NONEQUILIBRIUM TRANSPORT IN SUPERCONDUCTING FILAMENTS


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Abstract

The step-like current-voltage characteristics of highly homogeneous single-crystalline tin and indium thin filaments has been measured. The length of the samples $l \sim 1 \mu m$ was much greater than the nonequilibrium quasiparticle relaxation length $\Lambda$. It was found that the activation of successive i-th voltage step occurs at current significantly greater than the one, derived with the assumption that the phase slip centers are weakly interacting on a scale $l \gg \Lambda$. The observation of “subharmonic” fine structure on the voltage-current characteristics of tin filaments confirms the hypothesis of the long-range phase slip centers interaction.

INTRODUCTION

One of the most fundamental properties of superconductors is the vanishing of the electrical resistivity for direct current $I$ if its value does not exceed some critical threshold $I_C$. It was found that the destruction of superconductivity by the transport current in homogeneous bulk superconductor had a sudden feature. The phase transition could be described by the model of a spreading "hot spot". The normal phase nucleates within the field penetration layer $\sim \lambda(T)$ on the surface of the sample.

The above model does not hold for sufficiently thin superconducting wires with the transverse dimensions, comparable with the coherence length $\xi(T)$. In the latter case, only one supercurrent channel exist and therefore only an S-N-S boundary along the length of the filament could be formed. Due to strong temperature dependences of $\xi(T)$ and $\lambda(T)$ the requirement of quasi-one-dimensionality for clean I-type superconductors, as tin and indium, holds within few mK below $T_C$ for the samples of several mkm in diameter.

The first candidates for testing were tin whiskers of about 1 mkm$^2$ in cross section and with a distance between potential probes less or equal to 0.5 mm. In the early experiments [1] it was found that close enough to the critical temperature $T_C$ the $V(I)$ curves show a wide transition with a series of regular voltage steps. Later, the same results were obtained for whiskers and microbridges of various clean superconductors (Sn, In, Pb, Zn, Al) and alloys [2].

Irrespectively of the material and its purity several basic features were outlined. At least for the first few
voltage steps the differential resistance \((dV/dI)_i\) is a multiple of differential resistance \((dV/dI)_1\) after the first step. The back extrapolation of the \(V(I)\) curve for each voltage step gives a non-zero excess current \(I_0(i)\), corresponding to \(V=0\). Thus the \(V(I)\) curve could be approximated by a set of \(i\) voltage steps:

\[
V(I) = (dV/dI)_i[I-I_0(i)]
\]  

(1)

It was found that the increments of the differential resistance \(\Delta(dV/dI)_i\) and the excess currents \(I_0(i)\) for each \(i\)-th voltage step are approximately constants equal to the corresponding values for the first step.

The above experimental results indicate that the formation of the steps is a repetition of the same event. The differential resistance of each \(i\)-th voltage step could be associated with the destruction of superconductivity on a normal-like length \(L(i)\). The non-zero value of the excess current \(I_0(i)\) indicates that the naive model of normal domains nucleation is not legal.

At present moment it may be stated that the step-like peculiarities of the voltage-current characteristics in quasi-one-dimensional superconductors could be qualitatively described by the essentially nonequilibrium process of the phase slip centers (PSC) activation. Still there are some questions which are not clear enough. One of them is the problem of the interaction of the neighboring PSC.

The early experiments with whiskers and microbridges were performed for relatively short samples, where the length of the wire \(L\) was comparable with the normal-like length \(L(i)\leq 100\) mkm for clean superconductor (tin and indium). However, the predictions of some widely used theoretical models are valid only for the well-separated PSC. The assumption of the PSC isolation leads to the definite positions of the voltage steps activation \(I_0(i)\).

The precise experimental test of the theories needs for long highly homogeneous filaments, where the PSC are well-separated and are not pinned to the sample imperfections.

In this article we present the results of experimental study of transport properties of thin and long tin and indium filaments in glass cover. The objects studied have some remarkable features. All the filaments are single-crystalline with mean free path comparable with the diameter of the wire. The process of filament drawing permits to produce a homogeneous wire of hundred meters in length with highly uniform parameters. The last feature gives a possibility to study samples of various types (electrical probes, length, etc.) made of a single filament with almost the same parameters: diameter, mean
free path, critical temperature, etc. The high uniformity of the filaments opens a wide range of future applications.

EXPERIMENT

In the present work thin metal wires in glass cover were studied. The filaments were prepared by drawing of molten metal in glass capillary [3]. Depending upon the metal, type of the glass, temperature, cooling and speed of the wire spinning it is possible to produce filaments with diameter of metal core from 0.6\text{\,mm} to 40\text{\,mm} and with the external diameter of glass cover from 12\text{\,mm} to 45\text{\,mm}.

