Simulated Imaging Properties of a Series of Magnetic Electron Lenses

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Abstract—The paraxial lens data were determined for a series of symmetrical magnetic lenses of equal lens diameter but variable air gap width for a wide range of lens excitations using the threedimensional electromagnetic computer code MAFIA. The results are compared with a similar study done by Liebman and Grad wherein the field distributions within the lenses were measured experimentally with a resistance network analogue. Using these fields the lens data were obtained through numerical trajectory tracing. The utility of using MAFIA, instead of experimental methods for lens design is shown by the excellent agreement of the simulated results compared to experiment. Also demonstrated is the capability of using MAFIA to investigate aberration sources such as higher order off-axis magnetic field and space-charge effects.

I. INTRODUCTION

Paraxial lens characteristics were simulated for a series of common symmetrical magnetic lenses using the computer code MAFIA (MAnxwell’s Equations solved by the Finite Integration Algorithm). The results are compared with the experimentally obtained results published in [1] with excellent agreement. Thus, the accuracy of MAFIA indicates that time-consuming and costly experimental test measurements in lens design can be greatly reduced.

MAFIA is a powerful, three-dimensional electromagnetic code written in FORTRAN 77. It is used for the computer-aided design of fully 3D and 2D electromagnetic devices, magnets, RF cavities, waveguides, antennas, etc. The Finite Integration Technique (FIT) algorithm produces a set of finite-difference matrix equations for the electric and magnetic field vectors in the structure under study. The solution of these equations yields static, frequency-domain or time-domain solutions of Maxwell’s equations [2], [3]. The code includes nine interrelated modules.

The lens data of this report were calculated using the M (mesh generator), S (static solver), TS2 (two-dimensional (2D) particle-in-cell (PIC) solver), and P (postprocessor) modules of MAFIA. The remaining modules include an eigenmode solver, 3D PIC solver, 2D and 3D time domain solvers, and a frequency-domain eddy-current solver.

To test the utility of MAFIA, a study was conducted similar to [1] wherein a series of magnetic lenses was investigated. Since [1] indicates that the main geometrical parameter of a symmetrical lens is the ratio S/D (see Fig. 1), the dependence of the paraxial imaging properties of a series of lenses was computationally investigated where the coil diameter D was held constant and the air gap width S varied. The field distributions within the lenses calculated with the magnetostatic solver of MAFIA are compared to fields obtained experimentally with a resistance network analogue. The trajectories of electrons emitted from a cathode surface, accelerated through a constant potential and focused by the simulated magnetic electron lenses were calculated with the 2D PIC module of MAFIA and compared to paths calculated in [1] by numerical trajectory tracing. Aberration sources such as off-axis fields and space-charge effects are also computationally examined, demonstrating how these very important parameters in lens design are simulated using the MAFIA code.

II. ANALYSIS

The 2D MAFIA plot (generated using the M module) for the magnetic electron lens (axially symmetric about the z’z’ axis) is shown for the r-z plane in Fig. 1. A cathode and anode are placed at opposite axial ends of a stainless steel tunnel. The static solver module S has the ability to solve electrostatic and magnetostatic problems. The electrostatic solver was used to establish a constant potential V, between the cathode and anode and calculate the resulting electric field. The magnetostatic solver of MAFIA was used to calculate the magnetic field associated with the lens. A soft magnetic material was used for the magnetic circuit and a current density was defined across the cross-sectional winding area representing the current windings with ampere-turn product NI (See Fig. 1).

The MAFIA module TS2 computes the time integration of electromagnetic fields selfconsistently with the time integration of the equations of motion of charged particles that move under the influence of those fields. Since the fields caused by those moving charges are also taken into account, effects like space charge and magnetic forces between particles are fully simulated.

The electric fields calculated through the electrostatic solver and the magnetic fields calculated through the magnetostatic solver were loaded into the TS2 simulation. A particle with the physical properties of an electron was emitted from the cathode surface. Such properties as particle position and dynamic electric field were monitored. These monitored values can be displayed or subsequent calculations can be done on the fields using the postprocessor module P.
III. RESULTS

A. Static Calculations

The dimensionless excitation parameter $\beta$ is presented in Fig. 2 of [1] as a function of the relative pole piece gap width $S/D$, where

$$\beta = \frac{k^2 V_e}{(NI)^2}, \quad (1)$$

$$k^2 = 0.022 \frac{B_e^2 R^2}{V_e} \quad (2)$$

where $V_e$ is the relativistically corrected accelerating voltage [4], $R$ is the coil radius in m, and $B_e$ is the maximum axial field strength in Tesla. Fig. 2 compares the experimental $B_e$ calculated from (2) with the field strength calculated with MAFIA as a function of $S/D$. The agreement is very good, the small discrepancy probably due to the lack of information about the $(B,H)$ curve of the iron used for the experimental lens pole pieces, pole piece shape and exact lens dimensions. Fig. 3 compares the experimental halfwidth to the values calculated from the MAFIA field distribution. The halfwidth $a$ is defined as the $z$ value for which the field strength has fallen to $B_e/2$. Both Figs. 2 and 3 show that the on-axis field distribution for the series of coils is indeed accurately simulated with MAFIA.

