Simplified Data Reduction Methods for the ECT Test for Mode III Interlaminar Fracture Toughness

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ABSTRACT

Simplified expressions for the parameter controlling the load point compliance and strain energy release rate were obtained for the Edge Crack Torsion (ECT) specimen for mode III interlaminar fracture toughness. Data reduction methods for mode III toughness based on the present analysis are proposed. The effect of the transverse shear modulus, $G_{23}$, on mode III interlaminar fracture toughness characterization was evaluated. Parameters influenced by the transverse shear modulus were identified. Analytical results indicate that a higher value of $G_{23}$ results in a lower load point compliance and lower mode III toughness estimation. The effect of $G_{23}$ on the mode III toughness using the ECT specimen is negligible when an appropriate initial delamination length is chosen. A conservative estimation of the mode III toughness can be obtained by assuming $G_{23}=G_{12}$ for any initial delamination length.

KEYWORDS: composites, compliance, strain energy release rate, mode III fracture toughness.

Nomenclature

\begin{align*}
  a & \quad \text{delamination length, m} \\
  A_{ij} & \quad \text{extensional stiffness coefficients, N/m} \\
  b & \quad \text{specimen width, m} \\
  C & \quad \text{loading point compliance, m/N} \\
  d & \quad \text{loading arm, m} \\
  D_{ij} & \quad \text{bending and twisting stiffness coefficients, Nm}
\end{align*}
$E_{ii}$ Young's moduli, GPa
$f_1, f_2$ functions of ($A_{ij}, D_{ij}, b$ and $a$), m
$G_{ij}$ shear moduli, GPa
$G_T$ total strain energy release rate, kJ/m$^2$
$G_{III}$ mode III strain energy release rate, kJ/m$^2$
$G_{IIIc}$ critical mode III strain energy release rate (mode III toughness), kJ/m$^2$
$h$ sublamine thickness (half the specimen thickness), m
$H_{11}, H_{10}, I_{11}, I_{10}$ functions of ($A_{ij}, D_{66}, b$ and $a$), m
$l$ effective specimen length, m
$n$ number of ±45 and ±45 pairs in the sublamine
$P$ transverse load, N
$P_c$ transverse load at delamination growth, N
$\bar{Q}_{ij}$ 3-D transformed reduced stiffness, N/m$^2$
$s_0$ characteristic root for undelaminated region, 1/m
$s_1$ characteristic root for delaminated region, 1/m
$x, y, z$ Cartesian coordinates

**Greek Letters (Symbols)**

$\alpha_{ij}$ parameters that relate the end loading moments to bending and twisting curvatures, Nm$^2$
$\delta$ load point displacement, m
$\delta_c$ load point displacement associated with delamination growth, m
$\theta$ twisting angle per unit length, or twist, 1/m
$\phi$ fiber angle with respect to x axis for a ply in the laminate
INTRODUCTION

Delamination in composites is generally characterized in terms of three fracture modes: mode I (opening), mode II (in-plane shear) and mode III (anti-plane shear). Analysis and test techniques for delamination in mode I and mode II have been studied extensively. O'Brien and Martin [1] summarized the mode I standardization efforts. Mode II fracture toughness characterization [2,3] also received extensive attention, and standardization development is under way. However, relatively little work has been reported for mode III delamination testing. A mode III test is needed for complete characterization of the three fracture modes to completely describe mixed mode fracture processes.

Several test methods have been proposed for measuring mode III toughness. Donaldson proposed a split cantilever beam method for mode III delamination test [4]. Becht and Gillespie used a crack rail shear test for mode III interlaminar fracture toughness characterization [5]. Li and Armanios developed a shear deformation model for the analysis of torsion loaded unidirectional and cross-ply laminates with double mid-plane free edge delaminations, and the total strain energy release rate was found to be pure mode III [6]. Shear deformation is important not only because the shear modulus \( G_{23} \) of composites is relatively small in comparison with in-plane modulus but also because the out-of-plane warping due to torsion loading is predominate in rectangular cross-sectioned laminates.

