The Mosaic Structure of Plasma Bulk Flows in the Earth's Magnetotail

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The mosaic structure of plasma bulk flows in the Earth's magnetotail

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Abstract. Moments of plasma distributions observed in the magnetotail vary with different time scales. In this paper we attempt to explain the observed variability on intermediate timescales of ~10-20 min that result from the simultaneous energization and spatial structuring of solar wind plasma in the distant magnetotail. These processes stimulate the formation of a system of spatially disjointed, highly accelerated filaments (beamlets) in the tail. We use the results from large-scale kinetic modeling of magnetotail formation from a plasma mantle source to calculate moments of ion distribution functions throughout the tail. Statistical restrictions related to the limited number of particles in our system naturally reduce the spatial resolution of our results, but we show that our model is valid on intermediate spatial scales \( \Delta x \times \Delta z \sim 1 R_E \times 1000 \) km. For these spatial scales the resulting pattern, which resembles a mosaic, appears to be quite variable. The complexity of the pattern is related to the spatial interference between beamlets accelerated at various locations within the distant tail which mirror in the strong near-Earth magnetic field. Global motion of the magnetotail results in the displacement of spacecraft with respect to this mosaic pattern and can produce variations in all of the moments (especially the x-component of the bulk velocity) on intermediate timescales. The results obtained enable us to view the magnetotail plasma as consisting of two different populations: a tailward-Earthward system of highly accelerated beamlets interfering with each other, and an energized quasithermal population which gradually builds as the Earth is approached. In the near-Earth tail, these populations merge into a hot quasi-isotropic ion population typical of the near-Earth plasma sheet. The transformation of plasma sheet boundary layer (PSBL) beam energy into central plasma sheet (CPS) quasi-thermal energy occurs in the absence of collisions or noise. This paper also clarifies the relationship between the global scale where an MHD description might be appropriate and the lower intermediate scales where MHD fails and large-scale kinetic theory should be used.

1. Introduction

It is generally believed that the transport of magnetic flux, energy, and momentum throughout the solar wind-magnetosphere-ionosphere system can be described in terms of large-scale convection controlled by the magnetospheric electric field. However, observations have shown that the sunward convection flows in the magnetotail which result from this large-scale convection exhibit a great deal of variability, ranging from a few tens of kilometers per second up to \( \approx 1000 \) km/s. This variability has led researchers to conclude that the large-scale convection pattern is highly changeable and distorted and may even be turbulent.

During the 1970’s the variability of plasma flow in the Earth’s plasma sheet on timescales of < 1 hour was studied mainly for geomagnetically disturbed periods [Hones et al., 1972; Hones and Schindler, 1979; Lui et al., 1977; DeCoster and Frank, 1979]. DeCoster and Frank [1979] used IMP 7 and 8 measurements to show the occurrence of high-speed flows at the boundary of the plasma sheet, moving away from the neutral sheet at 11 km/s. These initial satellite measurements by DeCoster and Frank [1979] showed for the first time that the occurrence of high-speed flows is a spatial rather than a temporal effect. Parks et al. [1979] demonstrated that the vertical motion of the plasma sheet has velocities ranging from 10 to 60 km/s. This was later confirmed by Andrews et al. [1981], Forbes et al. [1981a, b], and Eastman et al. [1983]. In addition, Forbes et al. [1981a] used two-spacecraft data to show for the first time that the PSBL flows consist of beams with Earthward-flowing plasma near the lobes and beams with tailward (or mixed Earthward/ tailward) flowing plasmas nearer the CPS. They found that the highest velocities occur at the outer edges of the PSBL, with lower velocities towards the CPS. A number of studies [Möbius et al., 1980; Andrews et al., 1981; Williams, 1981; Eastman et al., 1984, 1985; Takahashi and Hones, 1988] have established that the PSBL is nearly always present, and velocities as high as 1000 km/s are detected even during quiet times [Takahashi and Hones, 1988]. Takahashi and Hones [1988] estimated that the outermost
Earthward-directed beam was 0.2 to 0.7 RE thick and found the total PSBL thickness to be substantially more than an Earth radius.

The detailed flow structure of the CPS is not as well known. ISEE observations [Eastman et al., 1985; Huang et al., 1987; Huang and Frank, 1986] show a CPS characterized by isotropic plasma and small flows (< -10 km/s when \( AE < 100 \) nT). This situation is interrupted by very high-speed flows (> 400 km/s) of short duration during disturbed times [Huang et al., 1987; Ohtani et al., 1992]. Using Active Magnetospheric Particle Tracer Explorer (AMPTE) observations, Baumjohann et al. [1989, 1990], and more recently Angelopoulos et al. [1992, 1993], also found low-velocity flows (averaging 30–60 km/s) in the CPS. However, although the average velocities they found were quite small, there was a large amount of scatter about the mean velocities. This scatter was interpreted as evidence for the existence of fluctuating electric fields and turbulent transport [Angelopoulos et al., 1993].

Spacecraft usually output data in terms of time series, and although time/space variations cannot be uniquely separated by using data from a single spacecraft, a number of scientists have attempted to use the time series to interpret the observed variability in the magnetotail flow pattern in terms of temporal effects. The main object of this paper is to present an alternative explanation and show that much of the observed variability on intermediate time scales of ~1–30 min by satellites crossing the PSBL and CPS at \( x \sim 10–20 \) RE (such as ISEE satellites) can be interpreted adequately in terms of spatial structure.

In order to introduce the high variability of the flow in the plasma sheet and particularly the spatial nature of the flow in the PSBL, we start by presenting in Figure 1 a typical example of multiple crossings of the lobe-plasma sheet interface. A number of exit-entry sequences of the ISEE 1 satellite can be seen in this figure in which we plot the moments of the distribution function (obtained every 128 s in the ~1 eV to 44 keV range by the University of Iowa LEPEDEA instrument) and the electron fluxes at two fixed energies, ~1.5 keV and 6 keV, measured simultaneously by ISEE 1 and 2 [Anderson et al., 1978]. Immediately after ~1158 UT, ISEE 1 enters the PSBL from the lobe for the first time. This entry is marked by a sharp increase in the plasma density and is characterized by a high-speed flow of ~200–300 km/s directed Earthward and dawnward for a duration of ~10 min, followed by a sequence of small tailward and Earthward flows up to ~1215 UT. This enhancement of the bulk

![Figure 1](image-url)

**Figure 1.** Plasma parameters and electron fluxes from ISEE 1 and 2 satellites, between 1100 and 1400 UT on April 19, 1978. From top to bottom, plasma ion and electron densities, flow velocities \((V_x, V_y, V_z)\) measured by ISEE 1, and 4-s averages of electron fluxes at two fixed energies, ~1.6 and ~6 keV, measured by ISEE 1 and 2 satellites using GSM coordinates (RE).
flow is marked by alternate changes in both the $v_x$ and $v_y$ directions. The second brief crossing of the PSBL occurs near $-1249 \text{ UT}$, as evidenced by the abrupt increase of the Earthward-directed bulk flow up to $\sim-450 \text{ km/s}$. ISEE briefly exists the PSBL (for $\sim10 \text{ min}$) and enters again around $-1312 \text{ UT}$. Two consecutive enhancements of the tailward and duskward flow can be detected then. Finally, during the last entry, occurring at $\sim1326 \text{ UT}$, once again a high-speed Earthward flow of $\sim-50 \text{ km/s}$ is encountered.

