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CENTER FOR COMPUTATIONAL STRUCTURES TECHNOLOGY

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Attention:
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DEPARTMENT OF CIVIL ENGINEERING
AND APPLIED MECHANICS

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ABSTRACT

The Center for Computational Structures Technology (CST) is intended to serve as a focal point for the diverse CST research activities. The CST activities include the use of numerical simulation and artificial intelligence methods in modeling, analysis, sensitivity studies, and optimization of flight-vehicle structures. The Center is located at NASA Langley and is an integral part of the School of Engineering and Applied Science of the University of Virginia. The key elements of the Center are: a) conducting innovative research on advanced topics of CST; b) acting as pathfinder by demonstrating to the research community what can be done (high-potential, high-risk research); c) strong collaboration with NASA scientists and researchers from universities and other government laboratories; and d) rapid dissemination of CST to industry, through integration of industrial personnel into the ongoing research efforts.

In addition to research, the activities of the Center include coordinating the activities of a consortium of NASA Centers, commercial software vendors, industrial firms, and universities on Advanced Analysis and Design Systems; organizing workshops and national symposia; as well as writing state-of-the-art monographs and NASA special publications on timely topics.

The Principal Investigator for this Cooperative Agreement is Dr. Ahmed K. Noor, Ferman W. Perry Professor of Aerospace Structures and Applied Mechanics, who is serving as the Director for the Center, and the NASA monitor is Mr. Gordon I. Johnston, Program Manager, Advanced Sensors and Instrument Systems, NASA Headquarters.

SUMMARY OF RECENT PROGRESS

During the last period (July 1, 1994-March 31, 1995), a total of twelve research scientists, one senior programmer/analyst, one program support technician and one executive secretary were supported by the Center. The list of the Center Staff is given in Appendix I. The accomplishments of the Center, under the present cooperative agreement
include completing the sixth and last volume of the Monograph on Flight Vehicle Materials, Structures and Dynamics; organizing one workshop and three symposia; publication of 20 journal articles, making 12 presentations; organizing 4 seminars; working cooperative agreements with five additional commercial software vendors as well as with the Center for Educational Computing Initiatives at M.I.T. These accomplishments are listed in Appendices III through VI and are briefly described subsequently.

1. Conducting research in the following general areas: a) innovative computational strategies for large-scale structural problems on new computing platforms; b) prediction and analysis of failure of structural components made of composite materials, and subjected to combined thermal and mechanical loads; c) sensitivity analysis for large structural systems; and d) nonlinear structural dynamics including impact-contact with friction.

A total of twenty-five research publications and twelve presentations have been made under the present cooperative agreement. A list of the publications and presentations are given in Appendix II. Also, the abstracts of the publications are included in Appendix V.

2. Organizing a seminar series by leading experts in the CST and related areas. Four seminars were given at Langley. The list of seminars is given in Appendix III.

3. Organizing one workshop at NASA Langley. The participants of the workshop came from NASA, other government laboratories, industry and universities. The proceedings of the workshop are being prepared for publication as a NASA CP.

4. Organizing three national symposia. Proceedings of these symposia were published by the American Society of Mechanical Engineers.

5. Completing the sixth and last volume of the Monograph on "Flight Vehicle Materials, Structures and Dynamics - Assessment and Future Directions."

FACILITIES

The computational and experimental facilities at NASA Langley Research Center will be used in performing part of this research. Other computational facilities (e.g., at NSF Illinois and San Diego Supercomputer Centers, CRAY Research, and the High-
Performance Computing Center at Vicksburg, MS) will be used by a special arrangement with Dr. Ahmed K. Noor at no cost to this cooperative agreement.

Appendix I - Center Staff

A. Research Scientists
6. Danielson, Kent T. (Ph.D.), Texas A&M University, College Station, TX (appointed June 14, 1993).
7. Watson, Brian C. (Ph.D.), Georgia Institute of Technology, Atlanta, GA (appointed July 6, 1993).
10. Xu, Kangming (Ph.D.), Northwestern University, Evanston, IL (appointed Jan. 3, 1994).

B. Supporting Staff
3. Mary L. Torian, Executive Secretary (appointed July 1, 1990).
Appendix II - Publications and Presentations

A. Publications

Books and Book Chapters


Journal Articles


B. Presentations


6. Noor, A. K., "Recent Advances in the Sensitivity Analysis of Composite Structures," 31st SES Annual Technical Meeting, College Station, TX, Oct. 10-12, 1994, College Station, TX.

7. Noor, A. K., "Hierarchical Sensitivity Analysis for Laminated Composites," 31st SES Annual Technical Meeting, College Station, TX, Oct. 10-12, 1994, College Station, TX.


Appendix III - Seminars, Workshops and Symposia

A. Seminars


B. Workshops

C. Symposia

1. Symposium on Computational Material Modeling, held at the ASME International Mechanical Engineering Congress and Exposition, in Chicago, IL, Nov. 6-11, 1994

2. Symposium on Durability and Damage Tolerance, held at the ASME International Mechanical Engineering Congress and Exposition, in Chicago, IL, Nov. 6-11, 1994