Recently, Lee proposed an Edge Crack Torsion (ECT) method for mode III toughness characterization [7]. The test is based on a \([90/(\pm45)_n/(\mp45)_n/90]_s\) laminate specimen, with one mid-plane free edge delamination, subjected to torsion. The torsional response and fracture toughness of the ECT specimen were analyzed on the basis of the Classical Lamination Theory (CLT). However, two outstanding issues associated with the ECT test must be addressed. First, the presence of off-axis plies
will induce twist-bending coupling, and consequently, the mode II contribution to
the total strain energy release rate needs to be identified. Second, the large difference
(16% and 40% for \(n=3\) and 4, respectively) between torsional stiffness predicted by
the CLT and that obtained from experiment [7] raises the need for a more rigorous
analytical treatment. To address these issues, Li and Wang developed a shear
deformation theory to analyze the ECT specimen [8]. The twist-bending coupling
effects on the torsional stiffness and the total strain energy release rate were isolated.
It was concluded that the mode II contribution to the total strain energy release for
the \([90/(\pm45)_n/\mp45)_n/90]_s\) class of laminates is negligible.

In the theory developed in reference 8, in addition to the in-plane elastic properties
of a typical unidirectional ply (laminae), the transverse shear modulus \(G_{23}\) is needed
to determine \(G_{III}\) of the ECT specimen. However, the transverse shear modulus is
much more complicated to determine than the in-plane properties, such as \(E_{11}, E_{22},\)
and \(G_{12}\). Both theoretical and experimental efforts have been devoted to the
determination of \(G_{23}\). Hashin and Rosen developed equations to calculate an upper
and a lower bound of \(G_{23}\) based on fiber and matrix properties [9]. Kurtz and Sun
tested thick laminates in torsion to determine the transverse shear modulus [10].
However, due to lack of complete fiber and matrix properties [9] and additional
efforts of special specimen preparation and testing [10], the transverse shear modulus
is not easily determined for most composite materials.

In this paper, the effect of variations in \(G_{23}\) on the shear deformation theory
prediction for the compliance and the mode III fracture toughness are investigated.
By taking advantage of the special lay-ups for mode III testing, the lengthy
expressions for compliance and strain energy release rate in reference 8 are simplified.
The simplified expressions are in terms of parameters with well-known physical
meanings. In addition, this paper also proposes data reduction methods using a single
specimen.
MODE III TOUGHNESS

A schematic of the ECT specimen is shown in Figure 1. The delamination is located at the midplane on one edge and runs through the entire length of the laminate. The specimen is supported at three points and is loaded by a transverse load ($P$) at one point as shown in Figure 1. The loading arm and the effective length of the laminate are denoted by $d$ and $l$, respectively. This loading condition introduces twisting to the specimen, and is approximated by the torsion loading shown in Figure 2. According to the St.-Venant principle, the solution of Figure 2 can be applied with satisfactory accuracy in region at some distance from the loading points on the specimen shown in Figure 1 [11]. The effect of local irregularity in stress distribution around loading points is neglected in the present analysis. This assumption appears to be justifiable because as shown in reference 8, the predicted load point compliance compared favorably to the experimental results. A three dimensional finite element analysis may be needed to determine the transition from the actual deformation and stress state near the loading points to the pure torsion state assumed in Figure 2.

The laminate in Figure 2 is under a generalized plane deformation [12], and a closed form solution for the problem in Figure 2 has been developed in reference 8 and is based on a shear deformation theory. The torsional response and total strain energy release rate are expressed as [8]

$$\frac{M_T}{\theta} = 8 \left( \alpha_{11} - \frac{\alpha_{12} \alpha_{21}}{\alpha_{22}} \right)$$

(1)

$$\frac{G_T}{4 \theta^2} = -\alpha_{11,a} + \frac{\alpha_{12,a} \alpha_{21} + \alpha_{12} \alpha_{21,a}}{\alpha_{22}} - \frac{\alpha_{12} \alpha_{21,a}}{\alpha_{22}^2}$$