Observations indicate that a majority of the PSBL crossings seen during the $90 \text{ min}$ interval were caused by the relative vertical motion of the boundary. High-resolution ($4 \text{ s}$) electron fluxes can be used to determine the vertical motion of the boundary. The time delays between the electron profiles at the onset of electron increases were used in conjunction with the known separation distance ($\sim150 \text{ km}$ in the $z$ direction) between the spacecraft to determine the velocity as being $\sim-12-20 \text{ km/s}$ in the case of the first crossing, occurring around $1200 \text{ UT}$. Assuming that this vertical velocity remains constant during the $\sim1200-1215 \text{ intervals}$ the thickness of each high-speed flow is of the order of $2300-3600 \text{ km}$. It is important to note here that the detailed ion spectrum between $1159 \text{ UT}$ and $1201 \text{ UT}$, during which period the boundary moved $\sim3000 \text{ km}$ (see Figure 11a of Parks et al. [1984]), is consistent with a mix of even narrower spatial components of different energies, flowing Earthward and tailward. The second encounter with the PSBL around $-1249 \text{ UT}$ occurs when the vertical velocity of the tail is much faster, $\sim482 \text{ km/s}$, so that the thickness of the outer high-speed beam can be estimated to be $\sim1 \text{ RE}$. At $-1326 \text{ UT}$, due to a partial loss of data on ISEE 2, the vertical velocity can only be estimated as being $\sim50 \text{ km/s}$; this corresponds to a vertical extent of $1 \text{ RE}$ for the outer beam.

This example clearly shows substructures in the PSBL flow. A crossing of the PSBL (and/or the CPS) that takes $10-30 \text{ min}$, in reality corresponds to a spacecraft having a relative motion through successive and narrow layers with thicknesses ranging from $0.3$ to $1 \text{ RE}$. On these scales the flow in the outer regions of the plasma sheet appears to be not as well organized and supports frequent changes in amplitude and direction. We refer to this timescale as intermediate because it is lower than MHD spatial and temporal scales but faster than the time scales of typical plasma waves and instabilities.

To investigate the structure of the plasma sheet we carried out a large-scale kinetic (LSK) simulation of magnetotail ion distributions. The main idea of the LSK calculations is to follow the trajectories of noninteracting particles exactly in given realistic B and E field models. By recording the position and velocity components of the particles we are able to calculate the distribution function and the resulting moments. Using the Tsyganenko [1989] magnetic field model, a constant dawn-dusk electric field, and assuming a plasma mantle source, Ashour-Abdalla et al. [1991a, b, 1993] used LSK to form a plasma sheet populated by hot plasma and composed of a well-defined CPS and PSBL. From these calculations, Ashour-Abdalla et al. [1993] studied the microphysics of the magnetotail, including the shape and detailed features of the ion distribution functions. They found intrinsic structures (beamlets) in the distribution functions which they attributed to the nonadiabatic interaction of the plasma with the current sheet. On the global scale, Ashour-Abdalla et al. [1994] took moments of the distribution functions throughout the tail and showed that the overall picture obtained agreed with the observed global plasma patterns. In that paper they also showed that although the model used was not self-consistent, ion distributions in the magnetotail approximately satisfied local stress balance conditions.

In this paper we will go one step further and look at the relationship between the large- and small-scale structures in the tail with emphasis on the structure of the convection system. We find an "intermediate" spatial scale that falls between the large-scale (MHD picture) and the small-scale and is on the order of $1 \text{ RE}$ in length and $\sim1000 \text{ km}$ in width. This intermediate scale results from the interaction and interference of numerous structures accelerated in different regions of the distant magnetotail. Following this introduction in section 2 we discuss the properties of large-scale flows ($\gtrsim3 \text{ RE}$) and show that in this limit the results agree in general with an MHD approach. As stated above, although we postulate the existence of beamlets [Ashour-Abdalla et al., 1993], we have not discussed their consequences in the magnetotail. We undertake this task in section 3 by first reviewing our understanding of the formation of beamlets and then giving an analytical estimate of the number of beamlets one would expect to see in the chosen magnetotail configuration. In section 3 we also address the natural concern of how these structures survive in the magnetotail after their formation and how long they keep their identity under the influence of numerous effects of smearing and scattering pertinent to the tail conditions.

The interference and overlapping of numerous plasma structures accelerated in different regions of the magnetotail leads to the formation of a complicated mosaiclike pattern of plasma bulk parameters. Numerical ramifications of these mosaic structures are presented in section 4. Since the plasma in the tail consists of two populations, one which has suffered a reasonable amount of scattering and one consisting of beamlet particles, we show in section 5 how these two populations coexist and eventually mix. Finally we summarize our results and argue that the basic physics of the tail requires the existence of these spatial structures.

2. Large-Scale Kinetic Modeling and Global Structure of Plasma Flows

2.1. Model

In this paper we use the approach of large-scale kinetics, which has been previously described in detail [Ashour-Abdalla et al., 1993]. In our study we use a two-dimensional (2D) reduction of the Tsyganenko [1989] magnetic field model with an $x$-type neutral line $100 \text{ RE}$ downtail (see Ashour-Abdalla et al. [1993] for details) along with a constant, dawn-to-dusk directed convection electric field of $0.1 \text{ mV/m}$, typical for quiet magnetospheric periods. Throughout this paper we assume a coordinate system with $z$ northward, $x$ antisunward, and $y$ toward dawn. In this coordinate system $x = 0$ is at Earth's center, and $z = 0$ coincides with the current sheet plane. The width of the tail in our model is $25 \text{ RE}$ with the dawn flank lying at $y = 12.5 \text{ RE}$ and the dusk flank of the tail at $y = -12.5 \text{ RE}$. The ion sources are located in the plasma mantle at $x = 15 \text{ RE}$, $z = \pm 3 \text{ RE}$ and extend from $y = -12.5 \text{ RE}$ to $y = +12.5 \text{ RE}$ in $1 \text{ RE}$ increments. Because of the small thickness of the plasma mantle, it is a good approximation to neglect its thickness in $z$. The finite spread would produce a velocity spread during the first crossing that is less than the thermal velocity and much less than the convection velocity in the current sheet. Each source distribution is a drifting Maxwellian with a temperature of $300 \text{ eV}$ and a field-aligned bulk flow velocity of $200 \text{ km/s}$. Particles are collected at a series of virtual detectors at various positions in $x$, $y$, and $z$. When
2.2. Global Structure of Plasma Flows

Before trying to determine the physics of smaller scale ( \( \leq R_E \) ) structures of the plasma sheet, it is necessary to first review the large-scale pattern of plasma bulk flows in the magnetotail region.

Figure 2 illustrates the large-scale distribution of plasma bulk flows in the equatorial region of the magnetotail. This plot was obtained by binning the bulk velocities of ions in \( 3 R_E \times 3 R_E \) x-y bins and averaging to obtain \( V_x, V_y, \) and \( V_z \) for each bin. Because of symmetry, \( V_z = 0 \) in this plane. The length of the arrows correspond to the magnitude of the velocity in the plane (scale shown at the top of the figure) and the direction of the arrow gives its angular orientation. As one can see the average flow velocities in the x-direction are Earthward in a large part of the magnetotail, and the typical magnitude of these flows (~50 km/s at \( x = 20 R_E \) ) corresponds fairly well to the value of the local convection velocity in the x direction \( V_c = E_y / B_0(x) \). Closer to the Earth (\( x \leq 15 R_E \) ) the x component of flow becomes very weak, and all flows are predominantly directed duskward. The duskward flow becomes overwhelming near the “wall” region [Ashour-Abdalla et al., 1992] where the magnetic field (and therefore pressure) have strong gradients. The duskward component of the average flow vectors is largest near the duskward flank of the model magnetotail. This average pattern conforms reasonably well with spacecraft data taken in the quiet CPS, especially near the noon-midnight meridian where the results are least influenced by the two-dimensionality of the model [e.g., Baumjohann et al., 1989; Angelopoulos et al., 1993]. The results shown here are averaged over large spatial bins which correspond to the procedure normally used in statistical analysis of satellite data [e.g., Angelopoulos et al., 1993]. In the next section, in which we deal with local results, we will see how rich in small-scale structure the detailed picture can be in comparison with this MHD-like large-scale pattern. We will also show that the level of variability in the flows (\( \Delta V / V \) ) is much larger than those in the density and pressure distributions.

