THE SOLAR DYNAMO

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The solar dynamo is the process by which the Sun's magnetic field is generated through the interaction of the field with convection and rotation. In this, it is kin to planetary dynamos and other stellar dynamos. Although the precise mechanism by which the Sun generates its field remains poorly understood in spite of decades of theoretical and observational work, recent advances suggest that solutions to this solar dynamo problem may be forthcoming.

The two basic processes involved in dynamo activity are fairly simple to understand. When the fluid stresses dominate the magnetic stresses (high plasma $\beta = 8\pi \rho/B^2$) shear flows can stretch magnetic field lines in the direction of the shear (the "$\omega$ effect") and helical flows (flows with $\nabla \times \mathbf{v} \neq 0$) can lift and twist field lines into orthogonal planes (the "$\alpha$ effect"). These two processes can be active anywhere in the solar convection zone but with different results depending upon their relative strengths and signs. How and where these processes do occur is one source of uncertainty about the solar dynamo. Other processes, such as magnetic diffusion and the fibril structure of the solar magnetic field, pose additional problems.

Several observational constraints must be explained by any prospective model of the solar dynamo. Observations of sunspots and solar activity since the mid 17th century show that solar activity associated with the Sun's magnetic field waxes and wanes with an approximate 11-year cycle. The number of sunspots and the area they cover rise rapidly from minima near zero to maxima 3 to 4 years later. The decline from maximum then progresses more slowly over the remaining years of each cycle. Most measures of solar activity show this asymmetric rise and decline but exhibit substantial variations from one cycle to the next. During the Maunder Minimum, a period of time from 1645 to 1715, the sunspot cycle seems to have ceased entirely. This nonlinear and sometimes chaotic behavior suggests that the dynamo is not a simple wave or oscillatory phenomenon.

Sunspots do not appear randomly over the surface of the Sun but are concentrated in two latitude bands. This is best illustrated by a "Butterfly Diagram" like that shown in Figure 1. This diagram marks the latitudes at which sunspots appear for each 27-day rotation of the Sun from May 1874 to June 1994. At the beginning of a cycle, sunspots appear only in the mid-latitudes near 30°. As the cycle progresses, the latitude bands widen and move toward the equator where they disappear at the next minimum. This equatorward
movement of the activity bands, known as Spörer's Law, suggests the presence of an underlying flow or wavelike propagation for the source of the activity. Sunspots tend to occur in groups that are strung out along a mostly east-west line. Spots within a group can be separated into preceding (as given by the Sun's rotation) and following spots. These groups usually have a characteristic tilt such that the preceding spots are closer to the equator than the following spots (Joy's Law).

Direct measurements of the Sun's magnetic field began in 1908 and show that sunspots are the sites of intense magnetic fields (3000 Gauss or more) which are cooler and therefore dimmer than their surroundings. Early magnetic measurements revealed Hale's Polarity Laws: (1) the preceding spots have one polarity while the following spots are of opposite polarity; (2) the polarity of the preceding spots in one hemisphere is opposite the polarity of the preceding spots in the other hemisphere; and (3) the polarities reverse from one 11-year sunspot cycle to the next to produce a 22-year cycle for magnetic activity.

Observations of weak magnetic fields provide additional details about the dynamo. After the strong fields erupt through the surface to form sunspots and active regions, the field elements spread out across the surface of the Sun. The field becomes concentrated in the network of downdrafts that outline the supergranule convection cells. As the supergranulation pattern evolves from day-to-day, the magnetic network evolves as well. The weak field observations reveal a slow poleward migration of these elements and the presence of weak (1-2 G) polar fields that reverse polarity at about the time of solar maximum. Measurements of these fields using spectral lines with different magnetic sensitivity indicate that the actual field has a fibril nature. In weak field regions the field is concentrated in small flux tubes (unresolved by modern instruments) that are surrounded by field-free regions.

Models of the solar dynamo involve fluid motions within, or adjacent to, the solar convection zone that comprises the outer 30% of the Sun. These models should be consistent with the observed motions. Doppler velocity measurements and feature tracking provide information on flows at or near the top of the convection zone while helioseismology provides information on flows in the interior.

The relevant flows include rotation, differential rotation (variations in rotation rate with latitude and radius), meridional circulations, and convection. The Sun rotates with a basic period of about 27 days but the equatorial regions rotate more rapidly (24 days) and the polar regions rotate more slowly (>30 days). Small variations on this rotation profile occur over the course of the solar cycle. The rotation tends to be slower near sunspot maximum, slower in the hemisphere with more spots, and slower in cycles with more spots. Rapidly and slowly rotating streams (torsional oscillations) are observed in conjunction with the sunspots. These streams move toward the equator like the sunspots but appear to start earlier and at higher latitudes. The meridional flows at the surface are weak and thus
difficult to measure but most observations indicate the presence of a flow of \(\sim 10-20\) m/s from the equator toward the poles. The convection observed at the surface indicates the presence of three convective flow patterns: granules with diameters of \(\sim 1000\) km, mesogranules with diameters of \(\sim 6000\) km, and supergranules with diameters of \(\sim 30,000\) km.