X-ray analysis show that all the wires studied are single crystals.

The observation of the samples with the SEM displayed no cuts neither of the metal, nor of the glass cover (fig.1). The diameter of the filaments was measured with SEM and the distance between the voltage probes with a light microscope.

The filament with length L=1\text{\,cm} was glued to sapphire stage excluding the regions for electrodes where the glass was removed with hydrofluoric acid.

After several experiments we succeeded in producing "windows" for potential probes of \(\sim 20\text{\,mm}\) wide.

The current probes were prepared by direct placing of silver paint or Wood's metal above the ends of the metal filament. The best results for potential probes were obtained by placing 8\text{\,mm} copper wire covered with a thin layer of conducting epoxy across the sample.

The temperature of the helium bath was measured with germanium thermometer and stabilized with accuracy \(\sim 1\text{\,mK}\) using PC as a PID controller. When necessary, the samples were placed inside a massive vacuum calorimeter with internal heater. The temperature of the stage was measured with additional thermometer and stabilized with the intellectual PID controller OXFORD ITC-4. The resulting typical temperature stability was about \(\pm 0.1\text{\,mK}\).

All measurements were performed using conventional 4-probe method with accuracy 1nV. The direct measure current could be monitored with accuracy 1nA. The Earth magnetic field was reduced to \(\leq 1\text{\,mOe}\) by superconducting shield.
THEORY

Soon after the experimental observation of a step-like \( V(I) \) structure of tin whiskers [1] several theoretical models were derived to describe the observed phenomena. One line of development [4-6] introduced the phenomenological conception of the PSC activation. Another line of work, culminating in the work [7], has been to explore the time dependent Ginzburg-Landau (TDGL) equations, and nonequilibrium extensions thereof. And the third line of theoretical investigations was summarized in [8], emphasizing the static solutions of equations taking full account of nonlinear effects of large supercurrent densities.

The phenomenological models [4-6] associates the voltage steps with activation of PSC along the one-dimensional superconductor. The PSC is a region of weakened superconductivity (the S-S'-S boundary) with the region S' of the weak core \( \sim t/T \). If the voltage \( V \) across the PSC is non-zero the phase \( \psi(r,t) \) of the superconducting order parameter \( \psi(r,t) = \psi_0 e^{i\psi(T,t)} \) increases as a function of time: \( d\psi/dt = 2eV/\hbar \). This leads to an increase of \( \nabla \psi(r,t) \) and thus increase of the supercurrent \( j_s \sim |\psi|^2 \nabla \psi \). However, with increasing \( j_s \) the absolute value of the order parameter \( |\psi|^2 \) will decrease. The critical value \( j_c \) will be reached as soon as \( |\psi|^2 = 2/3|\psi_0|^2 \), corresponding to \( \nabla \psi_0^2 = 1/3 \), and the superconductor will enter the normal conducting state.

The idea of PSC is related with the assumption that the phase \( \psi(r,t) \) is periodically reduced by \( 2\pi \) to compensate the monotonic growth of the time-dependent phase. In order to explain the co-existence of superconductivity and non-zero voltage along the one-dimensional sample in a wide range of transport currents \( I = I_c \) within the phenomenological models [4-6], one should involve the model of a two-fluid superconductivity. The superconducting and the normal current components are associated with non-stationary concentrations of Cooper pairs and quasiparticles. A few moments after the phase-slip event the normal current component \( I_n \) will carry the most part of a total current \( I = I_n + I_s \): \( \nabla \psi \approx 0 \) in the PSC core and thus the supercurrent \( j_s \sim |\psi|^2 \nabla \psi \) is close to zero. While just before the phase-slip event the supercurrent will be close to its critical value \( I_c \). In order the above process could be stationary in time the periodic variation of the phase \( \psi(r,t) \) by \( 2\pi \) should compensate the monotonic growth of the phase during one period \( \tau_{\text{slip}} \).
where $\langle V \rangle$ is the time-averaged voltage across the PSC. This leads to the well known Josephson relation: 
$\omega_{\text{slip}} = \frac{2\pi}{\tau_{\text{slip}}} = 2e\langle V \rangle/h$.

Skocpol, Beasley and Tinkham (SBT) [5] postulated that the phase-slip event occurs within the range $\sim L(T)$ of the PSC core, while the non-equilibrium quasiparticles charge imbalance relax on a length scale $\Lambda = (1/31V_Fr_{\text{in}})^{1/2}$, where $l$ is the mean free path, $V_F$ - the Fermi velocity and $r_{\text{in}}$ is the inelastic relaxation time for normal particles. The differential resistance $(dV/dI)_i$ is associated with the resistance of the normal-like section of length $L(i) = 2\Lambda$.