(2)

where the twisting moment $M_T$ in Figure 2 is equal to $Pd$, $\alpha_{11,a} = \partial \alpha_{11}/\partial a$, etc., $\theta$ is the twisting angle per unit length, or the twist, and $G_T$ denotes the total strain energy release rate. The parameters $\alpha_{ij}$ in Equations 1 and 2 are functions of the sublamine stiffness coefficients ($A_{ij}, B_{ij}, D_{ij}$) and specimen configurational parameters ($a, b, h$).
and are given in reference 8. For a valid mode III toughness test, the mode II contribution must be eliminated, or at least minimized. The bending-twisting coupling should be small for a desirable lay-up (i.e., $\alpha_{21}$ and $\alpha_{21,a}$ should be negligible). For such lay-ups, only the first term in the right side of Equations 1 and 2 need to be considered, and equations 1 and 2 may be simplified as follows

$$\frac{M_T}{\theta} \equiv 8\alpha_{11}$$

(3)

$$\frac{G_{III}}{4\theta^2} \equiv -\alpha_{11,a}$$

(4)

In addition, the sublaminates above and below the delamination plane should be symmetric about their own midplanes to eliminate any residual thermal stress contributions to strain energy release rate for delamination growth. For the symmetric sublaminates with negligible bending-twisting coupling, the parameter $\alpha_{11}$ can be simplified as [8]

$$\alpha_{11} = \left( D_{66} + \frac{h^2}{4} A_{66} \right) b - \frac{h^2}{4} A_{66} a + D_{66} f_1 + \left( D_{66} + \frac{h^2}{4} A_{66} \right) f_2$$

(5)

where $h$ is the sublaminates thickness (see Figure 2). In Equation 5, the first two terms represent the Classical Lamination Theory (CLT) solution, and the third and fourth terms contain functions $f_1$ and $f_2$ that represent the out-of-plane shear deformation contributions. These functions are given by

$$f_1 = -\left( H_{11} + H_{10} \right) \left( 1 - e^{s_1a} \right)^2 + \frac{1}{s_1} \left( 1 - e^{s_1a} \right)$$

(6)

$$f_2 = \left( I_{11} + I_{10} \right) \left( 1 - e^{-s_0(b-a)} \right)^2 + \frac{1}{s_0} \left( e^{-s_0(b-a)} - 1 \right)$$

(7)

where

$$H_{11} = D_{66} \left\{ 1 - \left[ 1 + \frac{s_1}{s_0} \tanh \left( s_0 (b - a) \right) \right] e^{s_1a} \right\} / \Delta$$

(8)

$$H_{10} = -\left( D_{66} + \frac{h^2}{4} A_{66} \right) \left\{ 1 + \left[ 1 + \tanh \left( s_0 (b - a) \right) \right] e^{-s_0(b-a)} \right\} / \Delta$$

(9)

$$I_{11} = \frac{H_{11} \left( 1 + e^{2s_1a} \right) + \frac{1}{s_1} e^{s_1a}}{1 + e^{-2s_0(b-a)}}$$

(10)
\[ I_{10} = \frac{H_{10} \left( 1 + e^{2s_1a} \right) - \frac{1}{s_0} e^{-s_0(b-a)} \left( 1 + e^{-2s_0(b-a)} \right)}{1 + e^{-2s_0(b-a)}} \]  

\[ \Delta = s_0 \left( D_{66} + \frac{h^2}{4} A_{66} \right) \left( 1 + e^{2s_1a} \right) \tanh \left( s_0 (b - a) \right) - s_1 D_{66} \left( 1 - e^{2s_1a} \right) \]  

\[ s_0 = \sqrt{\frac{A_{55} - \alpha_{45}^2}{A_{44}}} \]  

\[ s_1 = \sqrt{\frac{A_{55} - \alpha_{45}^2}{D_{66}}} \]  

For a sublaminate of thickness \( h \), the stiffness coefficients in the above equations are obtained from

\[ (A_{ij}, D_{ij}) = \int_{-(h/2)}^{(h/2)} \overline{Q}_{ij} \left( 1, z^2 \right) dz \]  

where \( \overline{Q}_{ij} \) is the 3-D transformed reduced stiffness as defined in reference [13].

