Preamble.

The early progress reports on this project (Radok 1991, 1992; Radok and Brown 1993) borrowed concepts and terms from the Neyman-Pearson theory of hypotheses and spoke of "inverting" the sequential analysis of Wald (1947) by determining decision probabilities from the data, rather than prescribing them. A reformulation of the project's basic idea for the fourth progress report (Radok and Brown, 1995) has led to the "change index", a simpler concept without equivalent in previous work. This development traded theoretical sophistication for expanded usefulness in statistical exploration and is reflected in the simpler title of this final report:

"Detecting change in progress"

by Uwe Radok and Timothy J. Brown*

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Summary

When one or more new values are added to a developing time series, they change its descriptive parameters (mean, variance, trend, coherence). A "change index (CI)" is developed as a quantitative indicator that the changed parameters remain compatible with the existing "base" data. CI formulae are derived, in terms of normalized likelihood ratios, for small samples from Poisson, Gaussian, and Chi-Square distributions, and for regression coefficients measuring linear or exponential trends. A substantial parameter change creates a rapid or abrupt CI decrease which persists when the length of the bases is changed.

Except for a special Gaussian case, the CI has no simple explicit frequency distribution that could be used to delineate critical regions for tests of hypotheses. However, its design ensures that the series sampled need not conform strictly to the distribution form assumed for the parameter estimates. The use of the CI is illustrated with both constructed and observed data samples, processed with a Fortran code "Sequitor" (Appendix B).
1. Introduction

Modern statistics came into being and developed, on the fields of the British agricultural research station Rothamsted, from carefully designed randomized experiments and results obeying strictly valid frequency distributions with well-defined population parameters. But from the start there was another reality, consisting of uncontrollable observational data from the natural and human environments - an untidy mess of scattered, trended, abruptly changing numbers with stretches of "statistically controlled" variation around quasi-constant central values. An example is shown in fig.1a. When a sufficiently large number of such data have been assembled an apparently orderly behavior may be constructed a posteriori (fig. 1b), but often the past is no reliable guide to a developing future, which then needs to be examined "sequentially" anew with each accruing observation.

A full analysis of time series with such short-lived quasi-steady regimes demands consideration of two disparate scales. A long time scale serves to define frequency distributions of regime lengths, regime sequences, and transitions between different regimes (cf. e.g. Olberg 1977). Here we shall be concerned exclusively with the other time scale: that of individual regimes, each of which needs to be considered on its own because its sample parameters constitute the full information available.

2. Probability argument.

The basic assumption made is that each observation has a probability \( p(x;\theta,\theta',\theta'\ldots) \) of occurring where the parameters \( \theta \) remain valid or change as new observations are added. For a sample of \( n \) observations the product of the \( n \) individual probabilities define its "likelihood" \( L(n;\theta,\theta',\theta'\ldots) = p_1 p_2 \ldots p_n \). In the simplest situation, two different estimates for a single parameter \( \theta \) constitute a "null hypothesis" \( H_0 (\theta=\theta_0) \) which will be rejected in favor of the alternative hypothesis \( H_1 (\theta=\theta_1) \) when the data fall into a "critical region" of the
Fig. 1a: Deviations of the Arctic annual mean temperature from its 1946-1960 mean (adapted from Raper et al. 1983). Note the abrupt changes of mean around 1920, 1955, and 1980; the trend changes around 1905 and 1940; and a variance change around 1968.

Fig. 1b: The original continuous interpretation of the same data.
n-dimensional sample space. The critical region is constructed to allow only a small probability $\alpha$ for $H_0$ in fact being true; $\alpha$ represents an "error of the first kind" and is also known as the "size" of the critical region. In the remainder of the sample space the acceptance of $H_0$, when in fact $H_0$ is true occurs with another probability $\beta$, known as "error of the second kind".

The likelihood ratio $L(n;\theta_1)/L(n;\theta_0)$ serves to define a best critical region if it exists. Its choice is to ensure that, for a prescribed small error of the first kind $\alpha$ (probability of rejecting $H_0$ though true), the error of the second kind (probability of rejecting $H_0$ and accepting $H_1$ in the remainder of the sample space) is as small as possible. This implies also that the probability of correctly accepting $H_0$ in the critical region becomes $1-\beta$.

A possible optimum critical region can be found, according to the Neyman-Pearson theory of hypotheses, by making the likelihood ratio $L(n;\theta_1)/L(n;\theta_0)$ larger than a constant $k$ in the critical region, and smaller than $k$ outside it. Intuitively this makes sense since the denominator is small in the critical region while the numerator (representing the probability $1-\beta$) should be as large as possible to minimize $\beta$.

The distribution of the likelihood ratio directly defines best critical regions but, as shown by Kendall and Stuart (1967, ch. 24), its form in most cases is known for large $n$ only. For one special case of practical importance (see Kendall and Stuart, example 24.1), however, the distribution of the likelihood ratio is an explicit function of Student's $t$; this will be further discussed below. Without such an explicit distribution function the likelihood ratio can still serve for exploratory tests which confirm $H_0$ when the likelihood ratio is near 1 and rejects it when the ratio is large. A sequential "change index" for such a test will now be described.

It employs four likelihoods $L(n;\theta_n)$, where $\theta_n$ is a parameter estimate derived from $n$ observations $x_i$, $i = 1,2,3...$ with probability $p(x_i;\theta_n)$ of occurring. Two of the likelihoods
use optimal ("maximum likelihood") parameter estimates, \( \theta_b \) for a base sample of \( m \) and \( \theta_{b,i} \) for one of a series of "augmented" samples of \( n = m+j \), \( j = 1, 2, \ldots \). The remaining two use \( \theta_b \) for the augmented sample, and \( \theta_{b,i} \) for the base sample. Thus \( L(m;\theta_b) = L_1 \), say; \( L(m+j, \theta_{b,i}) = L_1 \); \( L(m+j, \theta_b) = L_2 \); and \( L(m, \theta_{b,i}) = L_3 \).

The likelihood ratios \( q(m+j) = \frac{L_1}{L_2} \) and \( q(m) = \frac{L_1}{L_3} \) compare maximum likelihood estimates of \( \theta_b \), using the full samples, with "cross-over" likelihoods using parameter estimates based on more \( (L_4) \) or fewer \( (L_1) \) data, respectively. If the \( j \) new values represent a regime different from that of the initial \( m \) "base" values, both \( q(m+j) \) and \( q(m) \) will be larger than 1 and increase rapidly with increasing \( j \). Without a regime change both ratios remain near 1 or increase slowly as the parameter estimates for the augmented samples drift towards some population values ("return to normal").

The sum of the two likelihoods for a sample of \( m \) or \( m+j \) represents a measure of the probability that either parameter estimate is acceptable. For comparisons of different sample sizes these sums can be used to normalize their likelihoods. With \( L_1/(L_1 + L_4) = \varphi_b \), say, the other base likelihood becomes \( L_1/(L_1 + L_4) = 1 - \varphi_b \); that normalization leaves their ratio intact as \( L_1/L_4 = (1 - \varphi_b)/\varphi_b \). The corresponding normalization of the augmented-sample likelihoods \( L_2 \) and \( L_3 \) leads to \( L_2/L_3 = (1 - \varphi_{b,i})/\varphi_{b,i} \) . Writing the product of the two ratios as \( Q = L_1 L_2/L_1 L_4 = (1 - \varphi)/\varphi \) finally defines the "change index (CI)" \(^1\) in terms of the geometric average \( Q^{1/2} = [q(m)q(m+j)]^{1/2} \) as \[ \varphi = (1 + Q^{1/2})^{-1} \] (2.1)

Its maximum value, in absence of any parameter change, is 0.5, except in some special cases, discussed in section 3.3, when incompatible parameters can create values between .5 and 1.

\(^1\)Alternative names used during the project include "compatibility index" or "consistency index", and "no-change probability(NCP)"
3. Properties of the change index (CI).

Likelihood ratio products $Q$ are derived in Appendix A for the major probability laws (Poisson, Gaussian, Chi-square). For small samples of observational data the choice of law is somewhat arbitrary; it is then useful that the normalized likelihood ratios involved in $Q$ are less sensitive to the distribution form assumed than the likelihoods themselves. Except in the special case mentioned earlier and discussed in 3.2 below, the frequency distribution of the CI is unknown, but some of its properties have been determined experimentally, as described in the following sub-sections.

3.1. CIs from Poisson samples.

For Poisson samples of $m$ and $m+j$, with means $\bar{x}_b$ and $\bar{x}_{b+j}$, respectively, the normalized likelihood ratio product $Q$ is derived in Appendix A (equation A.1.6) as

$$Q = \exp \left[ \bar{x}_m \left\{ (m+j)y + j \log_e (1+y) - jy \right\} \right]. \quad (3.1)$$

where $y = (\bar{x}_{b+j} - \bar{x}_b) / \bar{x}_b$. This dependence of $Q$ on the base mean and the means difference scaled with the same base mean has been used by Radok and Brown (1993) to demonstrate the general dependence of the CI on the base sample mean and on the sample sizes $m$ and $m+j$ when all the new values $x_j$, $j=1,2,...$ equal the average of a Poisson distribution with mean $\bar{x}' = \bar{x}_b + \Delta = x_j$. Then with $g = \Delta / \bar{x}_b y$, say, $y = gj / (m+j)$, and equ. 3.1 takes the form

$$Q' = \exp \{ j\bar{x} (1+g) \log_e [1+jg/(m+j)] - gj/(m+j) \} \quad (3.1a)$$
Change indices have been calculated with (3.1a) for different combinations of the parameters \( m, x, j, \) and \( g \). The results are shown in figs. 3.1a and b. Longer bases (larger \( m \)) slow the response of the no-change probability to the change in mean but leave the curves essentially unaltered in shape. The decline in the change index starts at the change of mean and accelerates down to values of the order of 0.3 - 0.2 before becoming more gradual as small CIs are approached.

While of theoretical interest, these curves do not offer any help for assessing small samples which at best can be believed to have come from a Poisson distribution. The curves do however illustrate the basic tendency of the CI to decrease as a (Poisson) sample is being augmented.

3.2 CIs derived from Gaussian samples.

The product \( Q \) of the normalized likelihood ratios for Gaussian samples is derived in Appendix A (equation A.2.8) as

\[
Q = \exp \left[ \frac{m + j}{2} \log, \frac{\sigma^2_m}{\sigma^2_{m+j}} \right] 
\]

\[
+ \frac{m}{2} \log, \frac{\sigma^2_{m+j}}{\sigma^2_m} - m \left[ 1 - \frac{\sigma^2_m + (\Delta \mu)^2}{\sigma^2_{m+j}} \right] - \frac{m + j}{2} \left[ 1 - \frac{\sigma^2_{m+j} + (\Delta \mu)^2}{\sigma^2_m} \right] \]

The first two terms of (3.2) could be combined, but the form given will prove convenient for a development later in this section. The dependance of (3.2) on sample sizes and parameters cannot be evaluated in a manner analogous to that used for Poisson samples. Instead CI calculations have been carried out for small sets of independent values drawn at random from Gaussian populations with mean 10 and variances 4 and 25 \( \{N(10;4), N(10;25)\} \) as well as from the same data rendered
Fig. 3.1 a and b: The change index (CI; here called "no-change probability") of Poisson samples that result when the same new value $x + \Delta$ is added $j$ times to a base sample of $m$ with mean $\bar{x}$. The curves represent different values of $\bar{x}$, $m$, and $g = \Delta / \bar{x}$ (from Radok and Brown, 1993).
autoregressive with moving averages of lengths $\wedge = 3$ and $\wedge = 7$
\{N(10;4/3),N(1;4/7), etc.\}, creating non-zero lag correlations
$r_{\lambda} = (\wedge - \lambda) / \wedge, \lambda = 1, 2, \ldots (\wedge - 1)$.