B. Particle-In-Cell Calculations

The particle positions were monitored through time for an electron emitted into the focusing configuration of Fig. 1 at a radius $r = 0.1R$, parallel to the $zz'$-axis. Fig. 4 shows the focal points (defined as the intersection of the trajectory with the "optical" axis), calculated by numerical trajectory tracing in [1] versus those simulated with MAFIA as a function of the excitation parameter $k^2$ of (2). The focal points of MAFIA are very close to those calculated with numerical methods. It is seen that as the accelerating potential is increased ($k^2$ is decreased), the particle's focal point moves closer to the electron source ($z/R = 2.5$). The largest discrepancy occurs for excitation $k^2 = 100$. In this study, this corresponds to an accelerating potential of about 100 V. At such a low value of voltage, the magnetic effects of the lens coil on the trajectories is more prominent than the electrical effects. The discrepancy that was seen in Figs. 2 and 3 between the experimental magnetic flux density and the MAFIA calculated values will have the largest effect for this case. Furthermore, inaccuracies in the numerical trajectory tracing will also be most evident.
Fig. 5 shows the effects of spherical aberrations occurring when particles are emitted at different radii. From the MAFIA trajectory plots, it is seen that the particles emitted at a larger value of \( r/R \) are focused closer to the source. This effect can be explained by the high-order terms of the off-axis magnetic fields present in a high-density electron source, a particle bunch consisting of 2000 particles with current density 100 A/cm\(^2\) was emitted through the same magnetic lens configuration and its path monitored. Fig. 6 shows the MAFIA plot of the particle bunch and the arrowplot of the dynamic electric field, the size of the arrows proportional to the magnitude of the field. At this instant in time it can be seen that the particles have diverged from their initial radial positions and acquired various radial and axial velocities.

Fig. 4 MAFIA focal points and those obtained through numerical trajectory tracing using experimentally obtained field distributions for magnetic lens with gap width S/D=1.0 as a function of the excitation parameter \( k^2 \).

![Fig. 4](image1)

Fig. 5 MAFIA trajectory paths of electrons emitted separately at various radii through a magnetic lens with gap width S/D=1.0 and excitation \( k^2=5.0 \) illustrating spherical aberration effects.

![Fig. 5](image2)

In a large number of electron lens focusing applications, a high density electron source is used. With such a source, collisions between electrons, together with the Coulomb forces of repulsion tend to diverge the electrons. If there is no neutralization of this electron space charge by ions, the radial spread will introduce aberrations [4]. Thus, in this case the trajectory paths will not follow basic electron-optical theory as closely. To demonstrate the ability of MAFIA to simulate the space charge forces present in a high-density electron source, a particle bunch consisting of 2000 particles with current density 100 A/cm\(^2\) was emitted through the same magnetic lens configuration and its path monitored. Fig. 6 shows the MAFIA plot of the particle bunch and the arrowplot of the dynamic electric field, the size of the arrows proportional to the magnitude of the field. At this instant in time it can be seen that the particles have diverged from their initial radial positions and acquired various radial and axial velocities.

![Fig. 6](image3)

IV. CONCLUSIONS

A series of common symmetrical magnetic electron lenses was simulated with the computer code, MAFIA. The magnetic flux density was calculated with the magnetostatic solver of MAFIA for the series keeping the lens diameter constant and varying the air gap width for a large range of excitations. The MAFIA results are compared to experimental values calculated with a resistance network analogue with good agreement. The simulated magnetic field was subsequently loaded into the 2D PIC module of MAFIA and the trajectory paths through a constant accelerating field calculated. When compared with the focal lengths obtained from numerical trajectory tracing using the experimentally obtained field values, the focal lengths obtained with MAFIA showed good agreement with the largest discrepancy occurring at a potential of about 100 V. This deviation could be due to the difference in the simulated magnetic flux density of the coil compared with experiment or errors in the numerical trajectory tracing of [1] itself due to simplifications involved in the calculations. With more information about the specific lens parameters such as dimensions and magnetic material, it is believed that the accuracy could be improved.
Aberration sources such as high-order off-axis magnetic fields and space-charge forces were addressed, demonstrating the capability of MAFIA to simulate these sources which are often overlooked in basic electron-optical theory. Since these properties are fully simulated, the motion of particles in a bunch can be more closely monitored and focusing of magnetic coils can be better understood and controlled. This can save time and money in the design and testing of magnetic electron lenses. Also, such design concepts as varied pole piece shapes and non-symmetric coils can be modelled and tested easily with computer simulation instead of costly and time-consuming experimental testing.

REFERENCES

**Title and Subtitle**
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**Subject Terms**
Magnetic lens; Paraxial data; Electron-optical theory; Simulation; Electron trajectories