented by McAdam\textsuperscript{99} in terms $Nu_f$ vs. $Re_f$ and he recommended the following equation:

$$Nu_f = 0.32 + 0.43(Re_f)^{0.52}$$  \hspace{1cm} (20)

In comparison, King's equation with $Pr = 0.70$ is:

$$Nu_f = 0.3183 + 0.6676(Re_f)^{0.50}$$  \hspace{1cm} (21)

For Reynolds number equal to zero the two equations give essentially the same value for the Nusselt number. At higher values of Reynolds number King’s equation is about 40 percent higher than the values of Nusselt number presented by McAdam. A film temperature is often used as the temperature at which $k$, $\rho$ and $\mu$ are evaluated when correlating $Nu$ versus $Re$. However, $\rho$ is sometimes evaluated at the free stream static temperatures. The use of the film temperature for evaluating fluid properties has been questioned in reference 100. However, in this reference the density in the Reynolds number was evaluated at $T_f$, where as, in reference 99 this apparently was not the case.

**Stainback, P.C. and Nagabushana, K.A.**
Bradshaw\textsuperscript{27} notes that there is a difference of opinion throughout the hot-wire anemometry community about the usefulness of a universal correlation based on variables evaluated at a film temperature. These correlations provide a useful guide for plotting results and comparing mean flow results obtained by different investigators. However, if good accuracy is to be obtained for the fluctuations, individual calibration of probes is required.

Often attempts are made to measure mean velocities using hot-wire anemometry. It can be shown using equation (13) that the voltage across a wire is \( V = f(u, \rho, T_s, T_v) \). Therefore, to measure the mean velocity, the other variables must be held constant or a method must be used to correct the data for any variation in variables other than velocity. Because of this, the hot-wire anemometer is not a very good mean flow measuring device, even if one utilizes some of the corrections that have been developed. However, for limited variations in the independent variables, corrected velocity measurements were reported in reference 101.

**Very Low Velocities**

At very low velocities the heated wire can cause a relatively significant vertical movement of a fluid due to buoyancy effects on the lower density fluid adjacent to the wire. This results in a change in the effective velocity around the wire. Efforts were made to calibrate and use hot-wire anemometry at very low velocities\textsuperscript{102-107} where natural convection effects were present. The influences of natural convection are parameterized by the Grashof number. Experimental evidence\textsuperscript{108} indicated that if \( Gr < Re^3 \) then free convection effects were negligible. An example of the effect of low velocities on the Nusselt number is presented in figure 17.

King also provided an equation suitable for low speed flows:

\[
P = \frac{2 \pi L k (T_s - T_v)}{\log(b/c_w)}
\]  

(22)

where \( b = k e^{\gamma - 1}/c_p \rho u \) and \( \gamma = \) Euler's Constant = 0.57721. In terms of non-dimensional quantities, equation (22) becomes:

\[
Nu = \frac{2}{\log[2e^{\gamma - 1}]/(Pr Re)}
\]

(23)

Equation (22) is valid for \( ud_v < 0.0187 \) where \( U \) is in cm/sec and \( d_v \) is in cm. Equation (13) is valid for \( ud_v > 0.0187 \).

For velocities as low as 1.0 cm/sec, Haw and Foss\textsuperscript{102} attempted to correlate their data using King's equation in the form:

\[
E^2 = A + Bu^n
\]

(24)

A deviation of their data from a fitted curve was observed at \( u = 30 \) cm/sec. The diameter of the wire used in their experiment\textsuperscript{102} was not noted. However, if one assumes a value of 0.00015 inch or 0.00020 inch, the limits for the application of equations (13) and (22) indicate velocities of 49 cm/sec or 37 cm/sec, which are not too different from 30 cm/sec. The use of equation (22) would not improve the correlation presented in reference 102 since it can be shown that as \( u \rightarrow 0 \) in equation (22) \( E \rightarrow 0 \). The data of reference 102 indicates that at \( u = 0 \) the intercept of the curve is greater than the value indicated by the intercept in equation (24). Correlations obtained using the results of a theory based on Oseen\textsuperscript{108} flow would not improve the correlation since this approach gives results that are similar to those obtained using equation (22). For a heated wire tested in horizontal wind tunnels, \( u_d \) cannot reach zero since the effective velocity is:

\[
u_d^2 = u^2 + u_s^2
\]

(25)

and for a heated wire \( u_s \neq 0 \).

**SUBSONIC SLIP FLOW AND TRANSONIC FLOW**

**Theoretical Considerations**

These two flow regimes will be treated together since the experimental results are similar. Kovasznay\textsuperscript{6} extended hot-wire anemometry results to compressible flows and showed that there was a significant difference between the heat transfer in compressible and incompressible flows. Several experimenters obtained heat transfer measurements at low speeds and found an
apparent compressibility or Mach number effect\textsuperscript{35,64,109} at Mach numbers as low as 0.1. Spangenberg\textsuperscript{110} conducted extensive tests over a wide range of variables and determined that the apparent compressible flow effects at Mach number as low as 0.05 was really due to gas rarefaction (e.g., slip flow).

In this flow regime the heat transfer from the heated wire is generally given as:

\[ Q = P = \pi L k (T_\infty - \eta T_w) \frac{Nu}{\rho} \]

(26)

In transonic flow and subsonic slip flows the Nusselt number is no longer only a function of Reynolds number and Kings' law is no longer applicable. The most common functional relationship for the Nusselt number in these flow regimes is\textsuperscript{58}:

\[ Nu = f(M, Re, \theta) \]

(27)

since it was found that \( Nu \) is also a function of a temperature parameter. Another functional relationship that was used to analyze gas rarefaction effects is\textsuperscript{35}:

\[ Nu = f(M, Kn, T_r) \]

(28)

In subsonic compressible flows the recovery temperature of the wire can change and functional relationships for \( \eta \) are:

\[ \eta = f(M, Re) \quad \text{or} \quad \eta = f(M, Kn) \]

(29)

### Table I. Functional Relationships for \( Nu \) and \( \eta \).

<table>
<thead>
<tr>
<th>Dependent Variable</th>
<th>Independent Variable</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Nu )</td>
<td>( Re, M_a, \theta )</td>
<td>Morkovin\textsuperscript{58}</td>
</tr>
<tr>
<td>( Nu )</td>
<td>( Kn, M_a, T_r )</td>
<td>Baldwin\textsuperscript{35}</td>
</tr>
<tr>
<td>( Nu )</td>
<td>( Re, M_a, T_r )</td>
<td></td>
</tr>
<tr>
<td>( Nu )</td>
<td>( Kn, M_a, \theta )</td>
<td></td>
</tr>
<tr>
<td>( \eta )</td>
<td>( Re, M_a )</td>
<td>Morkovin\textsuperscript{58}</td>
</tr>
<tr>
<td>( \eta )</td>
<td>( Kn, M_a )</td>
<td>Vrebalovich\textsuperscript{111}</td>
</tr>
</tbody>
</table>

Baldwin chose \( Nu = f(M_a, Kn, r_e) \) and \( \eta = f(M_a, Kn) \). Independent variables that might be used to relate \( Nu \) and \( \eta \) to the dependent variables are presented in Table 1. Although Morkovin and Baldwin chose the variables of \( \theta \) and \( \Phi \), one could just as well have chosen the variables noted in \( \Phi \) or \( \Theta \). It will be shown later that the variables in \( \Theta \) might be the most efficient group to use.

