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Optimal Lunar Trajectories for a Combined Chemical-Electric Propulsion Spacecraft *

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I. Introduction

Spacecraft which utilize electric propulsion (EP) systems are capable of delivering a greater payload fraction compared to spacecraft using conventional chemical propulsion systems. Several researchers have investigated numerous applications of low-thrust EP including a manned Mars mission [1], scientific missions to the outer planets [2], and lunar missions [3]-[5]. In contrast, the study of optimal combined high and low-thrust spacecraft trajectories has been limited.

In response to the release of NASA's 1994 Announcement of Opportunity (AO) for Discovery class interplanetary exploration missions, a preliminary investigation of a lunar-comet rendezvous mission using a solar electric propulsion (SEP) spacecraft was performed. The Discovery mission (eventually named Diana) was envisioned to be a two-phase scientific exploration mission: the first phase involved exploration of the moon and second phase involved rendezvous with a comet. The initial phase began with a chemical propulsion translunar injection and chemical insertion into a lunar orbit, followed by a low-thrust SEP transfer to a circular, polar, low-lunar orbit (LLO). After scientific data was collected at the moon, the SEP spacecraft performed a spiral lunar escape maneuver to begin the interplanetary leg of the mission. After escape from the Earth-moon system, the SEP

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spacecraft maneuvered in interplanetary space and performed a rendezvous with a short-period comet.

An initial study that demonstrated the feasibility of using EP for the lunar and comet orbit transfer was performed under the grant NAG3-1581 [6]. This final report is a continuation of the initial research efforts in support of the Discovery mission proposal that was submitted to NASA Headquarters in October 1994. Section II discusses the lunar orbit transfer phase of the Diana mission which involves both chemical and electric propulsion stages. Section III discusses the chemical lunar orbit insertion (LOI) burn optimization. Finally, section IV presents the conclusions of this research effort.

II. Combined Chemical-Electric Propulsion Lunar Transfer

The initial phase of the Diana mission involves a ballistic lunar orbit transfer, followed by a LOI chemical propulsion burn, and finally an EP orbit transfer to a polar, low-lunar 100-km altitude orbit. The optimal lunar capture and circularization transfer using the solar electric propulsion (SEP) stage was outlined in the grant report NAG3-1581 [6]. In this section, the trajectory optimization study for the combined chemical-electric propulsion maneuver is presented.

Trajectory Optimization

The objective is to compute the minimum-fuel ballistic translunar trajectory from low-Earth orbit (LEO) to the optimal lunar orbit insertion (LOI) boundary conditions. The trajectory is shaped by two impulsive chemical burns at both ends. The complete optimal control problem is given below:

For the free end-time problem, find the orientation and magnitude of the chemical burn-out velocity vector $\vec{v}_{bo}$, the angular position of the spacecraft in LEO at translunar injection (TLI), and the magnitude of the chemical LOI ΔV which minimize

$$J = -m(t_f) = -m_{LOI} \tag{1}$$

subject to the unpowered three-body equations of motion

$$\dot{x} = u \tag{2}$$

$$\dot{u} = 2\omega v + \omega^2 x - \frac{\mu_e}{r_e^3}(x - x_e) - \frac{\mu_m}{r_m^3}(x + x_m) \tag{3}$$

$$\dot{y} = v \tag{4}$$

$$\dot{v} = -2\omega u + \omega^2 y - \frac{\mu_e}{r_e^3} y - \frac{\mu_m}{r_m^3} y \tag{5}$$

$$\dot{z} = w \tag{6}$$

$$\dot{w} = -\frac{\mu_e}{r_e^3} z - \frac{\mu_m}{r_m^3} z \tag{7}$$

with the initial conditions

$$X(0) = g(\theta_0, \psi_0, \vec{v}_{bo}) \tag{8}$$

and the terminal state constraints

$$\Psi[X(t_f), t_f] = \begin{pmatrix} h_p(t_f) - h_{p_f} \\ h_a(t_f) - h_{a_f} \\ i(t_f) - 90 \ \ \text{deg} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \tag{9}$$

The motion of the coasting spacecraft is governed by the restricted three-body problem dynamics as indicated by Eqs. (2-7). The differential equations are formulated in a rotating Cartesian frame with the origin at the Earth-moon system center of mass and the positive x-axis pointing to the Earth along the Earth-moon line. The x-axis is fixed with the Earth-moon line, the y-axis is in the Earth-moon orbit plane, and the positive z-axis is along the angular momentum vector of the Earth-moon system. The Earth and moon are assumed to revolve in circular orbits about their common center of mass. The constant Earth and moon distances from the center of mass are denoted by x_e and x_m, respectively. The constant angular rate of the Earth-moon system is ω and the gravitational parameters of

the Earth and moon are denoted by μ_e and μ_m, respectively. The position of the spacecraft in the rotating frame is denoted by (x, y, z) and the respective velocity components in the rotating frame are (u, v, w). The distances from the Earth and moon to the spacecraft are denoted by r_e and r_m, respectively.

