Axisymmetric Inlet Minimum Weight Design Method

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<td>Cross-sectional frame area</td>
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<td>$\bar{c}$</td>
<td>Mean geometric chord</td>
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<tr>
<td>$C_F$</td>
<td>Shanley's constant (ref. 5)</td>
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<td>$d$</td>
<td>Frame spacing</td>
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<td>$E_F$</td>
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<td>$f$</td>
<td>Cross force per unit length</td>
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<td>$F_c$</td>
<td>Sum of all shear forces produced by the cross forces ahead of the current analysis station</td>
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<td>$F_{cy}$</td>
<td>Material compressive yield strength</td>
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<tr>
<td>$F_h$</td>
<td>Force due to hammershock pressure</td>
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<tr>
<td>$F_{mat}$</td>
<td>Material constant (Equation 5.5)</td>
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<td>$F_p$</td>
<td>Force due to normal operating pressure</td>
</tr>
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<td>$F_s$</td>
<td>Material shear strength</td>
</tr>
<tr>
<td>$F_w$</td>
<td>Material tensile ultimate strength</td>
</tr>
<tr>
<td>FOD</td>
<td>Foreign Object Damage</td>
</tr>
<tr>
<td>$g$</td>
<td>Acceleration due to gravity</td>
</tr>
<tr>
<td>$I$</td>
<td>Moment of inertia</td>
</tr>
<tr>
<td>$I'$</td>
<td>Moment of inertia divided by the section thickness</td>
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<td>$I_F$</td>
<td>Moment of inertia of the frame cross-section</td>
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<td>$K_{F1}$</td>
<td>Frame stiffness coefficient, $I_F/A_F^2$</td>
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<td>$K_g$</td>
<td>Gust alleviation factor</td>
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<td>$l$</td>
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<td>$L$</td>
<td>Shell length for radial buckling</td>
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<tr>
<td>$m$</td>
<td>Buckling equation exponent</td>
</tr>
<tr>
<td>$M$</td>
<td>Longitudinal bending moment</td>
</tr>
<tr>
<td>$M_c$</td>
<td>Sum of the bending moments produced by all cross forces ahead of the current analysis station</td>
</tr>
<tr>
<td>$n$</td>
<td>Load factor; point for deflection calculation</td>
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<tr>
<td>$N^{+}$</td>
<td>Tensile stress resultant</td>
</tr>
<tr>
<td>$N_{x}$</td>
<td>Compressive stress resultant</td>
</tr>
<tr>
<td>$N_{x}^*$</td>
<td>Axial compressive stress resultant</td>
</tr>
<tr>
<td>$N_{x}$</td>
<td>Bending tensile stress resultant</td>
</tr>
<tr>
<td>$N_{x}$</td>
<td>Radial longitudinal compressive stress resultant</td>
</tr>
<tr>
<td>$N_{x}$</td>
<td>Shear stress resultant</td>
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\( N_r \) Radial hoop stress resultant
\( p \) Inlet internal pressure; normal operating pressure
\( P_h \) Hammershock pressure
\( P \) Radial component of the pressure
\( q_o \) Maximum dynamic pressure
\( r \) Shell radius
\( R \) Shell radius
\( R_i \) Small radius of a truncated cone
\( R_o \) Large radius of a truncated cone
\( R_e \) Axial force
\( R_n \) Resultant force for asymmetric hammershock condition
\( S \) Surface area
\( t \) Thickness
\( t_{gage} \) Equivalent isotropic thickness of shell
\( t_{buck} \) Minimum gage thickness of shell
\( t_{comp} \) Equivalent isotropic thickness of shell due to radial buckling
\( t_{long} \) Equivalent isotropic thickness of shell due to longitudinal buckling
\( t_{shear} \) Equivalent isotropic thickness of shell due to shear
\( t_{tension} \) Equivalent isotropic thickness of shell due to tension
\( U_{de} \) Derived gust velocity
\( |V_{max}| \) Absolute value of the maximum shear load
\( V_e \) Airplane equivalent airspeed
\( V_S \) Shear load
\( W \) Weight of the inlet centerbody structure ahead of station \( x \); weight in general
\( (W/S) \) Wing loading
\( W_f \) Frame weight
\( W_{nc} \) Weight of the nose cone
\( W_s \) Shell weight
\( W_{str} \) Ideal structural weight of the inlet centerbody
\( W_{structure} \) Ideal structural weight of the total inlet
\( W_{systems} \) Weight of inlet systems
\( W_{total} \) Total weight including structural and non-optimum weights
\( x \) Longitudinal centerbody coordinate
\( \bar{x} \) X location of the center of gravity of the structure ahead of station \( x \)
\( x_c \) X location of fixed point for cantilever beam calculations
\( \Delta x \) Longitudinal distance between \( x \) stations
\( x_{nc} \) Length of the nose cone
\( y \) Radial coordinate
\( y_{nc} \) Radial coordinate at the base of the nose cone
\( Z \) Non-dimensional shell geometry parameter
\( \alpha \) Cone angle
\( \beta \) Local slope angle
\( \gamma \) Maximum yaw or pitch angle
\( \delta \) Deflection
\( \varepsilon \) Shell buckling efficiency
\( \eta_r \) Non-dimensional shell geometry parameter
\( \theta \) Angle between face sheet and core in truss-core sandwich panel
\( \mu \) Poisson's ratio; also airplane mass ratio for calculation of gust loads
\( \pi \) 3.14159
\( \rho \) Density of the shell material
\( \rho_f \) Density of the frame material
\( \rho_{nc} \) Density of the nose cone material
\( \sigma_{cr} \) Design-allowable critical buckling stress
\( \sigma_y \) Yield hoop stress
Summary

An analytical method for determining the minimum weight design of an axisymmetric supersonic inlet has been developed. The goal of this method development project was to improve the ability to predict the weight of high-speed inlets in conceptual and preliminary design. The initial model was developed using information that was available from inlet conceptual design tools (e.g., the inlet internal and external geometries and pressure distributions). Stiffened shell construction was assumed. Mass properties were computed by analyzing a parametric cubic curve representation of the inlet geometry. Design loads and stresses were developed at analysis stations along the length of the inlet. The equivalent minimum structural thicknesses for both shell and frame structures required to support the maximum loads produced by various load conditions were then determined. Preliminary results indicated that inlet hammershock pressures produced the critical design load condition for a significant portion of the inlet. By improving the accuracy of inlet weight predictions, the method will improve the fidelity of propulsion and vehicle design studies and increase the accuracy of weight versus cost studies.

Introduction

The weight of an aircraft inlet is an important parameter in a conceptual design study and systems analysis. Accurate estimation of this weight, therefore, is desirable. Previous studies indicated a need for a design and weight analysis method applicable to high-speed inlets during conceptual level design, where a structural definition of the inlet is generally not available (ref. 1). To address this issue, a method for generating a minimum weight structural design for a high-speed inlet has been developed. The first stage of this method development effort focused on the design of an axisymmetric inlet, to be followed in the future by a method for the design of a two-dimensional inlet. In the method, a structural model of the inlet is created using the results from conceptual flowpath analysis. Analytical principles based on strength of materials, specifically Euler beam theory, are used to determine the minimum structure necessary to prevent failure due to specified load conditions. The weight of this structure is then determined. A non-optimum weight penalty is computed and added to the inlet structural weight. Finally, inlet system weights (bypass system, actuators, controls) are added to give an estimate of the total weight of the inlet.