The inverse of the loading point compliance, \( 1/C \), is defined as the division of the load point displacement, \( \delta = \theta ld \), by the transverse load, \( P \), and can be written using Equations 3 and 5 as

\[ \frac{1}{C} = \frac{P}{\delta} = \frac{8}{l d^2} \left\{ \left( D_{66} + \frac{h^2}{4} A_{66} \right) b - \frac{h^2}{4} A_{66} a + D_{66} f_1 + \left( D_{66} + \frac{h^2}{4} A_{66} \right) f_2 \right\} \]  

The mode III toughness, \( G_{IIIc} \), can be obtained by solving Equation 3 for \( \theta \) and substituting into Equation 4 as

\[ G_{IIIc} = -\left( \frac{P_c^2 a^2}{16} \right) \alpha_{11,a} \frac{\alpha_{11}^2}{\alpha_{11}^2} \]  

where \( P_c \) denotes the applied transverse load corresponding to delamination growth.

**TRANSVERSE SHEAR MODULUS G23 EFFECT**

The in-plane material properties and geometry of the ECT specimen used in reference 7 are presented in Table 1. The effect of \( G_{23} \) on the compliance (C) and toughness
are reflected through the parameter $\alpha_{11}$ and its derivative with respect to crack length $a$. The stiffness coefficients which depend on $G_{23}$ are $A_{44}$, $A_{55}$ and $A_{45}$ as seen from their (CLT) definitions,

$$A_{44} = \int_{-(h/2)}^{(h/2)} [G_{23} \cos^2 \phi + G_{13} \sin^2 \phi] dz$$  \hspace{1cm} (18)

$$A_{55} = \int_{-(h/2)}^{(h/2)} [G_{23} \sin^2 \phi + G_{13} \cos^2 \phi] dz$$  \hspace{1cm} (19)

$$A_{45} = \int_{-(h/2)}^{(h/2)} (G_{13} - G_{23}) \cos \phi \sin \phi dz$$  \hspace{1cm} (20)

where $\phi$ denotes the angle between the fiber direction in a ply and the $x$ axis.

These coefficients only appear in the expressions of the characteristic roots, $s_0$ and $s_1$, as seen in Equations 13 and 14. To investigate the $G_{23}$ effects on the predictions of the inverse of the compliance and mode III toughness, the $[90/(\pm45),/(-45),/90]_n$ laminates with $n=3$ and 4 were selected. Experimental results [7] showed that for the $[90/(\pm45),/(-45),/90]_n$ laminates, with $n=1$ (12 plies) and 2 (20 plies), the load versus deflection behavior was highly nonlinear, but with $n=3$ (28 plies) and 4 (36 plies), the load versus deflection behavior was linear up to the point of delamination growth. These lay-ups have negligible Mode II contributions as demonstrated in reference 8. To assess the influence of $G_{23}$, two extreme values of $G_{23}$ was examined; $G_{23}=0$ and $G_{23}=G_{12}$. In both cases, $v_{23}=v_{12}$ is assumed to isolate the effect of $G_{23}$ alone, even though the transverse isotropic condition, $v_{23}=E_{22}/(2G_{23})-1$, is violated.

The theoretical predictions of the inverse of the load point compliance, $1/C$, for both the $G_{23}=0$ and $G_{23}=G_{12}$ cases are plotted in Figures 3 and 4 for $n=3$ and 4, respectively, along with the test data from reference 7. For both lay-ups, $n=3$ and 4, $G_{23}=0$ yields a lower value for $1/C$, while $G_{23}=G_{12}$ gives a higher value for $1/C$. The mode III toughness predictions and the results from the Compliance Calibration (CC) method used in reference 7 are given in Table 2. The delamination length and critical load for each lay-up used in Table 2 were taken from actual tested ECT specimens in reference 7. In contrast to the inverse of the load point compliance, a higher value for
the mode III toughness is predicted when $G_{23}=0$ is used than when $G_{23}=G_{12}$ is used. The lower values of the predicted mode III toughness show good agreement with the CC results [7] for both $n=3$ and 4. Also appearing in Table 2 is a more realistic situation [10] where $G_{23}$ is 60% of $G_{12}$ and $v_{23}$ is calculated from the transverse isotropic condition. The calculated toughness value for this case is slightly higher than that corresponding to $G_{23}=G_{12}$. In conclusion, the approximation of $G_{23}=G_{12}$ appears to give a good conservative prediction of the mode III toughness. Hence, accurate determination of $G_{23}$ may not be mandatory to characterize the mode III interlaminar fracture toughness using the ECT specimen.

