The flux pinning properties in high temperature superconductors are strongly influenced by thermally activated flux motion. The scaling relation of the pinning force density and the irreversibility line in various high temperature superconductors are numerically analyzed in terms of the flux creep model. The effect of two factors, i.e., the flux pinning strength and the dimensionality of the material, on these properties are investigated. It is speculated that the irreversibility line in Bi-2212 superconductors is one order of magnitude smaller than that in Y-123, even if the flux pinning strength in Bi-2212 is improved up to the level of Y-123. It is concluded that these two factors are equally important in determination of the flux pinning characteristics at high temperatures.

INTRODUCTION
The critical current density $J_c$ is one of the most important parameters of high temperature superconductors in their application to various fields. However, the critical current density in these materials is empirically known to be significantly degraded with increasing magnetic field at high temperatures. Such a poor critical current property is ascribed to weak pinning strength in these materials and large thermal activation of fluxoids at high temperatures. The boundary between the magnetically irreversible region with nonzero $J_c$ and the reversible region with zero $J_c$ in the field($B$)-temperature($T$) plane is called the irreversibility line. The poor critical current property in high temperature superconductors is directly concerned with their low irreversibility line.

In order to quantitatively understand the critical current property at high temperatures, it is necessary to take into account the effect of the thermally activated fluxoid motion correctly. For this purpose the collective flux creep theory [1] seems to be useful. One of the important features of this theory is that the pinning potential $U_0$ can be estimated in terms of the virtual critical current density $J_{c0}$ in the creep-free case. The theoretical result of the summation of elementary pinning forces is applicable to this quantity for a superconductor with known pinning centers. On the other hand, in the case of unknown pinning centers as in most high temperature superconductors, the critical current density observed at sufficiently low temperatures where the thermal activation is not significant can be approximately used as $J_{c0}$. The irreversibility lines in various superconductors can be explained satisfactorily with respect to the dependences on temperature, flux pinning strength and superconducting materials [2]. Therefore, the flux pinning characteristics in the vicinity of the irreversibility line can also be expected to be analysed correctly in terms of the flux creep theory. In this paper theoretical results on various superconductors are discussed and the scaling laws of the pinning force density are compared with experimental results.

THEORY
According to the flux creep model [3], the induced electric field due to the thermally activated
fluxoid motion is given by

\[ E = B a \nu \exp \left( - \frac{U}{k_B T} \right), \]  

(1)

where \( a \) is the hopping distance of the flux bundle, \( \nu \) is the oscillation frequency of the flux bundle in the pinning potential, \( U \) is the activation energy caused by flux pinning and \( k_B \) is the Boltzmann constant. The factor of the exponential term gives the probability for flux bundle to jump over the energy barrier due to the thermal activation. From the fact that the situation changes almost periodically with respect to the displacement of fluxoids by their mean spacing \( a_f \), it is expected that \( a \) can be approximately replaced by \( a_f \). According to the theoretical analysis using the Fokker-Planck equation [4], the oscillation frequency is given by

\[ \nu = \frac{\zeta \rho_f J_{c0}}{2 \pi a_f B}, \]  

(2)

where \( \zeta \) is a constant dependent on the pinning center and takes \( 2\pi \) for point defects [5] and \( \rho_f \) is the flow resistivity.

The activation energy \( U \) varies when the current density \( J \) varies and its variation depends on the shape of the pinning potential. If we assume a sinusoidally varying washboard pinning potential, \( U \) is written as

\[ U(j) = U_0[(1 - j^2)^{1/2} - j \cos^{-1} j], \]  

(3)

where \( j = J/J_{c0} \) is the reduced current density.