The structural concept for the axisymmetric inlet designed by the method is based on a stiffened shell consisting of panels, longitudinal stiffeners, and ring frames. Nine different variations of this concept, including skin-stringer and truss-core sandwich shells, are included. A generic axisymmetric inlet was developed and used to test the method logic. This inlet model is described below. The method is then discussed beginning with the modeling of the geometry definition of the inlet using parametric cubic curves. Details of the loads analysis and the computation of stress resultants based on the maximum design loads are then presented. The minimum equivalent isotropic thickness of the shell/longitudinal stiffeners, and that of the ring frames, from the design stresses are then determined. The total weight of the inlet, including a non-optimum weight penalty and system weights, is then computed. The final inlet design is analyzed to find local and throat deflections. Finally, the method is validated using results from a study inlet developed by the Boeing Commercial Airplane Company and from a generic inlet design.
Generic Axisymmetric Inlet Description

In order to properly model an inlet for structural design, three types of information must be available. The gross geometry of the inlet, the forces applied to the inlet, and the material or materials from which the inlet will be constructed must be known. To facilitate the description of the minimum weight design method, a generic high-speed axisymmetric inlet has been modeled. All of the information presented for this inlet, with the exception of hammershock overpressures, is available from the results of conceptual level inlet aerodynamic analyses.

The gross geometry of the generic high-speed axisymmetric inlet appears in Figure 1. The inlet consists of three distinct main structures: centerbody, internal cowl, and external cowl. To simplify the minimum weight method, each of these main structures was analyzed and designed separate from, and without direct influence on, the others. Because of this, load transfer from one structure to the others is not considered. Secondary structure, shown in the upper right corner of Figure 1, would normally consist of a support tube running through a portion of the centerbody length and support struts located in the aft portion of the inlet. Design of the secondary structures was not included in the method development. It was assumed that the weight added by these members could be adequately accounted for through the independent design of the main structure and through systems and non-optimum weight penalties. This assumption was validated using a Boeing study inlet, discussed below.

Representative pressure distributions applied to a Mach 2.4 inlet at a given condition are taken from reference 1 and shown in Figure 2. They include: normal internal operating pressures, internal hammershock pressures (both asymmetric and axisymmetric), and external aerodynamic pressures. These pressures, when combined appropriately with inertial loads, determine the design loads for each major structural component of the inlet. It can be seen from Figure 2 that, in general, the hammershock pressures are considerably larger than the inlet normal operating pressures. As will be seen, these hammershock pressures play a critical role in the design of the inlet.

Finally, the material properties assumed for the generic axisymmetric inlet are listed in Appendix 1. Titanium alloy Ti-6Al-4V was assumed for all generic inlet structures. This is consistent with Mach 2.4 inlet design for the technology date of approximately 1995 (ref. 2).

Minimum Weight Design Method

The general strategy for minimum weight design of an axisymmetric inlet is similar to that used by Arde for the minimum weight design of arbitrary shaped bodies (ref. 3). Stiffened shell construction, i.e., shells or panels stiffened by longitudinal supports and ring frames, is assumed; truss-core sandwich shells are also considered. A complete list of the structural concepts available for design is found in Table 1. They include: unstiffened shells with and without frames; simply-stiffened shells designed for best buckling efficiency; Z-stiffened shells designed for best buckling efficiency, buckling efficiency and minimum gage compromise, and buckling efficiency and pressure compromise; and truss-core sandwich shells with and without frames designed for best buckling efficiency, and without frames designed for buckling efficiency and pressure compromise. These nine structural concepts were chosen based on their similarity to airframe structural designs.

The design approach is based on the calculation of the minimum equivalent isotropic, or "smeared", thickness of the shell/longitudinal stiffener structure to preclude failure. A loads analysis is performed at equally spaced stations along the length of the inlet. The minimum
allowable smeared material thickness is then calculated at each station based on the maximum loads for each failure mode. A comparison of all failure thicknesses is made to determine the minimum allowable equivalent thickness for the structure at that station. A smeared thickness for the ring frames is computed following the shell design. It is assumed that all structures behave elastically. The design process is necessarily iterative due to the contribution of inertial loads, which will change as the equivalent thicknesses change, to the structural design. All structures are designed for simultaneous failure by general instability and panel type failures.

Prior to performing the loads analysis and design, the geometry of the structural component must be represented in a level of detail sufficient for computing mass properties accurately. This process is described below, followed by details regarding the loads analysis, the computation of stress resultants, the determination of the minimum weight shell design, and the frame design. The computation of the total weight of the inlet, including a non-optimum weight penalty and systems weights, is also discussed.

Geometry and Mass Properties

As discussed earlier, one important requirement of the minimum weight inlet design method was for it to operate using the results available from conceptual level inlet design tools. Often the geometry definition available from such tools is coarse and consists of sparse data points such as those in Figure 3a. For the best possible design results, it is desirable to have as many data points as possible, or as precise a representation of the structure as possible. One method of accomplishing this is to use the sparse input data to create a parametric cubic curve that describes the surface of the structure. The result is a smooth curve, as shown in Figure 3b, that is described by many points (for example, 275 parametric points instead of 26 input points). The parametric points defining the curve are then used for further geometry related calculations such as computing mass properties (surface area, volume, center of gravity location) and interpolating to find the coordinates of the analysis stations.

The first operation performed using the parametric geometry is to interpolate to find the locations of the analysis stations along the inlet. The mass properties, including volume distribution, surface area, and center of gravity location, are then computed at each analysis station. Each segment of the structure is considered to be a shell with length equal to the distance between stations, and beginning and ending radii equal to the inlet radius at the bounding stations. The total initial volume, surface area, and center of gravity location are also calculated for the entire structural component. The initial weight distribution for each component is assumed to be directly related to its volume distribution. Initial values for the total weight of the components and the shell thicknesses are required to begin the iterative design process. The initial values used were based on study inlet designs and are: 200 lb for the centerbody weight, 400 lb each for the internal and external cowls weights, and .1 in for all shell thicknesses.

Loads Analysis

The inlet structure is subject to external and internal pressures, as well as inertial and aerodynamic loads. The loads and pressures that are applied to the inlet components are shown in Figure 4. They can be classified as axisymmetric (Figure 4a) and asymmetric loads (Figure 4b) and are determined at each analysis station along each structural component. The axisymmetric loads consist of the inlet normal operating pressure and hammershock pressure applied to the external surface of the inlet centerbody and the internal surface of the internal cowl structure. The external cowl structure is subjected to a leakage pressure from the inlet internal environment and to external pressure. Asymmetric loads applied to the centerbody include inertial loads, asymmetric hammershock pressure, the cross force produced by
aerodynamic maneuvers (see Appendix 2), and possible Foreign Object Damage (FOD) by bird strike, hail, etc., on the portion of the spike forward of the cowl leading edge. Asymmetric hammershock pressure, FOD at the cowl lip, and inertial loads are applied to the internal cowl structure. The external cowl structure is subjected to the resultant force produced by an asymmetric hammershock leakage pressure and external pressure in addition to inertial loads, aerodynamic cross forces, and FOD at the cowl lip. As will be discussed later, the FOD requirement is defined as a minimum gage specification.