The initial conditions of the spacecraft at TLI are given by Eq. (8) and are a function of the position in LEO (θ_0 and ψ_0) and the burn-out velocity $\vec{v}_{bo}$ as supplied by the upper stage of the Delta II launch vehicle. The initial velocity $\vec{v}_0$ of the spacecraft with respect to an Earth-centered inertial frame is calculated by the below vector equation

$$\vec{v}_0 = \vec{v}_{LEO} + \vec{v}_{bo} \tag{10}$$

where $\vec{v}_{LEO}$ is the velocity of the spacecraft in a circular, 185-km altitude low-Earth orbit. By defining the longitude θ_0 and heading ψ_0 at TLI, the initial velocity $\vec{v}_{LEO}$ can be computed. The initial LEO is assumed to be inclined 7 deg with respect to the Earth-moon orbit plane which corresponds to launch conditions in late 2000. By specifying the launch energy C_3 and the pitch and yaw orientation of the excess burn-out velocity, the additional velocity $\vec{v}_{bo}$ can be computed. Finally, the position and velocity components in LEO must be transformed to the rotating frame centered at the Earth-moon system center of mass.

The terminal state constraints as denoted by Eq. (9) require that the final lunar orbit after LOI be an elliptical polar orbit. Therefore, the first two terminal state constraints require perilune and apolune altitudes (h_p and h_a) match the desired values (h_{p_f} and h_{a_f}) and the third constraint requires inclination i be 90 deg at the final time t_f.

Solution Approach

Since our problem involves discrete control parameters, the optimal control problem is solved using a direct method. The optimal control problem is replaced with a nonlinear

programming problem (NLP) and the trajectory design variables are C_3, orientation of the spacecraft in LEO, orientation of $\vec{v}_{bo}$, coast time t_f, and magnitude of the LOI chemical ΔV. The NLP is numerically solved using sequential quadratic programming (SQP) which is a constrained parameter optimization method [7]. The SQP algorithm used here utilizes first-order finite differences to approximate the gradients and is due to Pouliot [8]. The SQP problem formulation involves seven optimization parameters and three equality constraints. The equations of motion are numerically integrated by using a standard fourth-order, fixed-step, Runge-Kutta integration scheme with 2000 steps.

The performance index to be minimized is the negative mass after LOI. This is equivalent to maximizing m_{LOI}. Spacecraft mass after LOI is computed by using the rocket equation:

$$m_{LOI} = m_{TLI} e^{-\Delta V/c} \tag{11}$$

where the ΔV is the velocity increment from the LOI burn and $c = I_{sp} g$ is the exhaust velocity of the chemical stage. Specific impulse I_{sp} of the chemical stage is 310 s. The injected mass m_{TLI} is computed by using a simple linear fit of the launch performance of the Delta II:

$$m_{TLI} = -27 C_3 + 1227 \quad \text{kg} \tag{12}$$

Results

Several optimal minimum-fuel lunar trajectories were readily obtained for a range of perilune and apolune altitudes by using the SQP optimization code. The optimal C_3 was found to be -2 km^2/s^2 for nearly every case which results in an injected mass of 1281 kg. Initially, the optimal *circular* LOI burn was obtained for a range of circular altitudes. That is, both h_{p_f} and h_{a_f} are equal and set at a wide range of altitudes from the lunar sphere of

influence (SOI) to the 100-km final low-lunar orbit (LLO). Since the optimal C_3 is nearly constant at -2 km^2/s^2, the impulsive ΔV required for the LOI burn has the most effect on m_{LOI} and the resulting optimal circular LOI ΔV's are presented in Fig. 1. The largest chemical propellant penalty is for a direct insertion into polar LLO (therefore bypassing the SEP transfer) as indicated by $\Delta V = 815$ m/s in Fig. 1. The curve shows a minimum at a circular altitude of about 13,000 km where $\Delta V = 580$ m/s.

Next, the optimal LOI burn for an elliptical lunar orbit was investigated. The desired apolune altitude h_{a_f} was set at 50,000 km which is within the lunar SOI (which has an altitude of roughly 64,500 km). A range of perilune altitudes h_{p_f} was utilized and the resulting optimal LOI ΔV is presented in Fig. 2. In this case, ΔV steadily decreases as perilune altitude is decreased and all cases showed better performance than the circular LOI maneuver. For safety purposes, the perilune altitude was chosen at 1000 km since trajectory simulations with a complete gravity model showed that a coasting trajectory with a perilune of 1000 km remained stable for several revolutions. The ΔV for this elliptical orbit is 223 m/s. Therefore, the 1000 $\times$ 50,000 km orbit is the starting point for the SEP circularization maneuver outlined in the previous final grant report [6].