### Examples of Data

Baldwin\textsuperscript{35} and Spangenberg\textsuperscript{110} investigated the heat transfer from wires over a wide range of \( Re, M \), and \( T_r \) in the slip flow and transonic flow regimes. Their results, presented in figures 18a-b, shows that \( Nu = f(Re, M) \) for Mach numbers ranging from 0.05 to 0.90 and Reynolds numbers ranging from 1 to about 100. The effects of wire overheat on the values of \( Nu \) were also determined by Baldwin and Spangenberg and examples of these effects are shown in figure 19. The values of \( Nu \) can increase or decrease with increased overheat depending on the Mach number and Knudsen number.

Results from theoretical calculations made for the effects of slip flow on heat transfer from wires were reported in reference 31. An example of these results is presented in figure 20. The levels of the calculated Nusselt number do not agree with measured results, however, the trends of the theoretical results agree with the experimental trends shown in figure 18.

### Supersonic Continuum Flow

General results for compressible flow shows that \( Nu = f(M, Re, \theta) \). However, it was experimentally\textsuperscript{21,112} determined that \( Nu \neq f(M) \) for Mach numbers greater than about 1.4. Typical heat transfer data for supersonic flow is presented in figure 21 to illustrate the approximate invariance of \( Nu \) with \( M \). At higher Mach number and relatively low total pressures, there is a high probability that much of the data presented for supersonic and hypersonic flows are in the slip flow regime.
FREE MOLECULAR FLOW

Standler, Goodwin and Creager computed the heat transfer from wires for free molecular flow and an example of their results along with measurements are presented in figure 22. A combination of continuum flow, slip flow and free molecular flow results are shown in figure 23. From this figure it can be seen that for continuum flow at large Reynolds number \( Nu = Re \). For free molecular flow \( Nu = Re \), and slip flow results smoothly connect the two regimes. Therefore, for slip flows, \( Nu \) varies with exponent of Reynolds number which range from \( \frac{1}{2} \) to 1.

RECOVERY TEMPERATURE RATIO

The recovery temperature ratio must be known to compute the heat transfer from heated wires. In general, the recovery temperature ratio is a function of Mach and Reynolds numbers or Mach and Knudsen numbers. However, for Mach number greater than about 1.4 the recovery temperature ratio is not a function of Mach number for continuum flow. A "universal" curve presented by Vrebalovich \(^{111}\) (figure 24) correlated the temperature recovery ratio with Knudsen number for all Mach numbers. Using the results presented in figure 24, the temperature recovery ratio for continuum flow and free molecular flow, curves of \( \eta \) vs. \( M \) and \( Kn \) can be calculated. An example of these calculations is presented in figure 25.

FLUCTUATION MEASUREMENTS

SUBSONIC INCOMPRESSIBLE FLOW

Theoretical Considerations

a. Constant Temperature Anemometer

For a constant temperature anemometer, King's equation can be expressed as:

\[
\frac{E^2}{R_e} = \left[ k_e + \sqrt{2 \pi k_e c_e \rho u^2} \right] T_e - \eta T_e
\]

(30)

where \( R_e \) and \( T_e \) are constants.

If one assumes that the changes in \( k_e \), \( c_e \), and \( \eta \) can be neglected, the change in \( E \) will be a function of \( \rho u \) and \( T_e \) as given by the following equation for small perturbations:

\[
\frac{e'}{E} = S_m \frac{m'}{m} + S_{T_e} \frac{T_e'}{T_e}
\]

(31)

where

\[
S_m = \frac{1}{4 \pi} \sqrt{\frac{2 \pi}{Re \cdot Pr}} \quad \text{and} \quad S_{T_e} = -\frac{1}{2} \frac{\eta}{T_e}
\]

(32)

From the above equation it can be seen that \( S_m \to 0 \) as \( Re \to 0 \) and \( S_{T_e} \to \frac{1}{2} \) as \( Re \to \infty \). For the temperature sensitivity, \( S_{T_e} \to -\infty \) as \( T_e \to 0 \) and \( S_{T_e} \to 0 \) as \( T_e \to \infty \). Equation (31) shows that \( E = f(M, T_e) \) where \( S_m = \partial \log E / \partial \log m \) and \( S_{T_e} = \partial \log E / \partial \log T_e \).

Since equation (31) shows that \( E = f(M, T_e) \), the fluctuation of mass flow and total temperature can be measured using a CTA. \(^{113-115}\) This can best be done by using two wires operated at different, but high overheats, digitizing the data, and solving two equations for \( m', T_e' \) and \( mT_e \) as functions of time.

If the total temperature and the Mach number varies significantly, then \( k_e \) and \( c_e \) must be differentiated with respect to \( T_e \) and \( \eta \) differentiated with respect to Mach number. Under these conditions it would be more appropriate to use the equation obtained by Rose and McDaid \(^8\) with the assumption that \( Nu = f(M) \).

Instead of using King's equation, consider equation (15) for measuring mass flow and total temperature fluctuation. For a CTA equation (15) becomes:

\[
\frac{E}{m} = \frac{2}{\pi} \frac{\partial \log B(T_e, m')}{\partial \log m'} \quad \text{and} \quad \frac{E}{m} = \frac{2}{\pi} \frac{\partial \log B(T_e, m')}{\partial \log m'}
\]

(33)

\[
\frac{d \log E}{d \log T_e} = \frac{\partial \log B(T_e, m')}{\partial \log T_e} \quad \text{and} \quad \frac{d \log E}{d \log T_e} = \frac{\partial \log B(T_e, m')}{\partial \log T_e}
\]

Stainback, P.C. and Nagabushana, K.A.
b. Constant Current Anemometer

For the CCA anemometer, Kings' equation becomes:

\[ El = \frac{L}{k_c} + \sqrt{2\pi \frac{k_c}{c_p}} \frac{\rho u d_u}{\left( T_u - T_e \right)} \]  

(34)

Again assume that \( k_c \), \( c_p \), and \( \eta \) are constant, the change in \( E \) is given by the following equation:

\[ \frac{\Delta E}{E} = \frac{m'}{m} + \Delta \frac{T'}{T_e} \]  

(35)

where

\[ S_e = \frac{k_c (1 - s)}{2 \sqrt{2 \pi} \sqrt{Re Pr}} \]  

and \( S_T = \frac{-k_c (1 - s)}{2 \sqrt{2 \pi} \sqrt{Re Pr}} \)  

(36)

If \( d \log I = 0 \) then

\[ S_e = \frac{k_c (1 - s)}{2 \sqrt{2 \pi} \sqrt{Re Pr}} \]  

and \( S_T = \frac{-k_c (1 - s)}{2 \sqrt{2 \pi} \sqrt{Re Pr}} \)  

(37)

Equation (36) and (37) shows that \( S_e = f(Re) \). If \( Re \rightarrow 0 \) then \( S_e \rightarrow 0 \), but if \( Re \rightarrow \infty \) then \( S_e \rightarrow \infty \). On the other hand, if \( T_e \rightarrow 0 \) then \( S_T \rightarrow 0 \) and if \( T_e \rightarrow \infty \) then \( S_T \rightarrow \infty \).