In order to describe the CI decrease with the number $j$ of
added new data, the medians $M_{\lambda}$ of the CI as functions of $j$ for
different base lengths $m$ were fitted with linear regression lines
of the form $M_{\lambda} = a_{\lambda} - b_{\lambda} j$. Close approximations to the actual
median CIs resulted in each case. The regression slopes $b_{\lambda}$,
representing the incremental CI change per new value added to the
sample, are given in table 3.1:

Table 3.1: Change rate $- b_{\lambda}$ per added datum of the CI medians
$M_{\lambda} = a_{\lambda} - b_{\lambda} j$ as functions of base length $m$:

Gaussian data set parameters $m = 5 \ 7 \ 10 \ 13$

<table>
<thead>
<tr>
<th>m</th>
<th>mean</th>
<th>variance</th>
<th>mov aver.</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>4</td>
<td>0</td>
<td>$0.021 \ 0.014 \ 0.011$</td>
</tr>
<tr>
<td>10</td>
<td>4</td>
<td>3</td>
<td>$0.050 \ 0.045 \ 0.037$</td>
</tr>
<tr>
<td>10</td>
<td>4</td>
<td>7</td>
<td>$0.079 \ 0.073 \ 0.073$</td>
</tr>
<tr>
<td>10</td>
<td>25</td>
<td>0</td>
<td>$0.035 \ 0.019 \ 0.013 \ 0.009$</td>
</tr>
<tr>
<td>10</td>
<td>25</td>
<td>3</td>
<td>$0.108 \ 0.058 \ 0.034$</td>
</tr>
<tr>
<td>10</td>
<td>25</td>
<td>7</td>
<td>$0.144 \ 0.080 \ 0.045 \ 0.052$</td>
</tr>
</tbody>
</table>

The entries in the table were obtained from 50 random
samples of 20 values each. As expected, the CI decreases in a
regular way when more values are added to the base sample, and
the rates of decrease are enhanced for the autocorrelated series
of moving averages. For quantiles lower (higher) than the median
the decrease is faster (slower), as demonstrated by the following
trends for samples from the $N(10,25)$ data set and from the 7-
term moving averages of the same data, with 13-term bases:

<table>
<thead>
<tr>
<th>Data set</th>
<th>Quantile: 20%</th>
<th>50%</th>
<th>80%</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N(10,25)$</td>
<td>$-b_{\lambda} = 0.015 \ 0.009 \ 0$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7-term mov.aver</td>
<td>$-b_{\lambda} = 0.091 \ 0.052 \ 0.025$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
A more general and complete description of Gaussian CI properties can be given for a test which allows for changes in means only. In that case, when \( \mu_{s+j} \) is used as alternative mean for the base sample of \( m \) values, the alternative variance \( \sigma^2_{s+j} \) becomes \( \sigma^2_s + (\Delta\mu)^2 \), where \( \Delta\mu = \mu_s - \mu_{s+j} \). Similarly, when \( \mu_s \) is to be tested as a potential alternative mean for the augmented sample of \( m+j \), the alternative variance becomes \( \sigma^2_{s+j} + (\Delta\mu)^2 \). With these variances inserted in (3.2), the last two terms cancel, and the remaining terms yield as likelihood ratio product

\[
Q' = \left[ 1 + \frac{(\Delta\mu)^2}{\sigma^2_{s+j}} \right]^{(m-1)/2} \left[ 1 + (\Delta\mu)^2 / \sigma^2_s \right]^{1/2} \quad (3.2a)
\]

With sample means \( \bar{x}_s \) and variances \( s^2_s = [n/(n-1)] \sigma^2_s \), the variable \( (\Delta\bar{x})^2 / [s^2_s (n-1)] \) represents the square of a "Student" variable \( t_{n-1} \) with \( n-1 \) degrees of freedom. Thus the product of the likelihood ratios becomes

\[
Q' = \left[ 1 + (m-1) t^2_{s-1} \right]^{1/2} \left[ 1 + (m+j-1) t^2_{s+j-1} \right]^{(n-1)/2} \quad (3.2b)
\]

The CI for this special case then has the explicit frequency distribution

\[
f(\delta') = [1 + (1+(n-1) t^2_{n-1})]^{-1} \quad (3.2c)
\]

in terms of a "Student" variate with \( m-1 < n < m+j-1 \) degrees of freedom.

Fig. 3.2a shows the cumulative form of equ. (3.2c) to be
Fig. 3.2a: Chance probabilities of exceeding change indices (CI) of Gaussian samples with the same variance but different means. The scale of cumulative probability has been determined with eqn. (3.2c) from "Student" t distributions with different degrees of freedom (d.f.).
similar for different degrees of freedom. With the probabilities
for \( n=10 \) in fig. 3.2a, a scale has been constructed on which
those probabilities fall along a straight line. That scale is
used in figs. 3.2b and 3.2c to examine the distribution of some
of the CIs obtained from random samples with the full Gaussian
equation (3.2). All the plots are approximately linear and
distorted only for high values of the CI. Critical regions for
small values of the change index might then be constructed with
the distribution function (3.2c) even when both mean and variance
are allowed to vary. But as in the case of Poisson samples, such
regions will be have little practical value when a small data
sample constitutes the complete information available.

3.3 CIs for regression coefficients.

The formulae derived in Appendix A.3 for the change index
of regression coefficients again clearly have no simple
interpretations. They have therefore been evaluated for some of
the Gaussian samples described in the preceding section. The
results are summarized in the following table for the independent
data with population mean 10 and population variance 25
(\( N(10;25) \)), as well as for the autocorrelated 7-term moving
averages of the same data, \( N(10;25/7) \):

Table 3.2 Frequency (%) of regression-coefficient change
indices for Gaussian samples. Frequencies in
parentheses relate to CIs < 0.4.

<table>
<thead>
<tr>
<th>CI Range</th>
<th>Sample size</th>
<th>m=5</th>
<th>m=10</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>m=5</td>
<td>10</td>
<td>15</td>
</tr>
<tr>
<td>Data set</td>
<td>j=5</td>
<td>10</td>
<td>15</td>
</tr>
<tr>
<td>( 0.5(0.4) &gt;\text{CI}&gt;0 ):</td>
<td>( N(10;25) )</td>
<td>62(36)</td>
<td>44(24)</td>
</tr>
<tr>
<td></td>
<td>( N(10;25/7) )</td>
<td>32(12)</td>
<td>36(20)</td>
</tr>
<tr>
<td>( \text{CI}&gt;0.5 )</td>
<td>( N(10;25) )</td>
<td>38</td>
<td>56</td>
</tr>
<tr>
<td></td>
<td>( N(10;25/7) )</td>
<td>68</td>
<td>64</td>
</tr>
</tbody>
</table>

The first two rows of frequencies in the table represent CIs in
the range \( 0.5<\text{CI}<0 \). For the longer of the two base lengths
tested, a majority of trend change indices for random samples
Fig. 3.2b and 3.2c: Cumulative probabilities of change indices determined with equ. (3.2) for 50 Gaussian samples with changing means and variances. Probabilities of CIs with changing means only (satisfying equ. (3.2c)) would form straight lines.
remain above 0.4. The corresponding frequencies for the moving-average (autocorrelated) series are smaller but increase in the same way for longer base samples.

The remaining two lines of the table refer to CIs that are larger than 0.5. Such unrealistic CIs arise from the violation of a limiting condition noted in the derivation of the regression formulae (A.3.7) and (A.3.8) in Appendix A.3; the use of the regression coefficient $b_{m+j}$ with the base sample of $m$ values, and of $b_s$ with an augmented sample of $m+j$ values must not result in a zero or negative residual sum of squares. This implies that a good regression fit with small residuals will narrow the limits for permissible alternative regression coefficients, increasing the sensitivity of the change index.

3.4 CIs from chi-square samples.

The product $Q$ of the normalized likelihood ratios for samples from a chi-square distribution with $\nu$ degrees of freedom (d.f., which represent also the mean and one half the variance of the distribution) is derived in appendix A (equation A.4.8) as

$$Q = \exp \left[ \frac{j}{2} (\nu_m - \nu_{m+j}) \log e + j \log e \left( \frac{\nu_m}{2} \frac{\Gamma\left(\frac{\nu_m}{2}\right)}{\Gamma\left(\frac{\nu_{m+j}}{2}\right)} + \frac{\nu_m + j - \nu_m}{2} \sum_{m+j}^n \log e \chi^2 \right) \right]$$ (3.4)

Some of the characteristics of this expression have again been determined experimentally for chi-square values calculated from the Gaussian data sets $N(10;25)$ and $N(10;25/7)$ introduced in section 3.2. For sample variances computed from 5 values, the chi-squares of the first data set should have 4 d.f. and those of the autocorrelated set, 1.14 d.f. (cf Radok 1992). The following
The table gives the percentage frequencies of CIs calculated for 50 samples of 100 in each case.

Table 3.3 Frequency (%) of change indices for chi-square samples. Bracketed frequencies relate to CIs < 0.4.

<table>
<thead>
<tr>
<th>Sample size</th>
<th>m=5</th>
<th>m=10</th>
</tr>
</thead>
<tbody>
<tr>
<td>CI Range</td>
<td>j=5</td>
<td>10</td>
</tr>
<tr>
<td>Data set</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.5(.4) &gt; CI &gt;0:</td>
<td>(N10;25) 76(58) 78(46) 74(50) 86(72) 78(62)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>N(10;25/7) 72(48) 70(34) 54(18) 48(38) 50(30)</td>
<td></td>
</tr>
<tr>
<td>CI&gt;0.5</td>
<td>N(10;25) 24 22 26 14 22</td>
<td></td>
</tr>
<tr>
<td></td>
<td>N(10;25/7) 28 30 36 52 50</td>
<td></td>
</tr>
</tbody>
</table>

These frequencies show a concentration of random-sample CIs in the 0.4 to 0.5 range but also a substantial fraction of unrealistic values exceeding 0.5, the largest value possible if the estimates and represent maximum-likelihood estimates for their respective samples of m and m+j. The Gamma functions in the chi-square likelihoods dominate the value of Q, especially in the region of small x where \( \Gamma(x) \) has a minimum around \( x = 1.46 \) and goes to infinity as \( x \) goes to zero. This suggests that chi-square CIs require samples larger than those used for table 3.3.

3.5 Limitations and scope.

The experimental results presented in the preceding sections despite their limited extent establish clearly that the change index and the underlying likelihood ratios have downward trends for growing samples even when the data have been drawn at random from a single population. The change index is not intended for finding critical regions for rejecting the null hypothesis — i.e. that the base and augmented sample estimates are equivalent, within a prescribed error probability. That represents a "confirmatory" task, as defined by Flueck and Brown (1993) for which standard tests (e.g. the "Student" t test) can be used. The
change index instead serves the "exploratory" purpose of detecting and measuring parameter changes that have resulted from the addition of even a single new observation, leaving the implications to be assessed from whatever information is contained in the small sample(s) under consideration.

The difference between the two types of statistical problem sketched above can be illustrated for one of the random Gaussian samples introduced in section 3.2 and plotted in fig. 3.3a. Inspection suggests that its first eleven values have a smaller variance than the nine values that follow, while both sections vary around similar means (as permitted by the characteristic independence of the two parameters in a Gaussian population).

To confirm that the data in fig. 3.3a come indeed from the same Gaussian population, the sample means and variance of m values from the first section and j values from the second have been compared with a t-variable of the form

$$t_{4;14} = \frac{(\bar{X}_j - \bar{X}_m)\{(m+2)m\}}{\{(m+j)[(m-1)s^2_m + (j-1)s^2_j]\}^{1/2}}$$

(3.5)

Probabilities $P$ of exceeding these $t$ values of the differences between the independent means $\bar{X}_j$ of three and more new observations and different base means $\bar{X}_m$ have been calculated with the corresponding sample variance estimates $s^2_m$ and $s^2_j$. The values of $P$ (+1 or 2 or 3, in order to separate the curves) are shown in fig. 3.3a for different bases $m$, with $j$ increasing from 3 to $20-m$ in each case. Except for $m=5$ the probabilities of exceeding $t$ lie well above the 5% significance level and vary little for the different $m$; a sudden temporary increase in $P$ occurs as the more variable section (and hence a smaller $t$ value) is encountered. The $t$ test therefore confirms, within
Fig. 3.3a: A Gaussian sample (heavy line), and probabilities that differences between the means of the initial m values and means of j=3,4...20-m subsequent values exceed tdiff, determined with equ.(3.5).
Fig. 3.3b: Change indices (CI) taking account of both means and variances of the Gaussian sample in fig. 3.3a (heavy line). Note the CI decrease signalling the onset of more variable values.
the limitations of a single 20-term sample, that the data in fig. 3.3a could indeed come from a single Gaussian population. The CIs for the same data are shown in fig. 3.3b and tell a very different story. For the same bases as before and \( j \) starting with 1 rather than 3 (reflecting an advantage the CI has over the t test), the CI drops abruptly to almost zero as soon as the more variable section is encountered with the 12th value. But after the first two values of that section have become part of the base, with \( m=13 \), the CI remains near 0.5 for the rest of the series. This exploratory test therefore responds sensitively to the sudden substantial increase in variance. It leaves open the question of whether that increase is compatible with a single Gaussian population or perhaps points to some other type of distribution - a question that may not have a firm answer if the data in question represent the full information available.