According to the flux creep theory [1] the pinning potential \( U_0 \) in bulk superconductors is expressed as

\[ U_0 = \frac{0.835 g^2 k_B J_{c0}^{1/2}}{\zeta^{3/2} B^{1/4}}, \]  

(4)

where \( g^2 \) is the number of fluxoids in the flux bundle. In the original flux creep theory [1], the number \( g^2 \) was expected to be determined by the elastic correlation length as

\[ g^2 = \frac{C_{66} a_f^2}{\zeta J_{c0} B} \equiv g_e^2, \]  

(5)

where \( C_{66} \) is the shear modulus of the fluxoid lattice. However, it has been clarified [6] that the practical \( g^2 \) values estimated from the observed irreversibility field are smaller than \( g_e^2 \) given by Eq. (5). This tendency is more remarkable for the larger \( g_e^2 \) value and for the case of the magnetic field parallel to the \( c \)-axis [7, 8]. This behavior can be understood as the result of the irreversible thermodynamics so as to minimize the energy dissipation. That is, if the flux bundle size becomes smaller, the flexibility of fluxoids to be pinned by distributed pinning centers increases, resulting in the stronger pinning, while it leads to the smaller pinning potential due to smaller volume, resulting in the larger effect of thermal activation. Thus, the optimal condition exists and \( g^2 \) is expected to be given by [7]

\[ g^2 = g_e^2 \left[ \frac{5 k_B T}{2 U_e} \ln \left( \frac{B a_f \nu}{E_c} \right) \right]^{4/3}, \]  

(6)

where \( U_e \) is the virtual pinning potential when \( g^2 \) is equal to \( g_e^2 \) and \( E_c \) is the electric field criterion for the definition of the critical current density.

In the case of magnetic field normal to the \( c \)-axis, \( g^2 \) is proportional to the coherence length along the \( c \)-axis. Hence, the flux bundle tends to have small sizes in two-dimensional superconductors. For the magnetic field parallel to the \( c \)-axis, the corresponding coherence length is that in the \( ab \) plane, and the obtained \( U_e \) is very large. However, this results in quite small \( g^2 \) value from Eq. (6) [8]. It is concluded, therefore, that the two-dimensional superconductors are more strongly influenced by the thermal activation in the both field directions.
In this paper the flux pinning property in the vicinity of the irreversibility line is investigated. In this region where the current density is small, the fluxoid motion in the direction opposite to the Lorentz force is also appreciable, since the activation energy in this direction is not largely different from that in the normal direction. Hence, Eq. (1) is replaced by

\[ E = B a_1 \nu \exp \left[ - \frac{U(j)}{k_B T} \right] \left[ 1 - \exp \left( - \frac{\pi U_0 j}{k_B T} \right) \right]. \] (7)

For the calculation of the \( E-J \) curve at desired temperature and magnetic field, it is necessary to express \( U_0 \) as a function of the temperature and the magnetic field. For this purpose, the empirical dependences are assumed for the virtual critical current density as

\[ J_{c0} = A \left[ 1 - \left( \frac{T}{T_c} \right)^2 \right]^m B^{\gamma - 1} \left( 1 - \frac{B}{B_{\text{c2}}} \right)^\delta, \] (8)

where \( A, m, \gamma \) and \( \delta \) are the pinning parameters. That is, \( A \) corresponds approximately to the critical current density at 0 K and 1 T and represents the flux pinning strength of superconductors. \( m \) and \( \gamma \) represent the temperature and magnetic field dependences, respectively. In the case of high temperature superconductors, the upper critical field \( B_{\text{c2}} \) is much larger than the irreversibility field, \( B_i \), and the parameter \( \delta \) is less important. In the numerical calculation, the flow resistivity is also necessary. In this paper, we assume the Bardeen-Stephen model [9], \( \rho_f = (B/B_{\text{c2}}) \rho_n \) where \( \rho_n(T) = (T/T_c) \rho_n(T_c) \) is the normal resistivity.

We calculate the pinning force in a melt-processed Y-123, a Bi-2223 tape wire and a melt-processed Bi-2212 in this paper. The assumed parameters are listed in Table 1. The case is treated where the magnetic field is applied normal to the \( c \)-axis. In the melt-processed Y-123, fine particles of the \( Y_2BaCuO_5 \) (211) phase of 2.0 \( \mu m \) in diameter and 20\% in volume fraction are assumed to work as the dominant pinning centers. In the Bi-2223 tape wire and the melt-processed Bi-2212, the results observed at low temperatures are approximately used for \( J_{c0} \) [6, 10]. Fine particles of