Appendix IV - Cooperating Organizations

1. The MacNeal-Schwendler Corporation
   815 Colorado Blvd.
   Los Angeles, CA 90041

2. ANSYS, Inc.
   Johnson Road
   P.O. Box 65
   Houston, PA 15342

3. Hibbitt, Karlsson and Sorensen, Inc.
   1080 Main Street
   Pawtucket, RI 02860

   3801 East Bayshore Road
   Palo Alto, CA 94303

5. Macsyma, Inc.
   20 Academy Street
   Arlington, MA 02174-6436

   100 Trade Center Drive
   Champaign, IL 61820-7237

7. ADINA R&D, Inc.
   71 Elton Avenue
   Watertown, MA 02172

8. Computational Engineering International (CEI, Inc.)
   P.O. Box 14306
   Research Triangle Park, NC 27709

9. Superscape, Inc.
   2479 East Bayshore #706
   Palo Alto, CA 94303
10. Center for Educational Computing Initiatives
Massachusetts Institute of Technology
Cambridge, MA 02139
Finite element buckling and postbuckling solutions for multilayered composite panels

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Abstract

A study is made of the buckling and postbuckling responses of flat, unstiffened composite panels subjected to various combinations of mechanical and thermal loads. The analysis is based on a first-order shear deformation von Karman-type plate theory. A mixed formulation is used with the fundamental unknowns consisting of the strain components, stress resultants and the generalized displacements of the plate. The stability boundary, postbuckling response and the sensitivity coefficients are evaluated. The sensitivity coefficients measure the sensitivity of the buckling and postbuckling responses to variations in the different lamination and material parameters of the panel. Numerical results are presented for both solid panels and panels with central circular cutouts. The results show the effects of the variations in the fiber orientation angles, aspect ratio of the panel, and the hole diameter (for panels with cutouts) on the stability boundary, postbuckling response and sensitivity coefficients.

1. Introduction

The physical understanding, and the numerical simulation of the buckling and postbuckling responses of laminated anisotropic plates have been the focus of intense efforts because of the extended use of fibrous composites in aerospace, automotive, shipbuilding and other industries, and the need to establish the practical limits of the load-carrying capability of structures made from these materials.

Extensive experimental and numerical studies have been performed on the buckling and postbuckling responses of composite panels (e.g. [1–10]). The numerical studies were based on either approximate analytical techniques [1,2,4,6,9,10], or the finite element methods [11–16]. Summaries of the many buckling and postbuckling studies reported in the literature are given in monographs [17–21] and survey papers [22,23]. Although these studies have contributed

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THREE-DIMENSIONAL SOLUTIONS FOR THERMOMECHANICAL STRESSES IN SANDWICH PANELS AND SHELLS

By W. Scott Burton and Ahmed K. Noor, Fellow, ASCE

ABSTRACT: Analytic three-dimensional thermoelasticity solutions are presented for static problems of simply supported sandwich panels and cylindrical shells subjected to mechanical and thermal loads. The panels and shells have laminated composite face sheets of arbitrary thickness separated by a core. Each of the individual layers of the face sheets and the core is modeled as a three-dimensional continuum. Analytic first-order sensitivity coefficients are evaluated to assess the sensitivity of the responses to variations in material parameters of the face sheets and the core, as well as to variations in the curvatures and thicknesses of the sandwich and face sheets. Also, the strain energy associated with various stress components in the face sheets and core are calculated and compared. The information obtained in the present study can aid the development and assessment of two-dimensional models for sandwich structures and illuminate the role of particular material parameters in an equivalent model for the core.

INTRODUCTION

The sandwich configuration is widely used in the construction of highly efficient lightweight load-carrying panels and structures. The key to the sandwich concept is the separation of relatively stiff face sheets by a lightweight and flexible core. Stress, free-vibration, and stability problems related to the application of this concept are presented in monographs (Allen 1969; Platema 1966), and some early work is reviewed in papers by Habib (1964; 1965). Emerging high-performance engineering systems such as high-speed civil transport and hypersonic aerospacecraft are likely to make use of sandwich structures with multilayer composite face sheets in meeting stringent stiffness, weight, and failure-resistance requirements while in use in a high-temperature environment.

If the core is modeled as an equivalent continuum, then from an analytical point of view a sandwich panel is no different from ordinary laminated structures for which a large body of literature exists [for modeling of laminated structures see, for example, Reddy (1990) or Noor and Burton (1990, 1992a)]. However, few three-dimensional analyses are available which explicitly examine thermally or mechanically stressed sandwich panels with composite face sheets. Thermal expansion effects of spherical sandwich shells were examined using three-dimensional elasticity theory by Hodges et al. (1985). The purpose of their analysis was to estimate the overall thermal expansion properties of a dish antenna that employed sandwich construction. Chamis et al. (1986) used the finite-element method to study sandwich plates with laminated composite face sheets. Their analyses in-
SENSITIVITY ANALYSIS OF THE NON-LINEAR DYNAMIC VISCOPLASTIC RESPONSE OF 2-D STRUCTURES WITH RESPECT TO MATERIAL PARAMETERS

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SUMMARY
A computational procedure is presented for evaluating the sensitivity coefficients of the viscoplastic response of structures subjected to dynamic loading. A state of plane stress is assumed to exist in the structure, a velocity strain-Cauchy stress formulation is used, and the geometric non-linearities arising from large strains are incorporated. The Jaumann rate is used as a frame indifferent stress rate. The material model is chosen to be isothermal viscoplasticity, and an associated flow rule is used with a von Mises effective stress. The equations of motion emanating from a finite element semi-discretization are integrated using an explicit central difference scheme with an implicit stress update. The sensitivity coefficients are evaluated using a direct differentiation approach. Since the domain of integration is the current configuration, the sensitivity coefficients of the spatial derivatives of the shape functions must be included. Numerical results are presented for a thin plate with a central circular cutout subjected to an in plane compressive loading. The sensitivity coefficients are generated by evaluating the derivatives of the response quantities with respect to Young’s modulus, and two of the material parameters characterizing the viscoplastic response. Time histories of the response and sensitivity coefficients, and spatial distributions at selected times are presented.