Fig. 3.4 serves to illustrate this type of uncertainty. Its data form come from a series of annual numbers of North Atlantic hurricanes (Case 1988) which will be shown in section 4.1 to approximate a Poisson distribution. As a consequence, sample means and variances would be expected to vary in a similar way, but this clearly is not the case in figure 3.4: the mean number for the first ten years (4.9) differs little from that for the remaining 5 years (5.2), whereas the sample variance increases from 0.89 for the first ten years to 6.56 for the last five years. The CI formula for Poisson samples considers the means only, and the Poisson CIs (full lines in fig. 3.4) remain in the 0.4 to 0.5 range throughout.

However, as noted earlier, independence of mean and variance is a defining characteristic of the Gaussian distribution; the data in fig. 3.4 in fact bear a close resemblance to the Gaussian sample of fig. 3.3. They have therefore also been analyzed with the Gaussian formulae of section 3.2. A set of Gaussian CIs calculated with equation (3.2a) considering only means changes is shown as a dotted curve and resembles the Poisson curves. By contrast the broken CI curves, calculated with equation (3.2),
take into consideration both the sample means and sample variances and drop to small values in 1980, when the first of the more variable numbers is encountered. After that year becomes itself part of the base set, the full Gaussian CIs approximate those derived with the Poisson formula.

This underlines the need for flexibility in the choice of the distribution form for the CI calculation when alternative forms are not firmly ruled out by the limited data available. The examples of CI tests to be presented next will therefore not be concerned with confirming the distribution forms assumed for the parameter estimates and likelihoods, but rather with the parameter changes themselves that resulted from the addition of extra observations, and with possible physical implications.

4. Examples.

The examples in this section have been chosen to illustrate potential alternative uses of the CI. The index is uniquely suited for assessing a single new observation which seems out of line with the quasi-controlled regime of its predecessors. In existing longer series the index can also be used to identify transition points between regimes, created by unknown causes or by suspected events such as a station shift or change in observational routine. Current data can be subjected to real-time monitoring with CIs calculated both retrospectively for the immediate past and progressively for each new observation that comes to hand.

4.1 Retroactive use of the CI.

The data of the first example are annual numbers of tropical hurricanes, reported for the North Atlantic by Case (1988) and updated to 1990; some of them were already used in section 3.5. Their frequency distribution (fig. 4.1) approximates a Poisson distribution (light shaded bars) with mean (and variance) 5.6. The annual numbers themselves are shown in fig. 4.2a and have been used to calculate sequences of progressive means shown in
Fig. 3.4: Annual numbers of North Atlantic tropical hurricanes and change indices assuming a Poisson distribution (full lines), a Gaussian distribution with constant variance (dotted line), and a Gaussian distribution with changing mean and variance (broken lines).
Fig. 4.1: Histogram of annual numbers of North Atlantic tropical hurricanes (Case 1988, updated). The light-shaded bars give the frequencies of a Poisson distribution with the observed mean number (5.6).
Fig. 4.2: (a) Time series of the annual number of North Atlantic tropical hurricanes; (b) Progressive means of the numbers in (a) starting with those of different base periods; (c) Change indices ("Gamma") starting from the ends of the base periods.
Fig. 4.2b. Each of these starts with the mean of a "base" section and continues until a parameter change is suggested by a substantial decrease in the change index CI (here called "no-change probability"), computed with equation 3.1 and shown in fig. 4.2c. This illustrate the use of the CI for delineating plausible "regimes" in an existing series, whereas the analysis of the 1970-1984 subset in section 3.5 simulated a real-time test initiated on the occurrence of a single anomalous value.

The first base period (1931-1935) had a mean value of 5.6 and was followed by several years with small hurricane numbers which produced a steady slow CI decrease. Starting anew with the numbers for 1937-1942 as base (mean 3.8) produced a sharper CI signal of a return to larger numbers in the late 1940s. The next base period (1948-1952) had an annual mean number of 7.6 hurricanes which decreased in the late 1960s to the original mean number of 5.6 for the next base period (1961-1965). Subsequent CIs showed no further change of control even after a new base period was adopted for the 1970s to sharpen the test.

The next illustration uses a composite annual mean temperature for four Alaskan airfields (Fairbanks, Anchorage, Nome, Barrow) reported by Bowling (1991). Fig. 4.3 shows the (cumulative) frequency distribution of the initial 23 values to approximate the Gaussian form, a straight line in the coordinate system of fig. 4.3. The annual mean temperatures themselves are shown in fig. 4.4a. To detect changes in mean and/or variance, three base periods of 7 values each are used, ending in 1960, 1967, and 1974. Cumulative means and variances following each of the three base periods are shown in fig. 4.4b and Fig. 4.4c, respectively.

The cumulative variance decreased steadily from 1960, the end of the first base period, while the cumulative mean remained near its base level. Both mean and variance increased rapidly after the 1977 change to higher winter temperatures reported by Bowling (1991).
Fig. 4.3: Probabilities of exceeding different composite annual mean temperatures at four Alaskan airfields (deviations from the 1954-1976 mean; Bowling 1991). The probability scale turns a Gaussian distribution into a straight line (cf. e.g. Radok 1992).
Fig. 4.4: (a) Time series of the Alaskan airfields mean temperature deviations in fig. 4.3, updated; (b) Progressive means starting with those of different base periods; (c) progressive variances; (d) Means/variances change indices ("Gamma") starting from the ends of base periods.
The corresponding CI values are shown in fig. 4.4d. The initial progressive decline in variance was reflected in a similarly gradual CI decline after each of the first two base periods. That CI decline was reversed temporarily by the 1977 temperature increase before continuing as a steeper CI descent. With the base period ending in 1974, the combined increases in both cumulative mean and variance produced a much steeper CI decrease after 1977, which became only a little more gradual when the entire set of 21 years was used as base period. CIs calculated with the temperatures for 1977-1981 as a new base suggested a return to slightly lower temperatures and unchanged small variance values during the 1980s, which have persisted through 1993 (Radok and Brown 1995).

The detection of trend changes can be illustrated with the first part of the same Alaska data, repeated in fig. 4.5 from fig.4.4a. Fig. 4.5 shows progressive regression coefficients starting with that for the 7-year base 1954-1960. The CIs in fig. 4.5 indicate the beginning of a distinctly different slope regime in 1964. The series of CI values is continued until two CI=1 are encountered, to illustrate what happens when the base regression coefficient $b_a$ exceeds the upper limit $b_{n,m}$ (the dotted line) for its use in an augmented sample.

None of the regression lines in this example explains more than 10% ($= r^2$, where $r$ is the temperature-time correlation) of the total variance. For a good regression fit the permissible range for $b_a$ as alternative for $b_{n,m}$ becomes very narrow. This is demonstrated in table 4.1 for an exponential approximation to annual mean atmospheric carbon dioxide concentrations C(PPM) observed at Mauna Loa, Hawaii (data from Boden et al, 1990):
Fig. 4.5: Change indices (CI) for the progressive trend $b_{s,i}$ in the Alaskan airfields mean temperature deviations (top curve) from the base trend $b_m$. Unrealistic values CI=1 arise after the $b_{s,i}$ curve descends below the value of $b$ (dotted line).
Table 4.1: Regression analysis of CO₂ concentrations C (ppm)
represented by $y = 100 \ (\log C - 5.8)$:

<table>
<thead>
<tr>
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<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>37</td>
<td>41</td>
<td>46</td>
<td>49</td>
<td>54</td>
<td>61</td>
<td>66</td>
<td>70</td>
<td>74</td>
<td>75</td>
<td>77</td>
</tr>
<tr>
<td>Change index CI</td>
<td>0.95</td>
<td>0.85</td>
<td>0.97</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Progressive regression coefficient $b_*$</td>
<td>4.78</td>
<td>4.55</td>
<td>4.31</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Upper limit $b_*$, for use in augm. sample:</td>
<td>4.80</td>
<td>4.57</td>
<td>4.35</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Residual variance $(1 - r^2)%$</td>
<td>0.7</td>
<td>0.5</td>
<td>1.1</td>
<td>1.8</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

The base regression of 4.83 leaves a mere 0.7 of the total variance unaccounted for. When the observations for the years 1991 through 1993 are added, this percentage remains almost the same, while the base regression coefficient immediately begins to exceed its permissible limits.

The final example, from Brown and Radok (1995), illustrates how the CI can be used to detect inhomogeneities in a climate series, caused by a station move or by a change in observational routine. Such discontinuities resemble sudden regime changes, or appear as outliers if the change was of short duration. An example is provided by temperatures observed at the San Francisco, California, National Weather Service airport station, which is known to have been moved further inland during the 1940s. By reducing the number of maritime air incursions this should have raised the mean temperatures for the summer months.

Fig. 4.6 shows those mean summer temperatures for the years 1933 through 1956. The CI calculations in this case used a moving ten-year base period advancing by two-year steps. For each of the first three bases, the gradual CI declines changed to a rapid drop with the 1948 temperature; this drop vanished as soon as the moving ten-year base reached and included the 1948 temperature itself. The station move in fact took place that year (Jón Eischeid, pers. comm.).
Fig. 4.6: Summer mean temperatures observed at the San Francisco Airport, and change indices for base periods extending between corresponding numbers. The CI decrease in 1948 arises from a change of station location.
4.2 Real-time CI analysis.

The first example of an (almost) real-time CI analysis also serves to demonstrate its broad applicability. The data used are shown as dots in Fig. 4.7; they represent Dow Jones 65-stock averages, as reported by the Wall Street Journal prior to and during the 1994 Congressional elections. Dow values of November 7 are used as an expanding base with \( m = 4 \) through 8. All the CIs decreased slowly during November 7 and early on November 8, until a noon increase in the Dow started a precipitous CI decrease to values near zero.

A possible form of real-time climatic monitoring is illustrated in tables 4.1 and 4.2 for global mean temperature anomalies \( y \) from their 1951-1980 averages, using data kindly provided by Dr. Henry Diaz. For each season 5 different bases extend backward from 1990, to provide retrospective CIs testing the statistical control or lack of it during the early 1980's. The same bases provide progressive CIs for 1991 through 1993. With the years 1985 through 1990 as base, the earlier anomalies for winter and, to a lesser degree, for summer appear to belong to different regimes, but all the most recent anomalies have remained within the framework of the statistical control established during the 1980s.

5. Alternatives.

Several other procedures have been proposed over the years for the specific purpose of detecting abrupt changes in time series (e.g. Oerlemans, 1979; Epstein, 1982; Goossens and Berger, 1987; Howell, 1995). All of them postulate some form of change to be confirmed by filtering. In the example shown in fig. 5.1a, Epstein (1982) postulated three possible change scenarios to account for the rise at the end of the 1970s of the annual mean temperatures recorded by a global network of radiosonde stations (Angell and Korshover 1977, updated to 1981). The parameter values of the scenarios were chosen to maximize Gaussian likelihood ratios, shown in fig. 5.1b. These ratios are
Fig. 4.7: Dow-Jones 65-stock averages (dots) and change indices before and during the November 1994 Congressional elections.
Table 4.1 Change in current global land temperature anomalies °C from 1951-1980 (H. Diaz, pers. comm).