3. Origin of Beamlet Structure

In the distant tail the conservation of the magnetic moment \( \mu \) breaks down and ions can no longer be described by guiding center trajectories. The behavior of ions there is principally governed by the parameter of adiabaticity

\[ \kappa = \left( \frac{R_{\text{curv}}}{R_{\text{max}}} \right)^{1/2} \]

where \( R_{\text{curv}} \) is the minimum radius of curvature of field lines achieved at \( z = 0 \) and \( R_{\text{max}} \) is the maximum value of the ion Larmor radius [Büchner and Zelenyi, 1989]. The parameter of adiabaticity \( \kappa \) can also be written as

\[ \kappa = \sqrt{\frac{R_{\text{curv}}}{\rho_{\text{max}}}} \left( \frac{e^2}{2m_i} \right)^{1/4} B_n R_0^{-1/2} L^{1/2} \tilde{W}^{-1/4} \]

Equation (2) relates \( \kappa \) with the plasma and magnetic field parameters. Here \( B_n \) is the normal magnetic field, \( B_0 \) is the lobe magnetic field, \( \tilde{W} \) is the energy of the particles in the de-Hoffman-Teller frame corresponding to convection of particles in the dawn-dusk electric field \( E_y \), and the circumflex indicates that we define the dimensionless parameters \( \kappa \) and \( W \) in a moving frame.

If the local convection velocity \( V_c = E_y / B_0(x) \) is small compared to the thermal energy of the particles, then \( \tilde{W} \approx W \). However, in the distant tail where \( B_n(x) \) is small, \( V_c \gg V \) and \( \tilde{W} \approx m_i V^2 / 2 = m_i E_y^2 / 2 B_0^2(x) \). \( \kappa \) acquires the approximate form [Ashour-Abdalla et al., 1993]

\[ \kappa = \left( \frac{e}{m_i} \right)^{1/2} B_n^{3/2} L^{1/2} R_0^{-1/2} \tilde{W}^{1/2} \]

When \( \tilde{\kappa} < 1 \), the Larmor radius is larger than \( R_{\text{curv}} \) and the guiding center approximation is violated [Büchner and Zelenyi, 1989]. However for \( \tilde{\kappa} < 1 \) the invariant,

\[ I_z = \tilde{\kappa} V_c \tilde{W} \]

[see Speiser, 1965; Sonnerup, 1971; Büchner and Zelenyi, 1989] is approximately conserved. In this paper we use \( I' \), which is a normalized dimensionless parameter related to \( I_z \) by

\[ I' = \left( \frac{3\pi}{8} \right)^{1/2} \left( \frac{eB_0}{m_i L} \right)^{1/2} \tilde{W}^{-3/4} I_z \]

where \( W \) is the energy of the particle in the given frame. The role of \( I' \) in quasi-adiabatic theory is similar to that of \( \tilde{\alpha} = \mu / \mu_{\text{max}} = \sin^2 \phi \) in guiding center theory (where \( \phi \) is the pitch angle of the particle) [Büchner and Zelenyi, 1989]. \( I' \) also can be written as a function of \( \alpha_0 \) and \( \beta_0 \), the angles of the particle’s velocity.
vector with respect to the equatorial plane (\(z = 0\)) during its equatorial crossing [Büchner and Zelenyi, 1989]:

\[
I' = \sin^{3/2} \sigma_0 \sqrt{\frac{\sin \beta_0}{2}}
\]  

(6)

where \(f(k) = (1-k^2)K(k) + (2k^2-1)E(k)\), and \(K(k)\) and \(E(k)\) are full elliptical integrals.

For certain values of \(\kappa\) called resonant values \((\kappa_R)\), \(I'\) remains small. Particles with \(\kappa - \kappa_R\) experience minimum scattering during their interaction with the current sheet. The resonant \(\kappa\) values are given by

\[
\frac{C(I')}{\kappa_R} = N + \frac{1}{2}
\]

(7)

where \(N\) is an integer and the function \(C(I')\) is given by [Ashour-Abdalla et al., 1993]

\[
C(I') = 4\sqrt{\frac{\pi}{\Gamma(3/4)}} \frac{\Gamma(3/4)}{\Gamma(1/4)} + O(I')
\]

(8)

The term with \(I'\) is small even for \(I' - 1\), so we can neglect the dependence on \(I'\) and use \(C(I') = \text{const} = 0.763\). This value is very close to the numerically obtained \(C = 0.8\) [Büchner, 1991] and \(C = 0.84\) from a corresponding expression found by Burkhard and Chen [1991]. For the present discussion it is important that these local conditions be written assuming a local deHoffman-Teller (HT) frame for each interaction. This gives

\[
\kappa_R = \frac{C}{N + 0.5}
\]

(9)

The occurrence of resonances is consistent with the results of Burkhart et al. [1991], who found fluctuations in density and current resulting from energization during neutral-line reconnection, and with Moses et al. [1993], who found that neutral line acceleration of ions resulted in the formation of bunchy distributions. The solid line in Figure 3 is a plot of \(\hat{\kappa}(x)\) for the T89 model, with the scale shown on the left of the figure. Using this curve and (9) we can obtain the locations \(X_N\) where resonance occurs. Figure 3 also shows the locations of resonance (marked \(X_1-X_7\)) and the regions of maximum trapping (marked \(X_2^*-X_6^*\)). The energy of the particles accelerated at each resonance location according to \(dW = 2mV^2(x)\) is also shown (dashed curve), with the scale on the right of the figure. We refer to each substructure accelerated at a given resonance \(N\) as beamlet number \(N\). The total number of beamlets is limited because higher order resonances overlap and are smeared into one.

\[
\hat{\kappa}(x)
\]

\[
W(x) \quad (\text{keV})
\]

Figure 3. Schematic showing \(\hat{\kappa}(x)\) (solid line, scale shown on left of figure) calculated for the T89 model, and \(W(x)\) from Lyons and Speiser, [1982] (dashed line, scale on right). The locations of resonance regions \((X_1-X_7)\) are shown with particle distributions; the maximum trapping regions \((X_2^*-X_6^*)\) are marked on the horizontal axis.
Let us now estimate the maximum number of resonances that might be obtained in a given magnetic field. The field reversal region of the magnetotail can be characterized by two main parameters: the thickness of the region, $L$ (which is assumed to be constant in the T98 model), and the magnetic field component normal to the current sheet plane ($B_D(x)$). Since the model contains a neutral line at $X_N = 100$ RE, $B_D(x)$ can be approximated by a power law dependence over the large region of the tail extending from $x \sim 40$ to $100$ RE:

$$B_n(x) = B_{np} \left( \frac{X_N - x}{X_N - X_p} \right)^q$$

where $B_{np} = B(x_p = 40$ RE) corresponds to the value of $B(x)$ at $X_p = 40$ RE. For the T98 model, $B_{np} = 1.4$ nT and $q = 1.59$.

Higher order resonances occur in the region of small $\hat{k}$, where the Larmor radius of the particles in the current sheet plane becomes larger. This provides a natural thickness for the resonance. Thus we assume that the resonances will overlap if

$$X_N - X_{N-1} = \hat{\rho} = \frac{\hat{V}}{\Omega \kappa (X_N^*)}$$

where $\hat{V}$ is the velocity and $\Omega \kappa$ is the gyrofrequency at a point between the resonance positions. Here we consider the problem again in the local HT frame and estimate $\hat{\rho}$ at some intermediate point $X_N^*$ between resonances number $N$ and $N-1$:

$$\hat{k}(X_N) = \frac{C}{N + 1/2}$$

$$\hat{k}(X_{N-1}) = \frac{C}{N - 1/2}$$

$$\hat{k}(X_N^*) = \frac{C}{N}$$

(12a)  (12b)  (12c)

The position $X_N^*$ is chosen rather arbitrarily but the exact position of $X_N^*$ has only a minor influence on the final results. The main variation in $\hat{k}(X)$ in both the (2) and the (3) regimes comes from the variation of $B_D(x)$, so below we assume that the other parameters ($B_D$, $L$) in the definition of $\hat{k}$ remain constant.

