THERMOMAGNETIC PHENOMENA IN THE MIXED STATE
OF HIGH TEMPERATURE SUPERCONDUCTORS

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1. Introduction

Galvano- and thermomagnetic phenomena in conductors are usually described with the help of kinetic coefficients which are determined with the following expressions for electric field \( E \) and the electron fraction of the heat flow \( q \) [1]:

\[
E = \rho_{xx} j + \rho_{xy} [k \times j] + \alpha \nabla T + \kappa \nabla B [k \times \nabla T], \tag{1}
\]

\[
q = \Pi j + E_B [k \times j] - \kappa \nabla T + L [k \times \nabla T]. \tag{2}
\]

Here \( \rho_{xx} \) and \( \rho_{xy} \) are components of the resistivity tensor; \( j \) is a current density; \( k = B/B \) is a unit vector parallel to magnetic field \( B \); \( \alpha, Q, \Pi, E, \kappa, \) and \( L \) are Seebeck, Nernst, Peltier, Ettingshausen, heat conductivity and Righi-Leduc coefficients, respectively. In accordance with the Onsager principle, some of these coefficients are interconnected:

\[
\Pi = \alpha T, \quad E_B = \kappa T. \tag{3}
\]

On the other hand, there is also a connection between the electric field and the heat flow in the superconductor mixed state. If one assumes that both the electric field and the heat flow arise only under the fluxoid motion then we have Josephson formula for the electric field:

\[
E = -\frac{1}{c} [v_f \times B] \tag{4}
\]

and Huebener relationship for the heat flow [2]:

\[
q = S_\phi T n_f v_f, \tag{5}
\]

where \( v_f, n_f = B/\Phi_0 \) are an average motion velocity and a fluxoids density (\( \Phi_0 = hc/2e \) is a magnetic flux quantum), and \( S_\phi \) is an entropy of a unit section of a fluxoid. It follows from Eqs. (4) and (5):

\[
q = T (c S_\phi / \Phi_0) [E x k]. \tag{6}
\]
For given values of the magnetic field and temperature the velocity of the fluxoid motion \( v_f \) depends only on the current density \( j \) and the temperature gradient \( \nabla T \). For a general case the dependence of \( v_f \) on \( j \) and \( \nabla T \) has the form:

\[
v_f = a_1 j + a_2 [j \times k] + a_3 \nabla T + a_4 [\nabla T \times k]
\]  

(a specific form of coefficients \( a_1, a_2, a_3, \) and \( a_4 \) depends on the choice of the fluxoid motion model, see below). Substituting Eq.(7) into (4) and (5), we obtain

\[
E = (a_2 B/c) j + (a_1 B/c) [k \times j] + a_4 B \nabla T - a_3 B [k \times \nabla T],
\]

\[
q = (n_f T S a_1) j - (n_f T S a_2) [k \times j] + a_3 \nabla T - a_4 [k \times \nabla T].
\]

The comparison of these relations with general expressions (1) and (2) for \( E \) and \( q \) and the use of the Onsager equation (3) lead us to

\[
\alpha_s = \left( \frac{c S_\phi}{\Phi_0} \right) \rho_{xy}^S,
Q_s B = - \left( \frac{c S_\phi}{\Phi_0} \right) \rho_{xx}^S,
\]

where index \( s \) marks kinetic coefficients relating to a superconductor in the mixed state. Eqs.(10) which are a consequence of the connection (6) between \( q \) and \( E \) in the mixed state of a superconductor forecast a certain correlation between "heat" and "current" coefficients (\( \alpha_s, Q_s \) and \( \rho_{xx}^S, \rho_{xy}^S \)) and give a possibility of checking the correctness of the conception involved.

The most detailed experimental researches of kinetic coefficients in the superconductor mixed state have been carried out for high temperature superconductors [3,4] where their value is essentially higher and the temperature range accessible for measurements is noticeably wider than those for traditional superconductors. The researches revealed the real connections between kinetic coefficients to be strongly different from those predicted by relations (10). Thus, for instance, Hall resistance \( \rho_{xy} \) in the mixed state is often a nonmonotonic and even sign-reversing function of temperature and magnetic field. Meanwhile Seebeck coefficient (thermal power) usually rises monotonically with the \( T \) or \( B \) increase and its behavior, as a rule, resembles that of the longitudinal resistance: \( \alpha \propto \rho_{xx} \).