Numbers in table are change indices CI*10

1) Winter (DJF)

<table>
<thead>
<tr>
<th>y</th>
<th>yr base: 5 6 7 8 9 10</th>
</tr>
</thead>
<tbody>
<tr>
<td>.437</td>
<td>81 000 465 454 488 488</td>
</tr>
<tr>
<td>.093</td>
<td>82 000 477 485 491</td>
</tr>
<tr>
<td>.449</td>
<td>83 000 493 484</td>
</tr>
<tr>
<td>-.001</td>
<td>84 000 488</td>
</tr>
<tr>
<td>-.380</td>
<td>85 000</td>
</tr>
</tbody>
</table>

.210 = base mean
.185 81-90 = base years
.197 82-90
.161 83-90
.188 84-90
.301 85-90
86-90
369 477 472 480 477 482
353 448 442 457 453 463
368 413 469 429 428 441

2) Spring (MAM)

<table>
<thead>
<tr>
<th>y</th>
<th>yr base: 5 6 7 8 9 10</th>
</tr>
</thead>
<tbody>
<tr>
<td>.356</td>
<td>81 315 429 476 491 492</td>
</tr>
<tr>
<td>-.025</td>
<td>82 301 427 478 478</td>
</tr>
<tr>
<td>.264</td>
<td>83 391 468 490</td>
</tr>
<tr>
<td>.070</td>
<td>84 408 485</td>
</tr>
<tr>
<td>.026</td>
<td>85 465</td>
</tr>
</tbody>
</table>

.242 = base mean
.221 81-90 = base years
.262 82-90
.261 83-90
.293 84-90
.346 85-90
86-90
486 488 489 489 489 490
462 467 471 475 476 479
419 430 439 447 450 456
Table 4.(cont'd): Change in current global land temperature anomalies °C from 1951-1980 (H.Diaz, pers. comm).
Numbers in table are change indices CI*10⁻³

### 3) Summer (JJA)

<table>
<thead>
<tr>
<th>y</th>
<th>yr base: 5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>.186</td>
<td>81</td>
<td>177</td>
<td>406</td>
<td>466</td>
<td>477</td>
<td>492</td>
</tr>
<tr>
<td>-.016</td>
<td>82</td>
<td>182</td>
<td>421</td>
<td>483</td>
<td>478</td>
<td></td>
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<tr>
<td>.226</td>
<td>83</td>
<td>305</td>
<td>475</td>
<td>490</td>
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<tr>
<td>.004</td>
<td>84</td>
<td>285</td>
<td>474</td>
<td></td>
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<tr>
<td>-.014</td>
<td>85</td>
<td>408</td>
<td></td>
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</tr>
</tbody>
</table>

- .174 = base mean
- .173 81-90 = base years

### 4) Fall (SON)

<table>
<thead>
<tr>
<th>y</th>
<th>yr base: 5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>.086</td>
<td>81</td>
<td>354</td>
<td>470</td>
<td>492</td>
<td>472</td>
<td>492</td>
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<tr>
<td>-.070</td>
<td>82</td>
<td>372</td>
<td>479</td>
<td>498</td>
<td>487</td>
<td></td>
</tr>
<tr>
<td>.367</td>
<td>83</td>
<td>436</td>
<td>497</td>
<td>481</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-.116</td>
<td>84</td>
<td>372</td>
<td>473</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-.081</td>
<td>85</td>
<td>462</td>
<td></td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

- .120 = base mean
- .124 81-90 = base years
equivalent to those in equ. (3.2a); however Bayesian scruples prevented the author from drawing firm conclusions about their significance.

Fig. 5.2 shows CI profiles for the same temperatures. CIs calculated with equ. (3.2a) (dotted line) suggested no change when means are considered alone; by contrast when the variance is taken into account as well, with equ. (3.2), the CIs in fig. 5.2 decrease rapidly after 1977 for a 6-year base extending from 1971 through 1976, and more gradually for a base including all data prior to 1977. The change index therefore would have provided an objective basis for the initial decision to postulate different change scenarios, but no guidance to their possible forms.

6. Conclusion.

The likelihoods computed for an existing sample, and for the same data augmented by a single new value or by a small number of such values, indicate changes in progress that can represent the end of a "regime" or a new beginning. The "change index (CI)" provides a quantitative measure of differences between one or more parameters estimated from the original and augmented samples. The normalisation of the likelihood ratios used in the CI reduce its sensitivity to the form of distribution assumed to govern the data, and makes the index an effective exploratory tool when a small amount of data represents the full information available.

Acknowledgement. Support for this work has been provided by NASA Grant NAGW-2706. Partial support for the second author was provided by NOAA's Climate and Global Change Program. We are grateful to Barbara Sloan who produced appendix A.
Fig. 5.1a: Global mean surface temperature anomalies (Angell and Korshover, 1977, top), and climatic change scenarios postulated by Epstein (1982).

Fig. 5.1b: Gaussian likelihood ratios for different parameter values of the scenarios in fig. 5.1a (Epstein 1982).
Fig. 5.2 The global mean surface temperature anomalies of fig. 5.1, and change indices testing the higher temperatures after 1976 against the previous 6 years (base 1) and against all observations prior to 1977 (base 2). The dotted CI line considers changes of mean only, the other two changes in mean and variance.
References.


Appendix A: Formulae

Subscripts will be used to indicate the number of values used for parameter estimates, and bracketed symbols the numbers used to calculate the likelihoods and their ratios. Thus \( L(m; \theta_m) = L_1 \) becomes \( L_m(m), L(m + j; \theta_m) = L_3 = L_m(m + j) \), etc.

A.1 Poisson mean (= variance)

With the mean number \( \bar{x} \) of occurrences the basic probability is

\[
p = \frac{\bar{x}^x}{x! \exp(\bar{x})} ,
\]

(A.1.1)

The logarithmic likelihood functions are

\[
\log_e L_m(m) = m \bar{x}_m \log_e \bar{x}_m - \sum_1^m \log_e x! - m \bar{x}_m ;
\]

(A.1.2a)

\[
\log_e L_{m+j}(m) = m \bar{x}_m \log_e \bar{x}_{m+j} - \sum_1^m \log_e x! - m \bar{x}_{m+j} ;
\]

(A.1.2b)

\[
\log_e L_{m+j}(m + j) = (m + j) \bar{x}_{m+j} \log_e \bar{x}_{m+j} - \sum_1^{m+j} \log_e x! - (m + j) \bar{x}_{m+j} ;
\]

(A.1.2c)

\[
\log_e L_m(m + j) = (m + j) \bar{x}_{m+j} \log_e \bar{x}_m - \sum_1^{m+j} \log_e x! - (m + j) \bar{x}_m ,
\]

(A.1.2d)

likelihood ratios

\[
q(m) = \exp \left[ m \left\{ \bar{x}_m \log_e \frac{\bar{x}_m}{\bar{x}_{m+j}} + (\bar{x}_{m+j} - \bar{x}_m) \right\} \right] ;
\]

(A.1.3)

\[
q(m+j) = \exp \left[ (m + j) \left\{ \bar{x}_{m+j} \log_e \frac{\bar{x}_{m+j}}{\bar{x}_m} - (\bar{x}_{m+j} - \bar{x}_m) \right\} \right] ,
\]

(A.1.4)

so that
\[ Q = \exp \left[ \left( (m+j)\bar{x}_{m+j} - m\bar{x}_m \right) \log_e \frac{\bar{x}_{m+j}}{\bar{x}_m} - j (\bar{x}_{m+j} - \bar{x}_m) \right] . \]  

(A.1.5)

with \( y = (\bar{x}_{m+j} - \bar{x}_m)^2 \) \( \bar{x}_m \)

\[ Q = \exp \left[ \bar{x}_m \left[ (m+j)y + jy \right] \log_e (1+y) - jy \right] . \]  

(A.1.6)

A.2 Gaussian means and variances

Gaussian distribution.

\[ p = (2\pi\sigma^2)^{-1/2} \exp \left[ \frac{-(x-\mu)^2}{2\sigma^2} \right] . \]  

(A.2.1)

For the population mean \( \mu \) and variance \( \sigma^2 \) we use the sample estimates \( \bar{x} = \frac{\sum x}{m} \) and

\[ s^2 = \frac{1}{(n-1)} \sum (x - \bar{x})^2 . \]

\[ \log_e L_m(m) = - \frac{m}{2} \log_e 2\pi - \frac{m}{2} \log_e \sigma^2_m - \sum \frac{(x - \mu_m)^2}{2\sigma^2_m} . \]  

(A.2.2)

with \( \sum (x - \mu)^2 = m \sigma^2_m \), the last term reduces to \(-m/2\).

Proceeding in the same way for \( L_{m+j} = L_m + j_m \) leads to

\[ \log_e L_{m+j}(m) = - \frac{m}{2} \log_e 2\pi - \frac{m}{2} \log_e \sigma^2_{m+j} - \sum \frac{(x - \mu_{m+j})^2}{2\sigma^2_{m+j}} ; \]  

(A.2.3a)

with \( \mu_{m+j} - \mu_m = \Delta \mu \), the numerator of the last term can be written as

\[ - \sum \frac{(x - (\mu_m + \Delta \mu))^2}{m} = - \sum \frac{(x - \mu_m)^2}{m} - \sum (-2x \Delta \mu + 2\mu_m \Delta \mu + \Delta \mu^2) \]

\[ = -m \sigma^2_m - 2m \mu_m \Delta \mu + 2m \mu_m \Delta \mu - m (\Delta \mu)^2 , \]

so that equation (A.2.3a) becomes
subtracting equation (A.2.2b) from equation (A.2.3) gives the log likelihood ratio

\[
\log_e q(m) = \log_e \frac{L_m(m)}{L_{m+j}(m)} = \frac{m}{2} \log_e \frac{\sigma_m^2}{\sigma_{m+j}^2} - \frac{m}{2} \left[ 1 - \frac{\sigma_m^2 + (\mu)^2}{\sigma_{m+j}^2} \right]
\]  

(A.2.4)

augmented set of \(m+j\) values proceeding in the same way for the yields first

\[
\log_e L_{m+j}(m + j) = -\left( m + \frac{j}{2} \right) \log_e 2\pi - \frac{m + j}{2} \log_e \sigma_{m+j}^2 - \sum_1^{m+j} \frac{(x - \mu_{m+j})^2}{2\sigma_{m+j}^2}
\]

(A.2.5)

where the last term, with \(\sum_1^{m+j} (x - \mu_{m+j})^2 = (m+j) \sigma_{m+j}^2\), reduces to \(-(m+j)/2\). Next,

\[
\log_e L_m(m + j) = -\left( m + \frac{j}{2} \right) \log_e 2\pi - \frac{m + j}{2} \log_e \sigma_m^2 - \sum_1^{m+j} \frac{(x - (\mu_{m+j} - \Delta\mu))^2}{2\sigma_m^2}
\]

(A.2.6a)

Expanding the last term as before yields

\[
\log_e L_m(m + j) = -\left( m + \frac{j}{2} \right) \log_e 2\pi - \frac{m + j}{2} \log_e \sigma_m^2 - \frac{m + j}{2} \left[ \frac{\sigma_{m+j}^2 + (\Delta\mu)^2}{\sigma_m^2} \right]
\]

(A.2.6b)

Subtracting equation (A.2.6b) from equation (A.2.5) we obtain the second log likelihood ratio

\[
\log_e q(m + j) = \frac{m + j}{2} \log_e \frac{\sigma_m^2}{\sigma_{m+j}^2} - \frac{m + j}{2} \left[ 1 - \frac{\sigma_m^2 + (\Delta\mu)^2}{\sigma_{m+j}^2} \right]
\]

(A.2.7)

\[
Q = q(m)q(m + j) = \exp \left[ \frac{m + j}{2} \log_e \frac{\sigma_m^2}{\sigma_{m+j}^2} + \frac{m}{2} \log_e \frac{\sigma_{m+j}^2}{\sigma_m^2} - \frac{m + j}{2} \left[ 1 - \frac{\sigma_m^2 + (\Delta\mu)^2}{\sigma_{m+j}^2} \right] - \frac{m + j}{2} \left[ 1 - \frac{\sigma_{m+j}^2 + (\Delta\mu)^2}{\sigma_m^2} \right] \right]
\]

(A.2.8)

A.3 Trends (least-square regression)