The translunar trajectory to the 1000 $\times$ 50,000 km elliptical orbit is presented in Fig. 3. Optimal transfer time is 5.6 days and the spacecraft "falls" into a vertical plane about the moon with a perilune altitude of 1000 km as shown by Figs. 4 and 5. Therefore, the optimal impulsive ΔV of 223 m/s is applied at perilune to produce the desired 1000 $\times$ 50,000 km elliptical lunar orbit.

Spacecraft Optimization with Chemical-EP Stages

The optimal chemical translunar trajectory problem was extended to include spacecraft system optimization. In this problem, the chemical TLI and LOI burns were optimized along with the subsequent EP transfer to polar, circular LLO and the EP transfer to escape

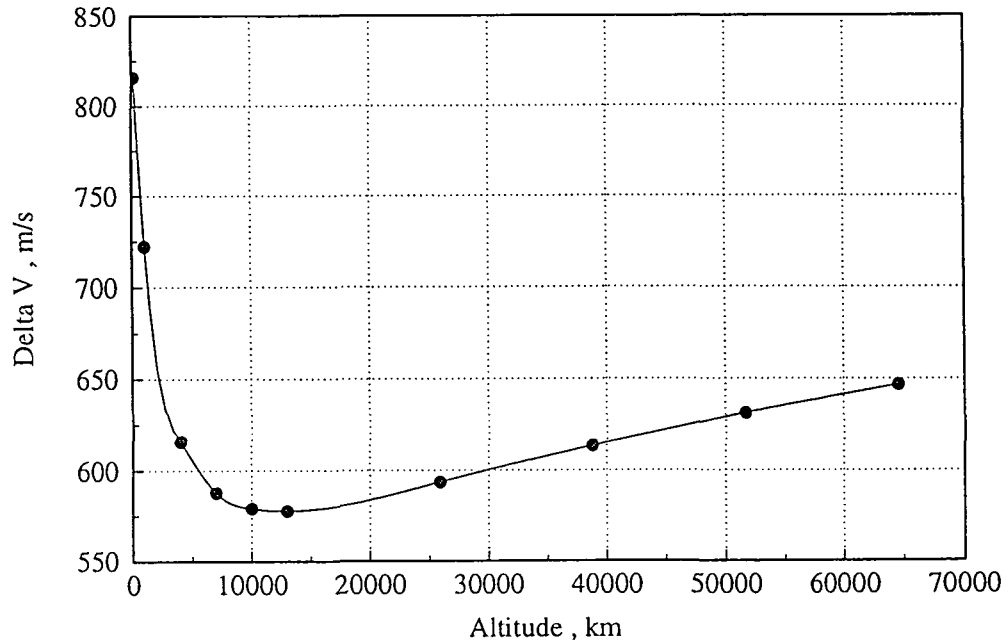

Figure 1: Optimal circular LOI burn ΔV vs altitude

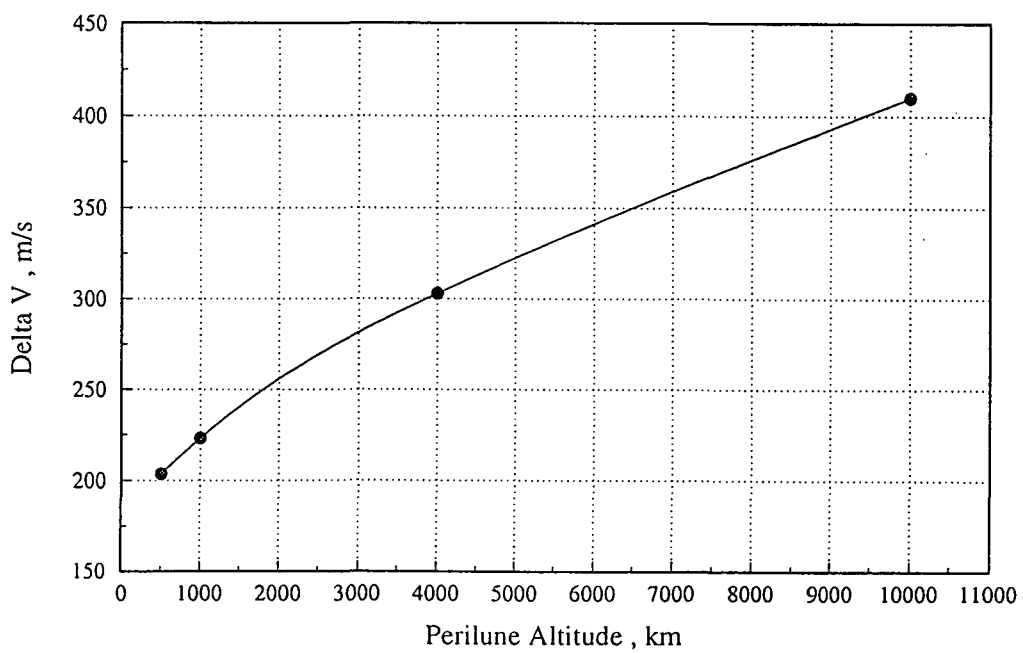

Figure 2: Optimal elliptical LOI burn ΔV vs perilune altitude

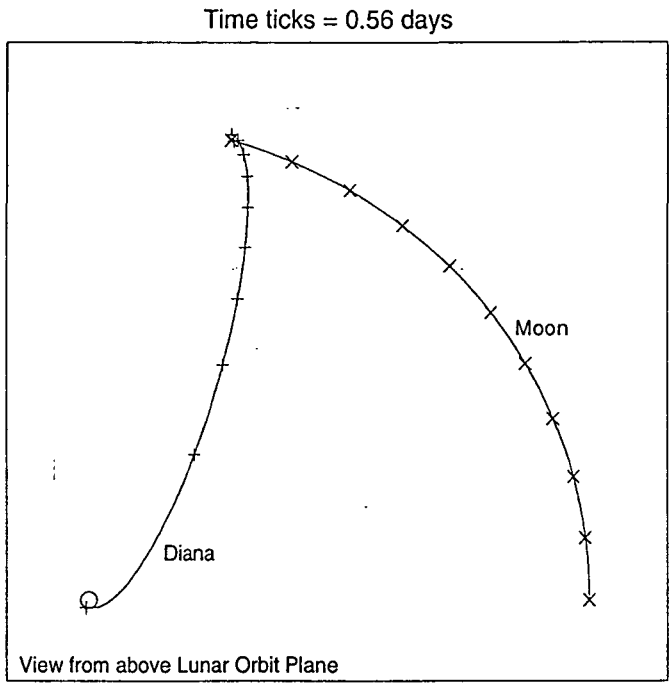

Figure 3: Minimum-fuel translunar trajectory - orbit plane

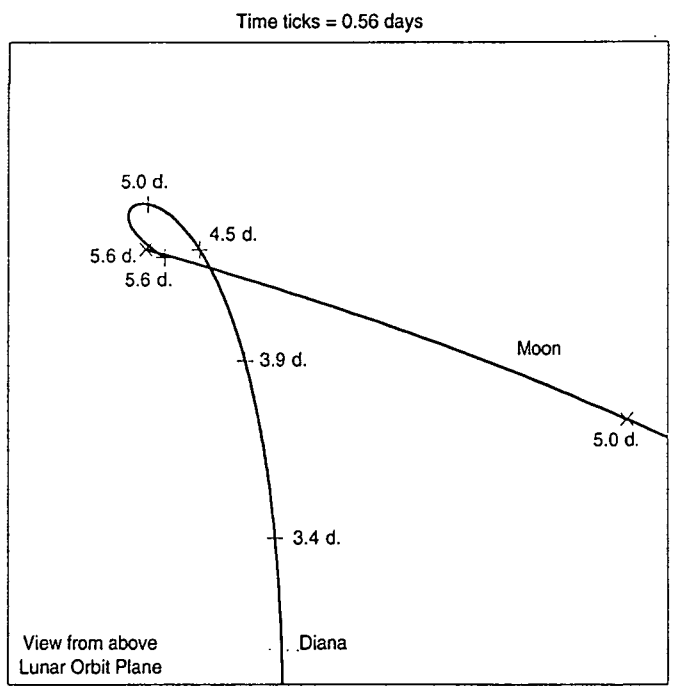

Figure 4: Translunar trajectory (near moon) - orbit plane

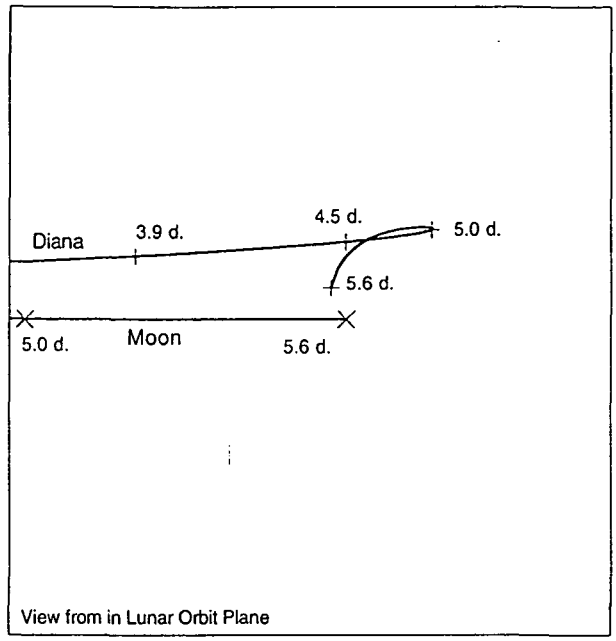

Figure 5: Minimum-fuel translunar trajectory - above orbit plane

conditions. The goal was to maximize the net mass of the spacecraft which is defined as the usable mass for payload plus basic spacecraft structural mass. Therefore, EP spacecraft system parameters such as I_{sp} and input power P are optimized in order to determine total low-thrust propellant mass, EP tankage mass, and EP power and propulsion system mass. The quasi-circular transfers are calculated by using Edelbaum's equations [9]. The results were published in a refereed journal article [10] which has been attached to this final report.