Besides, according to Eq.(10), the Seebeck coefficient goes to a zero on the normal state transition (since in this case entropy \( S_\phi \) goes to a zero), although this coefficient obviously should take the value peculiar for the normal state. All this testifies to the fact that Eq.(10) is based on incorrect premises. It is clear that these are only Eqs. (4) and (5) (ensuing from an assumption that the electric field generation and the heat
transfer and, consequently, all the kinetic phenomena in the superconductor mixed state are related solely to the fluxoid motion) may be erroneous.

This approach is applicable in full measure only far from the superconducting transition, where fraction \( N_n = n_n / N \) of normal quasiparticles is small (\( N \) and \( n_n \) are normal quasiparticles concentrations at \( T > T_c \) and \( T < T_c \), accordingly). In conformity with the weak coupling BCS theory such particles originate only due to their excitation via energy gap \( \Delta \), and hence in low temperature superconductors (with a weak electron-phonon interaction) normal quasiparticles appear in a noticeable quantity only in a direct proximity to \( T_c \) when the width of superconducting gap \( \Delta \) becomes close to (or smaller than) thermal energy \( kT \). Thus, this approach is valid almost everywhere but for a narrow temperature range near the transition.

A different situation occurs in high-temperature superconductors with a marked tendency towards a strong electron-phonon interaction resulting in the finite density of states within the energy gap [5]. In this case normal quasiparticles arise mainly owing to filling these states and their concentration is determined by a power-law dependence (e.g., by law \( N_n \propto (T/T_c)^4 \), known for a Gorter-Casimir two-liquid model [6]) rather than by temperature exponential dependence \( N_n(T) \propto \exp(-\Delta/kT) \). The temperature range, where fraction of normal quasiparticles is large, grows much broader and there is no ground to neglect their contribution into different kinetic phenomena. In Ref.[5] numerous references are made to the works which point out that the temperature dependences of thermal properties of high temperature superconductors near \( T_c \) are much better described by the Gorter-Casimir phenomenological model than by weak coupling BCS theory. In this connection the description of kinetic properties of the mixed state of high temperature superconductors requires a simultaneous and self-consisted account for the motion of fluxoids and normal quasiparticles.

The most straightforward way here is to proceed from an assumption that corresponding contributions into kinetic coefficients are independent and additive. And it exactly this way which is usually taken (see, e.g., Refs.[7 and 8]). This approach presupposes, in particular, that the presence of normal quasiparticles effects in no way the fluxoid motion (and, consequently, the electric field in a superconductor). This key

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1 Here by the term "normal quasiparticles" we understand not the particles which are "bound" in the fluxoid cores (with the concentration of \( N \)) but those which are outside the fluxoid cores and are "free" (their concentration is \( n_n < N \)).
assumption is corroborated with no due arguments and is simply postulated although it does not look absolutely obvious. So one needs to get rid of this limitation and to construct a self-consisted theory of galvano- and thermomagnetic phenomena in the superconductor mixed state within a frame work of a single model. Since at present the fluxoid motion is described approximately even in neglect for normal quasiparticles [9] a phenomenological approach analogous to that used in the classical work by Gorter-Casimir [6] might be used.