Observations made at equally spaced times \(t = 1, 2 \cdots n\) (\(n = m\) or \(m + j\)) are represented by
\[ y = A + Bt + e \]  

This also covers the case of exponential regression when \( y = \log x \). The residuals \( e \) are assumed to be normally distributed with zero mean and variance \( \sigma^2 \). Sample estimates of the regression coefficients \( A \) and \( B \) satisfying least-square requirements are

\[
a = \bar{y} + b\bar{t}; \quad b = \frac{\sum_{i=1}^{n}(y_i - \bar{y})(t_i - \bar{t})}{\sum_{i=1}^{n}(t_i - \bar{t})^2},
\]

where

\[
\bar{t} = \frac{(n+1)}{2}; \quad \frac{\sum_{i=1}^{n}(t_i - \bar{t})^2}{n(n-1)} = \frac{n(n^2-1)}{12}.
\]

The regression estimate \( \hat{y}' \) for a given \( t' \) is then \( \hat{y}' = \bar{y} + b(t' - \bar{t}) \), and the corresponding residual \( e_i = y'_i - \hat{y}'_i \) has a Gaussian distribution with zero mean and variance

\[
s_{e_i}^2 = \frac{1}{n-2} \left[ \sum_{i=1}^{n} \frac{(y_i - \bar{y_i})^2 - b_n^2 n(n^2-1)}{12} \right] \left[ \frac{n+1}{n} + \frac{(t - \frac{(n+1)}{2})}{\frac{n(n^2-1)}{12}} \right].
\]

The general form of the likelihood functions defined by the \( n \) residuals \( e_i \), \( i = 1, 2, \ldots, n (= m \) or \( m + j \) is

\[
L = \exp \sum_{i=1}^{n} \left\{ -\frac{(y_i - \bar{y_i})^2}{2s_{e_i}^2} \right\}.
\]

\[
\log_e L_m = \log_e L_1 = -\frac{m-2}{2[SS(m) - b^2_m d(m)]} \sum_{i=1}^{m} \frac{(y_i - \bar{y}_i)^2}{\left[ \frac{m+1}{m} + \frac{(t - \frac{m+1}{2})^2}{d(m)} \right]}.
\]
\[
\log_e L_m(m + j) = \log_e L_3 = -\left[ \frac{m + j - 2}{2[SS(m + j) - b_m^2 d(m + j)]} \right] \cdot \left[ \sum_{t=1}^{m+j} \frac{(y - \hat{y}_m)^2}{m + 1} + \frac{\left( \frac{t - m + j + 1}{2} \right)^2}{d(m + j)} \right]
\]

(A.3.6)

\[
\log_e L_m(m + j) = \log_e L_3 = -\left[ \frac{m + j - 2}{2[SS(m + j) - b_m^2 d(m + j)]} \right] \cdot \left[ \sum_{t=1}^{m+j} \frac{(y - \hat{y}_m)^2}{m + 1} + \frac{\left( \frac{t - m + j + 1}{2} \right)^2}{d(m + j)} \right]
\]

(A.3.7)

valid only with \( b_m < \sqrt{\frac{SS(m + j)}{d(m + j)}} \)

\[
\log_e L_{m+j}(m) = \log_e L_4 = -\left[ \frac{m - 2}{2[SS(m) - b_{m+j}^2 d(m)]} \right] \cdot \left[ \sum_{t=1}^{m} \frac{(y - \hat{y}_{m+j})^2}{m + 1} + \frac{\left( \frac{t - m + 1}{2} \right)^2}{d(m)} \right]
\]

(A.3.8)

valid only with \( b_{m+j} < \sqrt{\frac{SS(m)}{d(m)}} \),

where \( SS(n) = \sum_{i=1}^{n} (y - \bar{y})^2 \); \( d(n) = \frac{n(n^2 - 1)}{12} \); \( \hat{y}_n = a_n + b_n t \) when \( n = m \) or \( m + j \).

(A.3.9)

Finally as before

\[
Q = L_1 L_2 L_3 L_4
\]

(A.3.10)

A.4 Chi-square

The variances \( s^2 \) of samples from a Gaussian population with variance \( \sigma^2 \) define a chi-square variate
where \( h \) is the number of values in each sample. If these values are independent of one another, \( \chi^2 \) has \( h-1 \) degrees of freedom (d.f.). For "coherent" (autocorrelated) series the d.f. number (which also represents the mean of the chi-square distribution as well as one half its variance) is reduced (Radok, 1992) to

\[
\bar{\chi}^2 = v = h - \varepsilon(h),
\]

where

\[
\varepsilon(h) = 1 + \frac{2(h-1)}{h}r_1 + \frac{2(h-2)}{h}r_2 + \ldots + \frac{2}{h}r_{h-1}.
\]

Here the \( r_i \) are the autocorrelations of observations \( i \) values apart, and \( h-\varepsilon(h) \) represents the number of independent observations in each section, which equals \( h-1 \) when all autocorrelations are zero.

The basic probability for the chi-square distribution is

\[
p = \left[ 2^{v/2} \Gamma \left( \frac{v}{2} \right) \right]^{-1} (\chi^2)^{(v/2)-1} \exp \left( -\frac{\chi^2}{2} \right).
\]

Here \( v \) is the number of degrees of freedom which equals the mean as well as half the variance of the distribution. Then the logarithmic likelihood functions take the form

\[
\log_e L_m(m) = -\frac{m}{2} \frac{v_m \log_e 2 - m \log_e \Gamma \left( \frac{v_m}{2} \right)}{2} + \left( \frac{v_m}{2} - 1 \right) \frac{m}{2} \sum \log_e \chi^2 - \frac{1}{2} \frac{m}{2} \sum \chi^2 ;
\]

\[
\log_e L_{m+j}(m) = -\frac{m}{2} \frac{v_{m+j} \log_e 2 - m \log_e \Gamma \left( \frac{v_{m+j}}{2} \right)}{2} + \left( \frac{v_{m+j}}{2} - 1 \right) \frac{m}{2} \sum \log_e \chi^2 - \frac{1}{2} \frac{m}{2} \sum \chi^2 ;
\]
\[
\log_e L_{m+j}(m + j) = -\frac{m + j}{2} \nu_m \log_e 2 - (m + j) \log_e \Gamma \left( \frac{\nu_m + j}{2} \right) + \frac{\nu_m}{2} - 1 \right) + \sum \log_e \chi^2 - \frac{1}{2} \sum \chi^2 ;
\]  
(A.4.5c)

\[
\log_e L_m(m + j) = -\frac{m + j}{2} \nu_m \log_e 2 - (m + j) \log_e \Gamma \left( \frac{\nu_m}{2} \right) + \frac{\nu_m}{2} - 1 \right) + \sum \log_e \chi^2 - \frac{1}{2} \sum \chi^2 .
\]  
(A.4.5d)

The likelihood ratios become

\[
q(m) = 2^\frac{m}{2(\nu_m - \nu_m)} \left[ \frac{\Gamma \left( \frac{\nu_m + j}{2} \right)}{\Gamma \left( \frac{\nu_m}{2} \right)} \right]^m \exp \left[ \frac{1}{2} (\nu_m - \nu_m + j) \sum \log_e \chi^2 \right],
\]  
(A.4.6)

and

\[
q(m + j) = 2^\frac{m + j}{2(\nu_m - \nu_m)} \left[ \frac{\Gamma \left( \frac{\nu_m + j}{2} \right)}{\Gamma \left( \frac{\nu_m}{2} \right)} \right]^{m + j} \exp \left[ \frac{1}{2} (\nu_m + j - \nu_m) \sum \log_e \chi^2 \right],
\]  
(A.4.7)

so that

\[
Q = \exp \left[ \frac{j}{2} (\nu_m - \nu_m + j) \log_e 2 + j \log_e \Gamma \left( \frac{\nu_m}{2} \right) + \frac{\nu_m + j}{2} - \frac{\nu_m}{2} \sum \log_e \chi^2 \right].
\]  
(A.4.8)

The gamma functions can be evaluated with the Euler relation

\[
\Gamma(z)^{-1} = (z) e^{\frac{z}{z}} \prod_{i=1}^{\infty} \left( 1 + \frac{z}{i} \right) e^{-z/i},
\]  
(A.4.9)

where

\[
c = \lim_{i \to \infty} \left( 1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{i} \log_e i \right) = 0.5772156649 \ldots
\]  
(A.4.10)
However, a more efficient alternative procedure for determining the Gamma function has been provided by Press et al. (1986, 6.1). It has the form

\[
\Gamma(z + 1) = \left( z + \gamma + \frac{1}{2} \right)^{\frac{1}{2}} \exp \left( -z + \gamma + \frac{1}{2} \right) \sqrt{2\pi} \left[ c_0 + 1 + \frac{c_1}{z+1} + \frac{c_2}{z+2} + \cdots + \frac{c_N}{z+N} + \epsilon \right] (z > 0)
\]

(A.4.11)

The constants \( c \) are given in the Fortran routine, Appendix B.
Appendix B

Program "SEQUITOR" is designed to be an interactive program for analysis of Gaussian mean and variance, Poisson mean, chi-square (coherence), and linear (or exponential) trend changes in a sequential time series. The user typically will receive FORTRAN source code, providing an opportunity to make code changes as desired. For example, in the original code, data input is assumed to be free format. However, the user may desire to change this to a specific format. It may also be desirable for the user to add write statements that exclude headings, such that the results can then be easily imported into a graphics package.

An input/output flowchart is included in this appendix. Each square box represents an input step by the user, and an oval represents results output. A brief description of the input steps follows:

Enter input filename: This is the input data filename up to 80 characters.

Enter descriptive title: This a descriptive header of the data and/or the analysis up to 80 characters.

Enter number of values in series: This is the total number of rows in the input data file. It is assumed that the input data file contains a column of x-values (column 1) which represent an index or year for example, followed by n columns of y-values containing the actual series for analysis.

Enter missing value: This program allows for missing values. Enter a unique number (e.g., -999.) to represent missing values.

Enter column number: Input data files may contain multiple y-value columns. This entry should be from 1 to n depending upon which y-value column is desired for analysis. The very first column in the input data file is considered column 0.

Enter 0=continue, 1=reverse data input order: Often it is desirable to do the sequential monitoring analysis beginning with the most recent value and working backwards. This helps identify "regimes" in the time series. Enter either a 0 or 1.

Enter beginning and ending *-range values: Enter values separated by a comma. This range corresponds to the x-values in the very first column of the input data file. Since the x-values might represent an index or year, examples would be 10,18 or 1985,1992. Note that these values can represent a sub-set of the input data file.

Enter window size for sub-samples: In determining a regime, it is useful to examine smaller sub-sets of values. A typical sub-sample might contain 5 values. If the total number of cases in the series is not evenly divisible by the window size, the remaining values will be ignored in only the sub-sample analysis.

Enter analysis type: Here there are several options. Entering 1 through 4 places the user in the desired sequential analysis routine. Other options include changing the sub-sample size, changing
the column number, changing the data range, reversing the data order, or simply quitting the program.

Enter number of base period values: Within each analysis routine, the user is prompted for the number of base period values. These should typically be small, say 5 to 15 or so. Base period results is output at this point.

Enter 0=continue, 1=change number of base values: Upon examining the base period results, the user is given the option to change the base period size, or continue with the final analysis.

This program was written interactively because it is intended to be exploratory in nature. An attempt was made to allow the user to make changes during the analysis, instead of having to restart the program several times. Results are output to the screen and to a file named "sequitor.out", which is replaced each time the program is run.