<table>
<thead>
<tr>
<th></th>
<th>Y-123 (melt)</th>
<th>Bi-2223 (tape)</th>
<th>Bi-2212 (melt)</th>
<th>Bi-2212 (crystal)</th>
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<tr>
<td>( T_c (K) )</td>
<td>92.0</td>
<td>107.7</td>
<td>92.8</td>
<td>77.8</td>
</tr>
<tr>
<td>( B_{\text{c2}</td>
<td></td>
<td>} (0) (T) )</td>
<td>644</td>
<td>1000</td>
</tr>
<tr>
<td>( \rho_n(T_c) (\mu \Omega m) )</td>
<td>2.0</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>( AB_{\text{c2}</td>
<td></td>
<td>} (0) )</td>
<td>2.58 \times 10^9</td>
<td>2.54 \times 10^9/6.57 \times 10^8</td>
</tr>
<tr>
<td>( m )</td>
<td>1.5</td>
<td>3.0/1.5</td>
<td>2.25/1.5</td>
<td>3.1</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>0.5</td>
<td>0.63/0.5</td>
<td>0.79/0.5</td>
<td>0.98</td>
</tr>
<tr>
<td>( \delta )</td>
<td>2.0</td>
<td>2.0</td>
<td>2.0</td>
<td>2.0</td>
</tr>
<tr>
<td>( \zeta )</td>
<td>4</td>
<td>2\pi/4</td>
<td>2\pi</td>
<td>2\pi</td>
</tr>
<tr>
<td>( g_s^2 )</td>
<td>44.4</td>
<td>6.7/1.55</td>
<td>2.2</td>
<td>620</td>
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<tr>
<td>( g_s^2 )</td>
<td>6.0</td>
<td>4.0/1.55</td>
<td>2.0</td>
<td>14*</td>
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<tr>
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<td>6</td>
<td>10</td>
<td>11</td>
<td></td>
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<tr>
<td>assumption</td>
<td>( U_0 = (U_{01}^2 + U_{02}^2)^{1/2} )</td>
<td>( J_{c0} = (J_{c1}^2 + J_{c2}^2)^{1/2} )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Tab. 1. Superconducting and pinning parameters used in the numerical analysis. Two kinds of pinning centers are assumed to work in Bi-2223 tape wire and melt-processed Bi-2212. *\( g_s^2 = 14 \) is assumed so as to get a good fit, although \( g_s^2 = 4.0 \) is theoretically predicted.
Bi-free phases observed from the microscopic observation are assumed to contribute to the flux pinning in these specimens. The mean diameter and the volume fraction assumed are 5.0 μm and 10%, respectively. The E-J curves are calculated from Eq. (7) and the critical current density is estimated by the offset method from the point on the E-J curve at \( E = E_c = 1.0 \times 10^{-5} \text{ V/m} \). The irreversibility field is defined by the magnetic field at which the critical current density is reduced to \( 1.0 \times 10^5 \text{ A/m}^2 \).

RESULTS AND DISCUSSION

Figure 1(a) shows the results of numerical calculation of the irreversibility lines in various superconductors and the corresponding experimental results are given in Fig. 1(b) [12]. In these figures, the results on a Bi-2212 single crystal with very weak flux pinning strength are also shown for comparison. It can be seen from these figures that the agreement is fairly good for various superconductors. This means that the flux creep theory explains correctly the phenomena concerned with the thermal activation of fluxoids.

The results depicted in Fig. 1(a) and 1(b) clarify that the irreversibility line strongly depends on the flux pinning strength and the superconducting material. As for the latter factor, the superconductor with the larger two-dimensionality is inferior in the irreversibility. In order to clarify this point more quantitatively, we shall discuss the theoretical prediction from the flux creep theory [1]. According to this theory the irreversibility field is approximately estimated as

\[
B_i = \left( \frac{K}{T_c} \right)^{4/(3-2\gamma)} \left[ 1 - \left( \frac{T}{T_c} \right)^{2m/(3-2\gamma)} \right]
\]

in the vicinity of the critical temperature, where \( K \) is a constant given by

\[
K = \frac{0.835 g^2 A^{1/2}}{\zeta^{3/2} \ln(B_4 T_c/E_c)}.
\]