2. Seebeck and Nernst Effects

Average density $j_n$ of the normal quasiparticle current is determined by the standard relationship:

$$j_n = \sigma_{xx}^N E + \sigma_{xy}^N [E \times k] - \beta_{N1} \nabla T - \beta_{N2} [\nabla T \times k],$$

which takes into account that conductivity ($\sigma_{xx}^N$ and $\sigma_{xy}^N$) of a system of normal quasiparticles (metal) is proportional to their concentration $N_n$ while Seebeck ($\alpha_N$) and Nernst ($Q_N$) coefficients are independent on it. Here $\beta_{N1} = \alpha_N \sigma_{xx}^N + Q_N B \sigma_{xy}^N$ and $\beta_{N2} = -\alpha_N \sigma_{xy}^N - Q_N B \sigma_{xx}^N$. The electric field in a conductor with finite conductance is determined by well-known relations, namely, the Ohm's law

$$E = \rho_{xx}^N j_n + \rho_{xy}^N [k \times j_n^N] + \alpha_N \nabla T + Q_N B [k \times \nabla T]$$

for a normal metal and Josephson's formula (4) for a superconductor in the mixed state without normal quasiparticles. Here $\rho_{xx}^N$ and $\rho_{xy}^N$ are components of resistivity tensor of a normal metal. How could one generalize Eqs.(4), (12) corresponding to two limiting situations ($N_n = 0$ and $N_n = 1$) for a common case when $0 < N_n < 1$? In the absence of an exact answer to this question one may try the interpolation formula

$$E = f_s \left\{ \frac{1}{C} [v_f \times B] \right\} + f_n \left\{ \rho_{xx}^N j_n + \rho_{xy}^N [k \times j_n^N] + \alpha_n \nabla T + Q_n B [k \times \nabla T] \right\}$$

where weight factors $f_s$ and $f_n$ have to satisfy conditions

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2 In normal metal $\alpha, Q \propto kT/\epsilon_F$ [1] where Fermi energy $\epsilon_F$ is determined as the total concentration of charge carriers and is independent on $N_n$. 

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\[ f_s(N_n = 0) = 1, \quad f_s(N_n = 0) = 0, \quad f_n(N_n = 1) = 1 \]

which ensure the transfer of Eq.(13) into "correct" expressions (4) and (12) in the above cited limiting cases. The expression for an electric field which is obtained after substitution of Eq.(11) into (13) contains no terms proportional to \( \nabla T \) (it is required for obvious equality \( E = 0 \) to take place at \( v_f = 0 \)) and has the form

\[
E = -\frac{1}{C}[v_f \times B] \left( \frac{f_s}{1 - N_n f_n} \right). \tag{14}
\]

As it can be seen from Eq.(14), in order to determine an electric field it is necessary to find the velocity of fluxoid motion \( v_f \), for which purpose let us use the equation of motion suggested in Ref.[10] (when \( N_n = 0, \nabla T = 0 \)):

\[
\eta_m [v_s \times k] - \eta_m [v_f \times k] - \eta_\perp v_s - \eta_\parallel v_f = 0, \tag{15}
\]

where \( \eta_m = n_s e \Phi_0 / c \), \( n_s \) is a concentration of superconducting carriers; \( v_s \) is their (average) current velocity; and \( \eta_\perp \) and \( \eta_\parallel \) are viscosity coefficients. The existence of normal current \( j_n \) (for \( N_n = 0 \)) should have brought about the appearance of a \( v_n \)-dependent function \( \Lambda(v_n) \) in Eq.(17). Since at \( N_n \to 1 \) all the terms occurring in Eq.(17) go to a zero, the equation of fluxoid motion in this case come to relation \( \Lambda(v_n) = 0 \) coinciding with the equation of normal quasiparticles motion, i.e., to the Ohm's law (12). We may assume that at \( N_n < 1 \) variables \( v_s \) and \( v_n \) in the motion equation are also "divided", in other words, the equation is "split" into two expressions: \( \Lambda(v_n) = 0 \) (coincides with Eq.(11) ) and Eq.(15).

Now let us discuss the way of modification of the fluxoid motion equation (15) with the presence of the temperature gradient. Firstly, "thermodynamic" force \( F_{T1} = -S_\phi \nabla T \) must manifest itself in it. Here \( S_\phi = S_\phi(T) \) is a temperature dependent entropy of a fluxoid section of a unit length. This force is an analogy [2] for Lorenz force \( \eta_m [v_s \times k] \) and can be derive from the latter by substituting \( v_s \to v^* = (S_\phi / \eta_m)(\nabla T \times k) \). Simultaneously force \( F_{T2} \), analogous to "viscosity" force \( -\eta_\perp v_s \) from Eq.(15) should be introduced; it is derived from the latter by means of the same substitution \( v_s \to v^* : F_{T2} = -S_\phi (\eta_\perp / \eta_m)(\nabla T \times k) \).