It is assumed that the pressure at each analysis station can be interpolated linearly from the supplied pressure data. Hammershock pressures are also linearly interpolated and are applied only aft of the inlet cowl lip. Asymmetric hammershock overpressure occurs when the compressor stalls nonuniformly. While this condition is usually followed by full, axisymmetric stall (ref. 4), the asymmetric load produced initially will be severe and is therefore, considered as a separate loading condition. The force produced by an asymmetric hammershock is computed as the resultant of hammershock pressures in one section of the inlet and normal operating pressures in the opposite section (see Appendix 2).

Eight different design requirements incorporating the inlet pressures and loads are used to determine the maximum loads to which the structure will be designed. Listed in Table 2, these consist of five external (cases 1-5) and three internal (cases 6-8) load cases. These eight design requirements represent those most applicable to the design of an inlet in the Mach 2.4 class. Each is assumed to occur independent of the others. The maximum loads for all of the design load cases are based on ultimate load requirements. Details regarding each of these load cases can be found in Appendix 3.

Each of the design load requirements produce some combination of bending, shear, axial, and radial loads. For the calculation of bending moments and shear loads, it is assumed that the inlet structure acts like a cantilever beam. The fixed point for the cantilever was originally chosen to be at the compressor face, because this station was the physical end of the inlet structure. However, this location ignored the added stiffness produced by support struts, which would be present in an actual inlet (see Figure 1, upper right corner). The fixed location for the cantilever was therefore moved forward to a location approximating the centerline of the support struts. Based on study inlets (ref. 2), this was halfway in between the end of the centerbody spike and the compressor face. Axial loads and radial pressures are computed assuming the structure acts as a thin-walled cylindrical shell. Each load case is applied to the structure independently of the others. The final maximum loads at each analysis station are chosen by looking at all of the individual load case results. These loads are used to compute the stress resultants for the basis of the structural design, as discussed below.

**Stress Resultants**

The four types of maximum loads, bending moments, shear loads, axial loads, and radial pressures, contribute to the four stress resultants used to design the inlet structure. The first of these is the tensile stress resultant, which includes the influences of bending stresses, axial stresses, and longitudinal stresses caused by radial pressures. The tensile stress resultant is given by

\[
N_T = N_{sb} + N_{sa} + N_{sr}
\]  \hspace{1cm} (1)

where the bending, axial, and longitudinal stress resultants are given by

\[
N_{sb} = \frac{Mr}{I'}
\]  \hspace{1cm} (2)
for a circular cross-section. If the axial and radial stresses are compressive, then a conservative approach would be to neglect the positive influences of these stresses on the tensile resultant, giving

\[ N_s^* = N_{sB} \]  

It is assumed that the absolute value of \( N_{sB} \) is used in this calculation. The compressive stress resultant for the inlet centerbody is

\[ N_s^* = -N_{sB} - N_{sA} - N_{sR} \]  

where \( N_{sB}, N_{sA}, \) and \( N_{sR} \) are defined as above, and the absolute value of \( N_{sB} \) is assumed. If \( N_{sA} \) and \( N_{sR} \) are tensile, then the compressive stress resultant is equal to the bending stress resultant. Equations 1 through 7 also define the tensile and compressive stress resultants for both the internal and external cowl structures, with the exception that the radial stress resultant, which is indicative of pressure stabilization for the cowl, is not included.

The shear stress resultant is computed from the shear loads on the structure. It is given by

\[ N_{sy} = \frac{|V_{\text{max}}| \pi \tau^2}{2I'} \]  

The radial stress resultant is derived from the hoop stress that will result from the axisymmetric pressure on the inlet structure. The radial stress resultant is given by

\[ N_r = PyK_p \]  

where \( K_p \) is a geometric factor that accounts for the fact that only the skin, not the longitudinal stiffeners or truss core, is available for resisting hoop stress (ref. 3). \( P \) is either the normal operating pressure or the axisymmetric hammershock pressure.

**Minimum Weight Shell and Frame Design**

When determining the minimum weight design of the stiffened shell part of the inlet structure, five possible modes of failure are considered. These are tension, compression without buckling, shear, longitudinal buckling, and radial buckling. Radial buckling is only considered in the design of the inlet centerbody, since this structural shell component is subjected to an external pressure. Maximum stress failure theory was applied for all failure modes. Minimum gage
constraints, both due to manufacturing considerations and to FOD requirements, are considered as a separate design condition. Six minimum smeared thicknesses are therefore determined at each analysis station, one for each failure mode and one for minimum gage. Details regarding the development of these thicknesses can be found in Appendix 4. The final minimum smeared thickness of the shell for the section is determined by comparing those from each failure mode, or

$$t_{min} = \max(t_{minlong}, t_{long}, t_{shear}, t_{buck}, t_{ng})$$

(10)

The shell thicknesses to prevent longitudinal and radial buckling are dependent on the spacing of the ring frames. It is therefore necessary to determine this value before completing the design of the shell. However, the frame spacing is dependent on the buckling characteristics of the final shell design. The design of the shells and frames is therefore a multiple step process that begins with the determination of the optimum frame spacing for minimum weight. In general, the ring frames are sized and spaced based on the Shanley criterion (ref. 5). This assumes that the frames act as elastic supports for the wide column. It also assumes that the structure has an equal probability of failing by general instability or local panel failure (simultaneous failure modes). The location (spacing) and minimum equivalent isotropic thickness of the frames is then based on panel failure due to buckling, since this is the simpler approach of the two (ref. 5). If the frame spacing is not known, then it is possible to determine the optimum spacing that minimizes the weight of the frame and shell structure (ref. 3) for panel buckling failure. Assuming that the frame material is smeared over the same area as the shell material, the total weight per unit area of the structure will be given by (ref. 3)

$$W_s = \rho t_{min} + \rho_F t_F$$

(11)

Substituting the equation for the minimum smeared shell thickness and optimum frame thickness due to longitudinal or radial buckling (see Appendix 4) and minimizing with respect to the frame spacing, the initial optimum frame thickness and spacing are determined (see Appendix 5). This value for the frame spacing is used to compute the minimum smeared thicknesses due to longitudinal and radial buckling of the structure.