Let us start with the case $E_v = 0$ (or $V > V_c(x)$), which is the case for the $N = 1$ and $N = 2$ resonances in our model. From (2), (11) and (12) we obtain for the magnetic field profile approximated by (10):

$$N_{max} = \left( \frac{X_N - X_p}{L\rho_0} \right)^{q+1} \left( \frac{\hat{\rho}^{q+1}}{\kappa_p^{q+1}} \right)^{1/2}$$

$$E_v = 0, \text{ or } V \gg V_c(x)$$

(13)

Here $\rho_0$ is the Larmor radius of ions in the field $B_0$ and $(L\rho_0)^{1/2}$ is the thickness of the meandering region. For our model we estimate $X_N - X_p \sim 60$ RE, $(L\rho_0)^{1/2} = 0.5$ RE, and $\kappa_p - 0.7$, and for $C = 0.76, q = 1.5$, we get $N_{max} = 7$.

This calculation overestimates the number of beamlets because it neglects the additional smearing related to convection. In the region of overlapping higher-order resonances the effective Larmor radius significantly enhances $\hat{\rho} = (E/B_D(x))(eB_D(x)/m_i)$ due to the presence of this convective electric field, and this should influence the overlapping of neighboring resonances. However the values of $\hat{k}$ and $\kappa$ decrease simultaneously as a result of the increase in $E_v$. Therefore the final result is not easy to predict.

If we perform a transformation to the local HT frame and assume for simplicity that $V_c(x) > V_0$ (i.e., $\hat{k}(x)$ is described by (3)) we again get from (11) and (12) the expression for $N_{max}$:

$$N_{max} = \left( \frac{X_N - X_p}{L} \right)^{q+1} \left( \frac{\rho_0}{B_p} \right)^{q+1} \left( \frac{L}{\rho_\kappa} \right)^{q+1} \left( \frac{2}{3q} \right)^{q+1} C^{q+1}$$

$$E \neq 0, \text{ or } V_c(x) \gg V$$

(14)

Here we have the same parameters as in (13), except that $\rho_\kappa = (E,B_0)/(eB_0,L)$ is the Larmor radius in the $B_0$ field based on the convection velocity in lobe region. All terms except the first are very close to 1. Estimating $N_{max}$ for $X_N - X_p \sim 60$ RE and $L \sim 1$ RE gives

$$N_{max} \sim \left( \frac{X_N - X_p}{L} \right)^{q+1} \sim 60^{q+1} \sim 4 - 5$$

(15)

As one can see from (14), $N_{E=0}^{max}$ includes a very weak dependence on $E$ given by

$$N_{E=0}^{max} \sim E^{(1-\rho)(1/2^q+2)} \sim E^{-1/25}$$

This weak dependence means that the effects of the increase of $\hat{\rho}$ (which enhances the overlapping) and the reduction of $\hat{k}$ (which makes the separation between resonances larger) almost compensate each other. It is interesting to note that for the weak dependence of $B_D(x)$ on $x$, when $q < 1$, the number of structures could even grow with an increasing $E$.

With the assumption $V_c(x) > V_0$ the energy of each beamlet can be estimated:

$$W_{bN} = \frac{m_i V_c^2(X_N)}{2} = 2(N + 0.5)^4 \left( \frac{E_v}{B_p} \right)^2 \left( \frac{L}{\rho_\kappa} \right)^{2/3}$$

(16)

One very important point in the comparison of our results with those obtained by Huang et al. [1987] and Burkhardt and Chen [1991] is that they considered the one-dimensional current sheet to have a fixed $B_0$ so that in a resonant condition like that in (7) the only parameter they could vary was energy. There is no fundamental disagreement in the general physics of our results because, as we have shown above, even the numerical value of the resonant parameter $K_\rho$ could be reproduced with very good accuracy by using separatrix theory. The principal difference is in the application to the real magnetotail with its convection electric field. The energy of the particles interacting with the current sheet in the regions of effective acceleration $(m_i V_c^2(x)/2)$ is determined by the convection velocity, and is not a free parameter that can be varied arbitrarily. Formally it can be
defined as the difference between the usual energy dependent \( \chi'(W, R_0) \) defined by (2) and the \( \chi \) defined by (3), which is already energy independent and which should be used for regions in which accelerated structures occur. The structures we expect to obtain in the tail are spatial, not structures in velocity space, as was suggested by Huang et al. [1987]. Analytical theory predicts that the combined processes of acceleration and structuring which take place during the interaction of particles with the current sheet produces a finite number (4–6) of strongly accelerated ion beams highly collimated along field lines. Particles accelerated at locations in between resonances are scattered and form a second plasma population with distinct properties.

Analytical theory has limited applications in modeling a realistic tail (even 2D) because many simplifying assumptions are needed to reach analytically tractable results. The parameter \( \chi \) is usually not small enough for the assumptions we have used to be valid in the main part of the tail: \( \chi \) becomes greater than 1 as Earth is approached, so the diffusion-like picture based on small jumps of \( \Delta \mu / \mu \ll 1 \) usually fails for large regions of the tail. Thus the use of numerical techniques is unavoidable in analyzing this problem. Because the primary acceleration occurs in a region where \( B_\|= \) and the corresponding \( \chi \) remain small, analytical theory generalized to include the effects of convection and the gradients of tail parameters in \( \chi \) can be used as a guide to interpret modeling results. It is clear that because of the considerable number of simplifications made in setting up the analytical theory, one should practice caution when comparing the analytical and the numerical results.

4. Propagation and Interference Pattern of Beamlets

4.1. Propagation of Beamlet Particles

In this section we consider what happens to the structures (beamlets) after their formation at the \( X_N \) \((N = 1, 2, \ldots, 5)\) locations in the distant tail. Each of these beamlets has a different velocity (16), and during propagation to Earth they experience \( \mathbf{E} \times \mathbf{B} \) drift in the lobe \((-B_\|) \) magnetic field. The velocity filter effect [Green and Horwitz, 1986; Liu and Hill, 1986] influences their distribution in space in the course of their Earthward propagation. In Appendix 1 of Ashour-Abdalla et al. [1993] this problem was analyzed for the simplified model \( \mathbf{B} = B_0 \frac{x}{L} \mathbf{e}_x + B_0 \left( x, z \right) \mathbf{e}_z \), and it was shown that the velocity filter effect can result in an increased separation of beamlets with distance. Although in a realistic magnetic field model the distance will diminish due to convergence of field lines, the result is still valid that beamlets maintain their identities while propagating towards Earth (see (AB)–(AB) from Ashour-Abdalla et al. [1993]). Near Earth some beamlet particles are precipitated and others are reflected. Precipitated beamlet particles manifest themselves as velocity dispersed ion structures (VDIS) [Bosqued, 1987; Zelenyi et al., 1990; Ashour-Abdalla et al., 1992; Bosqued et al., 1993] characterized by a latitude-energy dispersion consistent with distant tail beamlet acceleration (higher energies at higher latitudes).

Nonprecipitating beamlet particles form a tailward-propagating beam, which in turn interacts with the current sheet and can be accelerated again in the manner discussed above. It is unlikely that this beam will hit the current sheet at another resonance location \( X_N \), and therefore after this and subsequent interactions with the current sheet it will gradually be scattered into the quasi-isotropic population (see section 5). Although some elements of this process could still be described analytically the picture becomes increasingly complicated and to understand it we should return to numerical results.

Plate 1 shows the trajectories of one particle from each beamlet and illustrates this pattern. As we will see below (section 5), the beamlet particles move as a group and do not diverge in space for a relatively long time. Although individual beamlets remain distinct, near-Earth reflection causes Earthward- and tailward-moving beamlets to intersect each other throughout the magnetotail. In the next subsection we discuss the consequences of the interference of beamlets.