In Eq. (10) the logarithmic term is almost constant and typically takes a value from 12 to 15. We shall compare the cases of melt-processed Y-123 and Bi-2212 superconductors as an example where the irreversibility field \( B_i \) is approximately different by a factor of 100. At high temperatures, such as the reduced temperature of \( T/T_c = 0.95 \) (i.e., \( 1 - (T/T_c)^2 \approx 0.1 \)), the nonsuperconducting particles are expected to be dominant pinning centers in both superconductors. In this case, the pinning force density at low fields calculated from the direct summation is

\[
J_{c0B} = \frac{\pi B_i^2 \eta D}{4 \mu_0 a \rho D},
\]

where \( \eta \) and \( D \) are the volume fraction and the mean diameter of the nonsuperconducting particles, respectively. This leads to \( m = 3/2 \) and \( \gamma = 1/2 \). Thus, we have \( B_i \propto A g^4 \). The \( A \) value in Bi-2212 is expected to be smaller by a factor of 7.8 than that in Y-123 due to the worse flux pinning efficiency of larger nonsuperconducting particles and their smaller concentration. In addition, the \( g^2 \) value in Bi-2212 is estimated to be smaller by a factor of 3 than that in Y-123 due to the smaller coherence length along the c-axis. This also leads to the smaller \( B_i \) by a factor of about 9. As a result, the \( B_i \) in Bi-2212 is expected to be smaller by a factor of about 70 than that in Y-123. This is in an approximate agreement with the experiments shown in Fig. 1(b).

It means that the two factors, i.e., the flux pinning strength and the dimensionality are equally important in determination of the irreversible property of superconductors. It seems to be fairly difficult to improve drastically the irreversibility field in Bi-2212 superconductors, although it can be improved to some extent by introduction of strong pinning centers in principle. That is, even if the flux pinning strength in Bi-2212 is improved to the level of present Y-123, its irreversibility field is expected to be still by one order of magnitude smaller than in Y-123.

Here we shall discuss the flux pinning property in the vicinity of the irreversibility line. Figure 2(a) and 2(b) are the calculated results of the normalized pinning force density in Y-123 and Bi-2212.
Fig. 1. (a) Calculated irreversibility lines and (b) observed irreversibility lines for melt-processed Y-123 (△), silver-sheathed Bi-2223 tape wire (●), melt-processed Bi-2212 (□) and TSFZ-processed Bi-2212 single crystal (■).
Fig. 2. Calculated scaling law of the pinning force density for melt-processed (a) Y-123 and (b) Bi-2212.

as a function of the magnetic field reduced by their irreversibility field, $b_i = B/B_i$. It is found that the results collapse approximately on a master curve in the both cases, as experimentally observed. The relation between the maximum pinning force density and the irreversibility field is shown in Fig. 3. From the results shown Figs. 2 and 3, the pinning property in the vicinity of the

Fig. 3. Relation between the maximum pinning force density and the irreversibility field calculated for melt-processed Y-123 and Bi-2212.
irreversibility line is expressed in the form of scaling law of

\[
F_p = A' B_i'^m(T) f(b_i),
\]

(12)

where \(A'\) is a constant and \(f\) is a function only of the reduced field \(b_i\) shown in Fig. 2. The obtained parameter \(m'\) representing the temperature dependence is 1.58 for Y-123 and 1.15 and 1.47 in the lower and higher temperature regions for Bi-2212. The reason why the two \(m'\) values are obtained for Bi-2212 is that the dominant pinning centers are different between the lower and higher temperature regions as can be seen from the two slopes in Fig. 1(a). These \(m'\) values are smaller than \(m + \gamma\) given in Table 1 representing the temperature dependence of the original pinning force density. This is caused by the fact that the temperature dependence of \(B_i\) is stronger than that of \(B_{c2}\) which is assumed as \(B_{c2} \propto 1 - (T/T_c)^2\). That is, if the temperature dependence of \(B_i\) is expressed as \([1 - (T/T_c)^2]^n\), \(n\) takes 1.5 for Y-123 and 3.2 and 1.5 in the lower and higher temperature regions for Bi-2212.