Another reason for appearance of "thermal" forces in the fluxoid motion equation is the origination of counterflow supercurrent in-between their normal cores [11]. If there is a temperature gradient inside the fluxoid core (that is in the
region where \( N_n = 1 \) there appears additional normal current with the density, according to Eq.(11), equal to \( j_T = -\beta_{N1} \nabla T - \beta_{N2}[\nabla T x k] \). This current is balanced with externally flowing countercurrent with an average density \( j_b = j_{bs} + j_{bn} = -j_T \), which is a sum of supercurrent \( j_{bs} \) and normal current \( j_{bn} \). The superconducting component of this current equals \( j_{bs} = N_s j_b = -N_s j_T \). Similarly to the transport supercurrent \( j_s = e_n v_s \) it is a source of forces effecting fluxoids [12]. Hence two more forces

\[
F_{T3} = -(\eta_m/Ne)(\beta_{N1}[\nabla T x k] - \beta_{N2}[\nabla T]),
\]

\[
F_{T4} = (\eta_{\perp}/Ne)(\beta_{N1} \nabla T + \beta_{N2}[\nabla T x k])
\]

should be introduced into fluxoid motion equation (15) related to the case when \( \nabla T = 0 \). These forces are derived from Lorenz force \( \eta_m[v_s x k] \) and viscous force \( -\eta_{\perp} v_s \) by means of substitution \( v_s \to -v^{**} \), where \( v^{**} = j_{bs}/e_n = -(1/eN) j_T^3 \).

Thus, in the presence of \( \nabla T \) the fluxoid motion equation acquires the form:

\[
\eta_m[v_s x k] - \eta_{\perp} v_s - \left[ S_{\phi}(\eta_m/Ne) + (1/eN)(\eta_{mN1} - \eta_{N2}) \right][\nabla T x k] -
\]

\[
- \left[ S_{\phi} - \left( 1/eN \right)(\eta_{\perp}N1 + \eta_{mN2}) \right] \nabla T - \eta_m[v_f x k] - \eta_{\parallel} v_f = 0 \quad (17)
\]

Using expression \( j_s = e_n v_s = j - j_n \) (where the normal current follows Eq.(11) ) and substituting relationship \( v_s = c[\nabla B]/(1 - N_n f_n)/f_s \) ensuing from Eq.(14) into Eq.(17) we obtain the equation for the electric field. The solution of that equation in respect to \( E \) gives the relationship determining the transport coefficients of superconductors in the mixed state: a Seebeck coefficient (thermal power)

\[3\]

The negative sign appeared in the \( v_s \to -v^{**} \) substitution is a result of the reverse (in respect to the normal current ) direction of supercurrent \( j_{bs} \). The nondiscrepancy of the fluxoid motion equation (17) thus obtained is confirmed by the fulfillment of the Onsager relationship for calculated kinetic coefficients (see below).
\[ \alpha = N_n \left[ \alpha_n \left( \sigma_{xx}^N + \sigma_{xy}^N \right) + Q_n B \left( \sigma_{xx}^N - \sigma_{xy}^N \right) \right] + \left( cS_\phi / \Phi_0 \right) \rho_{xy} \]  

(18)

and Nernst coefficient

\[ Q = N_n \left[ Q_n \left( \sigma_{xx}^N + \sigma_{xy}^N \right) - \left( \alpha_n / B \right) \left( \sigma_{xx}^N - \sigma_{xy}^N \right) \right] - \left( cS_\phi / B \Phi_0 \right) \rho_{xx} \]  

(19)

It is easy to see that for normal metal \((N_n = 1, S_\phi = 0)\) Eqs. (18) and (19) give \( \alpha = \alpha_n \) and \( Q = Q_n \), and in the superconducting state (no fluxoid motion, \( p < p_c \) and \( \rho_{xx} = \rho_{xy} = 0 \)), as it might be expected, \( \alpha = Q = 0 \). Thus, Eqs. (18) and (19) are true for a whole transition region of the resistive mixed state of a superconductor.