III. Optimal LOI Burn Maneuver

Once the optimal translunar trajectory has been computed using impulsive ΔV computations, a realistic LOI burn optimization study is performed to determine the optimal steering profile for the chemical burn and the proper sizing of the insertion engines. The boundary conditions for the trajectory optimization problem are determined from the optimal translunar trajectory from section II.

Trajectory Optimization

The objective is to compute the minimum-fuel, finite-time chemical LOI burn which results in the desired 1000 × 50,000 km elliptical lunar orbit. A range of thruster levels was investigated to determine the best trade-off between thruster size and gravity loss.

The complete optimal control problem is given below:

For the free end-time problem, find the pitch and yaw thrust steering angles $u(t)$ and $v(t)$, and the burn duration t_{burn} which minimize

$$J = -m(t_f) \tag{13}$$

subject to the three-body equations of motion

$$\frac{dr}{dt} = v_r \tag{14}$$

$$\frac{dv_r}{dt} = \frac{v_\theta^2}{r} + a_T \sin u \cos v + \nabla U_r \tag{15}$$

$$\frac{dv_\theta}{dt} = -\frac{v_r v_\theta}{r} + a_T \cos u \cos v + \nabla U_\theta \tag{16}$$

$$\frac{d\Omega}{dt} = \frac{\sin\theta}{v_\theta \sin i}(a_T \sin v + \nabla U_h) \tag{17}$$

$$\frac{di}{dt} = \frac{\cos\theta}{v_\theta}(a_T \sin v + \nabla U_h) \tag{18}$$