From the results shown in Fig. 2(a) and 2(b), it is found that the scaling form of the pinning force density is rather independent of the superconductor, although a slightly longer tail of the pinning force density is obtained for Bi-2212. Such a common feature of the pinning property is considered to be caused by the fact that the resultant pinning property does not remember the original pinning characteristics due to the significant effect of flux creep like a shape of sand hills attacked by waves. It is seen from Fig. 3 that the obtained \(B_i\) is smaller in Bi-2212 than in Y-123 even for the same maximum pinning force density. This also suggests that the influence of flux creep is larger in Bi-2212 with the larger two-dimensionality.

Similar theoretical result is obtained also for a Bi-2223 tape wire. Here this theoretical result is compared with experimental results. Figure 4(a) and 4(b) are the observed scaling behavior of pinning force density in the Bi-2223 tape wire in the magnetic field normal and parallel to the c-axis, respectively. The temperature dependence of the irreversibility is shown in Fig. 5. The anisotropy factor in this specimen is about 6.2 at 80 K. Figure 6 represents the relation of the maximum pinning force density and the irreversibility field in the two field directions. Using the formula given by Eq. (12), we have \(m' = 1.4\) for the field normal to the c-axis and \(m' = 1.5\) for the

Fig. 4. Observed scaling of pinning force density in Bi-2223 tape wire in the magnetic field (a) normal to the c-axis and (b) parallel to the c-axis.
field parallel to the c-axis. If the measurement is carried out to much higher temperature region, larger $m'$ values are expected to be observed. In this specimen the pinning parameters observed at sufficiently low temperatures are $m=3.96$ and $\gamma=0.73$. Thus, the $m'$ value is smaller than $m+\gamma$ as in the theoretical treatment for Y-123 and Bi-2212 in the above.

Fig. 5. Temperature dependence of irreversibility field in Bi-2223 tape wire in the two field directions.

Fig. 6. Relation between the maximum pinning force density and the irreversibility field in Bi-2223 tape wire.
Fig. 7. (a) Calculated scaling law of the pinning force density and (b) relation between the maximum pinning force density and the irreversibility line in Bi-2223 tape wire.

Numerically calculated results for the magnetic field normal to the c-axis, corresponding to the experimental results shown in Fig. 4(a) and Fig. 6, are given in Fig. 7(a) and 7(b), respectively. The reduced field at the maximum pinning force density is approximately the same and a similar scaling curve is obtained, although the decrease of the pinning force density at high field is not large as observed. The rapid decrease in the observed result may be attributed to the distributed pinning strength due to inhomogeneity. The parameter $m'$ obtained from Fig. 7(b) is 1.2 and is approximately the same with the observed $m' = 1.4$. Thus, the agreement between the theory and the experiment seems to be satisfactory.

The above argument shows that the flux creep theory explains systematically the phenomena concerned with the thermally activated motion of fluxoids. Satisfactory explanation can be given also on anomalous magnetic relaxation at low temperatures [7] and scaling of $E-J$ curves [13]. Hence, this theory is expected to be applicable for the estimation of the critical current density in a superconductor with artificially introduced pinning centers that will be fabricated in the future. This will be helpful for foreseeing the attainable characteristics of high temperature superconductors on which the possibility of application of these materials entirely depend.

**SUMMARY**

The flux pinning property of high temperature superconductors is investigated in the vicinity of the irreversibility line using the flux creep theory. The following results are obtained:

1. The irreversibility line depends on both the flux pinning strength and the dimensionality of superconductors due to their anisotropic structures. These factors are found to be equally important in determination of the irreversibility line. For example, the irreversibility field in Bi-2212 is estimated to be approximately one order of magnitude smaller than that in Y-123, even if the flux pinning strength in Bi-2212 is improved up to the present level of Y-123.

2. The scaling law of the pinning force density as observed usually is obtained for various superconductors. The scaling curve is fairly insensitive to the material. This means that the original flux pinning property dependent on the material is shaded off by the strong effect of flux creep. Agreements with experiments are obtained for the case of a Bi-2223 tape wire.
(3) From these points it can be concluded that the electromagnetic phenomena concerned with thermally activated flux motion can be correctly described by the flux creep theory. This theory is expected to be useful for foreseeing the pinning property in superconductors that will be fabricated in the future.

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