Helioseismology probes the interior of the Sun by measuring the characteristics of sound waves produced by the turbulent convective flows. These waves, or \(p\)-modes, are trapped inside the Sun by the rapid change in density at the surface and the increasing sound speed deeper inside the Sun. The internal rotation can be measured by comparing the frequencies of waves moving prograde and retrograde for \(p\)-modes that sample different latitudes and depths. These observations of the internal rotation show that the observed surface rate extends inward through the convection zone along radial lines for each latitude. At the base of the convection zone the latitudinal differential rotation disappears and the rotation becomes more uniform. An important aspect of this rotation profile is that radial gradients in the rotation rate occur primarily at the bottom of the convection zone with only very weak radial gradients throughout the bulk of the zone itself.

One final observation concerning the solar dynamo is the nature of activity cycles in other stars. The level of emission in the Ca II spectral absorption line at \(\lambda 393.4\) nm is another measure of solar activity. Emission features at certain wavelengths within this line are associated with the solar chromosphere. Variations in the emission levels are well correlated with sunspot and magnetic activity. These same spectral features are also observed in other solar-type stars and so provide a measure of the level of chromospheric activity in those stars. Observations over several years show that other stars have activity cycles much like the Sun's. For a given stellar type the level of activity increases with rotation rate. Cyclic behavior is found primarily in slow rotators like the Sun and amongst these a quarter to a third appeared to be inactive during the years of observation.

With these observations in mind, how are dynamos constructed that produce similar behavior? Early dynamo work showed what wouldn't work. Cowling produced an antidynamo theorem that showed an axisymmetric magnetic field could not be produced by any axisymmetric flows. It was later shown that axisymmetric flows such as differential rotation and meridional circulation can at best only lengthen the natural decay time of the magnetic field. Non-axisymmetric flows provide the key for unlocking a variety of possible dynamos. In one branch of dynamo theory (Mean-Field Electrodynamics) these non-axisymmetric flows are represented by an average of their dynamical properties. In another branch (Large Eddy Simulations) the largest of these non-axisymmetric flows are directly simulated. Each approach has its own advantages but at the present neither one produces a model in agreement with all the observations.
The basic equation of dynamo theory is the magnetic induction equation constructed from Maxwell's equations and Ohm's law:

\[ \frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B}, \tag{1} \]

where \( \mathbf{B} \) is the magnetic induction, \( \mathbf{v} \) is the fluid velocity, and \( \eta \) is the magnetic diffusivity. In Mean-Field Electrodynamics both the velocity and the magnetic induction are separated into mean and fluctuating parts. An average of the induction equation gives the mean-field equation that contains a new induction term given by the average of the cross product of the fluctuating velocity and magnetic induction. To first order this term is proportional to the magnetic induction and its curl so that

\[ \bar{\mathbf{v}} \times \bar{\mathbf{B}}' \approx \alpha \bar{\mathbf{B}} - \beta \nabla \times \bar{\mathbf{B}} \tag{2} \]

where the primes denote fluctuating quantities, the overbar denotes an average, the constant \( \alpha \) is proportional to the helicity in the fluctuation velocity field, and the constant \( \beta \) is proportional to the eddy diffusivity. Using spherical polar coordinates \((r, \theta, \phi)\), equation (1) can then be written in terms of the mean toroidal component of the magnetic induction, \( \mathbf{B}_\phi \), and the poloidal component, \( \mathbf{B}_p = (\mathbf{B}_r, \mathbf{B}_\theta) = \nabla \times \mathbf{A}_\phi \), where \( \mathbf{A} \) is the vector potential. This gives a pair of coupled equations with

\[ \frac{\partial \mathbf{B}_\phi}{\partial t} + (\mathbf{U}_p \cdot \nabla) \mathbf{B}_\phi = (\mathbf{B}_p \cdot \nabla) \mathbf{U}_p + \alpha \nabla \times \mathbf{B}_p + \beta \nabla^2 \mathbf{B}_\phi \tag{3} \]

and

\[ \frac{\partial \mathbf{A}_\phi}{\partial t} + (\mathbf{U}_p \cdot \nabla) \mathbf{A}_\phi = \alpha \mathbf{B}_\phi + \beta \nabla^2 \mathbf{A}_\phi \tag{4} \]

