Scanning micro-Hall probe mapping of magnetic flux distributions and current densities in YBa$_2$Cu$_3$O$_7$ thin films

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Mapping of the magnetic flux density $B_z$ (perpendicular to the film plane) for a YBa$_2$Cu$_3$O$_7$ thin-film sample was carried out using a scanning micro-Hall probe. The sheet magnetization and sheet current densities were calculated from the $B_z$ distributions. From the known sheet magnetization, the tangential $(B_{x,y})$ and normal components of the flux density $B$ were calculated in the vicinity of the film. It was found that the sheet current density was mostly determined by $2B_{x,y}/d$, where $d$ is the film thickness. The evolution of flux penetration as a function of applied field will be shown.

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I. Introduction

Stationary and scanning Hall probes have been used for evaluation of high temperature superconducting (HTS) thin films. The magnetic flux density $B_z$ normal to the film surface is usually measured. Algorithms were developed to calculate the sheet current density $J$ from the measured field above the sample (inverse problem). In our previous paper, the inverse problem was converted to a magnetostatic calculation using

$$J = \nabla \times M = (J_x, J_y)$$

$$= \left( \frac{\partial M}{\partial y}, -\frac{\partial M}{\partial x} \right) \quad (1)$$

where the sheet magnetization $M$ is normal to the film plane (parallel to $z$-axis).

The flux density $B_z$ at the point $(m,n,z)$ is given by

$$B_z = \sum_{i,j=1}^{N_1,N_2} \frac{\mu_0}{4\pi} M(i,j) \int_{S_{i,j}} \frac{3z^2-r^2}{r^5} dx' dy' \quad (2)$$

$$= \sum_{i,j=1}^{N_1,N_2} M(i,j)G(m,n,i,j,z)$$

where $r$ is the distance between the local sheet magnetization $M(i,j)$ and the field point $(m,n,z)$. $N_1$ and $N_2$ are the total number of scanning steps in the $x$ and $y$ directions, respectively. The integral in (2) is over the area of the grid cell $(i,j)$. Equation (2) can be written in matrix notation

$$B_z = G \cdot M. \quad (3)$$

where $G$ is a matrix of order of $N_1^2 \times N_2^2$ and $M$ and $B_z$ are column vectors of dimensions $N_1 \times N_2$. Equation (3) states that the sheet magnetization is uniquely determined by the measured $B_z(x,y)$. The sheet magnetization $M(i,j)$ allows one to calculate the flux density $B$ anywhere around the film including the tangential components $B_{x,y}$ of $B$, which are not obtainable directly from our scanning Hall Probe measurements. That way one can obtain a full picture of the flux penetration. This is a forward problem which does not require the time consuming solutions of inverse matrices.

The contributions to $J$ from the gradient of $B$ and from the curvature of $B$ were compared. The purpose of this paper is to illustrate graphically the evolution of flux penetration into a zero-field-cooled YBa$_2$Cu$_3$O$_7$ (YBCO) thin-film sample.
Ideal film: saturation remanent state

Fig. 1. (a) and (b) $B_0$, (c) and (d) $M$, and (e) and (f) $B_x$ distributions in the remanent state for an ideal film with uniform current distributions and for a YBCO thin film (sample Y259), respectively.
II. Experimental

The micro-Hall probe was patterned from a GaAs quantum-well heterostructure thin film (University of Bath). Its active area is $25 \times 25 \mu m$. The scanning micro-Hall probe system is a customized commercial device developed by Quantum Technology Corp. in collaboration with the participants from Simon Fraser University. Epitaxial quality YBCO thin-film samples were prepared on LaAlO$_3$ substrates by pulsed excimer laser ablation from a stoichiometric target of YBCO. We present the data for one of the YBCO films (Y259) in this paper. Sample Y259 had a critical temperature of 90 K, and lateral dimensions $= 1.08 \times 1.08 \text{ cm}$, and thickness $d = 300 \text{ nm}$. The Hall probe was maintained at a constant height of $z = 250 \mu m$ above the film surface while taking the lateral scans. The lateral scanning step was 0.3 mm. The experimental details can be found in Ref. 4.

III. Results and discussion

Fig. 1 compares the theoretical calculations for an ideal film (uniformly distributed currents flowing in concentric square paths) with the measurement results for the YBCO (Y259) thin film. Figs. 1(a) and 1(b) show the $B_z$ distributions for the ideal film and for the YBCO film, respectively. An external field $H_a$ ($\mu_0H_a = 30 \text{ mT}$) perpendicular to the film plane was applied to the YBCO film (zero-field-cooled), and then $H_a$ was switched off. The mapping of $B_z$ was carried out with the film in a remanent state. The applied flux $B_a$ penetrated fully into the film. The ideal film was chosen to have the same size as that of sample Y259. The sheet current for the ideal sample, $j (= J/d) = 2.5 \times 10^6 \text{ A/cm}^2$, was chosen to bring the overall dependence of $B_z$ close to that of the YBCO sample. The distribution of $M(x,y)$ was obtained by solving the matrix equations (2). The results are shown in Figs. 1(c) and 1(d). Fig. 1(d) demonstrates that the YBCO sample was in the saturated state. The contour lines of $M(x,y)$ represent the current stream lines, and the separation between stream lines is inversely proportional to the value of the sheet current density [Eq. (1)].