Again, it is possible to measure both \( \dot{m} \), \( T_e \), and \( Re \) using a CCA and the fluctuation diagram developed by Kovasznay. An example of fluctuation diagrams for two discrete frequencies measured with a CCA is presented in figure 26. Again if the total temperature and the Mach number varies significantly, then it would be more appropriate to use Morkovin's equation with the assumption that \( Nu = f(M) \).

Similarly, to measure mass flow and total temperature fluctuation, equation (15) for CCA becomes:

\[ \frac{d \log E}{d \log T_p} = \frac{k_c (1 - s)}{2 \sqrt{2 \pi} \sqrt{Re Pr}} \left[ \frac{\frac{\partial (\rho u)}{\partial T_p}}{\frac{\partial (\rho u)}{\partial T_p}} \right] \]  

(38)

Mass flow fluctuations measured in subsonic flows can be very misleading where there is a significant amount of far-field sound. The mean square value of mass flow fluctuation is:

\[ \left( \frac{m'}{m} \right)^2 = \left( \frac{\bar{n}}{\bar{m}} \right)^2 + 2R_e \left( \frac{\bar{n}}{\rho} \right) + \left( \frac{\bar{\rho}}{\rho} \right)^2 \]  

(39)

The magnitude of the mass flow fluctuation depends on \( \bar{u} \), \( \bar{\rho} \), and \( R_e \) where \(-1 \leq R_e \leq 1\). As an example, assume \( R_e = 1 \), indicating downstream moving sound, and \( \bar{u} = \bar{\rho} \). Under these assumption the mass flow fluctuation equals twice the velocity or density fluctuation. However, if \( R_e = -1 \), indicating upstream moving sound, and \( \bar{u} = \bar{\rho} \); the mass flow fluctuation are zero.

Examples of Data

Most of the measurements made using hot-wire anemometry were and still are being made in the subsonic, incompressible, continuum flow regime. An extensive amount of data was accumulated over the years in various flow fields. Some of the first fluctuation measurements made using hot-wire anemometers were obtained in the freestream of wind tunnels to help evaluate the effects of turbulence on the transition of laminar boundary layers. The purpose of this effort was an attempt to extend wind tunnel transition data to flight conditions in order that the on-set of transition might be predicted on full scale aircraft. Measurements in the freestream are also required to study the effect of freestream disturbances on laminar boundary layer receptivity. An example of measurements made in the freestream is presented in figure 27 for the Low Turbulence Pressure Tunnel located at the NASA Langley Research Center. The filled symbols represent data taken in the facility during 1940 and the curves represent measurements made in 1980. The agreement between the two sets of data is very good when it is noted that the data taken in 1940 was obtained at

Stainback, P.C. and Nagabushana, K.A.
\( p_e = 4 \) atmospheres and the low datum point at \( Re_e = 5 \times 10^5 \) is for \( M = 0.02 \).

Fluctuation measurements were also made in various location within wind tunnel circuits, predominantly in the settling chamber. Anemometry was used to evaluate the efficiency of contractions in reducing vorticity levels in the test section of wind tunnels. An example of the results obtained through a contraction is presented in figure 28. The absolute value of the velocity fluctuation in the direction of the flow was reduced through the contraction but the relative values were greatly reduced depending on the area ratio of the contraction. For example in figure 28 the velocity fluctuation downstream of the contraction are ratioed to the mean flow in the large section of the contraction where the local velocity is low. If these downstream velocity fluctuations were ratioed to the local mean velocity, these normalized fluctuations would be substantially smaller, i.e., \( \bar{u}/u_e = 2.6 \) vs. \( \bar{u}/u = 0.16 \).

b. Grids
It was found that screens or grids can effectively reduce vorticity fluctuations. Because of this attenuation, screens have been extensively investigated using hot-wire anemometry to optimize their characteristics for use in wind tunnels to reduce the vorticity levels in the test section. An example of these measurements is presented in figure 29 where the turbulent reduction factor is given as a function of the Ap/q across the screens. Therefore, the use of screens in the settling chamber along with a contraction of adequate area ratio, can substantially reduce velocity fluctuations in the test section due to vorticity. (Also see ref. 122-130).

c. Boundary Layers
Hot-wire measurements were made in turbulent boundary layers to measure the Reynolds stresses and other fluctuation quantities to furnish data for the development of turbulent boundary layer theories. An example of measurements made in the boundary layer on a flat plate is presented in figure 30a-b. Figure 30a shows the significant variation of the velocity fluctuation across the boundary layer while figure 30b shows an example of the local streamwise velocity fluctuation ratioed to the local velocity. Forming the ratio in this latter manner indicates that the velocity fluctuations can exceed 40 percent, a value that is too large for an accurate assumption of small perturbation.

Extensive measurements were made of turbulent flows in pipes to compare theoretical and measured results. From these measurements many statistical quantities were obtained including Reynolds stresses; triple and quadruple correlations; energy spectra; rates of turbulent energy production, dissipation, and diffusion; and turbulent energy balance. An example of the streamwise velocity fluctuations across a pipe is presented in figure 31a-b.

Hot-wire anemometry has been extensively used to investigate the characteristic of various boundary layer flow manipulators such as Large Eddy Break-up devices (LEBUS), Riblets, and roughness elements. Laminar boundary layer transition due to T-S waves, cross flow and Gortler vortices was extensively studied using the hot-wire techniques. Also the effects of heat addition, sound and vorticity on boundary layer characteristics have been investigated.

The hot-wire anemometer with a single wire cannot determine the direction of flow. However, a technique using a multi-wire, “ladder probe” was developed to study the separated boundary layer where a significant amount of reverse flow occurred. This technique was used to determine the location of the zero average velocity in a subsonic turbulent boundary layer. (Also see ref. 142-144).

d. Laminar Boundary Layer Receptivity
One of the major impediments to a thorough understanding of laminar boundary layer transition is the ability to predict the process by which freestream disturbances are assimilated into the boundary layer. These free stream disturbances can be either vorticity, entropy, sound or a combination of these fluctuations.