KEY WORDS
Sensitivity Non-linear dynamic response Viscoplasticity Finite elements Generalized plane stress

1. INTRODUCTION
Significant advances have been made in the development of effective computational strategies for the numerical simulation of the non-linear dynamic response of structures. However, the use of non-linear dynamic analysis in automated optimum design of structures, with rate-dependent inelastic material response, requires efficient techniques for calculating the sensitivity of the non-linear dynamic response to variations in the design variables. The sensitivity coefficients (derivatives of the response vector with respect to design variables) are used for the following:

(a) determine a search direction in the direct application of non-linear mathematical programming algorithms;
(b) generate an approximation for the dynamic response of a modified structure (along with a reanalysis technique);
(c) assess the effects of uncertainties, in the material and geometric parameters of the computational model, on the dynamic response; and
(d) predict the changes in the dynamic response due to changes in these parameters.

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Thermomechanical postbuckling of multilayered composite panels with cutouts

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The results of a study of the detailed thermomechanical postbuckling response characteristics of flat unstiffened composite panels with central circular cutouts are presented. The panels are subjected to combined temperature changes and applied edge loading (or edge displacements). The analysis is based on a first-order shear deformation plate theory. A mixed formulation is used with the fundamental unknowns consisting of the generalized displacements and the stress resultants of the plate. The postbuckling displacements, transverse shear stresses, transverse shear strain energy density, and their sensitivity coefficients are evaluated. The sensitivity coefficients measure the sensitivity of the postbuckling response to variations in the different laminate and material parameters of the panel. Numerical results are presented showing the effects of the variations in the hole diameter, laminate stacking sequence, fiber orientation, and aspect ratio of the panel on the thermomechanical postbuckling response and its sensitivity to changes in panel parameters.

NOTATION

\[
\begin{align*}
[A], [B], [D], [A]\text{,} & \text{ Matrices of the extensional, coupling, bending and transverse shear stiffnesses of the panel} \\
E_I, E_T & \text{ Elastic moduli of the individual layers in the direction of fibers and normal to it, respectively} \\
\{F\} & \text{ Linear flexibility matrix of the panel (see eqn (B2) in Appendix II)} \\
G_{II}, G_{IT} & \text{ Shear moduli of the individual layers in the plane of fibers and normal to it, respectively} \\
\{\tilde{G}(Z)\} & \text{ Vector of nonlinear terms of the panel (see eqn (1))} \\
h & \text{ Total thickness of the panel} \\
\{H\} & \text{ Vector of stress resultant parameters} \\
\{\tilde{K}\} & \text{ Global linear structure matrix (see eqns (1) and (B2) in Appendix II)} \\
L & \text{ Side length of the panel} \\
M_{1l}, M_{2l}, M_{12l}, M_{21l} & \text{ Bending stress resultants} \\
\tilde{M}(X, X_e), \tilde{N}(H, X, X_e) & \text{ Subvectors of nonlinear terms (see eqn (B3) in Appendix II)} \\
N_1, N_2, N_{12}, N_{21} & \text{ In-plane (extensional) stress resultants} \\
\tilde{N}_{12} & \text{ Applied in-plane edge shear stress resultant} \\
\{N_1i, [M_1i] & \text{ Vectors of inplane and bending stress resultants (see eqn (A1) in Appendix I)} \\
\{N_2i, [M_2i] & \text{ Vectors of thermal forces and moments in the panel (see eqn (A1) in Appendix I)} \\
N_l & \text{ Total number of layers in the panel} \\
q_e & \text{ Applied edge displacement} \\
q_{e,cr} & \text{ Critical value of } q_e \\
q_1, q_2 & \text{ Thermal strain and edge displacement parameters associated with } \{\tilde{Q}^{(1)}, \tilde{Q}^{(2)}\} \text{, respectively} \\
\{Q_1, Q_2\} & \text{ Transverse shear stress resultants} \\
\{Q\} & \text{ Vector of transverse shear stress resultants (see eqn (A1) in Appendix I)} \\
\{\tilde{Q}^{(1)}, \tilde{Q}^{(2)}\} & \text{ Vectors of normalized thermal and mechanical strains} \\
\{Q^{(1)}, \{Q^{(2)}\} & \text{ Matrices of the extensional and transverse shear stiffnesses of the } k\text{th layer of the plate (referred to as the } x_1, x_2, x_3 \text{ coordinate system)}
\end{align*}
\]
SENSITIVITY ANALYSIS FOR THE DYNAMIC RESPONSE
OF VISCOPLASTIC SHELLS OF REVOLUTION

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ABSTRACT

A computational procedure is presented for evaluating the sensitivity coefficients of the dynamic
axisymmetric response of viscoplastic shells of revolution. The analytical formulation is based on
Reissner's large deformation shell theory with the effects of transverse shear deformation, rotatory
inertia and moments turning around the normal to the middle surface included. The material model
is chosen to be isothermal viscoplasticity, and an associated flow rule is used with a von Mises
effective stress. A mixed formulation is used with the fundamental unknowns consisting of six
stress resultants, three generalized displacements and three velocity components.