$$B_{x,y} = \sum_{i,j=1}^{N_1N_2} \frac{\mu_0}{4\pi} M(i,j) \int_{S_{i,j}} \frac{3x(y)z}{r^5} \ dx' \ dy' .$$

Figs. 1(e) and 1(f) show that $B_x(x,y)$ is distributed over two opposing triangles. The sample symmetry requires that the component $B_y$ is distributed in two other triangles. In each triangular region, there are either $B_{x,z}$ or $B_{y,z}$ components of $B$. $B_x$ and $B_y$ vanish along the film diagonals.

The distributions of $B_x$, $B_z$, and $M$ for the YBCO film are very similar to those of the ideal film. However there are noticeable deviations of the flux distribution from the four-fold symmetry pattern which are caused by defects in the YBCO sample, see Figs. 1 and 2.
Fig. 3. Calculated $B_{x,z}$ values versus $z$ for the ideal film for different grid size of the sheet magnetization $M(x,y)$.

The calculations in Fig. 3 were used to demonstrate the dependence of $B_x$ and $B_z$ on the grid size, $w$, of the sheet magnetization $M(x,y)$. $B_z$ was calculated at the film center. $B_x$ was calculated at midpoint between the center and the edge of the film. The calculated values of $B_x$ are accurate if the distance above the sample surface $z \geq w$. $B_x$ decreases for $z < w$ and reaches zero when $z \to 0$. This is an artifact of the finite grid size. In an ideal sample $B_x$ should reach a constant value by approaching the film surface (Ampere’s law). The decrease in $B_x$ for $z < w$ is due to the change of the sign of $B_x$ across the mid-plane of the film. The calculated values of $B_z$ support that view. $B_z$ is continuous across the film thickness and the calculated $B_z$ show no noticeable dependence on $w$.

The application of Ampere’s Law

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I$$

(5)

to a path in x-z plane shown in Fig. 4 leads to

$$\frac{\Delta B_x}{d} + \frac{\Delta B_z}{\ell} \cdot \frac{t}{d} = \mu_0 j_y,$$

(6)

where $\ell$ and $t$ are the integral paths along the x and z directions and $\Delta B_z$ is the difference in the $B_z$ components which are separated laterally by $\ell$. $\Delta B_x$ is the difference between the $B_x$ components above and below the film, see Fig. 4. Since $d$ is comparable to $2\lambda$, where $\lambda$ is the London penetration depth, $j$ is considered uniform over the film thickness. (Otherwise, $j$ is the average value over the film thickness.) The first term in (6) is due to the curvature of $B$, $(BV \times \hat{\mathbf{B}}$, where $\hat{\mathbf{B}}$ is the unit vector along $\mathbf{B} = B \hat{\mathbf{B}})$ and the second term is due to the gradient of $B$, $(VB \times \hat{\mathbf{B}})$. Since $B_x$ and $B_z$ are comparable and $t$ can be close to $d$, the flux gradient term is much smaller (by approximately three orders of magnitude) than the flux curvature term. In other words, $j$ is mostly determined by the curvature term,

$$\frac{\Delta B_{x,y}}{d} = \frac{2B_{x,y}}{d} = \mu_0 |j_{y,x}|.$$  

(7)

This observation is in agreement with the theoretical analysis of Clem. From Eq. (7) and from the local sheet current $j_C = 3 \times 10^6$ A/cm$^2$ corresponding to $x = -5$ mm in Fig. 2, it can be shown that the calculated value of $B_x$ (for $z = 250 \mu m)$ is 13% smaller than that obtained from Eq. (7). In fact, the profile of the $B_x(x)$ curve in Fig. 2(b) is very similar to that of $j_y(x)$ for the YBCO film. This shows clearly that the local current density in superconducting thin films determines the parallel components of $B$ in the vicinity of the film. Fig 2(b) shows that $B_x$ does not reach the saturated value as in the case of an ideal sample. The observed slope of $B_x$ in Fig. 2(b) is a consequence of the dependence of $j_C$ on the local $B_x$, see Ref. 4.
Fig. 5. Measured $B_z$ of the YBCO film for $\mu_0H_a = (a) 2$ mT, (b) 4 mT, (c) 6 mT, (d) 8 mT, and (e) 0 mT.
Fig. 6. The sheet magnetization $M$ of the YBCO film for $\mu_0 H_a =$ (a) 2 mT, (b) 4 mT, (c) 6 mT, (d) 8 mT, and (e) zero mT.
Fig. 7. $B_x$ of the YBCO film for $\mu_0H_a = (a) \ 2 \ \text{mT}, \ (b) \ 4 \ \text{mT}, \ (c) \ 6 \ \text{mT}, \ (d) \ 8 \ \text{mT}, \ \text{and (e) } 0 \ \text{mT} ; \ (f) \ B_y$ for $\mu_0H_a$ = 6 mT.
app