The effect of freestream fluctuations on the stability of the laminar boundary can be investigated by making measurements in the freestream and in the boundary layer to evaluate the receptivity of the boundary layer to fluctuations in the freestream. An example of
fluctuations measured in a subsonic boundary layer on a flat plate for various frequency bands is presented in figure 32. Kendall presented three types of measurements made in a laminar boundary layer due to velocity fluctuations from the free stream. The first type is illustrated by the x's and consisted of broadband velocity fluctuation where the peak level occurs towards the inner part of the boundary layer. This type of measurement is noted as the Klebanoff's mode and is represented by the solid line. The results obtained when the data were filtered at the Tollmein-Schlichting (T-S) frequency are represented by circles. Although the frequencies were identical to those of T-S waves they were not T-S waves since the convection velocity was equal to the free stream value. The maximum level of these fluctuations occurred at the outer part of the boundary layer. The third type of fluctuation is represented by the dotted line. These were true T-S waves which occurred in packets and had a convective velocity of 0.35 to 0.4 of the free stream velocity. These peak fluctuation levels occurred near the wall. (Also see ref. 146-156).

e. Jets

Hot-wire measurements were obtained in jets to measure the Reynolds stresses associated with free shear layers and to help evaluate the RMS levels and frequencies associated with jet noise. An example of the velocity and temperature fluctuations measured across a heated jet is presented in figure 33. The two types of fluctuations were normalized by the maximum and the local mean values, respectively. (Also see ref. 158-160).

f. Wakes

Various statistical quantities were measured downstream of a heated cylinder by Townsend to obtain experimental results to help improve turbulent theories applicable to this type of flow. Some of the quantities obtained included turbulent intensities, sheer stress, velocity-temperature correlation, triple velocity correlation, diffusion rate and energy dissipation. Measurements were made from 500 to 950 diameter downstream of the cylinder where dynamical similarity was assumed to exist. An example of the mean curve fitted to the u-component of the velocity fluctuation is presented in figure 34 and shows good similarity. Uberoi and Freymuth made extensive spectral measurements downstream of a cylinder and their data indicated that only the spectra of large-scale turbulence were dynamically similar.

SUBSONIC SLIP FLOW AND TRANSONIC FLOW

Theoretical Considerations

In compressible flows the heat transfer from a wire is usually described by the following equation:

$$Q = \pi L k(T_e - \eta T_f) Nu$$

(40)

Differentiating the above equation for the case where $$Q = P$$ gives:

$$\frac{d \log P - \theta}{\tau_e} = \frac{d \log Nu - \eta}{\tau_o} = \frac{d \log \eta}{\tau_o} + \frac{d \log T_e}{\tau_e}$$

(41)

The terms on the right hand side of the above equation depend only on the functional forms assumed for Nu, and \( \eta \) (Table I) and the chosen independent variables (Table II). Ultimately these terms depend on the variation of Nu, and \( \eta \) with the flow variables along with the aerodynamic and thermodynamic properties of the flow. The final form for the left hand side of the equation depends on the type of anemometer used.

It was shown in reference 161 that, for a wire mounted normal to the flow, \( E = f(u,p,T_e,T_f) \). Morkovin and Baldwin related Nu, and \( \eta \) to the non-dimensional variables noted in Table I. However, recent results presented by Barre et al. suggested advantages from using the following variables: \( E = f(m,M_a,T_e,T_f) \) and \( E = f(p,m,T_e,T_f) \). In order to obtain the equation for the CCA, they transformed Morkovin's equations into their variables. This transformation is not necessary, since once the variables are chosen, the equations can be derived directly using a method similar to the one described by Anders.

The equations obtained using the variables of \( \Theta \) from table II gives the same results as those of Morkovin, namely \( E = f(m,T_e) \) under the condition where \( Nu \neq f(M_a) \) and \( S_e = S_p \). Using the variables in \( \Theta \), Barre et al. applied the resultant equations to measurements made in a turbulent boundary layer under the assumption.
that $\bar{p} = 0$ without assuming that $S_s = S_p$. They also extended the equation to the case of supersonic flow in the test section of wind tunnels by assuming that all the fluctuations were sound, again not assuming $S_s = S_p$.

Once the independent variables are chosen, it is not necessary to derive the equations using $\Nu$, and $\eta$. The "primitive" variables, $u$, $\rho$, $T$, etc., greatly simplifies the manipulation of the calibration data and can be used to correlate $E$ as a function of $u$, $\rho$, $T$, etc., without evaluating $\Nu$, and $\eta$. This technique might have advantages in the calibration of wires and the ease of operation of calibration facilities.

The possible sets of variables based on the above discussion is presented in Table II.

<table>
<thead>
<tr>
<th>Dependent Variable</th>
<th>Independent Variable</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E$, $\Nu$, $\eta$</td>
<td>$u$, $\rho$, $T$</td>
<td>Baldwin$^{35}$, Morkovin$^{58}$</td>
</tr>
<tr>
<td>$E$, $\Nu$, $\eta$</td>
<td>$m$, $M_\infty$, $T_e$</td>
<td>Barr, Quine and Dussauge$^{162}$</td>
</tr>
<tr>
<td>$E$, $\Nu$, $\eta$</td>
<td>$p_\infty$, $m$, $T_e$</td>
<td>Barr, Quine and Dussauge$^{162}$</td>
</tr>
<tr>
<td>$E$</td>
<td>$u$, $\rho$, $T_e$</td>
<td>Rose and McDaid$^{8}$</td>
</tr>
<tr>
<td>$E$</td>
<td>$M_\infty$, $T_e$</td>
<td>Stainback and Johnson$^{85}$</td>
</tr>
</tbody>
</table>

Table II. Various Independent Variables to Derive the Hot-Wire Anemometry Equations

Therefore, there are many forms for the hot-wire equations depending on the variables chosen and the anemometer used. One should choose the variables that are most convenient for the flow situation under investigation.

### a. Constant Current Anemometer

Using the heat transfer equation (40) and the functional relationship from equation (27) and (29), the change in voltage across a wire can be related to the changes in $u$, $\rho$, and $T_e$. An example of a set of equations obtained for a constant current anemometer was given by Morkovin$^{58}$ as:

$$ \frac{e'}{E} = -S_s \frac{\nu'}{u} = S_p \frac{\rho'}{\rho} + S_{T_e} \frac{T_e'}{T_e} \tag{42} $$

where

$$ S_s = \frac{\partial \log E}{\partial \log u} = E' \left( \frac{\partial \log \Nu}{\partial \log \nu} \right) \left( \frac{1}{\alpha} \frac{\partial \log \eta}{\partial \log \nu} \right) \frac{1}{\alpha} \frac{\partial \log \eta}{\partial \log \nu} \tag{43} $$

$$ S_p = \frac{\partial \log E}{\partial \log \rho} = E' \left( \frac{\partial \log \Nu}{\partial \log \rho} \right) \left( \frac{1}{\alpha} \frac{\partial \log \eta}{\partial \log \rho} \right) \frac{1}{\alpha} \frac{\partial \log \eta}{\partial \log \rho} \tag{44} $$