Peltier, Ettingshauseen and Righi-Leduc Effects

Heat flow \( q_s \), related to the entropy transfer with moving fluxoids, equals \( q_s = S_\phi T n_{ef} v_f \), where \( n_{ef} \) is effective density of moving fluxoids which is different from their real density \( n_f = B / \Phi_0 \). Nevertheless, rewriting Eq. (14) in the form \( E = -(1/c) [v_f \times B] (n_{ef} / n_f) \) we come to the relation

\[ q_s = T \left( cS_\phi / \Phi_0 \right) [E \times k] \]

which is an analogy to Eq. (6). As to the heat flow due to the normal quasiparticle motion, it is defined by the relationship

\[ q_n = \Pi_n j_n + E_n \kappa_n B [k \times j_n] + N_n \left[ -\kappa_n \nabla T + L_n B [k \times \nabla T] \right], \]

(20)

which accounts for \( \kappa_n \) and \( L_n \) being proportional to current carrier concentration \( N_n \) while \( \Pi_n \) and the product \( E_n \kappa_n \) are independent on it.

Thus, the electron component of the heat flow in a superconductor equals

\[ q = q_s + q_n = T \left( cS_\phi / \Phi_0 \right) [E \times k] + \Pi_n j_n + E_n \kappa_n B [k \times j_n] + N_n \left[ -\kappa_n \nabla T + L_n B [k \times \nabla T] \right]. \]

(21)

To deduce corresponding kinetic coefficients is possible by means of expressing \( q \) in terms of total transport current \( j \). First let us find the connection between normal and total currents substituting Eq. (1) into Eq. (11):

\[ j_n = N_n \left[ j (\rho_{xx}^N + \rho_{xy}^N) + [k \times j] (\rho_{xy}^N - \rho_{xx}^N) \right] - \]
where \( \beta_1 = (\sigma_{xx}^N\alpha + \sigma_{xy}^NQB), \) \( \beta_2 = (\sigma_{xy}^N\alpha - \sigma_{xx}^NQB). \) Next, substituting Eqs. (1) and (22) into (21) we come to expression (2) where Peltier coefficient

\[
\Pi = N \left[ \Pi_N (\rho_{xx}^N + \rho_{xy}^N\sigma_{xy}^N) + E_N \kappa_B (\rho_{xx}^N - \rho_{xy}^N\sigma_{xx}^N) \right] + T(cS_\phi/\Phi_0)\rho_{xy},
\]

Ettingshausen coefficient

\[
(E\kappa_B) = N \left[ E_N \kappa_N B (\rho_{xx}^N + \rho_{xy}^N) - \Pi_N (\rho_{xx}^N - \rho_{xy}^N\sigma_{xx}^N) \right] - T(cS_\phi/\Phi_0)\rho_{xx},
\]

heat conductivity coefficient

\[
\kappa = N \kappa_N + \Pi_N (\beta_1 - \beta_2) + E_N \kappa_B (\beta_2 - \beta_1) - T(cS_\phi/\Phi_0)\rho_{xy},
\]

and Righi-Leduc coefficient

\[
LB = N \Pi_N B + \Pi_N (\beta_2 - \beta_1) - E_N \kappa_B (\beta_2 - \beta_1) - T(cS_\phi/\Phi_0)\alpha.
\]

Proceeding from the Onsager equation for a normal metal \((\Pi_N = \alpha_N T, E_N \kappa_N = Q_N T)\) we make sure that Onsager equations (3) are also valid for above given Seebeck and Peltier coefficients (compare Eqs. (18) and (23)) as well as for Nernst and Ettingshausen coefficients (compare Eqs. (19) and (24)) for a superconductor in the mixed state. This is an evidence of inherent nondiscrepancy of the model suggested and, in particular, of a correct form of the fluxoid motion equation (17). The latter might be obtained without defining concretely expressions for "thermal" forces \( F_{T1} - F_{T4} \). It would be sufficient first to obtain relations (18) and (19) for \( \alpha \) and \( Q \), proceeding from Onsager relations and Eqs. (23) and (24) for \( \Pi \) and \( E \) coefficients, and then to "restore" the fluxoid motion equation (17).