where \( \mathbf{U} \) is the mean fluid velocity consisting of meridional flow, \( \mathbf{U}_p \), and differential rotation \( \mathbf{U}_\phi \). Neglecting for the moment the meridional flow, equation (4) shows us that the poloidal field is produced by the \( \alpha \)-effect in which the toroidal field is lifted and twisted by the nonaxisymmetric helical motions. Equation (3) shows us that the toroidal field is produced by both the \( \alpha \)-effect and by the \( \omega \)-effect in which the poloidal field is stretched out by the differential rotation. The relative strength of these different terms determines the nature of the resulting dynamo.
If the differential rotation is much weaker than the \( \alpha \)-effect then the \( \omega \)-effect term is dropped from equation (3) and an \( \alpha^2 \)-dynamo can be obtained. These dynamos tend to produce steadily growing fields. If the differential rotation is much stronger than the \( \alpha \)-effect then the \( \alpha \)-effect term is dropped from equation (3) and an \( \alpha \omega \)-dynamo can be obtained. These dynamos tend to produce oscillatory waves that propagate at right angles to the shear flow. The direction of propagation, toward the poles or toward the equator, depends upon the sign of \( \alpha \) and the direction of the velocity shear. If the \( \alpha \)-effect and the \( \omega \)-effect are of similar strength an \( \alpha^2 \omega \)-dynamo can be obtained. These dynamos also tend to produce oscillatory behavior but with periods that differ from those for \( \alpha \omega \)-dynamos depending upon the relative strength of the \( \alpha \)-effect.

Kinematic dynamos for the Sun have been constructed from these equations by taking a specified rotation profile, \( U_\phi(\theta,r) \), and a functional form for \( \alpha \). Dynamos produced in the 1970's by investigators such as Yoshimura and Stix reproduced many of the characteristics of the solar cycle. These were \( \alpha \omega \)-dynamos in which the Sun's differential rotation takes a poloidal magnetic field and shears it to produce a stronger toroidal field below the surface. This toroidal field is then lifted and twisted by the \( \alpha \)-effect to produce a poloidal field of reversed polarity. These steps are illustrated in Figure 2. The key ingredients in these dynamos were a rotation profile in which the rotation rate increases inward and left-handed helicity in the northern hemisphere. These conditions produce dynamo waves that propagate toward the equator in agreement with Spörer's Law. The problem with these dynamos is the constraints they place on the fluid flows. In order to produce a dynamo with a 22-year period the \( \alpha \) effect produced by the convection must be diminished enormously. Otherwise very short cycles result. In addition, the rotation profiles they use do not agree with the helioseismic profiles.

Non-axisymmetric flows with helicity are a natural consequence of the effects of rotation on convection. As fluid elements rise and expand the Coriolis force produces a clockwise rotation in the northern hemisphere giving left-handed helicity. Likewise, as fluid elements sink and contract a counter-clockwise rotation is produced which also gives left-handed helicity. Right-handed helicity would be produced in the southern hemisphere. This source of the \( \alpha \)-effect is illustrated in Figure 3.

Self-consistent magnetohydrodynamic dynamos were produced in the 1980's by Gilman and Glatzmaier. These Large Eddy Simulation models start with the equations of motion and the induction equation and calculate numerically both the velocity field and the magnetic field. With these models the convection itself explicitly produces both the differential rotation for the \( \omega \)-effect and the helicity for the \( \alpha \)-effect. While the calculated fields are self-consistent they are not consistent with the observations. The rotation profile produced in these models has rotation constant on cylinders. While the \( \alpha \) effect has the expected sign, the rotation rate decreases radially inward, contrary to the helioseismic observations, and the dynamo waves propagate toward the poles, contrary to Spörer's
Law. These dynamos also had short cycle periods due to the large magnitude of the \( \alpha \)-effect.

A major problem shared by both types of dynamos is the nature of the internal rotation profile as determined by helioseismology. Although the magnetohydrodynamical models produce surface rotation profiles in agreement with observations, the internal profiles disagree. Likewise, the internal profiles assumed to be present in the kinematic models disagree with the observations. This problem extends beyond dynamo theory itself. Dynamical models for the convection zone produce rotation profiles with surfaces of constant rotation rate lying on cylinders aligned with the rotation axis. The largest convection eddies (as yet unobserved) become elongated north to south to form banana shaped cells. Horizontal flows within these cells are turned by the Coriolis force so that eastward momentum is transported toward the equator to maintain the latitudinal differential rotation observed at the surface. While this process is well understood and produces the observed surface profile, the internal rotation profile is all wrong - both for the dynamo and for agreement with the observed internal profile. This remains an outstanding problem in convection zone dynamics.

Another problem shared by both types of dynamos was noted by Parker: magnetic flux tubes should be buoyant and not remain in the convection zone long enough for the fluid motions to work on them. The magnetic pressure within a flux tube requires a smaller contribution from the gas pressure inside to balance the gas pressure outside. If the tube is in thermal equilibrium with its surroundings this gives a lower gas density and makes the tube buoyant.