Later Kadin, Smith and Scocpol (KSS) developed a detailed model of a charge imbalance wave equation for a PSC connected to a transmission line [6]. The KSS model includes the SBT [5] as a limit of a diffusive decay of a charge imbalance. However, KSS showed that under some conditions the relaxation of charge imbalance may result in propagating of charge-imbalance waves on the scales much greater than the $\Lambda$ of SBT model. Therefore, the KSS model predicts the long-distance interaction of the PSC.

The above simplified review of the phenomenological models [4-6] outlines the main features of the PSC conception. Theoretically more strict developments [7,8] includes the phase-slip solutions as a limit.

Introducing the TDGL equations Kramer and Baratoff [9] obtained the following results:

1) When the current $I$ is less than some value $I_1$ the superconductor enters a uniform current-carrying state.

2) When the current $I$ is larger than the threshold value $I_2$ the superconducting current-carrying state transforms into an expanding domain of normal phase. The threshold current $I_2$ coincides with the stability limit of the normal-superconducting interface studied by Likharev [10].

3) In the interval between $I_1$ and $I_2$ there exist a solution which corresponds to phase slippage. The one-dimensional sample remains superconducting over the its length, but at some point local oscillations of the order parameter $|\Psi|^2$ take place. When $|\Psi|^2$ turns to zero the phase $\phi$ experiences a jump of $2\pi$.

The numerical solutions of the TDGL equations [11, 12] give for a PSC region size of the order $\sim L(T)^{1/2}$, where $L$ is the pair-breaking term: $L = (8\sqrt{5.79r_{\text{in}}T_c})(1-T/T_c)^{1/2}$. Thus the TDGL equations predict the long-range interaction. The distance of this interaction diverges rapidly at $T_c$. 

\[
2\pi = \phi(0) - \phi(\tau_{\text{slip}}) = 2e/h \int_0^{\tau_{\text{slip}}} V(t) dt = 2e/h \langle V \rangle \tau_{\text{slip}}
\]
RESULTS AND DISCUSSION

For all the wires studied the relation of the resistance at room temperature to the one at 4.2K gives the mean free path $l$, comparable to the filament diameter. All the samples displayed the superconducting transition. The typical width of the transition $\Delta T_C$ is about $\sim 0.01K$. The critical temperature $T_C$, determined from the slope of the function $I_C^{3/2}(T_C-T)$ correlates well with the one from the $R(T)$ transition.

The voltage-current characteristics at fixed temperatures close to $T_C$ display a wide transition with pronounced step-like structure (fig.2). The curvature of the actual $V(I)$ characteristics (especially for high currents) indicates the existence of heating effects.

Fig.3b (left axis) shows the temperature dependences of the normal-like lengths for the first and the second voltage steps $L(1)$, $L(2)$ corresponding to $V(I)$ transition of fig.2. Within experimental errors no temperature dependence could be found. The absence of temperature dependence and the values for $L(i,T)$ correlate well with existing results for tin whiskers [2].

According to SBT model [5] the relation of the excess current to the critical value is a constant equal to $I_0(I)/I_C(I)=0.65$. Within experimental errors our results give the value $I_0/I_C\approx 0.8$ for the first step (fig.3b, right axis) which correlate with the TDGL model [13].

It is remarkable that the height of the first voltage jump $V(1)=V(I_C(1))$ follows a straight line (fig.2, inset), which holds for all tin and indium samples. Since, according to (1), $V(1) = (dV/dI)\left[1-I_0(I)/I_C(I)\right]I_C(1)$ this observation indicates that the temperature dependence of $(dV/dI)\left[1-I_0(I)/I_C(I)\right]$ compensates each other so that their product is independent of the temperature and, therefore, is constant for different critical currents $I_C(I,T)$.

The above experimental results, dealing with quanti-
ties derived from the first step of the V(I) transition: \( L(1), I_0(1)/I_C(1), V(I_C(1)) \) correlate well with the corresponding data obtained for the short samples [2] and with theoretical models involving the isolated PSC.

However, to calculate the critical currents \( I_C(i) \) for \( i > 1 \), where the successive voltage steps build up, one should make some additional assumptions to improve the models of non-interacting PSC.

Tinkham [14] applied the ideas of SBT [5] to describe the current-voltage characteristics of an ideal homogeneous filament. The resulting step-like V(I) dependence is associated with successive activation of \( N \) independent PSC at critical currents \( I_C(1) < I_C(2) < \ldots < I_C(N) \):

\[
\frac{I_C(i)}{I_C(1)} = \frac{\cosh(L/2i\Lambda) - I_0(1)/I_C(1)}{\cosh(L/2i\Lambda) - 1}
\]

The interaction of the PSC is reduced to the activation of the successive PSC midway between existing ones. It was shown [14] that for sufficiently long filament \( L/2N\Lambda > 1 \) the PSC are well separated and weakly interacting. The general spacing of the predicted steps is in a qualitative agreement with experimental results, obtained for the short samples [2]. But the positions of the first steps for widely studied whiskers [2] are separated by inevitable inhomogeneities, which overwhelm the exponentially weak interaction of the ideal model [14].