After the minimum equivalent smeared thickness is found (Equation 10), the frame design is reconsidered. If the minimum smeared thickness was found to be equal to the thickness due to longitudinal or radial buckling, then the frame design values are optimum for the structure. If, however, the minimum equivalent smeared thickness was found to be due to a non-buckling mode of failure, then the frame design values are no longer optimum for the design based on the assumption of simultaneous failure modes. In other words, the design is no longer buckling critical. The frame spacing is therefore recomputed to give a buckling critical design based on the computed minimum shell thickness value. A new frame equivalent thickness is also computed. The final structural design of the segment is therefore buckling critical. The frame thickness, like the shell thickness, is also subject to a minimum gage constraint based on the frame cross-section geometry. If the computed thickness of the frame is less than the minimum gage, it is changed to be equal to the minimum gage. The frame spacing is recomputed again to ensure that optimum use is being made of the frame material.

Nose Cone Design

Due to manufacturing and maintenance considerations, the nose cone of the axisymmetric inlet centerbody is a solid structure. The length of the cone is determined by the cone angle and the specification of a maximum base diameter. Since the nose cone dimensions and material are known, no further design activity is required to determine the weight of the nose cone.
Total Inlet Weight

The total weight of the inlet consists of the structural weights of the centerbody, internal cowl, and external cowl, a weight penalty to account for non-optimum weight items, and the weight of inlet systems such as actuators, controls, and bypass mechanisms. Given the material properties and the equivalent minimum thicknesses of the shells and frames, the structural weight of each inlet component is easily computed. The total structural weight of the inlet centerbody is the summation of the minimum allowable nose cone, shell, and ring frame weights, or

\[ W_{str} = W_{ac} + W_s + W_j \] \hspace{1cm} (12)

where, in general, \( W = \rho \Delta S \). The weight of the nose cone is given by

\[ W_{nc} = \frac{1}{3} \pi \rho_{nc} \gamma_{nc} x_{nc} \] \hspace{1cm} (13)

The weights of both the centerbody shells and frames are summed over all analysis stations and are given by

\[ W_s + W_f = \pi \sum (\rho_{lms} + \rho_{lms})(y_i + y_{i+1}) \sqrt{(y_i - y_{i+1})^2 + \Delta x^2} \] \hspace{1cm} (14)

So, the total structural weight of the axisymmetric inlet centerbody is given by

\[ W_{str} = \frac{1}{3} \pi \rho_{nc} \gamma_{nc} x_{nc} + \pi \sum (\rho_{lms} + \rho_{lms})(y_i + y_{i+1}) \sqrt{(y_i - y_{i+1})^2 + \Delta x^2} \] \hspace{1cm} (15)

The total structural weights of the internal and external cowls are likewise found from Equations 12 and 14.

Additional weight is added to the total structural weight of the inlet components by multiplying the structural weight by a non-optimum factor. The non-optimum weight factor accounts for non-modeled structural items such as fasteners and bolts, extra weld material, uniform gages, etc., and also corrects for inaccuracies due to approximations and assumptions made in the analysis method. The total weight of the inlet without systems is then given by

\[ W_{structure} = W_{str}(1 + K_{non-opt}) \] \hspace{1cm} (16)

where \( K_{non-opt} \) is approximately 0.2 (ref. 2).

Finally, weight to account for inlet actuators, controls, and bypass systems are added to the inlet weight. The total weight of the inlet is

\[ W_{total} = W_{structure} + W_{systems} \] \hspace{1cm} (17)
Deflection Analysis

Optimum performance of a high-speed inlet is dependent on the flowpath geometry. It is important, therefore, to consider the possible deflections that might occur during inlet operation. After the design of the inlet is determined using the method described above, the local and throat deflections are computed and compared with acceptable limits. If the inlet deflections exceed these limits, then the inlet geometry must be redesigned to increase the stiffness of the structure, and thus reduce the deflections.

The Moment-Area method (ref. 15) was used to compute the deflection at each analysis station along the inlet centerbody and internal cowl. The deflections of the external cowl were not computed because they do not affect the internal performance of the inlet. The Moment-Area method of computing deflections was chosen because of its ability to analyze structure with discrete loads and varying geometry. Details of the deflection calculations can be found in Appendix 6. The deflection limits applied to the inlet design result from performance requirements during normal internal operation. Therefore, deflections are only computed for limit loads due to normal operating pressures. As in the inlet design, the inlet component structures are treated as cantilever beams for the deflection analysis.

Results and Discussion

The minimum weight inlet design method has been tested on various axisymmetric inlets, including the generic axisymmetric inlet described above and the Mixed Compression Translating Centerbody study inlet developed by the Boeing Commercial Airplane Company (ref. 2). Results from these studies are presented below. Future goals of the methods development effort will also be discussed.

Design of a Generic Axisymmetric Inlet

The generic axisymmetric inlet pictured in Figure 1 was designed for minimum weight based on the pressures shown in Figure 2. Load factors for each of the design load cases listed in Table 2 were taken from ref. 2. The resulting maximum bending moments at each analysis station for an inlet with structural concept 3 (Table 1) appear in Figure 5. As can be seen, the inlet centerbody is subjected to negative, or downward, bending moments, while both the internal and external cowl lips see positive moments. It can also be seen in this figure that most of the maximum moments computed for the inlet are due to the asymmetric hammershock load case.

The maximum bending moments, shear, axial, and radial loads were used to determine the minimum equivalent isotropic thickness required for a minimum weight design. The minimum equivalent shell thicknesses for the concept 3 inlet centerbody are shown in Figure 6. Also indicated in this figure are the critical failure modes. The initial segment of the centerbody is the solid nose cone. The section of the centerbody spike exposed to external flow aft of the nose cone is designed by Foreign Object Damage minimum gage requirements. Aft of the cowl lip, the centerbody design is influenced by radial buckling failure and compression without buckling. The final section of the centerbody, from the assumed centerline of support struts to the compressor face, is labeled as support. This is the aft section of the support tube and is assumed to have dimensions equal to those of the last analyzed segment of the centerbody. It is assumed that this final section does not influence the moments or deflections of the structure.

The critical failure mode is dependent not only on the applied loads and geometry of the structure, but also on the structural concept. Figure 7 shows the critical failure modes for the
centerbody designed with the various structural concepts listed in Table 1. As can be seen in this figure, longitudinal buckling is critical for most of the unstiffened shell, framed inlet structure (concept 1); radial buckling is critical for most of the unstiffened shell, unframed inlet structure (concept 7). There are some similarities between the different structural concepts, however. The possibility of Foreign Object Damage on the forward part of the centerbody spike forces the design of all of the inlet concepts in that region to the higher FOD minimum gage value except for the unstiffened, unframed concept (concept 7). This indicates the importance of this type of failure to the inlet design, regardless of structural concept. Likewise, radial and longitudinal buckling as well as compression failure modes are critical to most of the centerbody designs. None of the inlet centerbodies were designed in any part by tension or shear failure. Failure of the inlet cowl structures was found to be generally due to minimum gage constraints, both FOD and non-FOD minimum gage, longitudinal buckling, tension, and shear.

The ring frames used for the generic inlet design were assumed to be Z-shaped cross-section. This concept is an efficient design common in airframe structures. It was found, however, that the minimum equivalent frame thickness was determined to be minimum gage for the entire structure. This indicates that the Z-shaped frames may not be optimum for the inlet, resulting in an overdesigned structure. This may be improved by changing the cross-section of the frame to a less-efficient concept. This has not yet been tested.