$$\frac{d\theta}{dt} = \frac{v_\theta}{r} - \frac{\sin\theta \cos i}{v_\theta \sin i}(a_T \sin v + \nabla U_h) \tag{19}$$

where $$a_T = \frac{T}{m_{TLI} - \dot{m}t}, \quad 0 \le t \le t_f$$

with the initial conditions

$$r(0) = 52,250.54 \text{ km} \tag{20}$$

$$v_r(0) = -0.921677 \text{ km/s} \tag{21}$$

$$v_\theta(0) = 0.108347 \text{ km/s} \tag{22}$$

$$\Omega(0) = 268.95 \quad \text{deg} \tag{23}$$

$$i(0) = 89.18 \quad \text{deg} \tag{24}$$

$$\theta(0) = 5.98 \quad \text{deg} \tag{25}$$

and the terminal state constraints

$$\Psi[x(t_f), t_f] = \begin{pmatrix} h_p(t_f) - 1000 \quad \text{km} \\ h_a(t_f) - 50,000 \quad \text{km} \\ i(t_f) - 90 \quad \text{deg} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \tag{26}$$

The states are radial position r, radial velocity v_r, circumferential velocity v_θ, longitude of the ascending node angle Ω, inclination i, and in-plane longitude angle θ. The radius r is the distance from the center of the moon to the spacecraft and v_r and v_θ are the inertial velocity components measured in the instantaneous orbit plane. The ascending node angle Ω is measured counter-clockwise from the fixed $+x$ axis to the ascending node direction. The inertial $+x$ axis is initially pointing from the moon's center to the Earth at $t = 0$. The inclination i is with respect to the $x - y$ or Earth-moon orbit plane. Longitude angle θ is the in-plane angle measured from the ascending node to the spacecraft in the direction of motion. Therefore, θ is the sum of argument of perilune ω and true anomaly ν.