4.2. Interference of Streaming and Counterstreaming Beamlets

As we saw in Plate 1, multiple Earthward-moving beamlets should intersect with the tailward parts of the other beamlets. If we assume (and the assumption is supported by modeling results) that the beamlets retain their identity after at least two interactions with the current sheet (they may do so even longer), then we will have about \( 2N_{\text{max}} \) tailward-moving and \( 2N_{\text{max}} \) Earthward-moving filaments. Taking into account that the Earthward-moving part of the outermost beamlet does not intersect any returning beams, we can estimate that the number of such intersections in the northern hemisphere is about \( 2N_{\text{max}} - 1)^2 - 80 \). This rough estimate gives the impression that the appearance of such interference regions (about \( 100–200 \) of them) is a common feature of the magnetotail. One such interference region is identified in Plate 1(\( x = 40 \) \( R_E \), \( z = 4 \) \( R_E \)).

The velocity dispersion of PSBL beams is such that beam velocities are higher at higher \( z \) [e.g., Forbes et al., 1981a; Takahashi and Hones, 1988]. At the same time, convection moves reflected beams to a lower \( z \) than the Earthward-directed beams. As a beamlet moves Earthward or tailward \((\mathbf{E} \times \mathbf{B})\) converges toward the neutral sheet. A tailward-moving (reflected) beamlet may therefore intersect the adjacent Earthward-moving (original) beamlet. Assuming that the beamlets have approximately the same density, the net effect at an intersection (illustrated schematically in Figure 4) is a small but tailward directed bulk velocity. Each beamlet actually has a smooth profile \( n(z) \), and Figure 4 is highly simplified. Actually beamlets are spatial structures that have a density maximum in their centers. Positive and negative density gradients occurring at the edges of beamlets result in a magnetization current.

At interference regions, together with compensation for net bulk flow, one should see an enhanced (approximately doubled) plasma density that results from the overlapping of beamlets. This effect might be related to the experimental finding by Baumjohann et al. [1990] of an anticorrelation of bulk velocity and density in the PSBL. Each local enhancement of density (and therefore of pressure) should also result in peaks of \( V_\| \) where there are sharp local density gradients produced as a result of this interference.

We illustrate the interference of beamlets in the \( x-z \) plane in Figure 5. This figure is a schematic representation of an interference region (shown with box in Plate 1) and shows an Earthward propagating beamlet (beamlet \( k = 1 \)) interfering with the reflected portion of beamlet \( k \). Since beamlets in the PSBL and outer CPS (OCPS) are predominantly field-aligned and have a relatively small convection velocity, their interference should occur at small angles resulting in overlapping regions being long (in \( z \)) in comparison with the thickness of the beamlets.
Beamlet Particle Trajectories

Plate 1. Trajectories of five beamlet particles launched from the plasma mantle projected onto the x-z plane. Rectangular box denotes interference region.

It is clear from this schematic that the dimension in z, \( \Delta z \), is simply due to the spatial extent of the beamlet whereas the dimension in x, \( \Delta x \), results from the interference of beamlets. This interference pattern of beamlets leads to structuring on an "intermediate" scale. The intermediate scale is \( \Delta x \approx 1 \, R_E, \Delta z \approx 1000 \, \text{km} \).

As discussed above, a number of unusual effects appear within these regions, which can be well within the PSBL or the OCPS, such as a dramatic decrease of bulk velocity, an increase of density, localized peaks of \( V_x \), and variations in the \( B_x \) magnetic field due to enhanced pressure. The occurrence of such rapid changes in a dense region without large bulk flows could be misinterpreted, for example, as exits from the PSBL or different plasma clouds [e.g., Belova et al., 1987]. The beamlet interference region has two counterstreaming populations, a situation that is very unstable to the generation of a number of instabilities, especially broadband electrostatic noise (BEN) [Schriver and Ashour-Abdalla, 1990, 1991]. We might expect that regions of enhanced BEN would have a mosaiclike or patchy pattern in the PSBL.

4.3. Magnetotail Plasma Flows

As we argued in the preceding subsection, interference of multiple beam structures should result in a complicated patchy or mosaiclike pattern of plasma distributions in the magnetotail. Elements of this mosaic could be as small as the thickness of beamlets and therefore should scale as fractions of \( R_E \) (say 1000 - 2000 km). In previous publications we used virtual detectors at planes of constant x and interpolated to obtain an average pattern of plasma moments in the x-z plane [Ashour-Abdalla et al., 1993, 1994]. Although this method provided very convincing evidence of structuring along the z axis, the use of interpolation between detectors placed several \( R_E \) apart smeared all of the small-scale effects. The advantage of using \( x \) detectors was that we averaged numerical data in the y direction, thus improving our statistics and could prove the existence of beamlets.

To identify the fine-scale pattern in meridional planes without large-scale averaging we used a detector along the noon-midnight meridian (y = 0). This provided high spatial resolution of our structures, at the cost of poorer statistical results. The total number of particles in the simulation plays an important role in improving the results so that they are statistically reliable. Using only 100,000 or even 200,000 particles, we found large variations in the moment values in each bin. These values rapidly converged for results calculated with 300,000 or more particles in
the system. Figure 6 is a plot of $V_x$, $V_y$, and $n$ as a function of $x$ for different total numbers of particles in the calculations. Here we show results for 300,000 particles (dotted curve), 400,000 particles (dashed curve) and 500,000 particles (solid curve). We see that the results for all three cases are virtually identical and the intermediate scale emerges explicitly. In this particular case ($z = 0$) we find $\Delta x = 2 R_E$, which is equivalent to several bins in $x$. To be able to resolve the intermediate scale, we require a bin size several times smaller than $\Delta x$. In order to have statistically meaningful results, we use 500,000 particles in our simulation.

Plate 2a shows the density distribution in the $y = 0$ plane. On a large scale, the features of this pattern resemble those seen in our previous published work [Ashour-Abdalla et al., 1994, Plate 2] but the fine-scale structure is new and difficult to predict. On a smaller scale one would expect that beamlets in general and overlapping beamlets in particular would result in regions of enhanced density. In the PSBL, there are localized, sharp-edged density dropouts and increases, and this structuring is especially pronounced in the distant tail. This intermittency is manifested not only in the PSBL, but also, surprisingly, in the CPS and even in the current sheet near $z = 0$. After reflection in a stronger field near the Earth, beamlets supply an enhanced particle flux to certain locations in the CPS in addition to the quasi-isotropic scattered population from ions trapped in the CPS. Beamlets arriving in the current sheet cause regions of enhanced density, and when they leave for the PSBL there are density dropouts. On a larger scale there is a large density dropout in the CPS near $x = 60 R_E$, a position too far Earthward to be populated by particles from the mantle and too far tailward for particles leaving the CPS at larger $x$ to form beamlets to have returned. The picture also shows a clear transition from a thin to a thick plasma sheet at near $x = 70 R_E$.

The most surprising structures in Plate 2 are in the velocity profiles. The component of bulk velocity in the $x$-direction ($V_x(x,z)$) (Plate 2b) shows how variable and patchy the pattern of bulk flows in the magnetotail can be even for quiet times. One exception is the outermost edge of the PSBL, where the most highly accelerated particles are found. This large outermost Earthward beamlet does not experience interference from other beamlets and has a strong Earthward bulk velocity streaming with $V \sim 700$ km. The ISEE results [Forbes et al., 1981a; Takahashi and Hones, 1988] have shown that the highest speed Earthward beam is effectively the first seen when entering the surface layer of the PSBL from the lobe and is a characteristic signature of the PSBL. The pattern in this plate is even more patchy than that for density, because we have mixed regions of strongly negative (Earthward flow, blue), strongly positive (tailward flow, yellow) and small bulk velocities (when oppositely directed beams superpose, purple). We see the manifestations of distinct beamlets as well as the regions of overlapping beamlets. The small scale size of the elements of this mosaic, sometimes less than $1 R_E$ across, yield up to several tens of individual elements. The intermittency of bulk flow velocities is astonishing but is not due to numerical and/or statistical effects; instead it is the result of the physical picture described above. We can easily foresee that a spacecraft engulfed by this mosaiclike external plasma sheet will detect alternate sheets of Earthward/tailward flows as shown in the experimental example of Figure 1.