**COMPLIANCE CALIBRATION AND DATA REDUCTION METHODS**

The strain energy release rate can also be calculated from the compliance of the delaminated specimen [14] as

$$G_{\text{III}} = \frac{p^2}{2l} \frac{\partial C}{\partial a}$$

(21)

If the strain energy release rate is established experimentally from measuring the compliance of the cracked body, the method is usually called the compliance calibration method. Two procedures are often adopted in the experimental evaluation to approximate the derivative of $C$ with respective to $a$. In the first procedure, the delamination is extended by a small increment $\Delta a$ and the change in compliance $\Delta C$ is measured. Then the ratio $\Delta C/\Delta a$ is used to approximate the derivative in Equation 21. In the second procedure, the compliance $C$ as a function of crack length $a$ is established experimentally. A polynomial curve fit is applied to the experimental data. The compliance expressed in this way can then by substituted into Equation 21.

For the present specimen shown in Figure 1, it is difficult to generate a compliance curve from a single ECT specimen due to its unstable crack growth. A large number
of ECT specimens with different crack lengths need to be tested to obtain the compliance curve. Although, the shear deformation theory discussed above gives an effective prediction of the mode III toughness, it is not suited for data reduction purpose as it requires lamina elastic properties. However, it is possible to incorporate the shear deformation theory into the compliance calibration method for the ECT specimen. This may be achieved as follows.

Substitute the derivative of $\alpha_{11}$ with respect to $a$ (using Equation 5) into Equation 4 to obtain

$$\frac{G_{III}}{4\theta^2} = \frac{h^2}{4} A_{66} - D_{66} f_{1,a} - \left(D_{66} + \frac{h^2}{4} A_{66}\right) f_{2,a}$$

Equation 22 is plotted for the entire range of $a/b$ (from 0 to 1) in Figures 5 and 6 for the $[90/(\pm 45)_n/(\mp 45)_n/90]_s$ class of laminates with $n=3$ and 4. These results are plotted for three values of $G_{23}$; $G_{23}=G_{12}$, $G_{23}=0.6G_{12}$, $G_{23}=0$ in the figures. The dotted horizontal line in these figures represents the delamination size independent term ($h^2 A_{66}/4$) of Equation 22. For a wide range of $a/b$ (0.2 < $a/b$ < 0.5), the parameter, $G_{III}/(4\theta^2)$, is nearly independent of the crack length. Furthermore for the case when $G_{23}=G_{12}$ is assumed, there appears to be no variation when 0.2 < $a/b$ < 0.5. Therefore for delamination sizes within this range, the mode III strain energy release rate may be approximated by

$$G_{III} = \left(\frac{\delta_{c} h}{l d}\right) A_{66}$$

where, $\delta_{c}$ is the measured load point displacement associated with delamination growth, and $A_{66}$ represents the average shear modulus of the sublaminates times its thickness. If $A_{66}$ can be measured conveniently, the mode III toughness may be determined from Equation 23.
CONCLUDING REMARKS

Compliance and strain energy release rate are expressed in terms of well-known physical parameters for desirable mode III test lay-ups for the Edge Crack Torsion (ECT) test. The influence of the transverse shear modulus $G_{23}$ is introduced through the characteristic roots, $s_1$ and $s_0$, to the compliance and strain energy release rate of the ECT specimen. Analytical investigation of the mode III lay-up $[90/(\pm45)n/(\mp45)n/90]_s$ with $n=3$ and 4 indicates that higher value of $G_{23}$ results in lower load point compliance and lower mode III toughness estimation. In addition, the lower values of the predicted mode III toughness show good agreement with the compliance calibration results given in the literature. By assuming $G_{23}=G_{12}$, the present analysis gives a conservative estimation of the mode III toughness, $G_{III}$, for any initial delamination length. Furthermore, the parameter, $G_{III}$, normalized by the square of the twisting angle per unit length, $(G_{III}/\theta^2)$, is nearly independent of the crack length when the crack length-to-width ratio, $(a/b)$, is between 0.2 and 0.5. An alternative to the compliance calibration technique is proposed based on the delamination size independent term in the parameter, $G_{III}/(4\theta^2)$.