This is a general equation for a wire mounted normal to the flow in compressible flows where $S_s = S_p$. This is a single equation with six unknowns. In principal, this equation can be solved by operating a single wire at six overheats and solving six equations to obtain the three fluctuating quantities and their correlations. In the past, it was generally stated that the calibration of the wire cannot be made sufficiently accurate or the velocity and density sensitivities cannot be made sufficiently different to obtain a suitable solution using this technique. Demetriades$^{15}$ noted that the coefficient in equation (46) must occur to at least the fifth degree. This constraint, however, appears to be too restrictive. For example, assume that $s$ is a function of $q$ as follows:

$$ s = a_1 + a_2 q^4 + a_3 q^5 + a_4 q^6 \tag{47} $$

It can be shown that $b_1$, $b_2$ and $b_3$ can have any value provided that the substitution of the relationship for $s$ into equation (46) results in an equation having at least six terms. An analysis of data obtained at transonic Mach number by Spangenberg indicates that $s$ can be non-linearly related to $q$ and suggest

Stainback, P.C. and Nagabushana, K.A.
Horstman and Rose\textsuperscript{170} made measurements at transonic speeds where, for their flow condition, it was found that $S_* = S_p$. For this condition the transonic hot-wire problem degenerated to the supersonic flow problem where only $m'/m$, $T_e/T_o$ and $R_e$ could be measured. From their measurement of $m'/m$, the velocity and density fluctuations were computed by assuming that $f_o/T_e$ and $p/p$ were zero. An example of these results is presented in figure 40. In this figure Horstman and Rose's hot-wire results, represented by the circles, are compared with the thin film results obtained by Mikulla\textsuperscript{170}.

c. **Flight in Atmosphere**

Any attempts to extrapolate the effect of wind tunnel disturbances on laminar boundary layer transition to flight conditions requires some knowledge of the disturbance levels in the atmosphere. Much of the fluctuation data obtained in the atmosphere was measured using sonic anemometers on towers\textsuperscript{179}. There was a limited amount of data obtained in the atmosphere using hot-wire anemometry on flight vehicles\textsuperscript{49,180}. Otten et. al.,\textsuperscript{49} expanded the methods devised by Rose and McDaid by using a two wire probe. One wire was operated by a CCA at a low over heat to measure $T_e$. The other wire was operated with a CTA that was sensitive to $m$ and $T_e$. The results from these two wires were used to measure $m$ and $T_e$ in the atmosphere. An example of spectral data obtained in the atmosphere is presented in figure 41 and reveals the expected $-\frac{5}{3}$ slope, for $m$ and $T_e$.

d. **Subsonic Slip Flow**

For this regime $Nu = f(M_*, Re_*, R_e)$ and $S_* = S_p$. These results are identical to those in the transonic flow regime and attempts have been made to apply the three wire technique developed for transonic flows to subsonic slip flows. For tests in subsonic slip flows the three wires were of different diameters in addition to being operated at different over-heats. Some very preliminary data obtained using this technique in the Langley LTPT tunnel is presented in figure 42a-b where comparison with results obtained using King's equation are made.

### HIGH SUPERSONIC AND HYPERSONIC FLOW

#### Theoretical Consideration

a. **Constant Current Anemometer**

In the 1950's and 1960's hot-wire anemometry was extended into the high supersonic and hypersonic flow regime\textsuperscript{6,7,181,182}. For high supersonic flows it was found experimentally that $S_* = S_p$ and equation (42) becomes:

$$\frac{e'}{E} = -S_* \frac{m'}{m} + S_p \frac{T_e}{T_o}$$

Dividing equation (49) by the total temperature sensitivity, squaring, and then taking the mean results in the following equation:

$$\overline{\phi^2} = \frac{\overline{m'^2}}{m^2} - 2R_e (\frac{\overline{m/T_e}}{\overline{m}}) + \left(\frac{\overline{T_e}}{\overline{T_o}}\right)^2$$

This equation was derived by Kovasznay\textsuperscript{6} and used to generate fluctuation diagrams for supersonic flows. This equation was also used in references 167 and 168 for subsonic compressible flows. The general form of equation (50) is a hyperbola where the intercept on the $\phi$-axis represents the total temperature fluctuation and the asymptotes represent the mass flow fluctuation\textsuperscript{7}.

Kovasznay demonstrated that the basic linear perturbation in compressible flows consists of vorticity, entropy and sound. He termed these basic fluctuations as "modes". If the fluctuation diagram is assumed to consist of a single mode the diagrams were termed "mode diagrams". An example of a general fluctuation diagram and the various mode diagrams for supersonic flow are presented in figures 43 and 44.

b. **Constant Temperature Anemometer**

For this case equation (42) becomes:

$$\frac{e'}{E} = S_* \frac{m'}{m} + S_p \frac{T_e}{T_o}$$

This is a single equation in two unknowns and a two wire probe can be used to obtain $m'$, $T_e$ and $m'T_e$ similar to the compressible subsonic flow case\textsuperscript{61,183,184}.
Examples of Data

a. Freestream

In order to evaluate the relative "goodness" of supersonic wind tunnels and to relate the levels of disturbances in the test section to laminar boundary layer transition on models, a large amount of hot-wire measurements were made in the test sections of supersonic and hypersonic wind tunnels. In 1961 Laufer\textsuperscript{185} presented measurements made in the test section of the Jet Propulsion Laboratory 18 x 20 inch supersonic wind tunnel over a Mach number range from 1.6 to 5.0 using CCA. An example of the fluctuation diagrams obtained by Laufer is presented in figure 45. From these diagrams the mass flow and total temperature fluctuations were obtained. Examples of the mass flow fluctuations are presented in figure 46a. There was a significant increase of $\dot{m}/m$ with Mach number ranging from 0.07% at $M=1.6$ to about 1.0 to 1.35% at $M=5.0$, depending on Reynolds number. All of the fluctuation diagrams were straight lines and Laufer demonstrated that these results indicated that the fluctuations were predominantly pressure fluctuations due to sound. Examples of the calculated pressure fluctuations are presented in figure 46b. Laufer concluded that the pressure fluctuations originated at the turbulent boundary on the wall of the tunnel and because of the finite value of the temperature fluctuations the sound source had a finite velocity. An example of the sound source velocities is presented in figure 47.

A large amount of hot-wire data was taken in the freestream of various facilities to measure disturbance levels in efforts to develop quiet supersonic wind tunnels. A review of this effort was reported in reference 186.

Measurements in the freestream of the Langley Research Center Mach 20 High Reynolds number Helium Tunnel were performed by Wagner and Weinstein\textsuperscript{181}. All of their fluctuation diagrams were straight lines similar to the results obtained in supersonic flows. Examples of their measured mass flow and total temperature fluctuations are presented in figure 48. The mass flow fluctuations were substantially higher than the values measured by Laufer at $M=5.0$. Pressure fluctuation measurements presented in figure 49 indicate that at low total pressures the boundary layer on the nozzle wall was probably transitional at the acoustic origin of the sound source. Relative sound source velocities are presented in figure 50. The source velocities for the Mach 20 tunnel at the higher pressures are significantly higher than those measured by Laufer at Mach numbers up to 5. (Also see ref. 187).

b. Boundary Layer

Measurements were made in supersonic and hypersonic turbulent boundary layers to extend the range of Reynolds stress measurements needed in the development of turbulent boundary layer theories. Barre et al.,\textsuperscript{162} conducted hot-wire tests in a supersonic boundary layer where transonic effects were accounted for by using a transformation of equation (42-45) from $u, \rho, T_s$ to $\rho, m, T_s^0$. Using the assumption that $E = f(p,m,T_s)$ and $\dot{p}/\dot{m} = 0$, reduced their equation to $E = f(m,T_s)$. Under these conditions the fluctuation diagram developed by Kovasznavy was used to obtain $\dot{m}/m$, $\ddot{m}/\dot{m}$, and $R_m T_s^0$ without the assumption that $S_u = S_p$.