The gravity potential gradient ∇U for the combined Earth and moon gravity field is

$$\nabla U = -\frac{\mu_m \vec{r}}{r^3} - \mu_e \left[\frac{\vec{r}_e}{r_e^3} - \frac{\vec{r}_{e-m}}{D^2} \right] \tag{27}$$

where the gravitational parameters of the Earth and Moon are represented by μ_e and μ_m, respectively, $\vec{r}_e$ is the radius vector from the Earth to the spacecraft, $\vec{r}_{e-m}$ is the radius vector from the Earth to the moon, and D is the constant separation distance between the Earth and moon. The components of ∇U are

$$\nabla U_r = -\frac{\mu_m}{r^2} - \frac{\mu_e r_{e_r}}{r_e^3} + \frac{\mu_e r_{e-m_r}}{D^3} \tag{28}$$

$$\nabla U_\theta = -\frac{\mu_e r_{e_\theta}}{r_e^3} + \frac{\mu_e r_{e-m_\theta}}{D^3} \tag{29}$$

$$\nabla U_h = -\frac{\mu_e r_{e_h}}{r_e^3} + \frac{\mu_e r_{e-m_h}}{D^3} \tag{30}$$

where the subscripts r and θ correspond to components along the radial and circumferential in-plane directions and the subscript h corresponds to the direction normal to the instantaneous orbit plane.

The in-plane pitch thrust steering angle u is measured positive above the local horizon to the projection of the thrust vector onto the orbit plane. The out-of-plane yaw thrust steering angle v is measured positive above the orbit plane to the thrust vector and is between ± 90 degrees. The high-thrust acceleration of the spacecraft, a_T, is computed by dividing the constant thrust magnitude, T, by the current spacecraft mass. Thrust levels ranging from 5 to 100 lb_f were investigated. The mass of the spacecraft is denoted by m, and propellant mass flow rate $\dot{m}$ is considered positive out of the vehicle.

The initial conditions (20-25) represent a spacecraft state from the optimal translunar trajectory 13.7 hrs from perilune. At this given state, the spacecraft has crossed the SOI and has a moon-relative energy of 0.3368 km^2/s^2 and eccentricity of 1.378. The three terminal state constraints (26) define the desired 1000 $\times$ 50,000 km polar elliptical orbit. The goal is to find the thrust steering angles $u(t)$ and $v(t)$, and the duration of the finite-burn arc such that the final spacecraft mass $m(t_f)$ is maximized (or, equivalently, such that fuel is minimized) and the spacecraft terminates in the prescribed polar elliptical orbit. The optimal control problem is solved using SQP with each continuous-thrust steering angle replaced by a cubic-spline fit through six SQP control parameters.

Results

Optimal minimum-fuel LOI burn maneuvers were readily obtained for constant thrust levels ranging from 5 to 100 lb_f. The equivalent ΔV as computed by the rocket equation for the resulting finite burn is presented in Fig. 6. The performance greatly improves and asymptotically approaches the impulsive ΔV of 223 m/s as thrust level is increased. The

gravity loss can be computed as the difference between the finite ΔV and the impulsive ΔV. The "knee" of the curve is at a thrust range of about 20-35 lb_f where the gravity loss ranges from about 18 to 8 m/s. The optimal finite-burn duration is presented in Fig. 7 and a similar profile is observed with the burn duration ranging from 54 to 25 min for the "knee" of the curve. Using these plots, it was determined that a thrust level of 24 lb_f would provide sufficient performance. This thrust level could be easily supplied by three 8-lb_f thrusters that are currently available from TRW.

The resulting optimal pitch steering history for the 24-lb_f thruster is presented in Fig. 8. The pitch steering angle plotted here is measured from the velocity vector and shows a linear relation symmetric about the 180 deg steering angle. Therefore, the optimal pitch steering profile initially points opposite the velocity vector but with a slight positive radial velocity component. The thrust vector is then turned at a constant rate in the orbit plane and is directly opposite the velocity vector (180 deg) at the mid-point of the burn. The optimal yaw steering is effectively zero since the maneuver is essentially planar. The burn lasts for about 45 min. The optimal LOI perilune burn is presented in Fig. 9 as viewed from the moon-centered, inertial y-z vertical plane.

IV. Summary and Conclusions

A study of a translunar injection trajectory using a combined chemical-electric propulsion spacecraft has been performed. The study was in support of the Diana mission proposal for NASA's 1994 Announcement of Opportunity (AO) for Discovery class exploration missions. The trajectory optimization was performed using sequential quadratic programming (SQP), which is a direct method.