The observed V(i) characteristics for our samples show regular \( I_C(i) \) dependences contrary to the case of the widely studied whiskers with random PSC activation, determined by the sample imperfections.

Fig. 3 Temperature dependencies for the sample Sn-01 of: (a) the normalized critical currents \( I_C(i)/I_C(1) \); (b) the normalized excess critical currents \( I_0(1)/I_C(1) \) (right axis).

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Our results are in a disagreement with the model of the weakly interacting PSC [14]. The strong temperature and the step number i dependencies of the successive critical currents $I_c(i)/I_c(1)$ are observed (fig.3a). While according to eq. (3) neither of this dependencies should be detected.

Even utilizing the temperature dependence of the normal-like length $L(1,T)$, which is not observed in our experiments, but is reported in literature for indium whiskers [15], one can fit the $V(I)$ curves only by introducing the "effective" averaged normal-like length $L^*(T)\approx 81L(1,T)$ (fig.4, vertical bars). Experiments with highly homogeneous long filaments presented here permits one to neglect the pinning of the PSC to sample imperfections. Therefore increasing of the effective normal-like length $L^*(T)$ in comparison to the one derived from the differential resistance of the voltage steps: $L(i)=(L/R_n)(dV/dI)_i$, ($R_n$ - is the full normal resistance), indicates that for higher steps the PSC are "repulsed".

The activation of successive i-th PSC occurs at current $I_c(i)$ greater than the one derived with the assumption that the PSC are weakly interacting eq. (3) [14]. The physical mechanism of the observed enhancement of superconductivity is not clear.

One can make a surprising conclusion that the distance of the required long-range interaction is significantly greater than the normal-like length $L(i)\sim (dV/dI)_i\sim$const, corresponding to the quasiparticle relaxation length $\Lambda=L(i)/2$.

The calculations of the $I_c(i)$ dependencies, using the theoretical model of Galaiko and co-workers [16, 12], are in a qualitative disagreement with the experiment: the spacing between the voltage steps in I-scale should decrease with increasing current I, contrary to the observed $V(I)$ characteristics (fig.4, vertical bars).
The conception of the activation of similar PSC results in equal increments of the differential resistance $\Delta(dV/dI)_i = const$ and excess currents $I_0(i)^- const$ for all $i$ voltage steps of the same $V(I)$ characteristic. The above requirement has been confirmed at least qualitatively for a whole variety of whiskers and microbridges of different superconductors [2]. It also holds for the indium filaments studied at present work. But for tin samples we observed the "subharmonic" fine structure on the $V(I)$ curves (Fig.5). Along with the set $i$ of voltage steps with differential resistances $(dV/dI)_i = i*(dV/dI)_1$ and with constant value of the excess current $I_0(i)^- I_0(1) =$, there exist the subset $n_i$ of the steps with equal differential resistances $(dV/dI)_{n_i} = const$ and rational values of the excess currents $I_0(n_i)/I_0(1)$ (Fig.5).

We consider the above result as an additional manifestation of the PSC interaction. The mysterious extra long-range ($\approx 300$ mkm) influence of the PSC has been observed already on tin whiskers [17]. The physical mechanism of such an interaction is not clear.

As a preliminary hypothesis we may propose the model of the PSC interacting not only via nonequilibrium quasiparticles on a scales $=A$, but additionally through the ac irradiation of the active PSC. To our knowledge, while the sensitivity of the PSC to the external rf has been observed (the ac Josephson effect) [5], the inverse effect of the rf generation has not been detected yet. Thus the above hypothesis needs for further experimental confirmation.
SUMMARY

The step-like current-voltage characteristics of highly homogeneous single-crystalline tin and indium thin filaments has been measured. The length of the samples L~1cm was much greater than the nonequilibrium quasiparticle relaxation length Λ.

The quality of the filaments gives the possibility to neglect the pinning of the PSC to the sample imperfections. This assumption is confirmed by the reproducibility of the results for the samples cut from different parts of the same wire and by the observation of definite dependence of the critical currents Ic(i) versus the step number i.

Quantitative values, obtained from the first voltage steps of the V(I) curves (the activation of the solitary PSC) agree with the experiments reported previously for whiskers and microbridges [2] and correlate with corresponding calculations [5, 12, 15].

The observed enhancement of superconductivity for higher voltage steps and the "subharmonic" fine structure could not be understood within the existing theories of weakly interacting PSC.

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