Examples of their results are presented in figures 51 and 52. Figure 51 shows that the quantity $\sqrt{\langle u' \rangle^2/\rho_w}$ is greatly underestimated if the assumption is made that $S_u = S_p$ when the velocity in the boundary is transonic and $S_u \neq S_p$. Figure 52 show the variation of $R_m$ with the local Mach number through the boundary layer. The expected value for $R_m$ is -0.85 and the data obtained for $S_u \neq S_p$ agrees well with this value. However, data evaluated where $S_u$ was assumed to be equal to $S_p$ were substantially higher at the lower transonic Mach numbers.

Fluctuations in a hypersonic boundary layer were made by Laderman and Demitrades\textsuperscript{182} and reported in reference 188. An example of the mass flow and total temperature fluctuations measured across the boundary layer is presented in figure 53. The velocity, density, static temperature, and pressure fluctuations were calculated using the mass flow and total temperature fluctuations and various assumptions. An example of these measurements is presented in figures 54 and 55.

Additional measurements were made in hypersonic boundary layers by Laderman\textsuperscript{189} and Stainback, P.C. and Nagabushana, K.A.
on an ogive cylinder by Owen and Horstman\textsuperscript{59} at $M = 7.0$. The measurements made by Owen and Horstman included not only data for $\bar{m}$, $\bar{I}$, and $R_\infty$, but integral scales and microscales, probability density distributions, skewness, kurtosis and intermittancy distribution across the boundary layer. A summary paper by Owen\textsuperscript{190} presents additional data which included space-time correlation, convective velocities, disturbance inclination angle, and turbulence life time distributions. (Also see ref. 87, 191-196).

c. Wake

An example of fluctuation diagrams measured in the wake behind a 15° half angle wedge at $M = 15.5$ is presented in figure 56. From these results Wagner and Weinstein concluded that the predominant fluctuation in this wake was entropy since the fluctuation diagrams were straight lines that intersected the $r$-axis at approximately $-\alpha$.

d. Downstream of Shock

In supersonic and hypersonic flows the disturbances measured in the freestream of the test section are not necessarily the disturbances that can affect the transition of the laminar boundary layer on a body. The passage of sound waves through shocks will result in a combination of vorticity, entropy, and sound downstream of the shock\textsuperscript{197}. Because of this the fluctuation diagram will no longer be a straight line but a general hyperbola\textsuperscript{48}. An example of this result is presented in figure 57. (Also see ref. 198-201).

CONFIRMATION OF THEORETICAL RESULTS

Hot-wire anemometry was used extensively to validate or confirm theoretical results. Some examples of these efforts are presented below.

Theoretical studies of the stability of laminar boundary layers to small disturbances were initially performed by Tollmein\textsuperscript{202,203} and Schlichting\textsuperscript{204}. These calculations indicated that disturbances of a given frequency could decrease, remain constant or be amplified depending on the frequency chosen and the Reynolds number. The first experimental verification of the theory was made by Schubauer and Skramstad\textsuperscript{135}. An example of recent experimental and theoretical results for the determination of the neutral stability boundary\textsuperscript{205} in a laminar boundary layer is presented in figure 58.

Hot-wire measurements were made downstream of "grids" to evaluate the theory for the decay of turbulence. Tests conducted by Kistler and Vrebalovich\textsuperscript{206} to evaluate the "linear" decay law is presented in figure 59 and confirm this law for large values of the Reynolds number. At lower Reynolds number\textsuperscript{25} the exponent can be closer to 1.20 - 1.25. Measurements of spectra for velocity fluctuations downstream of grids was also made and compared with theory. Two examples of these spectra are presented\textsuperscript{206,207} in figure 60a-b. The spectra in figure 60a had an insignificant amount of energy in the expected inertial sub-range indicated by a slope of $-5/3$. This result was attributed to the low Reynolds number of the flow. The spectra presented in figure 60b was measured in a high Reynolds number flow and revealed a significant amount of energy in the inertial sub-range. Theoretical calculations were made for the temperature spectra in a heated jet and downstream of a heated grid. An example of the theory and measurements\textsuperscript{207} is presented in figure 61. Attempts have been made to predict the influence of measured freestream fluctuations on laminar boundary transition. An example of this efforts\textsuperscript{208} is presented in figures 62.

A considerable amount of data was obtained by Stetson et. al.,\textsuperscript{209} in hypersonic flow to study the stability of laminar boundary layer. An example of these results are presented in figure 63 and indicates the existence of first and second mode instabilities in the laminar boundary layer. These results were in agreement with those obtained by Kendall\textsuperscript{155} and Demetriades\textsuperscript{210} and was in qualitative agreement with the theoretical results obtained by Mack\textsuperscript{156}.

OTHER APPLICATIONS

Hot-wire anemometry was used in shock tubes in an attempt to check the frequency response of probes and to trigger other events\textsuperscript{52,211}.

Theoretical results indicated that there would be "temperature fronts or steps" in cryogenic
wind tunnels due to the injection of liquid nitrogen into the circuit. Measurements were made using a hot-wire anemometer to determine the possible occurrences of these thermal steps.

A pulsed hot-wire was used to measure the velocities and flow angle in low speed flows. A wire, which was operated by an anemometer, was placed in the wake of a second wire which could be alternately heated. The time lag between heating the forward wire, this pulse being detected by the second wire and the distance between the wires was used to compute the velocity of the flow. In reference 213 a somewhat different technique was described that used two CCA's and a CTA to measure the velocity and flow angle.

Hot-wire anemometry was used to obtain the location of transition in a laminar boundary in addition to obtaining some information on the fluctuations in the laminar, transitional and turbulent boundary layers. In some flows where the fluctuation levels are high, such as a jet, a moving hot-wire probe was found to improve the accuracy of the results. This technique is usually noted as flying hot-wire anemometry. (Also see ref. 217-221).

Hot-wire anemometry was used to measure the focal points of the laser beams for Laser Transit anemometry (LTA). Using a traversing mechanism and a CTA, the distance between the two beams was determined by measuring the difference between the two maximum voltage outputs from the anemometer. It was noted that additional information could be obtained such as the mean value of the beam intensity intersected by the wire, laser beam power, beam separation, beam diameter, beam divergence, cross-sectional beam-intensity distribution and relative beam intensity.