Plate 2c shows the distribution of duskward and dawnward $V_y(x,z)$ components of the bulk velocity. This pattern is different from that for $V_x$ or density as the scale of the patches is smaller than for the $n(x,z)$ or the $V_x(x,z)$ pattern. The spikes in $V_y$ should result from ion gyroradius effects at the edge of each beamlet in which there is a local peak in density, and they should occur twice as often as the local density peaks. These duskward flows are important even though they are small in comparison with the $V_x$ flows in the PSBL, because they can become the dominant component of flow deeper in the CPS where $V_x$ is small on average. Another interesting feature of the $V_y(x,z)$ pattern is the very distinct diamagnetic effect occurring in a very narrow strip at the outermost edge of the PSBL, that is, at the edge of the strongest Earthward beamlet where there is a large gradient in density. Here the duskward flow occurs because of the

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Figure 5. Schematic of beamlet interference representing the area in the rectangular box shown in Plate 1. In this picture $\Delta x \sim 1 R_E$ and $\Delta z \sim 1000$ km.
noncompensated ion gyration in this beamlet. Its thickness can be used to estimate the density gradients between the PSBL and the tail lobes.

We will now explain a large-scale feature evident in Plate 2, but not manifested in our previous results. This is a throatlike transition from a thin plasma sheet to a thick one occurring in Plates 2a–2c at $x = 65 R_E$ which shows a strong dominant duskward velocity at large $x$ (Plate 2c) and a reversal in the $V_x$-velocity at $x = 80 R_E$, as shown in Plate 2b. The schematic shown in the top panel of Plate 3 illustrates that in the distant tail where the primary acceleration takes place, particles traverse very large distances across the current sheet in a meandering mode of motion when they are trapped within the region. Because the gyration of meandering ions near $z = 0$ is directed duskward in the current sheet, we see strong duskward $V_y$ flow in this region.

The position of the throat (at $x = 65 R_E$), that is, the location where the blue (Earthward) beam starts to emerge from the current sheet in Plate 2b, can be understood from the following considerations. In order for particles to leave the current sheet plane at $y = 0$, they must complete their roughly semicircular rotation in the equatorial magnetic field, that is, $2 \beta (x_{entry}) < 12.5 R_E$ where $x_{entry}$ is the x location at which the ion enters the current sheet. Particles entering the current sheet beyond the region where the condition can be met do not become beamlet particles at $y = 0$. Particles that meet this condition can cross the $y = 0$ detector as beamlet particles, but only at lower $x$, because during their gyration in the $B_n$ field fast convection (large $E/B_n$) will bring them to significantly smaller $x = x_{entry} - x_{exit} \sim (E/B_n)/(\Omega_n)$. The reversal in $V_x$ flow, as one can see from Plate 2b for $y = 0$ occurs at $x = 80 R_E$, that is, at larger distances than the “throat,” but significantly closer to Earth than the position of the X line. As one can see in the bottom panel of Plate 3, a plot of the $x$ component of bulk velocity in the $z = 0$ plane, the position of the reversal (visible as a black line between orange and blue areas), has the same value ($x \sim 80 R_E$) at $y = 0$, but its location moves tailward in the $-y$ direction. Near the dusk flank more meandering particles exit from the current sheet because the room for their gyration near $z = 0$ is larger and the
Plate 2. Moments of the distribution function in the $y = 0$ plane (noon-midnight) obtained by collecting plasma data in $5800 \times 500$ km bins. (a) For the density plot the color coding extends from 0.0 (blue) to 0.6 cm$^{-3}$ (red) on a linear scale. (b) For the $x$ component of the bulk velocity the color coding extends from -500 (yellow) to +500 (blue) km/s. (c) For the $y$ component of the velocity the color coding extends from -200 (yellow) to +200 (blue) km/s.
reversal in $V_x$ shifts to larger $x$ toward dusk. In the vicinity of the $X$ line, particles have such a large $\beta$ that they move roughly in the $-y$ direction and the bulk flow is determined by incoming tailward moving mantle particles and is directed tailward. The result is a reversal of the bulk flow velocity well Earthward of the nominal position of the $X$ line (100 RE in our model). This may shed light on some controversies in interpreting ISEE 3 plasma and magnetic field data which give different locations for the distant tail neutral line from plasma and magnetic field measurements. Our point is that due to nonadiabatic ion orbits, these locations of the flow reversal and the $B_n(x)$ reversal should not coincide.

Despite the fact that we used a 2D magnetic field model, the plasma distributions we obtained have an appreciable $y$ dependence, and the pattern obtained at another $y$ detector might be quantitatively different from that shown above, although the main features we observe at $y = 0$ RE should appear in a wide range of $y$ values. The pattern we get at more duskward $y$ values ($y < 0$ RE) might be even more complex because the fastest beamlets moving in the PSBL have access to this detector (acceleration is equivalent to displacement in $y$ in our model). On the other hand, the dawnside, which is populated with less energetic plasma (i.e., with fewer beamlets), should be less structured and might look more homogeneous. In the last section of this paper we will show some initial results of three-dimensional (3D) modeling and demonstrate that a mosaic-like pattern of magnetotail bulk parameters exists in a 3D system as well as in a 2D system, and therefore is not an artifact of the two-dimensionality of the model discussed thus far.

5. Coexistence and Transformations of Chaotic and Regular Populations in the Magnetotail

The results of 1D models used by Chen and Palmadesso [1986] and Büchner and Zelenyi [1986] that have been used to chart the development of chaos in a Hamiltonian system cannot be automatically applied to the realistic magnetotail. These models assume that there is unlimited time for the evolution of the system. However, as we have argued in a previous publication [Ashour-Abdalla et al., 1991b], the presence of even a weak convection electric field limits the number of interactions
of particles with the current sheet in the $\hat{x} < 1$ region and therefore hinders the development of chaos in the system. In this section we concentrate on the basic interaction of ions with the current sheet and delineate differences in ion behavior that come about because of local parameters of the current sheet where the first interaction occurs. These interactions are important because there are very few resonant positions of scattering in $I'$ and an interaction that occurs between these positions could produce a large scatter in $I'$. Thus it is possible to be very specific in speaking about the chaotization of ions in the magnetotail.

A well-known quantitative characteristic of chaotic behavior is the Lyapunov exponent. This method was successfully used by Martin [1986] to describe the chaotization near the X line. The Lyapunov exponent $\lambda$ gives the divergence of trajectories in phase space with time as

$$|\Delta d(t)| = |\Delta d_0| \exp(\lambda t)$$  \hspace{1cm} (17)

where $\Delta d_0$ is the distance in phase space between two neighboring particles at an initial moment and $|\Delta d(t)|$ is the change of this distance with time. For particles on neighboring integrable orbits this parameter should remain very small, since $|\Delta d(t)| - |\Delta d_0|$, while for particles on chaotic trajectories (i.e., those sensitive to initial conditions) it is the rate of their divergence.

Plate 4 illustrates the difference between the properties of particles accelerated at a "resonant" location (i.e., beamlet particles) and those of particles accelerated in the gaps between them. The top panel of Plate 4 shows the $x$-$z$ projection of the orbits of two beamlet particles. One particle in this panel is colored red, and the second particle is colored blue. Both ions are launched from the plasma mantle and cross the current sheet with a separation of $0.1 R_E$ (initial points are shown in the top panel of Plate 4). This plot clearly shows that the beamlet ions follow very similar trajectories for a long period of time. The lower panel in Plate 4 shows the $x$-$z$ projection of the orbits of two nonbeamlet particles. One particle in this panel is colored red, and the second particle is colored blue. Both ions are launched from the plasma mantle and cross the current sheet with a separation of $0.1 R_E$, the nonbeamlet ions quickly diverge in space. We can therefore conclude qualitatively that beamlet ions stay together and are capable of traveling in a coherent bunch to the PSBL, while nonbeamlet ions are trapped in the CPS, are scattered and quickly diverge in space.