The minimum weight design process is necessarily iterative due to the dependence of the weight on inertial loads, which change as the structural thickness is modified. Results from each iteration required for the generic inlet design using structural concept 5 appear in Figure 8. As can be seen, the solution converges at a rapid rate. This does not occur because the initial weight guess is very close to the final result. The number of iterations does not change dramatically unless the initial guess is orders of magnitude away from the final result. Instead, this highlights the more significant influence of the applied pressure and aerodynamic loads as compared to the inertial loads in determining the inlet structural design. Therefore, reducing the pressures on the inlet has a more significant impact on decreasing the inlet design weight than reducing the size of the inlet.

Figure 9 shows the weight breakdown of the generic axisymmetric inlet for the different structural concepts. As can be seen, the inlet systems (bypass, actuators, controls) contribute a large percentage of the overall inlet weight. For this exercise, these weights were estimated from the INSTAL program (ref. 6). Non-optimum weights were computed as 20 percent of the structural weight. Monocoque shell design, concepts 1 and 7 in Table 1, resulted in inlets with the largest total and structural weights. This is to be expected, since these concepts do not have the benefit of longitudinal supports. While the truss-core concept with frames (concept 6 in Table 1) is heavier than the concepts with supports (concepts 2 through 5), the truss-core concepts without frames (concepts 8 and 9) are considerably lighter than the unstiffened shell without frames (concept 7). Again, this is an expected result since the truss-core provides longitudinal stiffness with less material than a monocoque structure. The fact that the truss-core concept with frames is heavier than the stiffened shell concepts with frames indicates that ring frames may not be required with truss-core panels. This may not hold true, however, with a less-efficient frame design.

The deflection of the concept 3 inlet centerline is plotted in Figure 10. The maximum deflection of 0.028 in occurs at the tip of the centerbody. The location of the inlet throat is indicated in the figure. Neither the local deflections along the inlet, nor the deflections at the inlet throat, exceeded specified limits (obtained from ref. 2).
Boeing MCTCB Inlet Design

The minimum weight design method was used to model the Boeing Mixed-Compression Translating Centerbody (MCTCB) axisymmetric inlet. Figure 11 shows a comparison of the weights for two inlets designed using the new method with the weight as reported by Boeing (ref. 2). Both method inlets were modeled using structural concept 3 in Table 1. The first method, "Method w/INSTAL", used estimates for the inlet systems weights from the INSTAL program, the same as the generic axisymmetric inlet discussed above. The total weight for this inlet was less than 1 percent lower than the MCTCB weight. The weight breakdown was similar, but discrepancies in the system weights were noticed. It was determined that the INSTAL weights were not accurately modeling the actual systems used in this inlet, leading to an overprediction in the system weights. Substituting the Boeing systems weights for the INSTAL weights gave a better estimate of this weight group ("Method w/Boeing Systems" in Figure 11). Comparison of the structural weights shows that the centerbody weight computed with the design method is lower than that presented by Boeing; the cowl weight, however, is heavier. The difference in the structural weights of the method inlet and the Boeing inlet is 4 percent. This is good agreement for this level of design. Likewise, the difference between the total weights of the two inlets is approximately 8 percent, also a reasonable result for conceptual design.

The results discussed above indicate that the method works well for inlets that can be reasonably modeled by stiffened shell structure, such as the MCTCB inlet. They also indicate that the assumption made concerning the inclusion of support structure in the analysis was a reasonable one, though the small differences between the computed weights and the Boeing numbers may be, in part, a result of this assumption. Another possible cause of discrepancies is the lack of any interaction between the inlet centerbody, internal cowl, and external cowl in the analysis. Load transfer from one component to another and stiffness resulting from the connection of one component to the next cannot therefore be accounted for. Comparison of method design results with inlets that are constructed using slats and seals, or that contain a large amount of mechanism for varying geometry, are not well modeled by this method.

Future Goals

While many high-speed inlet designs are based on an axisymmetric geometry, many others are based on two-dimensional design. The method for the minimum weight design of axisymmetric inlets will therefore be extended to the design of a two-dimensional inlet structure. Variable geometry design considerations are also important. Additional design requirements, such as those associated with inlet unstart and buzz, will be explored.

Conclusion

An analytical method for determining the minimum weight design of high-speed axisymmetric inlets has been developed. The method requires input from conceptual level inlet design tools, and is therefore applicable during the conceptual or preliminary design phases. After performing a loads analysis, the minimum required equivalent isotropic shell and frame thicknesses are determined for all possible failure modes. For a generic Mach 2.4 axisymmetric inlet, it was found that asymmetric hammershock pressure determined many of the maximum loads used for the design. Critical failure modes for the generic inlet were Foreign Object Damage, radial buckling, longitudinal buckling, and compression for the centerbody, and Foreign Object Damage, minimum gage, longitudinal buckling, tension, and shear for the cowl. Deflection analysis using the Moment-Area method indicated minimal deflections as compared to required design limits. Comparison of method results with the Boeing MCTCB study inlet showed total weight prediction within 8 percent, which is acceptable.
agreement for conceptual level design tools. In addition, the method produced the correct trends when changes to the inlet structural concept were made.

The minimum weight design method developed for axisymmetric inlets will improve the accuracy of the weight predictions used in propulsion and vehicle design studies. Weight versus cost studies will, similarly, have greater accuracy, which will allow designers to assess competing concepts with more confidence. The extension of the axisymmetric inlet design method to the design of two-dimensional inlet structures will increase its usefulness and potential to aid in conceptual and preliminary design studies.
References


Appendix 1. Material Data

Ti-6Al-4V: (Temperature: 350 F, ref. 7)

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<th>Property</th>
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</tr>
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</table>
Appendix 2. Applied Loads

Asymmetric Hammershock Resultant Load

The resultant force for the asymmetric hammershock load case is computed by assuming that the compressor stalls in the upper two quadrants of the inlet. Normal operating pressure is maintained in the lower two quadrants. This produces a "worst case" scenario, because the resultant asymmetric hammershock force acts in the same direction as the inlet weight, resulting in high bending moments and shear loads. Integrating around the top half of the cross-section gives

\[ F_h = -2p_h c \cos \beta (x - x_{i-1}) \]  

(2.1)

for the force due to the hammershock pressure. Similarly, the force due to normal operating pressure on the bottom half of the cross-section is given by:

\[ F_p = 2p c \cos \beta (x_i - x_{i-1}) \]  

(2.2)

The resultant force is

\[ R_h = F_h + F_p = 2r c \cos \beta (p - p_h)(x_i - x_{i-1}) \]  

(2.3)

Aerodynamic Maneuver Cross Force

Some aerodynamic maneuvers, for example, pitch and yaw, cause separation of the flow at the inlet lip, resulting in an asymmetrical flow (ref. 8). This distorted flow results in a loss of pressure recovery and produces an asymmetric force on the surfaces of the inlet exposed to the flow. This asymmetric force can be reasonably modeled as the cross force on a slender body of revolution at small angle of inclination with respect to a flow (ref. 9). Computed as a function of the dynamic pressure and the rate of change in cross-sectional area along the body, the cross force per unit length on the inlet is given by (ref. 9)

\[ f = 4 \pi c \gamma \frac{dy}{dx} \gamma \]  

(2.4)

where \( \gamma \) is the maximum yaw or pitch angle.
Appendix 3. Design Load Requirements

The design load requirements listed in Table 2 are discussed briefly below. Design loads produced are bending moments, $M$, shear loads, $V$, axial forces, $F_a$, and radial pressures, $P$. All structures are designed for ultimate load. The ultimate load factors associated with each design requirement described below are equal to the limit load factors multiplied by a factor of safety. The ultimate load factor will vary with design load requirement and applied load.