plied fields (e.g. $\mu_0 H_a = 2 \text{ mT}$), the induced supercurrents effectively shield the interior of the film from the external flux $B_a$. However, the tangential components $B_{x,y}$ are present over the entire film surface. The exclusion of $B_z$ requires screening currents across the whole film surface, and the presence of $B_{x,y}$ in vortex-free region is a consequence of Ampere’s law. However note that the values of $B_{x,y}$ in the vortex-free region are smaller than those in the vortex-penetrated regions [corresponding to the plateaus in $B_x(x,y)$ in Fig. (7)] where the supercurrent reached its critical value, see Fig. 8.

Fig. 9(c) shows a noticeable bending of the flux lines over the film surface. The Lorentz force density $F = j \times B$ has two components. The tangential component (due to $B_z$) points inwards and is responsible for moving the flux lines; this force density is balanced by the pinning force density. The perpendicular component (due to $B_{x,y}$) points downwards at the top and upwards at the bottom of the film. This part of the Lorentz force density causes the tilting of vortex lines.

It is interesting to note that the distortion of the applied flux is clearly visible only for $z \leq 5 \text{ mm}$, see Fig. 9(c), which is approximately equal to the half length of the film edge.

As the applied field progressively increases, the flux penetrates deeper into the film and the region in which flux lines appreciably deviate from the external flux is more and more confined to the center of the film, see Figs. 9(d)-9(f);

Fig. 9(g) shows the flux line patterns in the remanent state after the applied field $\mu_0 H_a = 8 \text{ mT}$ was switched off. The trapped flux shows four hilltops, see Fig. 5(e). It is worthwhile to point out that at the hilltops (maxima of $B_z$) both $B_{x,y}$ and $J$ are zero [Figs. 8, 9(b), and 9(g)]. The induced currents flow in closed loops under each hilltop and enclose the hilltops along the sample edges [Fig. 6(e)]. This multiply-connected current distribution can be reconstructed using a linear superposition principle with appropriate "virgin" states$^{1,2}$.

Figs. 5-9 show that for relatively low applied fields (e.g. $\mu_0 H_a = 2 \text{ mT}$), the induced supercurrents effectively shield the interior of the film from the external flux $B_a$. However, the tangential components $B_{x,y}$ are present over the entire film surface. The exclusion of $B_z$ requires screening currents across the whole film surface, and the presence of $B_{x,y}$ in vortex-free region is a consequence of Ampere's law. However note that the values of $B_{x,y}$ in the vortex-free region are smaller than those in the vortex-penetrated regions [corresponding to the plateaus in $B_x(x,y)$ in Fig. (7)] where the supercurrent reached its critical value, see Fig. 8.

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Fig. 9. Vector field plots of the flux density $B$ in x-z plane (a) and (b) for the ideal and the YBCO films in the saturation remanent state, and for the YBCO film for $\mu_0 H_a = (c) 2 \text{ mT}, (d) 4 \text{ mT}, (e) 6 \text{ mT}, (f) 8 \text{ mT}$, and (g) 0 mT.
IV. Conclusion

The mapping of $B_z$ near the superconducting film surface is shown here to include complete and detailed information of the current distributions in the film, and the flux density in the film vicinity. The tangential components $B_{x,y}$ of the flux density $B$ were calculated from the sheet magnetization $M$ which was determined from the normal component of $B$, $B_z$. $B_z$ was measured by a scanning micro-Hall probe. The minimum height at which the tangential components $B_{x,y}$ can be evaluated is determined by the scan step size. The application of Ampere's law indicates that the induced supercurrents in the YBCO thin-film sample are nearly entirely determined by $2B_{x,y}/d$, which is due to the curvature of $B$.

The Lorentz force has two components, one due to $B_z$, the other due to $B_{x,y}$. In the stationary state, the Lorentz force caused by $B_z$ is compensated by the restoring force of pinning sites and the Lorentz force caused by the tangential flux densities $B_{x,y}$ is responsible for the tilting of flux lines. The calculation of $B$ in the vicinity of the film revealed several interesting features: (a) the extent of screening by supercurrents in the vertical direction is comparable to that in the horizontal direction. With an increasing applied field, the flux penetrates deeper into the film and the region in which flux lines appreciably deviate from the external field is more and more confined to the center of the films; (b) in the remanent state, the areas with a maximum trapped flux density have no supercurrents; (c) the tangential components $B_x$ and $B_y$ are mostly present in separate and mutually perpendicular triangular regions. A significant presence of $B_x$ and $B_y$ in the same region, is a consequence of local defects; (d) the tangential flux densities $B_x$, $B_y$ are nearly zero along the film diagonals.

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