The Hall conductivity (resistance) is known to be sufficiently lower than the longitudinal conductivity (resistance) at any temperature [3, 4]. It gives a possibility to simplify noticeably the expression obtained for kinetic coefficients:

\[
\alpha = \Pi/T \propto N\alpha_N (\rho_{xx}/\rho_N) + (cS_\phi/\Phi_0)\rho_{xy} \propto N\alpha_N (\rho_{xx}/\rho_N),
\]

\[
Q = (E/\kappa T) \propto N\alpha_N (\rho_{xx}/\rho_N) - (cS_\phi/\Phi_0)\rho_{xx} \propto - (cS_\phi/\Phi_0)\rho_{xx},
\]

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The first two of them were derived in earlier works in a not quite correct way [1, 2, 11, 13].

Let us emphasize that the expressions derived for coefficients $\alpha$, $\Pi$, $E$, $K$, and $L$ are universal, i.e., independent on the choice of coefficients $f_s$ and $f_n$. The latter determine only the connection between total conductivity of a superconductor in the mixed state and "partial conductivities", related to normal quasiparticles and fluxoids [14].

From the expressions obtained for kinetic coefficients $\alpha$, $\Pi$, $E$, $K$, and $L$ of a superconductor in the mixed state it is seen that each of them consists of two terms, one of which is proportional to concentration $N_n$ of normal quasiparticles and the other is to the $S_\phi$ entropy of fluxoids. These two parts may be conditionally called "normal" and "fluxoid" contributions. Their role in different kinetic coefficients is not identical. Longitudinal effects (described with coefficients $\alpha$, $\Pi$, and $K$) are defined basically with the "normal" contribution, while transverse effects (coefficients $Q$, $E$, and $L$) are governed by the "fluxoid" one. It is due to the following:

1) Seebeck and Peltier coefficients $\alpha$, and $\Pi$: a relatively minute value of the fluxoid entropy ($cS_\phi P N / \alpha N \Phi_0 \sim R \ll \rho_{xx} / \rho_{xy}$) and a absolutely small Hall angle ($\rho_{xx} / \rho_{xy} \gg 1$); the vortices are almost immovable along the temperature gradient and transfer small (in comparison with normal electrons) energy;

2) Nernst and Ettingshausen coefficient: a relatively large value of the fluxoid entropy ($cS_\phi P N / \alpha N \Phi_0 \sim R \gg 1$) and a relatively small Nernst coefficient $Q_N$ in the normal state ($Q_NB / \alpha N \sim \gamma b \ll 1$); fluxoid transfer high (as compared to normal electrons) energy in the direction perpendicular to the temperature gradient;

3) Heat conductance and Righi-Leduc coefficients $\kappa$ and $L$: all the above-said conditions and a relatively high electron heat conductance in the normal state ($\kappa N \rho N / T c \alpha N \^2 \gg 1$); the vortices are

4The conventionality of this division can be seen if only from the fact that the "normal" contribution into kinetic coefficients is related to the longitudinal resistivity $\rho_{xx}$, which, at least to a certain extent, is determined by the fluxoid motion.
almost immovable along the temperature gradient and transfer small (in comparison with normal electrons) energy.

A good agreement with experimental data for HTSC of different compositions has been achieved for a different selection of interpolation factors of the form $f_s = N^\mu_s, f_n = N^\nu_n$ [14]. The values $\mu = \nu = 1$ for $YBa_2Cu_3O_{7-\delta}$ and $\mu = \nu = 3/2$ for Bi-2223 were shown to provide the best fit. A possible reason for these different values is the different degree of anisotropy of the two compounds in question. This problem, however, is to be further analyzed.

REFERENCES

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