1. Landing: The critical load during landing is the inertial weight of the inlet. This load is asymmetric and will produce bending moments and shear loads. The bending moment of the structure ahead of the current analysis station is given by

   \[ M = Wn(x_c - \bar{x}) \]  

   and the shear load by

   \[ V_s = Wn \]  

   where all symbols are defined in Appendix 7. Both positive and negative moments and forces due to landing are analyzed.

2. Yaw Maneuver: The critical load for the computation of bending moments and shear loads generated during a yaw maneuver is also the inertial weight of the inlet. The design loads are therefore given as above, in equations 3.1 and 3.2.

3 - 4. Lateral Load With Nose Left Yawing Moment and Vertical Load With Nose Down Pitching Moment: These load conditions result from aerodynamic maneuvers that produce a cross force on the inlet. The cross force is combined with the inlet weight to determine the design bending moments and shear loads. These load conditions are only applicable on the portion of the centerbody structure exposed to the airstream and on the external cowl structure. The internal distortion produced by these maneuvers are believed to be less significant than the asymmetric hammershock condition and are, therefore, not considered (ref. 10).

The cross force per unit length is given in Appendix 2. The bending moment and shear loads are given by

   \[ M = Wn(x_c - \bar{x}) \pm M_c \]  

   and

   \[ V_s = Wn \pm F_c \]  

   where $M_c$ and $F_c$ are the sums of the bending moments and shear loads, respectively, produced by the cross force at each analysis station ahead of the current station, or

   \[ M_c = \sum_i f_i n(x_c - x_i) \]

   \[ F_c = \sum_i f_i n \]
5. Wind Gust: The load factors given by the wind gust condition act on the weight of the inlet and are found from (ref. 11)

\[ n = 1 \pm \frac{K_z U_w V_c C_{L_w}}{498(W/S)} \]  

where

\[ K_z = \frac{88\mu}{5.3 + \mu} \]  

and

\[ \mu = \frac{2(W/S)}{g\bar{c}pC_{L_w}} \]

\( U_w \) is given by the gust load design requirements. The bending moment and shear force are given by equations 3.1 and 3.2, above.

6. Asymmetric Hammershock: The resultant force on the inlet centerbody caused by an asymmetric hammershock condition combine with the inlet weight to produce bending moments and shear loads. These are given by equations 3.3 and 3.4, where \( M_c \) and \( F_c \) are replaced by the bending moments and shear loads, \( M_h \) and \( F_h \), respectively, that are produced by the resultant asymmetric hammershock force (see Appendix 2).

7-8. Normal Operating Pressure and Axisymmetric Hammershock: The critical applied loads for these cases are the internal pressure, normal or hammershock, and the inlet inertial load. All four design loads will be produced: bending and shear from the inertial loads, and axial and radial loads from the pressures. Bending moments and shear loads are given by equations (3.1) and (3.2) above. The axial force is given by

\[ R_s = 2\pi r_{\min} p n \sin \beta \]

where \( \beta \) is the local slope of the segment surface and \( r \) is the radius at the current analysis station. \( p \) is either the normal operating or hammershock pressure, depending on the current load case. The radial pressure is found from

\[ P = p n \cos \beta \]
Appendix 4. Minimum Weight Shell Design Equations

The equations used for the minimum weight design of a stiffened shell are described below.

**t-tension:** The minimum allowable smeared thickness for withstanding the tensile stress resultant is found by comparison with the tensile ultimate strength of the material:

\[
\tilde{t}_{\text{tension}} = \frac{\max(N_x^+, N_y)}{F_a}
\]  
(4.1)

if \(N_y\) is tensile.

**t-compression:** The minimum allowable equivalent isotropic (smeared) thickness for withstanding the compressive stresses, assuming no buckling, is found by comparing the maximum of the compressive and radial stress resultants with the compressive yield strength of the material:

\[
\tilde{t}_{\text{comp}} = \frac{\max(N_x^-, N_y)}{F_c}
\]  
(4.2)

if \(N_y\) is compressive.

**t-longitudinal buckling:** The minimum allowable smeared thickness for withstanding longitudinal buckling due to the compressive stresses for all structural concepts except the truss-core sandwich without frames is found by assuming the structure behaves as a wide column under compression. This is a reasonable assumption given that the shell thickness is small compared to its diameter (\(t / D < 0.1\), ref. 13). For \(t / D > 0.1\), this assumption does not hold. Induced bending moments from the axial force would also need to be considered for large \(t / D\). The truss-core sandwich concept without frames is modeled as a cylinder.

The general equation for the minimum weight design of a wide column with frames to withstand longitudinal buckling is (ref. 12)

\[
\tilde{t}_{\text{long}} = \left(\frac{N_x^- l}{E \varepsilon} \right)^{1/m}
\]  
(4.3)

where \(\varepsilon\) is a buckling efficiency factor dependent on the type of construction used, \(m\) is the buckling equation exponent, and \(l\) is a length parameter. Specific application of Equation 4.3 to the structural concepts listed in Table 1 are presented below.

1. Unstiffened wide column (concept 1) - While this is not a very efficient type of design, the equation for the minimum weight design of an unstiffened wide column is included for comparison (ref. 12). The equation is

\[
\tilde{t}_{\text{long}} = \left(\frac{N_x^- d^2}{.823E} \right)^{1/3}
\]  
(4.4)
The length parameter for this concept is the square of the frame spacing.

2. Unflanged, integrally stiffened wide column (concept 2) - The cross-section of an unflanged, integrally stiffened wide column is shown in Figure A4-1. The minimum weight equation for the maximum buckling efficiency ($\varepsilon=.656$) is (ref. 12)

$$\bar{t}_{\text{long}} = \left( \frac{N_d}{.656E} \right)^2$$

where the length parameter is equal to the frame spacing.

3. Z-stiffened wide column (concepts 3, 4, and 5) - The cross-section of a Z-stiffened wide column is shown in Figure A4-2. The minimum weight equation for the maximum buckling efficiency of this concept is Equation 4.5, with the buckling efficiency equal to .911, .760, and .760 for concepts 3, 4, and 5, respectively (ref. 12).