The optimal ballistic translunar trajectory was obtained using SQP and impulsive burns. A three-body dynamic model was used for the governing equations of motion. A complete vehicle sizing study for the combined chemical-electric propulsion spacecraft was performed

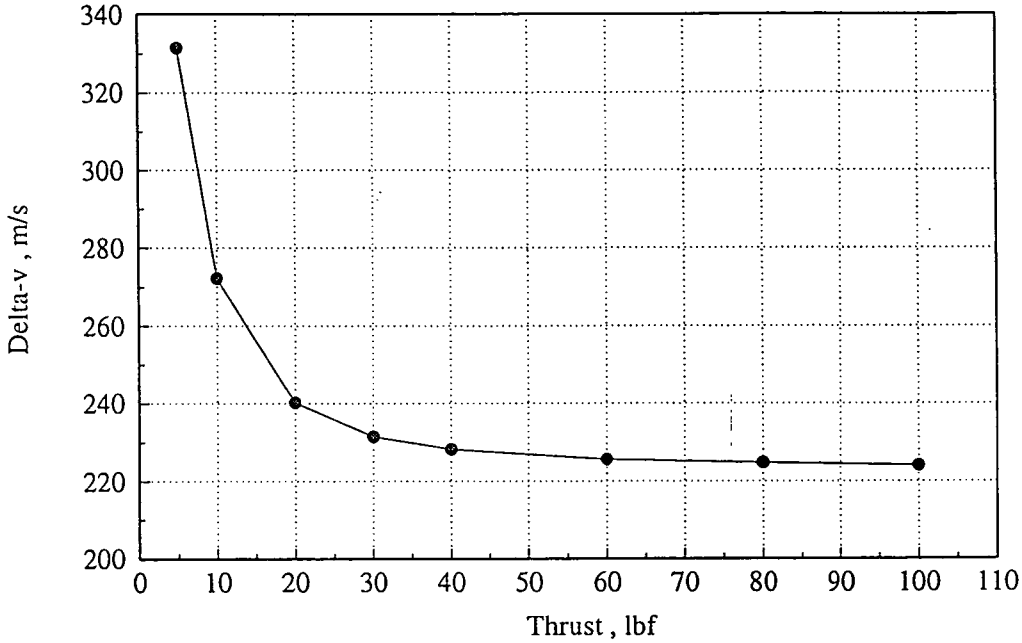

Figure 6: Optimal ΔV vs chemical thrust level

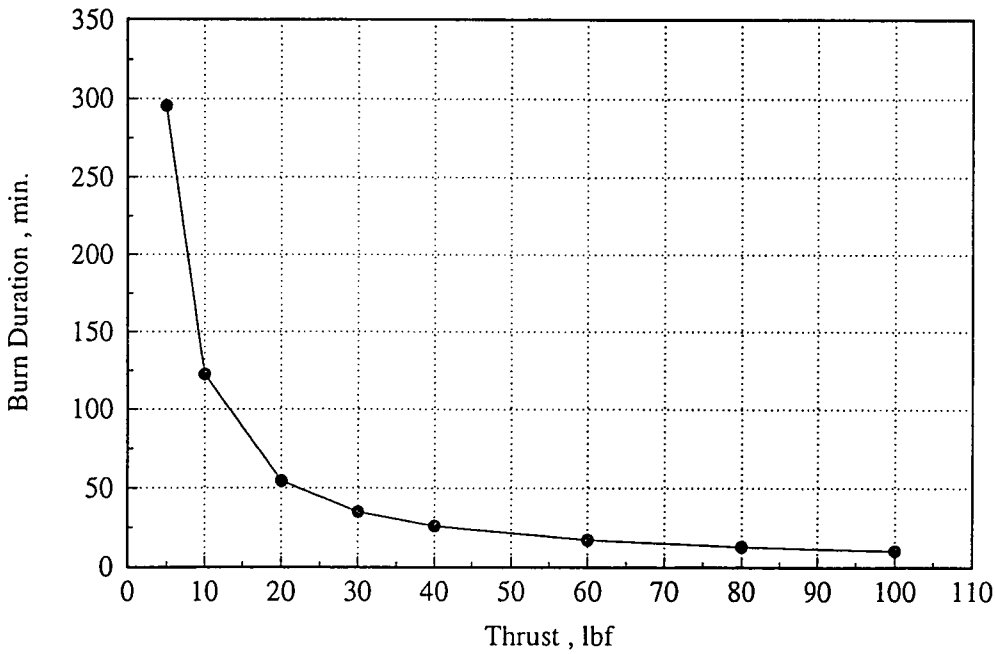

Figure 7: Optimal burn duration vs chemical thrust level

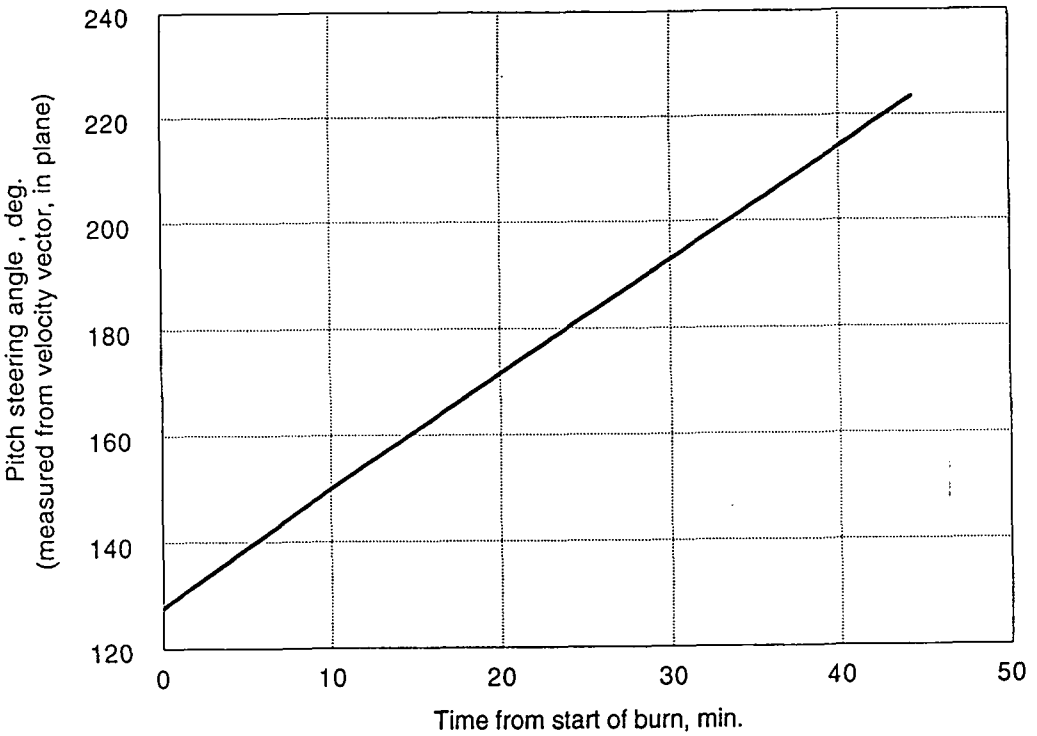

Figure 8: Optimal thrust steering for LOI burn (T=24 lb_f)

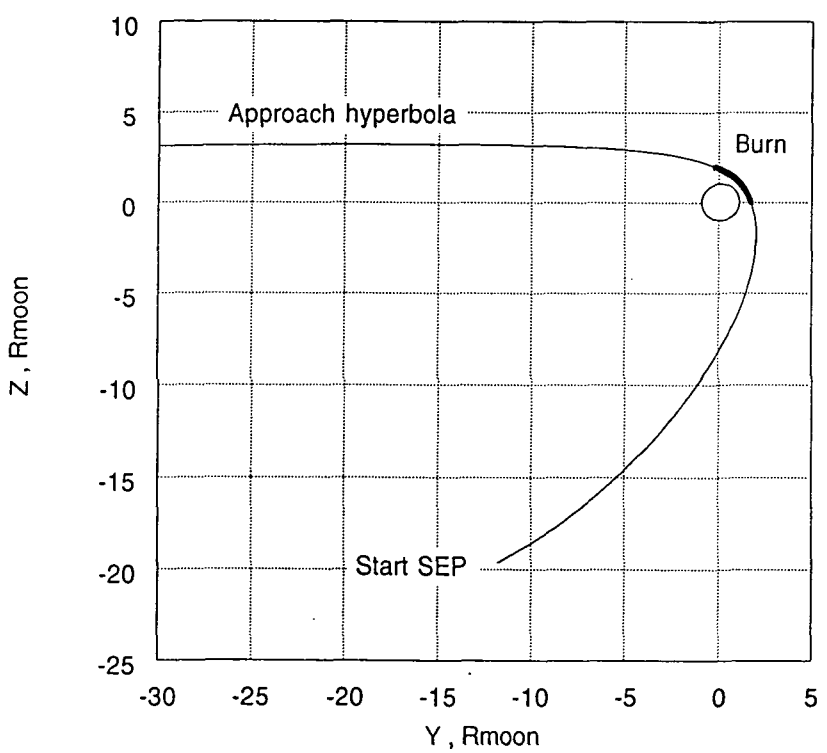

Figure 9: Optimal polar LOI burn maneuver

and the results were published in a refereed journal.

A detailed study optimizing the lunar orbit insertion burn was performed. A finite-duration perilune burn was used and a range of thrust levels was investigated in the context of the complete three-body equations of motion. A good trade-off between thruster size and gravity loss was identified and a 24-lb_f thruster was selected. The optimal thrust steering history was obtained and the result is a simple linear pitch profile.

Acknowledgments

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