Acknowledgment This work was performed while the first author was a National Research Council Research Associate at NASA Langley Research Center.

REFERENCES


Table 1. Elastic properties and configurational parameters of Carbon/Epoxy composite [7]

<table>
<thead>
<tr>
<th>Properties</th>
<th>Configurational Parameters</th>
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<tr>
<td>$E_{11} = 165$ GPa</td>
<td>$b=38.1$ mm (Width)</td>
</tr>
<tr>
<td>$E_{22} = E_{33} = 10.3$ GPa</td>
<td>$d=31.8$ mm (Moment arm)</td>
</tr>
<tr>
<td>$G_{12} = G_{13} = 5.5$ GPa</td>
<td>$l=76.2$ mm (Effective length)</td>
</tr>
<tr>
<td>$v_{12} = v_{13} = 0.28$</td>
<td>$t=0.13$ mm (Ply thickness)</td>
</tr>
<tr>
<td></td>
<td>$h=\text{Half laminate thickness}$</td>
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Table 2. Comparison of mode III fracture toughness for $[90/(\pm 45)_n/(\mp 45)_n/90]_s$ lay-ups.

<table>
<thead>
<tr>
<th>$n$</th>
<th>$a/b$</th>
<th>$P_c$ (N)</th>
<th>$G_{IIIc}$ (kJ/m$^2$)</th>
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<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Shear Deformation Theory</td>
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<td></td>
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<td></td>
<td>$G_{23}=0$</td>
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<tr>
<td>3</td>
<td>0.353</td>
<td>943</td>
<td>1.14 ±0.22</td>
</tr>
<tr>
<td>4</td>
<td>0.507</td>
<td>934</td>
<td>0.93 ±0.08</td>
</tr>
</tbody>
</table>
Figure 1. ECT specimen and loading.
Figure 2. Idealized ECT specimen under torsion loading.
Figure 3. Inverse of compliance as a function of normalized crack length $a/b$ for $[90/(\pm45)_n/(\mp45)_n/90]_s$ lay-up with $n=3$. 
Figure 4. Inverse of compliance as a function of normalized crack length $a/b$ for $[90/(\pm 45)_n/(\pm 45)_m/90]_s$ lay-up with $n=4$. 

$G_{23} = 0$ 

Experimental Data [7] 

$n = 4$
Figure 5. Strain energy release rate parameter as a function of normalized crack length $a/b$ for $[90/(\pm 45)_n/(\mp 45)_n/90]_s$ lay-up with $n=3$. 
Figure 6. Strain energy release rate parameter as a function of normalized crack length $a/b$ for $[90/(\pm 45)_n/(\mp 45)_n/90]_s$ lay-up with $n=4$. 
13. ABSTRACT (Maximum 200 words)

Simplified expressions for the parameter controlling the load point compliance and strain energy release rate were obtained for the Edge Crack Torsion (ECT) specimen for mode III interlaminar fracture toughness. Data reduction methods for mode III toughness based on the present analysis are proposed. The effect of the transverse shear modulus, \( G_{23} \), on mode III interlaminar fracture toughness characterization was evaluated. Parameters influenced by the transverse shear modulus were identified. Analytical results indicate that a higher value of \( G_{23} \) results in a lower load point compliance and lower mode III toughness estimation. The effect of \( G_{23} \) on the mode III toughness using the ECT specimen is negligible when an appropriate initial delamination length is chosen. A conservative estimation of mode III toughness can be obtained by assuming \( G_{23} = G_{12} \) for any initial delamination length.

14. SUBJECT TERMS

Composites; Compliance; Strain energy release rate; Mode III fracture toughness

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