Spatial discretization is performed using finite elements, with discontinuous stress resultants
across element interfaces. The temporal integration is performed by using an explicit central
difference scheme (leap frog method) with an implicit constitutive update. The sensitivity
coefficients are evaluated using a direct differentiation approach. Numerical results are presented
for a spherical cap subjected to step loading, and a circular plate subjected to impulsive loading. The
sensitivity coefficients are generated by evaluating the derivatives of the response quantities with
respect to the thickness, mass density, Young's modulus, and two of the material parameters
characterizing the viscoplastic response. Time histories of the response and sensitivity coefficients
are presented, along with spatial distributions of these quantities at selected times.

NOMENCLATURE

\( a \) radius of circular plate
\( \dot{a} \) characteristic strain rate
\([C]\) matrix of material stiffness coefficients, see Eq. B14, Appendix B
\(E\) Young's modulus
\([F]\) global linear flexibility matrix of the shell
\([F_{\text{ext}}]\) vector of nodal external forces
\([F_{\text{int}}]\) vector of nodal internal forces
\([G(X)]\) vector of nonlinear contributions
Nonlinear Vibrations of Thin-Walled Composite Frames

A reduced basis technique and a computational procedure are presented for generating the nonlinear vibrational response, and evaluating the first-order sensitivity coefficients of thin-walled composite frames. The sensitivity coefficients are the derivatives of the nonlinear frequency with respect to the material and lamination parameters of the frame. A mixed formulation is used with the fundamental unknowns consisting of both the generalized displacements and stress resultants in the frame. The flanges and webs of the frames are modeled by using geometrically nonlinear two-dimensional shell and plate finite elements. The computational procedure can be conveniently divided into three distinct steps. The first step involves the generation of various-order perturbation vectors, and their derivatives with respect to the material and lamination parameters of the frame, using the First-Order Poincaré perturbation technique. The second step consists of using the perturbation vectors as basis vectors, computing the amplitudes of these vectors and the nonlinear frequency of vibration, via a direct variational procedure. The third step consists of using the perturbation vectors, and their derivatives, as basis vectors and computing the sensitivity coefficients of the nonlinear frequency via a second application of the direct variational procedure. Numerical results are presented for semicircular thin-walled frames with I and J sections, showing the convergence of the nonlinear frequency and the sensitivity coefficients obtained by both the reduced basis and perturbation technique.

INTRODUCTION

Significant advances have been made in the development of effective analytical and numerical techniques for the nonlinear vibration analysis of structures. Reviews of some of the techniques developed for beam and plate structures are contained in survey papers (Bert, 1982; Sathyamoorthy, 1982a, 1987; Kapania and Yang, 1987; Chia, 1988; Kapania and Raciti, 1989) and two monographs by Nayfeh and Mook (1979) and Chia (1980). However, to our knowledge, none of the reported studies considered the nonlinear vibrations of thin-walled composite frames. Moreover, except for a recent study (Noor et al., 1993), no studies have been reported on the sensitivity of the nonlinear vibrational response to variations in the material and geometric parameters of the structure.

The present study is an attempt to fill this void. Specifically, the objective is to summarize the results of a recent study on the nonlinear vibrational response of thin-walled composite frames and the effects of variations in the material parameters of the individual layers on the nonlinear frequencies of vibration. The frames considered are semicircular, made of thin-walled graphite-epoxy material with I and J sections, and have a 36-in. radius (see Fig. 1).

The reduced basis technique was first pre-
EFFECT OF MESH DISTORTION ON THE ACCURACY OF TRANSVERSE SHEAR STRESSES AND THEIR SENSITIVITY COEFFICIENTS IN MULTILAYERED COMPOSITES

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SUMMARY

A study is made of the effect of mesh distortion on the accuracy of transverse shear stresses and their first-order and second-order sensitivity coefficients in multilayered composite panels subjected to mechanical and thermal loads. The panels are discretized by using a two-field degenerate solid element, with the fundamental unknowns consisting of both displacement and strain components, and the displacement components having a linear variation throughout the thickness of the laminate. A two-step computational procedure is used for evaluating the transverse shear stresses. In the first step, the in-plane stresses in the different layers are calculated at the numerical quadrature points for each element. In the second step, the transverse shear stresses are evaluated by using piecewise integration, in the thickness direction, of the three-dimensional equilibrium equations. The same procedure is used for evaluating the sensitivity coefficients of transverse shear stresses. Numerical results are presented showing no noticeable degradation in the accuracy of the in-plane stresses and their sensitivity coefficients with mesh distortion. However, such degradation is observed for the transverse shear stresses and their sensitivity coefficients. The standard of comparison is taken to be the exact solution of the three-dimensional thermoclasticity equations of the panel.

NOTATION

\[
\begin{align*}
\mathbf{\bar{B}} & : \text{strain displacement matrix} \\
[\mathbf{C}], [\mathbf{\bar{C}}] & : \text{effective and reduced stiffness matrices of the panel, respectively; the reduced stiffness matrix is based on neglecting the coupling between the transverse normal strain and the extensional strain components} \\
\mathbf{\bar{C}}_{\text{dyn}} & : \text{reduced stiffnesses of the material} \\
E_1, E_T & : \text{elastic moduli of the individual layers in the direction of fibres and normal to it, respectively} \\
\{\mathbf{E}\} & : \text{vector of average mechanical strain parameters through the thickness} \\
G_{LT}, G_{TT} & : \text{shear moduli of the individual layers in the plane of fibres and normal to it, respectively} \\
\{\mathbf{H}\}_1 & : \text{vector of normalized thermal forces} \\
h & : \text{total thickness of the panel} \\
h_i, h_i, h_{i-1} & : \text{distances from the top and bottom surfaces of the } i \text{th layer to the middle surface; see Figure 1} \\
[\mathbf{K}] & : \text{generalized stiffness matrix; see Appendix A} \\
L & : \text{side length of the panel} \\
NL & : \text{total number of layers in the panel} \\
\{\mathbf{P}\} & : \text{vector of nodal mechanical forces}
\end{align*}
\]