These two problems, flux tube buoyancy and the internal rotation profile, have lead to the suggestion that the dynamo acts in the interface layer at the base of the convection zone. Flux tubes are less buoyant there due to the stable stratification. Helioseismology results show that strong radial shear in the rotation profile occurs in this layer. It is also expected that the more vigorous convective motions will overshoot and penetrate into this layer. Although, for the equatorial region, the rotation rate decreases inward the \( \alpha \)-effect should still have the correct sign. In this interface layer sinking fluid should expand as it spreads out along the bottom while rising fluid should contract as the fluid converges in updrafts. This gives right-handed helicity in the northern hemisphere and thus produces dynamo waves that propagate in accordance with Spörer's Law. In the higher latitudes where the rotation rate increases inward these waves should move in the opposite direction. Details concerning dynamos in this interface layer have been examined by several investigators including Parker, Gilman, DeLuca, and Choudhuri. These models solve some problems associated with the convection zone but produce other problems of their own. In particular, it may be difficult to produce sufficient magnetic flux within this thin layer.
One final problem with current models of the solar dynamo concerns magnetic diffusion. For any of these dynamos to work diffusion is needed so that magnetic fields can reconnect to form new topologies. Ultimately this reconnection must take place in small scale diffusive processes. The problem is that vigorous small scale turbulence should amplify the magnetic field to levels that would prohibit the flows from moving the field any further. This limits the amplitude of the mean fields to values less than those observed. This remains one of the fundamental problems with dynamo theory and is actively being investigated.

Many of the current efforts in solar dynamo theory are associated with the dynamics of magnetic flux tubes themselves. Choudhuri, Gilman, D'Silva, Fan and others have examined how buoyant flux tubes move through the convection zone. Weak fields tend to rise parallel to the rotation axis and emerge at high latitudes. Fields with strengths of \(~100\text{kG}\) at the base of the convection zone are required to produce sunspots at the observed latitudes. Other investigators are studying the interactions between fluid flows and fibril magnetic field structures. The difficulty of including thin tubes with strong magnetic fields in global models is a severe computational problem for solar dynamo theory.

Observationally we still need to know more about the dynamics of the solar convection zone. Are we missing details about the internal rotation profile? What meridional flows exist within the convection zone? What is the structure of the convective flows and how do these flows interact with the axisymmetric flows and the magnetic field? Helioseismology is our best hope for obtaining answers to these questions. The Global Oscillations Network Group (GONG) will start fielding its instruments in late 1994 and early 1995. In July of 1995 ESA and NASA will launch the Solar and Heliospheric Observatory (SOHO) with an array of helioseismology instruments. These new instruments promise to tell us much more about the solar interior, convection zone dynamics, and the solar dynamo.
BIBLIOGRAPHY


FIGURE CAPTIONS

Figure 1. Sunspot areas and positions from 1874 to 1994. In the upper panel the latitudinal positions of sunspots are marked for each rotation of the Sun. This illustrates the equatorward movement of the active latitude band over each solar cycle. In the lower panel the average daily sunspot area, expressed as a percentage of the area of the visible hemisphere, is plotted for each rotation of the Sun. This illustrates the 11-year sunspot cycles and shows the cycle-to-cycle and rotation-to-rotation variations in total sunspot area.

Figure 2. The two basic dynamo processes: the $\omega$-effect and the $\alpha$-effect. With an $\omega\alpha$-dynamo the $\omega$-effect shown in Fig. 2a is produced by differential rotation shearing a poloidal field line and wrapping it around the solar interior to produce a strong toroidal field. The $\alpha$-effect shown in Fig. 2b is produced by helical motions that lift and twist the toroidal field to produce a new poloidal field of opposite polarity.

Figure 3. Helicity production by convection in rotating layers. The Coriolis force acting on convective flows produces left-handed helicity in the convection zone in the north and right-handed helicity in the south. Converging flows in downdrafts spin counter-clockwise in the north and clockwise in the south while diverging flows in updrafts spin in the opposite directions. The opposite sense of helicity is produce in the interface layer where the flows in downdrafts diverge and flows in updrafts converge.
DAILY SUNSPOT AREA AVERAGED OVER INDIVIDUAL SOLAR ROTATIONS

SUNSPOT POSITIONS IN EQUAL AREA LATITUDE STRIPS

DATE

AVERAGE DAILY SUNSPOT AREA (% OF VISIBLE HEMISPHERE)

DATE

NASA/MSFC/HATRAWAY 6/94
a) The $\omega$-effect

b) The $\alpha$-effect
a) Northern Hemisphere

b) Southern Hemisphere