The program contains minimal comments, but variables are defined at the beginning of each subroutine to help the user understand the program; a sample analysis is included in this appendix.
ENTER INPUT FILE NAME: gauss.dat
ENTER DESCRIPTIVE TITLE: Detection of change in Gaussian mean and variance, Test I
INVERSE SEQUENTIAL MONITORING (PROGRAM <SEQUITOR>)
ENTER NUMBER OF VALUES IN SERIES: 15
ENTER MISSING VALUE: -999
ENTER COLUMN NUMBER: 1
ENTER 0=CONTINUE, 1=REVERSE DATA INPUT ORDER: 0
ENTER BEGINNING AND ENDING X-RANGE VALUES: 1978,1992
FULL SAMPLE UNIVARIATE STATISTICS:
NUMBER OF VALUES = 15
NUMBER OF MISSING VALUES = 0
SAMPLE MEAN = 6.240
SAMPLE VARIANCE = 40.034
SAMPLE SLOPE = 0.466
ENTER WINDOW SIZE FOR SUB-SAMPLES: 5
SUB-SAMPLE PARAMETERS:
INDEX X-VALUE NUMBER OF MISSING
RANGE RANGE VALUES VALUES MEAN VARIANCE CHI-SQUARE
1- 5 1978.-1982. 5 0 3.640 71.713 7.165
6- 10 1983.-1987. 5 0 8.220 52.852 5.281
11- 15 1988.-1992. 5 0 6.860 1.723 0.172
ENTER ANALYSIS TYPE
1=GAUSS, 2=POISSON, 3=CHI-SQUARE, 4=LINEAR,
0=CHANGE SUB-SAMPLE SIZE, 6=CHANGE COLUMN NUMBER,
7=CHANGE DATA RANGE, 8=REVERSE DATA ORDER, 9=QUIT: 1

<table>
<thead>
<tr>
<th>TEST FOR CHANGE IN GAUSSIAN MEAN AND VARIANCE</th>
</tr>
</thead>
</table>

ENTER NUMBER OF VALUES IN BASE PERIOD: 5
BASE PERIOD PARAMETERS:
NUMBER OF VALUES = 5
NUMBER OF MISSING VALUES = 0
BASE MEAN = 3.640
BASE VARIANCE = 71.713

ENTER 0=CONTINUE, 1=CHANGE NUMBER OF BASE VALUES: 0
PROGRESSIVE PARAMETERS:
X Y DELTA
INDEX OBSERVATIONS MEAN VARIANCE GAMMA GAMMA
<table>
<thead>
<tr>
<th>Year</th>
<th>Value1</th>
<th>Value2</th>
<th>Value3</th>
<th>Value4</th>
<th>Value5</th>
<th>Value6</th>
</tr>
</thead>
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<td>1983</td>
<td>2.60</td>
<td>3.467</td>
<td>57.551</td>
<td>0.486</td>
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<td></td>
</tr>
<tr>
<td>1984</td>
<td>6.60</td>
<td>3.914</td>
<td>49.361</td>
<td>0.456</td>
<td>0.030</td>
<td></td>
</tr>
<tr>
<td>1985</td>
<td>2.50</td>
<td>3.737</td>
<td>42.560</td>
<td>0.410</td>
<td>0.047</td>
<td></td>
</tr>
<tr>
<td>1986</td>
<td>20.20</td>
<td>5.567</td>
<td>67.353</td>
<td>0.453</td>
<td>-0.044</td>
<td></td>
</tr>
<tr>
<td>1987</td>
<td>9.20</td>
<td>5.930</td>
<td>61.189</td>
<td>0.422</td>
<td>0.031</td>
<td></td>
</tr>
<tr>
<td>1988</td>
<td>6.80</td>
<td>6.009</td>
<td>55.139</td>
<td>0.394</td>
<td>0.028</td>
<td></td>
</tr>
<tr>
<td>1989</td>
<td>8.80</td>
<td>6.242</td>
<td>50.775</td>
<td>0.351</td>
<td>0.043</td>
<td></td>
</tr>
<tr>
<td>1990</td>
<td>5.10</td>
<td>6.154</td>
<td>46.644</td>
<td>0.322</td>
<td>0.029</td>
<td></td>
</tr>
<tr>
<td>1991</td>
<td>6.70</td>
<td>6.193</td>
<td>43.078</td>
<td>0.280</td>
<td>0.042</td>
<td></td>
</tr>
<tr>
<td>1992</td>
<td>6.90</td>
<td>6.240</td>
<td>40.034</td>
<td>0.236</td>
<td>0.044</td>
<td></td>
</tr>
</tbody>
</table>

Enter 0=continue, 1=change number of base values:

0

Enter analysis type:
1=GAUSS, 2=POISSON, 3=CHI-SQUARE, 4=LINEAR,
0=change sub-sample size, 6=change column number,
7=change data range, 8=reverse data order, 9=quit:

9

End of Sequitor run.
PROGRAM SEQUITOR

AUTHOR: TIMOTHY J. BROWN

INVERSE SEQUENTIAL PROGRAM

------- REVISION HISTORY -------

<table>
<thead>
<tr>
<th>LEVEL</th>
<th>AUTHOR</th>
<th>DATE</th>
<th>DESCRIPTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>.01A</td>
<td>TJB</td>
<td>93/04/28</td>
<td>ORIGINAL VERSION.</td>
</tr>
<tr>
<td>.01B</td>
<td>TJB</td>
<td>93/05/07</td>
<td>REMOVED UNUSED VARIABLE 'CFLAG' FROM OUTPUT IN ROUTINES SGAUSS, SCHI, AND SPOISS.</td>
</tr>
<tr>
<td>.01C</td>
<td>TJB</td>
<td>93/08/26</td>
<td>CHANGED ALGORITHM FOR COMPUTING CHI2; CHANGED ALGORITHM FOR COMPUTING GAMMA FUNCTION IN CHI2 CALCULATION. ADDED OUTPUT OF BASE CHI-SQUARE MEAN.</td>
</tr>
</tbody>
</table>

SIX SUBROUTINES ARE ATTACHED TO THE MAIN PROGRAM:

'SGAUSS' COMPUTES CHANGE IN GAUSSIAN MEAN AND VARIANCE.
'SPOISS' COMPUTES CHANGE IN POISSON MEAN.
'SCHI' COMPUTES CHANGE IN CHI-SQUARE DEGREES OF FREEDOM.
'SLINEAR' COMPUTES CHANGE IN LINEAR TREND.
'UNIVAR' COMPUTES UNIVARIATE STATISTICS MEAN, VARIANCE, AND SUM.
'RCOEFF' COMPUTES LLS REGRESSION B0 AND B1 COEFFICIENTS.

INPUT IS ASSUMED TO BE FREE-FORMAT, BUT USER CAN CHANGE AS DESIRED.

THE PARAMETER STATEMENT AND COMMON BLOCK IS LOCATED IN ALL SUBROUTINES. THE USER SHOULD CHANGE 'NDIM' AS REQUIRED.

THE FOLLOWING ARRAYS AND VARIABLES ARE USED IN THE COMMON BLOCK:

'FINDEX' INDEX VALUE (1, 2,...N) ASSOCIATED WITH EACH Y-VALUE.
'FXVAL' INPUT X-VALUES.
'FYVAL' INPUT Y-VALUES.
'XDATA' WORK ARRAY FOR X-VALUES.
'YDATA' WORK ARRAY FOR Y-VALUES.
'FMISS' NUMBER REPRESENTING MISSING VALUES.
'NDIM' DIMENSION SIZE FOR DATA AND WORK ARRAYS.
'NCASE' NUMBER OF FULL SAMPLE VALUES WITHIN INDEX RANGE.
'OUNIT' OUTPUT UNIT NUMBER.

THE FOLLOWING ARRAYS AND VARIABLES ARE USED IN THE MAIN PROGRAM:

'FDATA' HOLDS THE Y-VALUES WHEN THEY ARE INPUT; SHOULD BE DIMENSIONED >= NUMBER OF COLUMNS IN INPUT FILE.
'FXDATA' HOLDS THE ORIGINAL X-VALUES OR REVERSED ORDER VALUES.
'FYDATA' HOLDS THE ORIGINAL Y-VALUES OR REVERSED ORDER VALUES.
'XWORK' WORK ARRAY FOR X-VALUES.
'YWORK' WORK ARRAY FOR Y-VALUES.
'B0' INTERCEPT FROM LLS REGRESSION.
'B1' SLOPE FROM LLS REGRESSION.
'CFILE' INPUT DATA FILE NAME.
'CTITLE' DESCRIPTIVE TITLE.
'H' NUMBER OF VALUES WITHIN EACH SUB-SAMPLE.
'I' DO LOOP COUNTER.
'II' INDEX COUNTER.
'IBEG' INDEX COUNTER.
INDEX COUNTER.
DATA DIRECTION FLAG (1=REVERSE DATA ORDER, 0=CONTINUE).
ANALYSIS TYPE.
INPUT UNIT NUMBER.
DO LOOP COUNTER.
COUNTER.
DO LOOP COUNTER.
BEGINNING INDEX NUMBER FOR INDEX RANGE.
ENDING INDEX RANGE FOR INDEX RANGE.
COLUMN NUMBER OF Y-VALUES TO BE ANALYZED.
THIS IS USEFUL FOR FILES CONTAINING MULTIPLE COLUMNS OF DATA.
X-VALUES ARE ASSUMED TO BE IN COLUMN ONE.
NUMBER OF MISSING VALUES.
NUMBER OF POPULATION VALUES.
NUMBER OF SUB-SAMPLES.
NUMBER OF NON-MISSING VALUES.
POPULATION VARIANCE FROM FULL SAMPLE.
BEGINNING VALUE OF X-RANGE.
ENDING VALUE OF X-RANGE.
MEAN OF Y-VALUES.
SUM OF Y-VALUES.
VARIANCE OF Y-VALUES.
PARAMETER (NDIM=10000)
COMMON /WORK/ FXVAL(NDIM), FYVAL(NDIM), XDATA(NDIM),
+ YDATA(NDIM), FINDEX(NDIM), NCASE, NSUB, H,
+ OUNIT, FMISS
REAL FDATA(15)
REAL FXDATA(NDIM), FYDATA(NDIM), XWORK(NDIM), YWORK(NDIM)
INTEGER H, OUNIT
CHARACTER*80 CTITLE, CFILE
DATA IUNIT, OUNIT / 1, 2 /

---------------------------------------------------------------

THIS SECTION REQUESTS THE INPUT INFORMATION, OPENS FILES, INPUTS
THE DATA, AND COMPUTES FULL SAMPLE UNIVARIATE STATISTICS.

WRITE(*,801)
READ(*,101) CFFILE
OPEN(IUNIT,FILE=CFFILE,STATUS='OLD')
OPEN(OUNIT,FILE='sequitor.out')
WRITE(*,802)
READ(*,101) CTITLE
WRITE(OUNIT,900)
WRITE(*,900)
WRITE(OUNIT,901) CTITLE
WRITE(*,803)
READ(*,*) NPOP
WRITE(*,804)
READ(*,*) FMISS
3 CONTINUE
WRITE(*,805)
READ(*,*) NCOL
REWIND IUNIT

C INPUT THE DATA AND FILL WORK ARRAYS. REVERSE DATA ORDER IF REQUESTED.

DO 13 I = 1, NPOP
   READ(IUNIT,*) FXDATA(I), (FDATA(J), J=1, NCOL)
   FINDEX(I) = FLOAT(I)
   FYDATA(I) = FDATA(NCOL)
13 CONTINUE

WRITE(*,806)
READ(*,*) IDIR

IF( IDIR .EQ. 1 ) THEN
   K = 0
   DO 14 I = NPOP, 1, -1
      K = K + 1
      XWORK(K) = FXDATA(I)
      YWORK(K) = FYDATA(I)
14 CONTINUE

   DO 15 I = 1, NPOP
      FXDATA(I) = XWORK(I)
      FYDATA(I) = YWORK(I)
15 CONTINUE
END IF

C FILL WORK ARRAYS WITH DATA WITHIN SELECTED INDEX RANGE AND COLUMN.