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SENSITIVITY ANALYSIS OF FRICTIONAL CONTACT RESPONSE OF AXISYMMETRIC COMPOSITE STRUCTURES

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(Received 13 June 1994)

Abstract—A computational procedure is presented for evaluating the sensitivity coefficients of the static frictional contact response of axisymmetric composite structures. The structures are assumed to consist of an arbitrary number of perfectly bonded homogeneous anisotropic layers. The material of each layer is assumed to be hyperelastic, and the effect of geometric nonlinearity is included. The sensitivity coefficients measure the sensitivity of the response to variations in different material, laminate and geometric parameters of the structure. A displacement finite element model is used for the discretization.

The normal contact conditions are incorporated into the formulation by using a perturbed Lagrangian approach with the fundamental unknowns consisting of nodal displacements, and Lagrange multipliers associated with the contact conditions. The Lagrange multipliers are allowed to be discontinuous at interelement boundaries. Tangential contact conditions are incorporated by using a penalty method in conjunction with the classical Coulomb's friction model. The Newton Raphson iterative scheme is used for the solution of the resulting nonlinear algebraic equations, and for the determination of the contact region, contact conditions (sliding or sticking), and the contact pressures. The sensitivity coefficients are evaluated by using a direct differentiation approach. Numerical results are presented for the frictional contact of a composite spherical cap pressed against a rigid plate.

**NOTATION**

- $r, z, \theta$ orthogonal coordinate system, see Fig. 1
- $F, R$ force residual and normal contact residual vectors, respectively
- $\mu$ meridional distance
- $s, l^i, h^j, m^k$ components of the second Piola-Kirchhoff stress tensor
- $u, v, w$ normal component of contact tractions (pressures)
- $x^T$ tangential components of contact tractions (pressures) in the radial and circumferential directions, respectively
- $Y, \delta$ total strain energy
- $\psi, \gamma, \delta, \xi_5$ strain energy density (strain energy per unit volume)
- $\varepsilon, \eta$ displacement components in the radial, circumferential and axial directions, respectively
- $\phi$ volume of the structure
- $\xi_5, \eta_5$ initial and current coordinates of a generic point, respectively
- $\gamma, \delta$ increment of normal contact traction
- $\xi_5$ increment of nodal displacement vector
- $\phi$ penalty parameter in the normal direction
- $\epsilon_5$ penalty parameter in the tangential direction
- $\theta$ fiber orientation angle of the $i$th layer
- $\mu$ typical lamination or material parameters of the structure
- $\alpha$ coefficient of friction
- $\psi_5, \gamma_5$ Poisson's ratios of the individual layers
- $\psi_5$ contact consistency condition
- $\delta$ total potential energy
- $\phi_5$ Coulomb's friction function, see eqn (3)
- $\alpha_5$ components of the Cauchy stress

- $c_{ij}$ not

- $e_{ij}$ italics

- $e_i, e_j, e_z$ unit vectors in the radial, axial and circumferential, $r, z$ and $\theta$ directions, respectively
- $E, E_i$ strain energy of the individual layers in the fiber and transverse directions, respectively
- $e_{ij}$ components of the Green-Lagrange strain tensor
- $F$ contact traction vector
- $T$ tensor of deformation gradients
- $F_{ij}$, $F_{ij}$, $T_{ij}$ vectors of contact, external and internal forces, respectively
- $G_{ij}$ gap associated with a contact node in the axial direction
- $G_{ij}$ shear moduli of the individual layers in the plane of fiber and normal to it, respectively
- $G_{ij}$ total thickness of the structure
- $J$ vector associated with Lagrange multipliers
- $J^*$ Jacobian of the deformation gradient tensor
- $K^*$ tangent stiffness matrix
- $K^*$, $K_{ij}$ tangent stiffness matrix associated with the tangential contact conditions
- $K^*$, $K_{ij}$ symmetric and antisymmetric parts of the tangent stiffness matrix $K^*$, respectively, see eqn (29)
- $K^*$, $R$ effective stiffness matrix and force residual, respectively, see eqns (24) and (25)
- $P$ normal component of the total contact force
- $Q, R$ tangential stiffness matrices associated with the normal contact conditions, see eqns (B3) and (B4)
- $A$ change to 23681
- $B$ change to 23681

**ORIGINAL PAGE IS OF POOR QUALITY**
NONLINEAR STRUCTURAL ANALYSIS ON DISTRIBUTED-MEMORY COMPUTERS

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ABSTRACT

A computational strategy is presented for the nonlinear static and postbuckling analyses of large complex structures on massively parallel computers. The strategy is designed for distributed-memory, message-passing parallel computer systems. The key elements of the proposed strategy are: (a) a multiple-parameter reduced basis technique, (b) a nested dissection (or multilevel substructuring) ordering scheme, (c) parallel assembly of global matrices, and (d) a parallel sparse equation solver. The effectiveness of the strategy is assessed by applying it to thermo-mechanical postbuckling analyses of stiffened composite panels with cutouts, and nonlinear large-deflection analyses of HSCT models on Intel Paragon XP/S computers. The numerical studies presented demonstrate the advantages of nested dissection-based solvers over traditional skyline-based solvers on distributed memory machines.