**CONDITIONAL SAMPLING**

Organized motion or structures in a turbulent boundary layer has been extensively studied using the concept of conditional sampling. The flow of a turbulent boundary layer over a concave surface was studied in reference 226 to search for organized motion in the boundary layer. The possible existence of organized motion is illustrated in figure 64 from measurements made with two hot-wire probes located 0.1δ apart. The conditional sampling technique was used to determine the characteristic shape of the mass flow signal during the passage of an organized motion. An example of these results are presented in figure 64. Figure 65 shows that the measured event at an upstream and a downstream station had the same general characteristic shape.

**COMPARISON OF HOT-WIRE MEASUREMENTS WITH OTHER TECHNIQUES**

In the past the hot-wire anemometer, with all its limitations, was the only instrument available that was capable of measuring fluctuations with adequate fidelity. To some extent, this is no longer the case as other techniques such as LV, LIF, CARS, and Raman scattering are now available for measuring various mean flow and fluctuating quantities. In inviscid flow where the fluctuations can be low, the anemometer is presently the only reliable instrument available. Compared to other techniques the anemometer is still relatively simple to operate and relatively inexpensive. Because of its long history, the results obtained using anemometry is still often used as a standard for evaluating measurements obtained using other techniques. The extent to which the anemometer can maintain these advantages depends on the continued development of the other techniques.

Tests were conducted in turbulent boundary layers to compare hot-wire results with other techniques to validate hot-wire/laser velocimeter (LV) and hot-wire/laser-induced fluorescence (LIF) techniques. An example of velocity fluctuations obtained using hot-wire anemometer and a LV system is presented in figure 66 for streamwise velocity fluctuations. The agreement between the two sets of data is very good. Measured density and static temperature fluctuations measured with a hot-wire and a LIF system in a turbulent supersonic boundary layer is presented in figure 67. Again, except for two values for the density fluctuation obtained with the LIF system, the agreement between the two sets of data is very good. Various experiments were made using LIF and LIF/RAMAN techniques to measure $\tilde{T}/\bar{T}$ and $\tilde{\rho}/\rho$ where results were compared with hot-wire
measurements. An example of these results is presented in figure 68. The only disagreement between the hot-wire and the other results was attributed to a shock that apparently did not cross the hot-wire probe.

CONCLUDING REMARKS

A review was made to illustrate the versatility of hot-wire anemometry in addition to noting some of its limitations. The review included examples of results obtained in the various flow regimes and various types of flow fields. Examples of data were presented for the subsonic incompressible flow regime that were used to evaluate the flow quality in the test section of wind tunnels, to obtain measurements in turbulent boundary layer, and to substantiate or validate various theories of turbulence.

Recently attempts to extend hot-wire anemometry into the transonic and subsonic slip flow regimes were presented for cases where \( S^* = S^* \) and \( S^* \neq S^* \). Examples of data obtained at high supersonic and hypersonic Mach numbers were presented. These results revealed that the fluctuation diagrams measured in the test section were a straight line, indicating that the disturbances in the test sections were due to sound mode.

Examples were also presented to illustrate measurements made to substantiate turbulent theories, to compare with other techniques, and to illustrate the concept of conditional sampling.

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Stainback, P.C. and Nagabushana, K.A.


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34  Stainback, P.C. and Nagabushana, K.A.
Figure 1. Mach number vs. Reynolds number for lines of Constant Total Pressure and Constant Knudsen Number; \(d_0 = 0.00015 \text{ inch}; T_i = 560^\circ \text{F} \).

Figure 2. Frequency Compensation for CCA.

Figure 3. Schematic Representation of Constant Current Hot-Wire System; (ref. 62).

Figure 4. Schematic Representation of Constant Temperature Hot-Wire System; (ref. 62).

Figure 5. Schematic Representation of Constant Voltage Hot-Wire System; (ref. 57).

Figure 6. Modal Analysis that Compares the CTA and CCA Systems; \(y/\delta = 0.23; \) (ref. 59).

Stainback, P.C. and Nagabushana, K.A.
(a) Low Overheat;  
Total Temperature Fluctuations
Figure 7. Normalized Power Spectra that Compare the Constant Temperature Anemometer (CTA) and Constant Current Anemometer (CCA) Systems; \( y/\delta = 0.23 \); (ref. 59).

Figure 7. Concluded.

(b) High Overheat;  
Mass-Flow Fluctuations
Figure 7. Concluded.

Figure 8. Typical Temperature Distribution Along a Convection Controlled Hot-Wire; (ref. 21).

Figure 9. Hot-Wire Spectral Transfer Function; (ref. 143).

Figure 10. Auto-Correlation Coefficient \( R_m(\tau) \); (ref. 25).

Stainback, P.C. and Nagabushana, K.A.
(a) Third Moment; Skewness
Figure 11. Distribution of the Mass-Flow and Total Temperature Fluctuations Across the Boundary Layer; (ref. 59).

(b) Fourth Moment; Flatness
Figure 11. Concluded.

Figure 12. Composite Third-Order Auto-Correlation Coefficient $R_3(t)$; (ref. 25).

Figure 13. Examples of Filtered Space-Time Correlation Coefficient - 4 kHz; (ref. 87).

Figure 14. Decay of Energy of the Spectral Components; (ref. 88).

Stainback, P.C. and Nagabushana, K.A.
Figure 15. Voltage vs. Velocity for a Wire Operated with CTA; (ref. 93).

Figure 16. Summary of Heat Loss from Circular Cylinders Over a Wide Range of Reynolds Number in Continuum Flow; (ref. 21).

Figure 17. Interaction of Free and Forced Convection; (ref. 104).

Figure 18a. Nusselt Number Correlation for Cylinders in Subsonic Slip Flow; $T_e = 80^\circ F$; $T_w = 584^\circ F$; (ref. 35).

Figure 18b. Spangenberg's Reported Heat Loss Measurements from Electrically Heated Wires in Air; (ref. 171).
Figure 19. Normalized Nusselt Number Variation with Overheat for 0.00015 inch Diameter Platinum Wire Containing 10% Rhodium; (ref. 110).

Figure 20. Predicted Nusselt Number Correlation from Approximate Slip-Flow Theory; (ref. 35).

Figure 21. Summary of Supersonic Heat Transfer from Transverse Cylinders in Rarefied Air Flow; (ref. 21).

Figure 22. Comparison of Heat-Transfer Data with Free-Molecule-Flow Theory Using $\alpha = 0.57$; (ref. 33).

Figure 23. Correlation of Convective Heat Transfer from Transverse Cylinders; (ref. 10).
Figure 24. Normalized Recovery Temperature Ratio vs.
Free Stream Knudsen Number; (ref. 21).

Figure 25. Recovery Temperature Ratio vs. Mach
Number for Constant Values of
Knudsen Number.

Figure 26. Fluctuation-Diagram of the Filtered
Signals (Single-Wire); (ref. 97).

Figure 27. Velocity Fluctuations Measured in the
Test Section of LaRC Low Turbulence
Pressure Tunnel; (ref. 37).