WRITE(*,807)
READ(*,*) XBEG, XEND

DO 18 I = 1, NPOP
   IF( FXDATA(I) .EQ. XBEG ) NBEG = I
   IF( FXDATA(I) .EQ. XEND ) NEND = I
18 CONTINUE

IF( NBEG .LT. 1 ) THEN
WRITE(*,810)
GOTO 4

END IF
C---------
C---------
IF( NEND .GT. NPOP ) THEN

WRITE(*,810)
GOTO 4

END IF
C---------
NCASE = 0
C---------
DO 16 I = 1, NPOP
C---------
IF( I .GE. NBEG .AND. I .LE. NEND ) THEN

NCASE = NCASE + 1

FXVAL(NCASE) = FXDATA(I)
FYVAL(NCASE) = FYDATA(I)
XDATA(NCASE) = FINDEX(I)
YDATA(NCASE) = FYVAL(NCASE)

END IF
C---------
16 CONTINUE
C---------
C COMPUTE FULL SAMPLE STATISTICS AND OUTPUT RESULTS
C
CALL UNIVAR( NCASE, NVALS, NMISS, YBAR, YVAR, YSUM )
CALL RCOEFF( NCASE, B0, B1 )

IRANGE = NEND - NBEG + 1

WRITE(OUNIT,902) FXVAL(1), FXVAL(IRANGE), NVALS, NMISS, YBAR,
+ YVAR, B1
WRITE(*,902) FXVAL(1), FXVAL(IRANGE), NVALS, NMISS, YBAR,
+ YVAR, B1
1 CONTINUE

WRITE(*,808)
READ(*,*) H

WRITE(OUNIT,903)
WRITE(*,903)
C
C COMPUTE SUB-SAMPLE STATISTICS.
C
POPVAR = YVAR
NN = 0
K = 0
N = 0
NSUB = 0
IBEG = -(H) + 1
C---------
DO 17 I = NBEG, NEND
   K = K + 1
   IBEG = IBEG + 1
   N = N + 1
C------------------
   IF( FYVAL(N) .GT. FMISS ) THEN
      NN = NN + 1
      YDATA(NN) = FYVAL(N)
   END IF
C------------------
C------------------
   IF( K .EQ. H ) THEN
      CALL UNIVAR( NN, NVALS, NMISS, YEAR, YVAR, YSUM )
C OUTPUT SUB-SAMPLE STATISTICS.
C
II = (I-H) + 1
IEND = IBEG + H - 1

WRITE(OUNIT,904) II, I, FXVAL(IBEG), FXVAL(IEND), NVALS, NMISS, YEAR, YVAR
WRITE(*,904) II, I, FXVAL(IBEG), FXVAL(IEND), NVALS, NMISS, YEAR, YVAR

NSUB = NSUB + 1
NN = 0
K = 0
END IF
C------------------
17 CONTINUE
C------------------
C BRANCH OFF TO APPROPRIATE SUBROUTINE, CHANGE SUB-SAMPLE SIZE,
C CHANGE COLUMN NUMBER, OR STOP PROGRAM.
C
2 CONTINUE
WRITE(*,809)
READ(*,*) ITYPE
C------------------
   IF( ITYPE .EQ. 0 ) THEN
      GOTO 1
   ELSE IF( ITYPE .EQ. 1 ) THEN
      WRITE(OUNIT,1001)
      WRITE(*,1001)
      CALL SGAUSS
   ELSE IF( ITYPE .EQ. 2 ) THEN
      WRITE(OUNIT,1002)
      WRITE(*,1002)
CALL SPOISS
ELSE IF( ITYPE .EQ. 3 ) THEN
    WRITE(OUNIT,1003)
    WRITE(*,1003)
    CALL SCHI
ELSE IF( ITYPE .EQ. 4 ) THEN
    WRITE(OUNIT,1004)
    WRITE(*,1004)
    CALL SLINEAR
ELSE IF( ITYPE .EQ. 6 ) THEN
    GOTO 3
ELSE IF( ITYPE .EQ. 7 ) THEN
    GOTO 4
ELSE IF( ITYPE .EQ. 8 ) THEN
    GOTO 5
ELSE IF( ITYPE .EQ. 9 ) THEN
    WRITE(OUNIT,907)
    WRITE(*,907)
    GOTO 999
ELSE
    WRITE(*,905)
    GOTO 2
END IF
C----------------
GOTO 2

101  FORMAT(A)

801  FORMAT( ' ENTER INPUT FILE NAME:' )
802  FORMAT( ' ENTER DESCRIPTIVE TITLE:' )
803  FORMAT( ' ENTER NUMBER OF VALUES IN SERIES:' )
804  FORMAT( ' ENTER MISSING VALUE:' )
805  FORMAT( ' ENTER COLUMN NUMBER:' )
806  FORMAT( ' ENTER 0=CONTINUE, 1=REVERSE DATA INPUT ORDER:' )
807  FORMAT( ' ENTER BEGINNING AND ENDING X-RANGE VALUES:' )
808  FORMAT( ' ENTER WINDOW SIZE FOR SUB-SAMPLES:' )
809  FORMAT( ' ENTER ANALYSIS TYPE',\,/
+      ' 1=GAUSS, 2=POISSON, 3=CHI-SQUARE, 4=LINEAR',\,/
+      ' 0=CHANGE SUB-SAMPLE SIZE, 6=CHANGE COLUMN NUMBER',\,/
+      ' 7=CHANGE DATA RANGE, 8=REVERSE DATA ORDER, 9=QUIT' )
810  FORMAT( ' RANGE EXCEEDS TOTAL NUMBER OF CASES' )
FORMAT('INVERSE SEQUENTIAL MONITORING (PROGRAM <SEQUITOR>)')

FORMAT(//,A)

FORMAT(/'FULL SAMPLE UNIVARIATE STATISTICS:/',
         'X-VALUE RANGE = ',F5.0,'-',F5.0,/, 
         'NUMBER OF VALUES = ',I3,/, 
         'NUMBER OF MISSING VALUES = ',I3,/, 
         'SAMPLE MEAN = ',F8.3,/, 
         'SAMPLE VARIANCE = ',F8.3,/, 
         'SAMPLE SLOPE = ',F8.3)

FORMAT(/'SUB-SAMPLE PARAMETERS:',/, 
         'INDEX  X-VALUE  NUMBER OF MISSING',/,' ', 
         'RANGE  RANGE  VALUES  VALUES  MEAN  VARIANCE',/, 
         '-------  =======  ==========  =======  =======  =======')

FORMAT(I3, '-', I3, 2X, F5.0, '-', F5.0, 2X, I9, 2X, I7, 2(2X, F8.3))

FORMAT(//'»> NOT A VALID SELECTION <<<',/)

FORMAT(//'END OF SEQUITOR RUN')

SUBROUTINE SGAUSS

C SUBROUTINE TO COMPUTE CHANGE IN GAUSSIAN MEAN AND VARIANCE.
C
C VARIABLES USED IN SUBROUTINE NOT DESCRIBED IN MAIN PROGRAM:
C 'BARM' MEAN OF BASE PERIOD VALUES.
C 'BARMJ' PROGRESSIVE MEAN.
C 'DGAMMA' CHANGE IN GAMMA FROM PREVIOUS VALUE.
C 'GAMMA' GAMMA VALUE.
C 'GAMOLD' PREVIOUS VALUE OF GAMMA.
C 'J' PROGRESSIVE VALUE INDEX.
C 'JJ' DO LOOP COUNTER.
C 'J1' STARTING VALUE FOR DO LOOP COMPUTING PROGRESSIVE VALUES.
C 'J2' ENDING VALUE FOR DO LOOP COMPUTING PROGRESSIVE VALUES.
C 'M' REAL VALUE OF NUMBER OF BASE VALUES.
C 'MBASE' NUMBER OF BASE PERIOD VALUES.
PARAMETER (NDIM=10000)
COMMON /WORK/ FXVAL(NDIM), FYVAL(NDIM), XDATA(NDIM),
+ YDATA(NDIM), FINDEX(NDIM), NCASE, NSUB, H,
+ OUNIT, FMISS
REAL J, M
INTEGER OUNIT, H

C FILL WORK ARRAYS WITH BASE PERIOD VALUES, COMPUTE BASE PERIOD
C STATISTICS, AND OUTPUT RESULTS.

1 CONTINUE

WRITE(*,801)
READ(*,*) MBASE
WRITE(OUNIT,1000)
WRITE(*,1000)

DO 20 I = 1, MBASE

XDATA(I) = FINDEX(I)
YDATA(I) = FYVAL(I)

WRITE(OUNIT,1001) I, XDATA(I), YDATA(I)
WRITE(*,1001) I, XDATA(I), YDATA(I)

20 CONTINUE

CALL UNIVAR( MBASE, NVALS, NMISS, YBAR, YVAR, YSUM )

--- IF( NVALS .GT. 0 ) THEN

WRITE(OUNIT,1002) FXVAL(1), FXVAL(MBASE), NVALS, NMISS, YBAR,
+ YVAR
WRITE(*,1002) FXVAL(1), FXVAL(MBASE), NVALS, NMISS, YBAR,
+ YVAR

ELSE

WRITE(*,1003)
RETURN

END IF

--- WRITE(*,802)
READ(*,*) ITYPE
IF( ITYPE .EQ. 1 ) GOTO 1

WRITE(OUNIT,1004)
WRITE(*,1004)
INITIALIZE AND COMPUTE FIXED VALUES; BEGIN PROGRESSIVE VALUE LOOP.

BARM = YBAR
SDM = SQRT(YVAR)
SDM2 = YVAR
M = FLOAT(MBASE)
GAMOLD = FMISS
J1 = MBASE + 1
J2 = NCASE

FILL PROGRESSIVE WORK ARRAYS, COMPUTE PROGRESSIVE STATISTICS, AND OUTPUT RESULTS.

DO 22 JJ = J1, J2
   XDATA(JJ) = FINDEX(JJ)
   YDATA(JJ) = FYVAL(JJ)
   CALL UNIVAR(JJ, NVALS, NMISS, YBAR, YVAR, YSUM)

COMPUTE GAMMA. SEE TEXT FOR EQUATION DETAILS.

IF( NVALS .GT. 0 ) THEN
   BARMJ = YBAR
   SDMJ = SQRT(YVAR)
   SDMJ2 = YVAR
   J = FLOAT(JJ) - M
   T1 = M * ALOG(SDM / SDMJ)
   T2 = ((M - 1.) / 2.) * (1. - (SDM2 / SDMJ2))
   T3 = (M / (2. * SDM2)) * (BARMJ - BARM)**2
   QM = EXP(T1 + T2 - T3)
   T1 = (M + J) * ALOG(SDM / SDMJ)
   T2 = ((M + J - 1.) / 2.) * ((SDMJ2 / SDM2) - 1.)
   T3 = ((M + J) / (2. * SDM2)) * (BARMJ - BARM)**2
   QMJ = EXP(T1 + T2 + T3)
   Q = QMJ / QM
   GAMMA = 1. / (1. + SQRT(Q))
   DGAMMA = GAMOLD - GAMMA

IF( GAMOLD .EQ. FMISS ) THEN
   WRITE(OUNIT,1005) JJ, FXVAL(JJ), YDATA(JJ), YBAR, YVAR, GAMMA
   WRITE(*,1005) JJ, FXVAL(JJ), YDATA(JJ), YBAR, YVAR, GAMMA
ELSE
   WRITE(OUNIT,1005) JJ, FXVAL(JJ), YDATA(JJ), YBAR, YVAR, GAMMA, DGAMMA
   WRITE(*,1005) JJ, FXVAL(JJ), YDATA(JJ), YBAR, YVAR, GAMMA, DGAMMA
END IF

GAMOLD = GAMMA

ELSE

WRITE(OUNIT,1006) JJ, FXVAL(JJ), YDATA(JJ)
WRITE(*,1006) JJ, FXVAL(JJ), YDATA(JJ)
GAMOLD = FMISS

END IF

22 CONTINUE

WRITE(*,802)
READ(*,*) ITYPE
IF( ITYPE .EQ. 1 ) GOTO 1

801 FORMT(//,' ENTER NUMBER OF VALUES IN BASE PERIOD:')
802 FORMT(//,' ENTER 0=CONTINUE, 1=CHANGE NUMBER OF BASE VALUES:')

1000 FORMT('BASE PERIOD INPUT VALUES:'
   + 'INDEX  X      Y'
   + '===== ======== ========'

1001 FORMT(I5,2X,F8.3,2X,F8.2)

1002 FORMT('BASE PERIOD PARAMETERS:',/
   'X-VALUE RANGE = ',F5.0,'-',F5.0,/, 
   'NUMBER OF VALUES = ',I3,/, 
   'NUMBER OF MISSING VALUES = ',I3,/, 
   'BASE MEAN = ',F8.3,/, 
   'BASE VARIANCE = ',F8.3,/) 

1003 FORMT(' ALL VALUES IN BASE PERIOD MISSING')

1004 FORMT('PROGRESSION PARAMETERS:',/
   'INDEX  X      Y  DELTA',/
   'INDEX  OBSERVATIONS  MEAN  VARIANCE  GAMMA  GAMMA',/
   '===== ======== ===== ====='


1006 FORMT(I5,2X,F8.3,1X,F7.2)

RETURN

END

SUBROUTINE SPOISS

SUBROUTINE TO COMPUTE CHANGE IN POISSON MEAN.