INTRODUCTION

The increasing speed and capacity of present day distributed-memory parallel computers, such as Kendall Square KSR-1, Thinking Machines Corporation CM-5, Intel Paragon, and Cray T3D, have brightened the prospects of performing large-scale postbuckling and nonlinear analyses of complex structures on these platforms. In recent years, intense efforts have been devoted to the development of parallel computational strategies and numerical algorithms for large-scale finite element computations (see, for example, Refs. [1-4]). Much attention has been focused on implementing efficient linear equation solvers on distributed-memory computers. This has led to the development of a
THREE-DIMENSIONAL SOLUTIONS FOR COUPLED THERMEOLECTROELASTIC RESPONSE OF MULTILAYERED PLATES

Kangming Xu¹, Ahmed K. Noor², and Yvette Y. Tang³

ABSTRACT

Analytic three-dimensional solutions are presented for the coupled thermoelectroelastic response of multilayered hybrid composite plates. The plates consist of a combination of fiber-reinforced cross-ply and piezothermoelastic layers. Both the thermoelectroelastic static response and its sensitivity coefficients are computed. The sensitivity coefficients measure the sensitivity of the response to variations in different mechanical, thermal and piezoelectric material properties of the plate. A linear constitutive model is used, and the material properties are assumed to be independent of the temperature and the electric field. The plates are assumed to have rectangular geometry and special material symmetries.

A mixed formulation is used with the fundamental unknowns consisting of the three transverse stress components; three displacement components; transverse component of the electric displacement field; electric potential; transverse heat flux component, and temperature change. Each of the fundamental unknowns is expressed in terms of a double Fourier series in the Cartesian surface coordinates. A state space approach is used to generate the static response and to evaluate the sensitivity coefficients. Extensive numerical results are presented showing the effects of variation in the geometric parameters of the plate on the different response quantities and their sensitivity coefficients.

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Recent Advances in the Sensitivity Analysis for the Thermomechanical Postbuckling of Composite Panels

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Abstract

Three recent developments in the sensitivity analysis for the thermomechanical postbuckling response of composite panels are reviewed. The three developments are: effective computational procedure for evaluating hierarchical sensitivity coefficients of the various response quantities with respect to the different laminate, layer, and micromechanical characteristics; application of reduction methods to the sensitivity analysis of the postbuckling response; and accurate evaluation of the sensitivity coefficients of transverse shear stresses. Sample numerical results are presented to demonstrate the effectiveness of the computational procedures presented. Some of the future directions for research on sensitivity analysis for the thermomechanical postbuckling response of composite and smart structures are outlined.

Introduction

Significant advances have been made in the development of computational models and strategies for the numerical simulation of the thermomechanical buckling and postbuckling responses of composite panels (see, for example, Refs. 1–5 and the review article, Ref. 6). More recently, attempts have been made to extend the domain of sensitivity analysis to the thermomechanical postbuckling response and to evaluate the sensitivity of the response to variations in the panel characteristics (see Refs. 5 and 7). The sensitivity coefficients (derivatives of the response quantities with respect to design variables) can be used to 1) determine a search direction in the direct application of nonlinear mathematical programming

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A HYBRID NEUROCOMPUTING/NUMERICAL STRATEGY FOR NONLINEAR STRUCTURAL ANALYSIS

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Abstract

A hybrid neurocomputing/numerical strategy is presented for geometrically nonlinear analysis of structures. The strategy combines model-free data processing capabilities of computational neural networks with a Padé approximants-based perturbation technique to predict partial information about the nonlinear response of structures. In the hybrid strategy, multilayer feedforward neural networks are used to extend the validity of solutions by using training samples produced by Padé approximations to the Taylor series expansion of the response function. The range of validity of the training samples is taken to be the radius of convergence of Padé approximants and is estimated by setting a tolerance on the diverging approximants. The norm of a residual vector of unbalanced forces in a given element is used as a measure to assess the quality of network predictions. To further increase the accuracy and the range of network predictions, additional training data are generated by either applying linear regression to weight matrices or expanding the training data by using predicted coefficients in a Taylor series. The effectiveness of the hybrid strategy is assessed by performing large-deflection analysis of a doubly-curved composite panel with a circular cutout, and postbuckling analyses of stiffened composite panels subjected to an in-plane edge shear load. In all the problems considered, the hybrid strategy is used to predict selective information about the structural response, namely the total strain energy and the maximum displacement components only.
POSTBUCKLING AND LARGE-DEFLECTION NONLINEAR ANALYSES ON DISTRIBUTED-MEMORY COMPUTERS

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(Received 1)

Abstract—A computational strategy is presented for postbuckling and nonlinear static analyses of large complex structures on distributed-memory parallel computers. The strategy is designed for message-passing parallel computer systems. The key elements of the proposed strategy are: (a) a multiple-parameter reduced basis technique, (b) a nested dissection (or multilevel substructuring) ordering scheme, (c) parallel assembly of global matrices, and (d) a parallel sparse equation solver. The effectiveness of the strategy is assessed by performing thermomechanical postbuckling analyses of stiffened composite panels with cutouts, and nonlinear large-deflection analyses of High Speed Civil Transport models on three distributed-memory computers. The numerical studies presented demonstrate the advantages of nested dissection-based solvers over traditional skyline-based solvers on distributed-memory machines.