To make the preceding argument more quantitative, we calculated the parameter $\lambda$ in (17) by considering the spatial separation of the pairs of particles as a function of time. The results of this calculation are shown in Figure 7, where $\lambda$ is
plotted as a function of time. We see that for the beamlet particles \( \lambda \) is more than 1 order of magnitude less than that for the pair of nonbeamlet particles, that is, the nonbeamlet particles diverge at a much faster rate than the beamlet ions. For large times, both pairs of ions approach Earth, and the separation distances become small because of the compression of magnetic field lines. Although \( \lambda \) calculated by such a method is not exactly the classic value of the Lyapunov exponent (our \( \lambda \) is for divergence only in geometrical space), it has a similar physical meaning. So we see that due to nonmonotonic properties of particles scattering in the tail, two distinct accelerated populations form during the first interaction of the mantle ion flow (which can also be considered a "coherent" stream) with the current sheet. Accordingly, the destiny of these two populations will be quite different. Scattered ions (which comprise the chaotic part of plasma population) have a sufficiently large \( l' \) value to be trapped in the vicinity of the current sheet on short cucumber-shaped orbits and convect towards Earth, and one may speculate that the behavior of this part of the population might resemble that indicated by MHD analysis (we do not attempt to construct a rigorous proof of this speculation). Moreover as the period of quasidiabatic orbits scales as \((l')^{-2}\) [e.g., Buchner and Zelenyi, 1989], initially scattered orbits with large values of \( l' \) have smaller orbital periods, and these ions interact with the current sheet much more often during their Earthward convection than do beamlet particles. This again enhances their scattering, and this part of the distribution becomes rather quickly randomized over the angles. The situation with the part of the distribution is analogous to the evolution of the distribution of guiding center particles experiencing scattering in pitch angles. Since pitch angle diffusion tends to restore the angular symmetry of the distribution by making it isotropic, one could argue that the same effect should produce scattering over the values of \( l' \). To verify this hypothesis numerically we calculated the values of \( l' \) for each particle at the moment it crossed the equatorial plane, that is, our \( z = 0 \) detector. For these calculations we used the analytical expression (6) relating the value of \( l' \) with the angles \( \theta_0 \) and \( \beta_0 \) for the moment when the particle crossed the equatorial plane. This enabled us to obtain the distribution function \( f(l') \) all over the magnetotail equatorial plane. The results of this calculation are plotted in Figure 8, which shows \( f(l') \) averaged over \( \Delta x = 10 \, R_E \) bins. Accordingly, the top panel in Figure 8 represents the average for the interval \( 10 \, R_E < x < 20 \, R_E \), and the bottom panel represents the average over the interval \( 90 \, R_E < x < 100 \, R_E \). Of course, this averaging is too rough to reveal individual structures associated with resonances, but it gives us a very interesting picture of the overall evolution of \( f(l') \) towards an isotropic angular distribution. The dotted line in each of these plots corresponds to \( f(l') \) for an isotropic angular distribution calculated by Ashour-Abdalla et al. [1991]. In the distant tail (bottom panel), where mantle ions are first incident on the neutral sheet, there is an absence of ions with large \( l' \) values. Closer to Earth, between \( x = 70 \) and \( 90 \, R_E \), the distribution shifts towards larger \( l' \) values, as ions with small \( l' \) (beamlet particles) are ejected from the current sheet. Earthward, the beamlet particles remain to the current sheet (60 \( R_E < x < 70 \, R_E \)), and the distribution slowly becomes isotropic (compare this with the dotted line showing \( f(l') \) for a 1-keV Maxwellian distribution). This is consistent with observations reporting that the particle distributions in the middle and near-Earth parts of the magnetotail are nearly isotropic.

Figure 8 clearly illustrates the complicated structure of magnetotail plasma as basically containing two populations: (1) a bunched (collimated, "coherent") field-aligned part of the population associated with beamlets; energy gained by beamlet particles during their interactions with the current sheet is manifested in the form of the kinetic energy of their directed (roughly field-aligned) motion. This can be referred to as acceleration of beamlet particles, (2) As for other "scattered" parts of the population, the energy gained during the particles' interactions with the current sheet (as we argued above on an average it should have a value equal to approximately half of the beamlet energy) is manifested in a "randomized" form resembling thermal energy. Therefore in this case it is more accurate to call this process the energization of the population. As we will discuss in another paper the energy distribution created in the course of such acceleration should contain an essential non-Maxwellian tail, which might be described by a \( \gamma \) distribution with a particular value of \( \gamma \) depending on the topology of the magnetic field.

The "scattered" (chaotic) and regular (beamlet) populations coexist in the magnetotail until the regular (beamlet) population gradually converges to the scattered (chaotic) one as a result of the unavoidable scattering which occurs during each subsequent interaction with the current sheet.

The answer to the question of the extent to which deterministic chaos influences the formation of the ion distribution function in the magnetotail is two-fold: (1) these effects are very important for the scattered part of the magnetotail population and rather quickly randomize it; (2) for the beamlet population, which experiences far fewer scattered interactions, these effects become important only in the near-Earth tail and do not prevent the existence of very well defined structures in the vast regions of the distant and middle parts of the tail. Of course, both the strong chaotic scattering and almost regular (Speiser-type) acceleration are just two aspects of the same nonlinear process of ion interaction with a magnetic field reversal.

Plate 5 shows the near-Earth transition to the almost completely randomized system. The plate has four panels with distribution functions \( f(x, z) \) taken at four neighboring \( x \) detectors \( x = 30, 20, 12, 10 \). In this plate the ion distribution is plotted in \( z-x \) space at different \( x \) locations. At \( x = 30 \) the magnetotail distributions have the "usual" form with a rather hot (\( W \sim 6-7 \, keV \)) central plasma sheet and a very well defined PSBL with streaming-counterstreaming accelerated ion beams having large energies.

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**Figure 7.** Plot of \( \lambda \) versus time (analog of Lyapunov exponent) for the two pairs of beamlet and nonbeamlet particles shown in Plate 4.
or wave particle interactions which usually are assumed as transformation of directed energy into the randomized energy of absolutely necessary ingredients of such a transformation. We PSBL the quasi-thermal energy of the tail is reached with its strong magnetic field, more and more process may be referred to as a purely collisionless "geometrical" simple geometrical reasons. By the time the near-Earth part of the (up to 16 keV in our model). In moving from x = 30 to x = 10 the PSBL streams gradually become narrower and lose energy, while the CPS becomes denser and hotter. These plots illustrate how the very large directed energy of PSBL flows is transformed to the quasi-thermal energy of CPS plasma. As our model does not assume the existence of specific dissipation mechanisms, this process may be referred to as a purely collisionless "geometrical" transformation of directed energy into the randomized energy of the CPS plasma.

The heating of the CPS occurs in the absence of any collisions or wave particle interactions which usually are assumed as absolutely necessary ingredients of such a transformation. We called this transformation geometrical because it occurs for rather simple geometrical reasons. By the time the near-Earth part of the tail is reached with its strong magnetic field, more and more PSBL (beamlet) particles get reflected and experience multiple interactions with the current sheet until all the particles get scattered and all the beamlets are entangled and mixed into one hot quasi-isotropic CPS population. In convecting further towards Earth this distribution acquires a new anisotropy \((P_x > P_z)\) as a result of the predominance of betatron acceleration over Fermi acceleration [Cowley and Ashour-Abdalla, 1975; Ashour-Abdalla et al., 1994].

This collisionless transformation of the directed kinetic energy of PSBL beams (or beamlets) into a quasi-isotropic CPS population resembles the process of chaositization of trajectories by multiple reflections from the walls in so-called mathematical billiards. This phenomenon definitely deserves much more theoretical attention, but here we address only its role in beamlet evolution, because it provides a sort of sink (CPS plasma) where eventually almost all beamlet particles bring their energies, get mixed, and lose their identities.