4. Truss-core sandwich wide column with frame (concept 6) - The cross-section of a truss-core stiffened wide column is shown in Figure A4-3. The minimum weight equation for the maximum buckling efficiency is also given by Equation 4.5, with $\varepsilon$ equal to .605 (ref. 12).

$$\bar{t}_{\text{long}} = \left( \frac{N_d I}{.605E} \right)^2$$

5. Unstiffened wide column without frame (concept 7) - The minimum weight equation for an unstiffened shell structure without ring frames is given by (ref. 3)

$$\bar{t}_{\text{long}} = \left( \frac{N_d r^2}{7.26E} \right)^{1/3}$$

The length factor for this concept is equal to the square of the shell radius.

6. Truss-core sandwich without frame (concepts 8 and 9) - The minimum weight equation for the truss-core sandwich column with no frame and maximum buckling efficiency ($\varepsilon=.4423$ and .3615 for concepts 8 and 9, respectively) is (ref. 3)

$$\bar{t}_{\text{long}} = \left( \frac{N_d r}{.4423E} \right)^{1/6.67}$$

where the structure is modeled as a cylinder and the length factor is equal to the shell radius.

\textbf{t-shear:} The minimum allowable smeared thickness for withstanding the tensile stress resultant is found by comparing the shear stress resultant with the allowable shear stress of the material:

$$\bar{t}_{\text{shear}} = \frac{N_{gy}}{F_s}$$
**t-radial buckling:** To calculate the minimum allowable smeared thickness for radial buckling, the structure is analyzed as a thin-walled shell with an external pressure. It was assumed that the shells were simply supported; this is felt to be more realistic than fixed edges since the ring frames can deflect elastically. Elastic buckling is assumed for all components. The minimum thickness is found by comparing the calculated yield stress with the design-allowable critical buckling stress:

\[ \sigma_y = \sigma_{cr} \]  
\[ \sigma_y = \frac{Pr}{t_{buck}} \]

and \( \sigma_{cr} \) is given by one of the cases below. Minimum weight equations are available for only three construction types. These are monocoque (no stiffeners), truss-core sandwich, and integral ring-stiffened. Only the first two are considered due to the complexity of the integrally ring-stiffened shell design. Stiffened shells are analyzed as monocoque. This is appropriate because it is the smeared thickness shell between the ring frames that will resist radial buckling. The smeared thickness computed is multiplied by \( K_p \), again to adjust for the fact that only the shell skin is available for resisting hoop stresses.

1. **Monocoque shell** - The design-allowable critical buckling stress for an unstiffened cylindrical shell subject to uniform external pressure is given by (ref. 12)

\[ \sigma_{cr} = \frac{1.1\pi^2}{12(1-\mu^2)^{3/4}} \left( \frac{L}{R} \right)^{3/2} \]

and the minimum thickness is given by

\[ t_{buck} = K_p R \left[ \frac{12(1-\mu^2)^{3/4} (L/R)}{1.1\pi^2} \right]^{2/5} \]

The above efficiency equation can be extended to a truncated cone by using the equivalent cylinder approach (refs. 12 and 14). The following modifications are dictated by this approach (ref. 14):

\[ Z = \sqrt{1-\mu^2} \left( \frac{l}{R_2} \right)^2 \left( \frac{R_2}{t} \right) \cos \alpha \]

\[ \frac{R}{t} = \frac{\eta_r}{\cos \alpha} \left( \frac{R}{t} \right) \]

\[ \eta_r = 0.60 \left( 0.70 + \frac{R}{R_2} \right) \]

The critical buckling stress and minimum thickness are then given by

\[ (4.10) \]

\[ (4.11) \]

\[ (4.12) \]

\[ (4.13) \]

\[ (4.14) \]

\[ (4.15) \]
\[
\sigma_{cr} = \frac{1.1\pi^2}{12(1-\mu^2)^{1/4}} \left( \frac{L}{R_2} \right) \left( \frac{i_{\text{buck}}}{R_2} \right) \left( \frac{\cos \alpha}{\eta_r} \right)^{1/2}
\]

and

\[
\bar{i}_{\text{buck}} = K_p R_2 \left[ \frac{12(1-\mu^2)^{1/4} \left( \frac{L}{R_2} \right) \left( \frac{\eta_r}{\cos \alpha} \right)^{1/2}}{1.1\pi^2} \right]^{3/5}
\]

(4.16)

2. Truss-core sandwich shell - The design of truss-core sandwich shells for radial buckling assumes that the shells act like rings under uniform pressure. The minimum thickness to preclude buckling is given by (ref. 12)

\[
\bar{i}_{\text{buck}} = K_p R_2 1.076 \sqrt{\frac{P}{E}} \frac{1}{\tan \theta} \left[ 2 + \frac{0.572 \left( \frac{P}{E} \right)^{1/2}}{\left( \tan \theta \right)^{3/8} \cos^2 \theta} \right]
\]

Minimizing with respect to the truss-core angle \( \theta \) and letting \( L/R \to \infty \) results in the minimum weight equation

\[
\bar{i}_{\text{buck}} = K_p R \left( \frac{P}{0.146 E} \right)^{1/6}
\]

for optimum \( \theta \).

For a truncated cone, the same modifications are applied as for the monocoque shell. This results in the minimum weight equation

\[
\bar{i}_{\text{buck}} = K_p R_2 1.076 \left( \frac{\eta_r}{\cos \alpha} \right) \sqrt{\frac{P}{E}} \frac{1}{\tan \theta} \left[ 2 + \frac{0.572 \left( \frac{P}{E} \right)^{1/2}}{\left( \tan \theta \right)^{3/8} \cos^2 \theta} \right]
\]

(4.17)

(4.18)

Minimizing with respect to \( \theta \) and letting \( L/R \to \infty \) as above results in

\[
\bar{i}_{\text{buck}} = K_p R_2 \left( \frac{\eta_r}{\cos \alpha} \right) \left( \frac{P}{0.146 E} \right)^{1/6}
\]

(4.19)

(4.20)

**t-minimum gage:** Two minimum gage constraints are applied to the structure, one based on manufacturability considerations, and one influenced by requirements for Foreign Object Damage (FOD). The minimum allowable thickness based on these minimum gage constraints is given by

\[
\bar{t}_{\text{ug}} = K_m t_{\text{g}}
\]

(4.21)
$K_{nt}$ is a shell minimum gage factor based on the structural concept used (ref. 3).
Appendix 5. Minimum Equivalent Frame Thickness Design Equations

The equations used for the calculation of the minimum equivalent isotropic thickness of the frames and for the frame spacing appear below. It is assumed that the frame thickness is smeared along the length of the centerbody according to $A_F = \bar{t}_F \bar{d}$ (ref. 3).