INTRODUCTION

Distributed-memory parallel computers, such as Thinking Machines Corporation CM-5, Intel Paragon, CRAY T3D, and IBM SP2, have the potential to provide the increased speed and capacity required to perform large-scale postbuckling and nonlinear analyses of complex structures. This potential can only be realized if the computational strategies used in such analyses take advantage of the unique characteristics of these computers. In recent years, intense efforts have been devoted to the development of parallel computational strategies and numerical algorithms for large-scale finite element computations (see, for example, Refs 1–4). Much attention has been focused on implementing efficient linear equation solvers on distributed-memory computers. This has led to the development of a number of direct and iterative numerical algorithms for the solution of large sparse linear systems of equations (see Refs 5–11).

Most parallel strategies are related to the "divide and conquer" paradigm based on breaking a large problem into a number of smaller subproblems, which may be solved separately on individual processors. The degree of independence of the subproblems is a measure of the effectiveness of the algorithm since it determines the amount and frequency of communication and synchronization. The numerical algorithms developed for structural analysis can be classified into three major categories, namely: element-wise algorithms; node-wise algorithms; and domain-wise algorithms.

The element-wise parallel algorithms include element-by-element equation solvers and parallel frontal equation solvers. The node-wise parallel equation solvers include node-by-node iterative solvers as well as column-oriented direct solvers. The domain-wise algorithms include nested dissection-based (substructuring) techniques and domain decomposition methods. The first two categories of numerical algorithms allow only small granularity of the parallel tasks, and require frequent communications among the processors. By contrast, the third category allows a larger granularity, which can result in improved performance for the algorithm.

Nested dissection ordering schemes have been found to be effective in reducing both the storage requirements and the total computational effort required of direct factorization.12 The performance of nested dissection-based linear solvers on distributed-memory parallel computers depends on balancing the computational load across processors in a way that minimizes interprocessor communication. Several nested dissection ordering schemes have been developed which differ in the strategies used in partitioning the structure and selecting the separators. Among the proposed partitioning strategies are: recursive bisection strategies13–14 (e.g., spectral graph bisection, recursive coordinate bisection, and recursive graph bisection); combinatorial and design-optimization-based strategies15 (e.g., the simulated annealing algorithm, the genetic algorithm and neural-network-based techniques); and heuristic strategies16 (e.g., methods based on geometric projections and mappings; and algorithms based on embedding the problem in Euclidean space). For highly irregular and/or three-dimensional structures the effectiveness of nested dissection-based schemes may be reduced. However, this is also true for most other parallel numerical algorithms.
REDUCTION TECHNIQUE FOR TIRE CONTACT PROBLEMS

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Abstract

A reduction technique and a computational procedure are presented for predicting the tire contact response and evaluating the sensitivity coefficients of the different response quantities. The sensitivity coefficients measure the sensitivity of the contact response to variations in the geometric and material parameters of the tire. The tire is modeled using a two-dimensional laminated anisotropic shell theory with the effects of variation in geometric and material parameters, transverse shear deformation, and geometric nonlinearities included. The contact conditions are incorporated into the formulation by using a perturbed Lagrangian approach with the fundamental unknowns consisting of the stress resultants, the generalized displacements, and the Lagrange multipliers associated with the contact conditions. The elemental arrays are obtained by using a modified two-field, mixed variational principle.

For the application of the reduction technique, the tire finite element model is partitioned into two regions. The first region consists of the nodes that are likely to come in contact with the pavement, and the second region includes all the remaining nodes. The reduction technique is used to significantly reduce the degrees of freedom in the second region.

The effectiveness of the computational procedure is demonstrated by a numerical example of the frictionless contact response of the space shuttle nose-gear tire, inflated and pressed against a rigid flat surface.

Notation

$b_0, b_1, b_2, b_3$ parameters used in defining the cord end counts (epi); see Eq. 7 and
SENSITIVITY ANALYSIS FOR THE DYNAMIC RESPONSE OF THERMOVISCOPLASTIC SHELLS OF REVOLUTION

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Abstract

A computational procedure is presented for evaluating the sensitivity coefficients of the dynamic axisymmetric, fully-coupled, thermoviscoelastic response of shells of revolution. The analytical formulation is based on Reissner's large deformation shell theory with the effects of large-strain, transverse shear deformation, rotatory inertia and moments turning around the normal to the middle surface included. The material model is chosen to be viscoplasticity with strain hardening and thermal hardening, and an associated flow rule is used with a von Mises effective stress. A mixed formulation is used for the shell equations with the fundamental unknowns consisting of six stress resultants, three generalized displacements and three velocity components. The energy-balance equation is solved using a Galerkin procedure, with the temperature as the fundamental unknown.

Spatial discretization is performed in one dimension (meridional direction) for the momentum and constitutive equations of the shell, and in two dimensions (meridional and thickness directions) for the energy-balance equation. The temporal integration is performed by using an explicit central difference scheme (leap-frog method) for the momentum equation; a predictor-corrector version of the trapezoidal rule is used for the energy-balance equation; and an explicit scheme consistent with the central difference method is used to integrate the constitutive equations. The sensitivity coefficients are evaluated by using a direct differentiation approach.

Numerical results are presented for a spherical cap subjected to step loading. The sensitivity coefficients are generated by evaluating the derivatives of the response quantities with respect to the thickness, mass density, Young's modulus, two of the material parameters characterizing the
Dynamic Sensitivity Analysis of Frictional Contact/Impact Response of Axisymmetric Composite Structures

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Abstract

A computational procedure is presented for evaluating the sensitivity coefficients of the dynamic frictional contact/impact response of axisymmetric composite structures. The structures are assumed to consist of an arbitrary number of perfectly bonded homogeneous anisotropic layers. The material of each layer is assumed to be hyperelastic, and the effect of geometric nonlinearity is included. The sensitivity coefficients measure the sensitivity of the response to variations in different material, lamination and geometric parameters of the structure.