6. Summary and Discussion

In this paper we develop further the idea of spatial structuring of magnetotail plasmas which is intrinsically related to the nonlinear properties of the acceleration of solar wind protons during their interactions with the current sheet. We started with the example of ISEE measurements indicating the existence of beamlets (spatial structuring) in the near-Earth \((-20 R_E\) plasma sheet boundary layer and continued with a numerical analysis of their interference and evolution in the magnetotail. Let us first summarize our findings.

1. The PSBL is formed of well-defined spatially separated beamlets, each characterized by strong Earthward or tailward flows. At the edges of the beamlets diamagnetic effects can produce significant dawn or duskward flows. The spatial distribution of the beamlets and their velocity dispersion are fully consistent with the separatirix theory of nonlinear acceleration and structuring.

2. The presence of beamlets also results in the significant structuring of the CPS. Although the tailward/Eathward velocities in the CPS are much lower than in the PSBL due to the geometry of the field density, inhomogeneities created by the beamlets can result in significant duskward/dawnward flows in this region.

3. We have shown that beamlets keep their identities until they have undergone up to a few interactions with the current sheet, and the interference of tailward- and Earthward-moving parts of various beamlets create a complicated mosaiclike spatial distribution of plasma bulk parameters almost throughout \((z > 65 R_E)\) the magnetotail.

4. Magnetotail plasma is characterized by the coexistence of two well-defined populations: (1) an accelerated "ordered" population with small \(I'\) values having its energy mainly in the form of the kinetic energy of field-aligned motion and (2) a "scattered" energized population with large \(I'\) having its energy mainly in a quasi-thermal form and produced by the chaotic scattering of ions during their interaction with the current sheet. The relative contribution of these populations varies along and across the magnetotail. The distant CPS is formed mainly by the second population; nearer Earth, reflected beamlet particles from the first population also make an important contribution to the CPS population. The PSBL consists predominately of accelerated population (first population) ions.

5. In the near-Earth tail the accelerated and the energized populations finally intermix with each other and form a hot, almost isotropic plasma sheet. The kinetically directed energy of the majority of the beamlet particles (except a small precipitated
fraction) is converted to the thermal energy of the CPS. The
distribution function over \( l' \) evolves into the distribution characteristic for the isotropic one.

In all the results listed above the key element is the presence of spatial structuring found in our large-scale kinetic model. However, each model has its limitations, and one could argue that this principal effect will survive further generalizations of the model.

The important question, which will require more theoretical effort and which cannot be analyzed in the present modeling setup, is in what way do self-consistent wave particle interactions and particularly broadband electrostatic noise (BEN) contribute to the smearing of the beamlets. Results from previous studies [Schriver and Ashour-Abdalla, 1991] indicate that the intensity of BEN is usually low enough so that it does not cause appreciable spatial smearing of the beamlets and may result in some widening of their initially very narrow velocity distribution.

Another concern might be related to the two-dimensionality of the model discussed above. Since the mechanisms responsible for the formation of structures are local, the generalization of our results to 3D should not alter the basic picture, even though the spatial distribution and velocity dispersion of three-dimensional structures might be quite different than those for two-dimensional space. This conjecture was tested in our new series of three-dimensional runs of the LSK model. The model itself and the main result drawn from it will be discussed in detail in another


**X Bulk Velocity in the Noon-Midnight Meridian**

Plate 6. The x component of the bulk velocity calculated in the noon-midnight meridian for a 3D simulation using the Tsyganenko magnetic field model and an electric field derived from the Heppner and Maynard [1987] ionospheric potentials.

The existence of the mosaic pattern of the bulk plasma parameters in the tail motivates very important consideration of the relationship between the MHD and the kinetic descriptions of the magnetotail plasma. One of the most striking results of LSK modeling of plasma distributions is how the plasma in the magnetotail can differ on certain spatial scales from the convecting fluid implied by the MHD approximation. As we have shown recently, we know from calculating moments of plasma distribution functions [Ashour-Abdalla et al., 1994] after spatial averaging, that the LSK picture is basically consistent with the results of the MHD approach. The pattern of large-scale flows shown in Figure 2 also shows this agreement. However, a multitude of new physical effects emerges when we look at this distribution with a "magnifying glass" and retain the finer-scale spatial effects. As we have argued above, the plasma in the tail consists of a combination of a quasi-isotropic (scattered) population, which may be described by a fluid approximation and a separate system of interpenetrating interfering beams weakly interacting with each other. There are no mechanisms for the relaxation of such a structure on the timescale of their convection toward the Earth until, after numerous reflections in the inner central plasma sheet, they become naturally geometrically entangled and isotropized. By this process they collisionlessly convert their directed energy into quasithermal energy. MHD definitely fails to describe this process.

This situation in a way resembles that of collisionless shocks. On an average (at very large scales) they should always satisfy the MHD Rankine-Hugoniot conditions, but in collisionless plasmas they have very complicated internal structures which provide the real physical mechanisms of plasma relaxation and conversion of directed energy into heat.

A similar consideration might be relevant to essentially non-Maxwellian and nonisotropic magnetotail plasmas, which can be viewed as a convecting fluid only in a global spatial scale, but at intermediate spatial scales these plasmas should already have a rather sophisticated internal structure.

As we have shown above, results from the large-scale averaging of the bulk flow pattern in the CPS shown in Figure 2 conforms well with the E x B convection motion of the plasma and therefore agrees with the experimental data similarly averaged over large scales [e.g., Angelopoulos et al., 1993]. However, an analysis of the same pattern for smaller (intermediate in our notation) scales demonstrates a completely different picture.

Figure 9 shows the scatter plots of the bulk velocity obtained from our modeling results at $x = 30\ R_E$ in the $\Delta z \leq 1.5\ R_E$ interval around the equatorial plane. To produce these plots we averaged the value of $V_x$ inside the small pixels (5800 × 1600 km) from which our mosaic pattern is produced. The values of $V_x$ are similar in few neighboring pixels (which altogether form the patchy mosaic with the given value of $V_x$). As we can see the scatter is large, of the order of the average value of $V_x$. Angelopoulos et al. [1993] obtained similar results in the quiet CPS but interpreted them as oscillations of the electric field.

We, however, propose in this paper a different physical mechanism for explaining this scatter or variability of $V$. Each velocity vector shown in our plot corresponds to the actual bulk velocity measured in the particular spatial location coinciding with our pixel, and that should be exactly the value of bulk velocity that will be "instantly" measured at this location by spacecraft.
This scatter therefore reflects the real complexity of the magnetotail plasma and the existence of structures in it, that is, principally the non-MHD behavior. (Recall that in our model, flow is stationary ($\partial B/\partial t = 0$) and $E = E_y = \text{const.}$) Differences between the MHD and large-scale kinetic descriptions become especially dramatic in the very distant tail where due to the weakness of $B_L(x)$, the regions of non-MHD behavior could become huge: in the tens of $R_E$. We have discussed a very pronounced example of it at the end of section 4 in relation with the $20-R_E$ difference in reversals of $v_y$ and $B_L(x)$. This is a purely non-MHD effect related to rather "exotic" properties of ion trajectories in this region (very large local Larmor radius). The same effect on smaller (but still relatively large scales) occurs at other parts of the tail.

As we demonstrated here, the required variability of the plasma sheet on intermediate scales can be maintained by a constant and homogeneous electric field, and only in situ direct measurements of $E$ are capable of giving its local value; that is, it cannot be directly derived from MHD equations at small and intermediate scales. So although on large spatial scales the magnetopause as well as its numerical model behave like an MHD object, it may be meaningless to use MHD equations for making direct comparisons with local in situ measurements (unless they are averaged over large spatial regions).

Thus the existence of structuring brings new physical mechanisms to intermediate (below MHD) scales. The local picture now appears to be quite different from the large-scale one. Each process has its intrinsic scales and in moving to finer and finer resolution we should see quite new effects, which should not be surprising. One could even speculate that the hierarchy of scales is not exhausted at the large-scale kinetic mosaic pattern. By developing more sophisticated models we might see even smaller scale effects. The construction of this hierarchy (with our mosaic being just one of its steps) is truly a challenging goal, but one which is beyond the scope of this particular paper.

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