If the critical failure mode driving the structural design is longitudinal buckling (i.e., \( \bar{t}_{\text{min}} = \bar{t}_{\text{long}} \)), then the smeared frame thickness for a circular cross-section is given by (ref. 3)

$$\bar{t}_F = \left[ \frac{\pi C_F}{K_F E^3} \right]^{1/8} \left[ \frac{2\rho^3 N_r^2 r^2}{27\rho^3} \right]^{1/4}$$

and the optimum frame spacing is (ref. 3)

$$d = \left( \frac{6r^2 \rho_F E}{K_F E_F} \right)^{1/2}$$

where: \( r \) is the radius of the centerbody at station \( x \); \( C_F \) is Shaney's constant, which is based on the required frame stiffness to prevent general instability failure (ref. 5); and \( K_F \) is a shape parameter that relates the frame cross-sectional area with its moment of inertia \( (K_F = I_F / A_r^2) \). The value of \( C_F \) is taken as 6.25x10^{-5} (refs. 3 and 5). The value of \( K_F \) was found in reference 3 by looking at various aircraft frame structures.

If the inlet component under consideration is the inlet centerbody, then the optimum frame thickness and spacing must also be computed for radial buckling critical design. This applies only if the shell is not a truss-core concept. No dependency on frame spacing exists for truss-core concepts that are radial buckling critical (see Equation 4.18). The equations for optimum frame design when radial buckling critical are

$$\bar{t}_F, = 2r^2 \sqrt{\frac{C_F N_r \pi}{K_F d_r E_F}}$$

where

$$d_r = \left[ \frac{15}{2} \frac{r^{3/2} \rho_F C_F N_r \pi}{K_F (F_{ma})^{3/2} \rho (E F_K)} \right]^{1/2}$$

and

$$F_{ma} = \frac{12(1-\mu^2)\gamma}{1.1\pi E}$$

For the centerbody frames, the maximum of the two frame thicknesses, longitudinal and radial buckling, is chosen for the design with the associated frame spacing.
If the critical failure mode driving the structural design is not longitudinal or radial buckling, \( \tilde{t}_{\text{min}} \neq \tilde{t}_{\text{long}} \) or \( \tilde{t}_{\text{min}} \neq \tilde{t}_{\text{buck}} \), then the frame spacing is changed to force \( \tilde{t}_{\text{min}} = \tilde{t}_{\text{long}} \) or \( \tilde{t}_{\text{min}} = \tilde{t}_{\text{buck}} \). The new frame spacing for longitudinal buckling critical is given by (ref. 3)

\[
d = \frac{Ee \tilde{t}_{\text{min}}^2}{N_x}
\]

and the smeared thickness of the frames by Equation 5.3 above. The new frame spacing for radial buckling critical is given by

\[
d_r = \frac{r}{F_{\text{net}}p} \left( \frac{\tilde{t}_{\text{min}}}{K_F r} \right)^{\frac{1}{2}}
\]

The new frame thickness is given by Equation 5.3.

If the computed frame thickness is less than minimum gage, it is reset to the minimum gage value for the frame geometry. The new frame spacing will then be found from

\[
d = \left( \frac{C_F N_x \pi}{K_F E_F} \right)^{\frac{1}{2}} \left( \frac{2r^2}{t_{\text{frame}}} \right)^{\frac{1}{2}}
\]

for either longitudinal or radial buckling critical design.

For designs where the frame spacing is specified, the frame thicknesses will be given by Equation 5.3.
Appendix 6. Deflection Analysis Equations

The Moment-Area method was used to compute the deflections at each analysis station along the inlet centerbody and internal cowl. The Moment-Area method is based on the fact that the deflection of a beam can be determined from the moment of the area under the bending moment diagram divided by the flexural rigidity (ref. 15).

In general, the deflection at a point \( n \) is given by

\[
\delta_{o,n} = \int_{0}^{n} \left( \frac{M}{EI} \right) dx
\]  

(6.1)

Assuming constant \( E \), integration gives the following equation for the deflection at any analysis station along the inlet:

\[
\delta_{i} = \sum_{j=1}^{n} \frac{M_{j}}{EI_{j}} \left( x_{j} - x_{j-1} \right) \left( \frac{x_{j} + x_{j-1}}{2} - x_{i} \right)
\]  

(6.2)

where the moment of inertia for a circular shell cross section based on its average radius is given by

\[
I_{i} = \pi \left( \frac{y_{i-1} + y_{i}}{2} \right)^{3} t_{i}
\]  

(6.3)
Structural Concept

1 Unstiffened shell with frames
2 Simply-stiffened shell with frames, best buckling
3 Z-stiffened shell with frames, best buckling
4 Z-stiffened shell with frames, buckling/min. gage compromise
5 Z-stiffened shell with frames, buckling/pressure compromise
6 Truss-core shell with frames, best buckling
7 Unstiffened shell without frames
8 Truss-core shell without frames, best buckling
9 Truss-core shell without frames, buckling/pressure compromise

Table 1. Inlet Structural Concepts

Load Cases:
1. Landing
2. Yaw maneuver
3. Lateral load with nose left yawing moment
4. Vertical load with nose down pitching moment
5. Wind gust
6. Asymmetric hammershock
7. Normal operating pressure
8. Axisymmetric hammershock

Additional Constraints:
1. Minimum gage
2. Foreign Object Damage
3. Deflection limits

Table 2. Structural Design Load Requirements
Figure 1. Generic Axisymmetric Inlet Geometry

Figure 2. Generic Axisymmetric Inlet Pressures
Figure 3. Parametric Geometry From Input Data:
(a) Input Data; (b) Final Parametric Inlet Centerbody Geometry
Figure 4. Inlet Applied Loads: (a) Axisymmetric; (b) Asymmetric
Figure 5. Maximum Moments on Generic Axisymmetric Inlet

Figure 6. Minimum Equivalent Shell Thickness For Axisymmetric Inlet Centerbody Using Structural Concept 3
Figure 7. Critical Failure Modes For Generic Inlet Centerbody

Figure 8. Generic Axisymmetric Inlet Iteration Data For Structural Concept 5
Figure 9. *Generic Axisymmetric Inlet Weight Breakdown*

Figure 10. *Generic Axisymmetric Inlet Deflections For Structural Concept 3*
Figure 11. Weight Comparison Between Method and MCTCB

Figure A4-1. Cross-Section Geometry of an Unflanged, Integrally Stiffened Wide Column
Figure A4-2. Cross-Section Geometry of a Z-Stiffened Wide Column

Figure A4-3. Truss-Core Shell Geometry
**Abstract (Maximum 200 words)**

An analytical method for determining the minimum weight design of an axisymmetric supersonic inlet has been developed. The goal of this method development project was to improve the ability to predict the weight of high-speed inlets in conceptual and preliminary design. The initial model was developed using information that was available from inlet conceptual design tools (e.g., the inlet internal and external geometries and pressure distributions). Stiffened shell construction was assumed. Mass properties were computed by analyzing a parametric cubic curve representation of the inlet geometry. Design loads and stresses were developed at analysis stations along the length of the inlet. The equivalent minimum structural thicknesses for both shell and frame structures required to support the maximum loads produced by various load conditions were then determined. Preliminary results indicated that inlet hammershock pressures produced the critical design load condition for a significant portion of the inlet. By improving the accuracy of inlet weight predictions, the method will improve the fidelity of propulsion and vehicle design studies and increase the accuracy of weight versus cost studies.