A displacement finite element model is used for the discretization. The normal contact conditions are incorporated into the formulation by using a perturbed Lagrangian approach with the fundamental unknowns consisting of the nodal displacements, and the Lagrange multipliers associated with the contact conditions. The Lagrange multipliers are allowed to be discontinuous at interelement boundaries. Tangential contact conditions are incorporated by using a penalty method in conjunction with the classical Coulomb's friction model. Temporal integration is performed by using Newmark method. The Newton-Raphson iterative scheme is used for the solution of the resulting nonlinear algebraic equations, and for the determination of the contact region, contact conditions (sliding or sticking), and the contact pressures. The sensitivity coefficients are evaluated by using a direct differentiation approach. Numerical results are presented for the frictional contact/impact response of a composite spherical cap impacting on a rigid plate.
BUCKLING AND POSTBUCKLING OF COMPOSITE PANELS WITH
CUTOUTS SUBJECTED TO COMBINED EDGE SHEAR
AND TEMPERATURE CHANGE

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ABSTRACT

The results of a detailed study of the buckling and postbuckling responses of composite panels with central circular cutouts are presented. The panels are subjected to combined edge shear and temperature change. The panels are discretized by using a two-field degenerate solid element with each of the displacement components having a linear variation throughout the thickness of the panel. The fundamental unknowns consist of the average mechanical strains through the thickness and the displacement components. The effects of geometric nonlinearities and laminated anisotropic material behavior are included.

The stability boundary, postbuckling response and the hierarchical sensitivity coefficients are evaluated. The hierarchical sensitivity coefficients measure the sensitivity of the buckling and postbuckling responses to variations in the panel stiffnesses, and the material properties of both the individual layers and the constituents (fibers and matrix). Numerical results are presented for composite panels with central circular cutouts subjected to combined edge shear and temperature change, showing the effects of variations in the hole diameter, laminate stacking sequence and fiber orientation, on the stability boundary and postbuckling response and their sensitivity to changes in the various panel parameters.

NOTATION

\([A], [B], [D], [A_1]\) matrices of the extensional, coupling, bending and transverse shear stiffnesses of the panel
SENSITIVITY ANALYSIS FOR LARGE-DEFLECTION AND POSTBUCKLING RESPONSES ON DISTRIBUTED-MEMORY COMPUTERS

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ABSTRACT

A computational strategy is presented for calculating sensitivity coefficients for the nonlinear large-deflection and postbuckling responses of laminated composite structures on distributed-memory parallel computers. The strategy is applicable to any message-passing distributed computational environment. The key elements of the proposed strategy are: (a) a multiple-parameter reduced basis technique; (b) a parallel sparse equation solver based on a nested dissection (or multilevel substructuring) node ordering scheme; and (c) a multilevel parallel procedure for evaluating hierarchical sensitivity coefficients. The hierarchical sensitivity coefficients measure the sensitivity of the composite structure response to variations in three sets of interrelated parameters; namely, laminate, layer and micromechanical (fiber, matrix, and interface/interphase) parameters. The effectiveness of the strategy is assessed by performing hierarchical sensitivity analysis for the large-deflection and postbuckling responses of stiffened composite panels with cutouts on three distributed-memory computers. The panels are subjected to combined mechanical and thermal loads. The numerical studies presented demonstrate the advantages of the reduced basis technique for hierarchical sensitivity analysis on distributed-memory machines.

INTRODUCTION

Nonlinear large-deflection and postbuckling analyses of large-scale structures can require immense computational resources. Distributed memory parallel computers, such as the Intel Paragon, the Cray T3D, and the IBM SP2, have the potential to provide the
Computational strategies are presented for the modeling and analysis of tires in contact with pavement. A procedure is introduced for simple and accurate determination of tire cross-sectional geometric characteristics from a digitally scanned image. Three new strategies for reducing the computational effort in the finite element solution of tire-pavement contact are also presented. These strategies take advantage of the observation that footprint loads do not usually stimulate a significant tire response away from the pavement contact region. The finite element strategies differ in their level of approximation and required amount of computer resources. The effectiveness of the strategies is demonstrated by numerical examples of frictionless and frictional contact of the space shuttle Orbiter nose-gear tire. Both an in-house research code and a commercial finite element code are used in the numerical studies.
ASSESSMENT OF COMPUTATIONAL MODELS FOR THERMOELECTROELASTIC MULTILAYERED PLATES

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ABSTRACT

A study is made of the accuracy of the steady-state (static) thermoelectroelastic response of multilayered hybrid composite plates predicted by five modeling approaches, based on two-dimensional plate theories. The plates consist of a combination of fiber-reinforced and piezothermoelastic layers. The standard of comparison is taken to be the exact three-dimensional thermoelectroelastic solutions, and the quantities compared include gross response characteristics (e.g., strain energy components, and average through-the-thickness displacements); detailed, through-the-thickness distributions of displacements and stresses; and sensitivity coefficients of the response quantities (derivatives of the response quantities with respect to material parameters of the plate).

The modeling approaches considered include first-order theory; third-order theory; discrete-layer theory (with piecewise linear variation of the in-plane displacements, temperature and electric potential, in the thickness direction); and two predictor-corrector procedures. Both procedures use first-order theory in the predictor phase, but differ in the elements of the computational model being

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