Figure 28. Effect of 16:1 Contraction on Turbulence
Generated by 2-inch Square Mesh Grid;
$R_{ML} = 3710$; (ref. 118).

Stainback, P.C. and Nagabushana, K.A.
Figure 29. Measured Turbulence Reduction Factor for Various Screens and Combinations of Screens with Honeycomb Compared With Different Theories; (ref. 120).

(a) Zero Pressure Gradient
Figure 30. Relative Turbulence Intensities in a Boundary Layer Along a Smooth Wall; (ref. 131).

(b) Constant-Stress Layer
Figure 30. Concluded.

(a) Across the pipe
Figure 31. Distribution of velocity fluctuation in a pipe; (ref. 40).
2.4
1.1
0.8
0.6
0.4
0.2
0.1
0
0.02
0.04
0.06
0.08
0.1

(b) Near the wall

Figure 31. Concluded.

Figure 32. Types of Fluctuation Simultaneously Present in Boundary Layer; (ref. 145).

Figure 33. Distribution of Relative Intensity of Turbulent Velocity and Temperature Fluctuations in a Hot Round Free Jet; (ref. 157).

Figure 34. Relative Turbulence Intensity ($u$-component) in the Wake of a Circular Cylinder; $Re_{d_w} = 1360$; (ref. 43).

Figure 35. General Fluctuation diagram for Subsonic Compressible Flow; (ref. 168).
(a) Disturbances from Single Sound Source
Figure 36. Fluctuation Diagram; \( M = 0.76 \); (ref. 167).

(b) Combination of Vorticity and Entropy Modes Downstream of a Cylinder
Figure 36. Concluded.

Figure 37. Nusselt Number as a Function of \( \rho \); (ref. 110).

Figure 38. Spangenberg Data; (ref. 163).

Figure 38. Continued.

Stainback, P.C. and Nagabushana, K.A.
(a) Total Temperature Fluctuations
\[ T_f = 303°K; \rho = 0.0008 \text{ gm/cm}^3. \]

Figure 38. Concluded.

(b) Density Fluctuations

Figure 39. Continued.

(c) Velocity Fluctuations

Figure 39. Disturbances Measured in the LaRC 8' Transonic Pressure Tunnel using a Three Element Hot-Wire Probe; (ref. 86).

(c) Total Temperature Fluctuations

Figure 39. Continued.
Figure 39. Concluded.

(d) Mass Flow Fluctuations

Figure 40. Normalized RMS Velocity and Density Fluctuation Distribution Across the Boundary Layer; (ref. 170).

Figure 41. Spectra of Mass Flux and Total Temperature Fluctuations; 3.6 km; $M = 0.57$; Feb. 79; (ref. 49).

Figure 42. Fluctuations Measured in the LaRC Low Turbulence Pressure Tunnel; $p_s = 15$ psia; (ref. 172).

Stainback, P.C. and Nagabushana, K.A.
(b) Mass Flow Fluctuations

Figure 42. Concluded.

Figure 43. Generalized Fluctuation Diagram for Supersonic Flow.

Figure 44. Generalized Mode Diagram for Supersonic Flow.

Figure 45. Mode Diagram Measured in Test Section for $M = 5.0$; (ref. 185).

(a) Mass Flow Fluctuations

Figure 46. Variation with Tunnel Mach Number; (ref. 185).
(b) Pressure Fluctuation

Figure 46. Concluded.

Figure 47. Variation of Source Velocity with Tunnel Mach Number; (ref. 185).

Figure 48. Root-Mean-Square Mass-Flow and Total-Temperature Fluctuations in the LaRC 22" Hypersonic Helium Tunnel; (ref. 181).

Figure 49. Pressure Fluctuation in the LaRC 22" Hypersonic Helium Tunnel; (ref. 181).

Figure 50. Relative Source Velocities in the LaRC 22" Hypersonic Helium Tunnel; (ref. 181).
Figure 51. Velocity Fluctuation Measurements in Boundary Layer; Influence of the Sensitivity Coefficient to Mach Number; Shaded Zone - Range of the Available Velocity Fluctuation Data in Supersonic Boundary Layers, Including Hot-Wire and Laser-Doppler Measurements; (ref. 162).

Figure 52. Variation of \( R_{\varphi} \) with Mach Number Through Boundary Layer; (ref. 177).

Figure 53. Variation of Wide-Band RMS Mass-Flux and Total-Temperature Fluctuations Across the Boundary Layer. Kistler's Results at \( M = 4.76 \) (Solid Line) are Included for Comparison; (ref. 188).

Figure 54. Variation of Wide-Band RMS Velocity and Static Temperature Fluctuations Across the Boundary Layer; \( p' \neq 0 \); (ref. 188).
Figure 55. Variation of Wide-Band RMS Pressure and Density Fluctuations Across the Boundary Layer; \( p' \neq 0 \); (ref. 188).

Figure 56. Mode Diagrams in Wake of 15° Half-Angle Wedge; (ref. 181).

Figure 57. Fluctuation Mode Plots; \( M = 21; \)
\[ p_r = 12.41 \times 10^6 \, N/m^2; \] (ref. 48).

Figure 58. Neutral Disturbance in Studies of Laminar Boundary Layer Stability; (ref. 205).
Figure 59. Decay of $\frac{u^2}{\nu^2}$ and $\frac{v^2}{\nu^2}$ Downstream of a Grid; (ref. 206).

(a) Power Spectral Density of $u^2$; $u = 10 \text{ m/s}$; (ref. 88)

(b) Spectral Energy; (ref. 206)

Figure 60. One-Dimensional Spectrum.

Figure 61. One-Dimensional Temperature Spectrum; (ref. 207).

Stainback, P.C. and Nagabushana, K.A.
Figure 62. Effect of Free-Stream Turbulence on Boundary Layer Transition; (ref. 208).

Figure 63. Amplification Rate vs. Frequency, Comparison with Others; (ref. 209).

Figure 64. Typical Fluctuating Mass Flow Data from Two Hot-Wires Indicating Similarity Between the Signals; (ref. 226).

Figure 65. Ensemble Averaged Positive Events in the Weak Perturbation Case; (ref. 226).
Figure 66. Comparison of Streamwise Velocity Fluctuations Measured by Hot-Wires and Laser Velocimeter; (ref. 227).

Figure 67. A Comparison of the Distribution of RMS Fluctuation Amplitudes in Static Temperature and Density within the Boundary-Layer obtained using a Hot-Wire at Single and Multiple Overheat Ratios with Direct Measurements Obtained using Laser-Induced Fluorescence (LIF). The LIF data have been Corrected for Instrument Noise; (ref. 229).

Figure 68. Comparisons of Temperature and Density RMS Fluctuation Amplitudes Obtained with LIF Techniques or Implied by Hot-Wire Anemometer Data. Flow Conditions are the Same as in Figure 66. For Each Variable, the Fluctuation Amplitude is Normalized by the Local Time-Averaged Value; (ref. 230).