VARIABLES USED IN SUBROUTINE NOT DESCRIBED IN MAIN PROGRAM:
'BARM' MEAN OF BASE PERIOD VALUES.
'BARMJ' PROGRESSIVE MEAN.
'DGAMMA' CHANGE IN GAMMA FROM PREVIOUS VALUE.
'GAMMA' GAMMA VALUE.
'GAMOLD' PREVIOUS VALUE OF GAMMA.
'J' PROGRESSIVE VALUE INDEX.
'JJ' DO LOOP COUNTER.
PARAMETER (NDIM=10000)
COMMON /WORK/ FXVAL(NDIM), FYVAL(NDIM), XDATA(NDIM),
                   YDATA(NDIM), FINDEX(NDIM), NCASE, NSUB, H,
                   OUNIT, FMISS
REAL J, M
INTEGER OUNIT, H

C FILL WORK ARRAYS WITH BASE PERIOD VALUES, COMPUTE BASE PERIOD
C STATISTICS, AND OUTPUT RESULTS.

1 CONTINUE
WRITE(*,801)
READ(*,*) MBASE
WRITE(OUNIT,2000)
WRITE(*,2000)
DO 20 I = 1, MBASE
   XDATA(I) = FINDEX(I)
   YDATA(I) = FYVAL(I)
   WRITE(OUNIT,2001) I, XDATA(I), YDATA(I)
   WRITE(*,2001) I, XDATA(I), YDATA(I)
20 CONTINUE

CALL UNIVAR( MBASE, NVALS, NMISS, YBAR, YVAR, YSUM )

IF( NVALS .GT. 0 ) THEN
   WRITE(OUNIT,2002) FXVAL(1), FXVAL(MBASE), NVALS, NMISS,
                  YBAR, YVAR, YSUM
   WRITE(*,2002) FXVAL(1), FXVAL(MBASE), NVALS, NMISS, YBAR,
                  YVAR, YSUM
ELSE
   WRITE(*,2003)
   RETURN
END IF

WRITE(*,802)
READ(*,*) ITYPE
IF( ITYPE .EQ. 1 ) GOTO 1
WRITE(OUNIT,2004)
WRITE(*,2004)

C INITIALIZE AND COMPUTE FIXED VALUES; BEGIN PROGRESSIVE VALUE LOOP.
BARM = YBAR
M = FLOAT(MBASE)
GAMOLD = FMISS
J1 = MBASE + 1
J2 = NCASE

C FILL PROGRESSIVE WORK ARRAYS, COMPUTE PROGRESSIVE STATISTICS, AND
C OUTPUT RESULTS.

C-----
DO 22 JJ = J1, J2

XDATA(JJ) = FINDEX(JJ)
YDATA(JJ) = FYVAL(JJ)

CALL UNIVAR( JJ, NVALS, NMISS, YBAR, YVAR, YSUM )

C COMPUTE GAMMA. SEE TEXT FOR EQUATION DETAILS.

C-----
IF( NVALS .GT. 0 ) THEN

BARMJ = YBAR
J = FLOAT(JJ) - M

T1 = (BARMJ * (M + J)) - (BARM * M)
T2 = ALOG(BARMJ / BARM)
T3 = (BARMJ - BARM) * J
Q = EXP(T1 * T2 - T3)
GAMMA = 1. / (1. + SQRT(Q))
DGAMMA = GAMOLD - GAMMA

C-----
IF( GAMOLD .EQ. FMISS ) THEN

WRITE(OUNIT,2005) JJ, FXVAL(JJ), YDATA(JJ), BARMJ, YVAR,
                 YSUM, GAMMA
WRITE(*,2005) JJ, FXVAL(JJ), YDATA(JJ), BARMJ, YVAR,
                 YSUM, GAMMA

ELSE

WRITE(OUNIT,2005) JJ, FXVAL(JJ), YDATA(JJ), BARMJ, YVAR,
                 YSUM, GAMMA, DGAMMA
WRITE(*,2005) JJ, FXVAL(JJ), YDATA(JJ), BARMJ, YVAR,
                 YSUM, GAMMA, DGAMMA

END IF

C-----
GAMOLD = GAMMA

ELSE

WRITE(OUNIT,2006) JJ, FXVAL(JJ), YDATA(JJ)
WRITE(*,2006) JJ, FXVAL(JJ), YDATA(JJ)
GAMOLD = FMISS

END IF

C-----
22 CONTINUE
C--------
WRITE(*,802)
READ(*,*) ITYPE
IF( ITYPE .EQ. 1 ) GOTO 1

801 FORMAT(/, ' ENTER NUMBER OF VALUES IN BASE PERIOD: ')
802 FORMAT(/, ' ENTER 0=CONTINUE, 1=CHANGE NUMBER OF BASE VALUES: ')

2000 FORMAT(/, 'BASE PERIOD INPUT VALUES: '
    + '/ INDEX   X       Y'
    + '/==== ======== ========

2001 FORMAT(I5,2X,F8.3,2X,F8.2)

2002 FORMAT(/, 'BASE PERIOD PARAMETERS: ',/,
    + ' X-VALUE RANGE = ',F5.0,'-',F5.0,/, 
    + ' NUMBER OF VALUES = ',I3,/, 
    + ' NUMBER OF MISSING VALUES = ',I3,/, 
    + ' BASE MEAN = ',F8.3,/, 
    + ' BASE VARIANCE = ',F8.3,/, 
    + ' BASE SUM = ',F6.0,/) 

2003 FORMAT(' ALL VALUES IN BASE PERIOD MISSING') 

2004 FORMAT(/, 'PROGRESSIVE PARAMETERS: ',/,
    + ' X Y ' 
    + ' DELTA',/,
    + 'INDEX OBSERVATIONS MEAN VARIANCE SUM GAMMA', 
    + ' GAMMA',/,
    + '/==== ============== =========== ========= ===========


2006 FORMAT(I5,2X,F8.3,1X,F7.0)
RETURN
END

C-------------------
SUBROUTINE SCHI
C
C SUBROUTINE TO COMPUTE CHANGE IN CHI-SQUARE DEGREES OF FREEDOM.
C
C VARIABLES USED IN SUBROUTINE NOT DESCRIBED IN MAIN PROGRAM:
C 'DGAMMA' CHANGE IN GAMMA FROM PREVIOUS VALUE.
C 'FCHI2' CHI-SQUARE VALUE FOR EACH SUB-SAMPLE.
C 'GAMMA' GAMMA VALUE.
C 'GAMNUM' VALUE OF THE GAMMA FUNCTION FOR THE BASE PERIOD.
C 'GAMNUMJ' VALUE OF THE GAMMA FUNCTION FOR THE PROGRESSIVE PERIOD.
C 'GAMOLD' PREVIOUS VALUE OF GAMMA.
C 'GNUM' BASE PERIOD MEAN FOR GAMMA FUNCTION.
C 'GNUMJ' PROGRESSIVE MEAN FOR GAMMA FUNCTION.
C 'H' NUMBER OF SUB-SAMPLES.
C 'J' PROGRESSIVE VALUE INDEX.
C 'JJ' DO LOOP COUNTER.
C 'J1' STARTING VALUE FOR DO LOOP COMPUTING PROGRESSIVE VALUES.
C 'J2' ENDING VALUE FOR DO LOOP COMPUTING PROGRESSIVE VALUES.
C 'K' INDEX COUNTER.
C 'KK' INDEX COUNTER.
C 'M' REAL VALUE OF NUMBER OF BASE VALUES.
PARAMETER (NDIM=10000)
COMMON /WORK/ FXVAL(NDIM), FYVAL(NDIM), XDATA(NDIM),
+ YDATA(NDIM), FINDEX(NDIM), NCASE, NSUB, H,
+ OUNIT, FMISS
REAL J, M, NUM, NUMJ, FCHI2(NDIM)
INTEGER OUNIT, H

C FILL WORK ARRAYS WITH BASE PERIOD VALUES, COMPUTE BASE PERIOD
C STATISTICS, AND OUTPUT RESULTS.
C
1 CONTINUE
WRITE(*,801)
READ(*,*) MBASE
WRITE(OUNIT,3000)
WRITE(*,3000)

C---------
DO 20 I = 1, MBASE
   XDATA(I) = FINDEX(I)
   YDATA(I) = FYVAL(I)
   WRITE(OUNIT,3001) I, XDATA(I), YDATA(I)
   WRITE(*,3001) I, XDATA(I), YDATA(I)
20 CONTINUE
C---------
NBASE = MBASE * H
C---------
DO 21 I = 1, NBASE
   YDATA(I) = FYVAL(I)
21 CONTINUE
C---------
CALL UNIVAR( NBASE, NVALS, NMISS, YBAR, YVAR, YSUM )
C---------
IF( NVALS .GT. 0 ) THEN
   WRITE(OUNIT,3002) FXVAL(1), FXVAL(NBASE), NVALS, NMISS,
   + YBAR, YVAR
   WRITE(*,3002) FXVAL(1), FXVAL(NBASE), NVALS, NMISS, YBAR,
   + YVAR
   YVARM = YVAR
ELSE

WRITE(*,3003)
RETURN
END IF

C------
C  COMPUTE CHI2 VALUES.
C
  KK = 0
C------
  DO 22 JJ = 1, NSUB

    K = 0
    XDATA(JJ) = FLOAT(JJ)
C------
  DO 24 I = 1, H

    K = K + 1
    KK = KK + 1
    YDATA(K) = FYVAL(KK)

24 CONTINUE

C------
  CALL UNIVAR( H, NVALS, NMISS, YBAR, YVAR, YSUM )

  YVARC = YVAR
C------
  IF( JJ .GT. MBASE ) THEN

    KK = JJ * H
C------
  DO 26 I = 1, KK

    YDATA(I) = FYVAL(I)

26 CONTINUE

C------
  CALL UNIVAR( KK, NVALS, NMISS, YBAR, YVAR, YSUM )

  FCHI2(JJ) = (H - 1.) * YVARC / YVAR
ELSE

  FCHI2(JJ) = (H - 1.) * YVARC / YVARM
END IF

C------
22 CONTINUE

C------
  DO 28 I = 1, MBASE

    YDATA(I) = FCHI2(I)

28 CONTINUE

C------
  CALL UNIVAR( MBASE, NVALS, NMISS, YBAR, YVAR, YSUM )
C COMPUTE GAMMA FUNCTION FOR THE BASE PERIOD.
C
NUM = YBAR

WRITE(OUNIT,3004) NUM
WRITE(*,3004) NUM
WRITE(*,802)
READ(*,*) ITYPE
IF( ITYPE .EQ. 1 ) GOTO 1

WRITE(OUNIT,3005)
WRITE(*,3005)

GNUM = YBAR / 2.
CALL GAMMLN( GNUM, GAMNUM )

C INITIALIZE AND COMPUTE FIXED VALUES; BEGIN PROGRESSIVE VALUE LOOP.
C
J1 = MBASE + 1
J2 = NSUB
M = FLOAT(MBASE)
GAMOLD = FMISS
SUMCHI2 = 0.

C FILL PROGRESSIVE WORK ARRAYS, COMPUTE PROGRESSIVE STATISTICS, AND
C OUTPUT RESULTS.
C
-----
DO 30 JJ = J1, J2
     SUMCHI2 = SUMCHI2 + ALOG(FCHI2(JJ))
-----
DO 32 I = 1, JJ
     YDATA(I) = FCHI2(I)
     32 CONTINUE

CALL UNIVAR( JJ, NVALS, NMISS, YBAR, YVAR, YSUM )

IF( NVALS .GT. 0 ) THEN

C COMPUTE GAMMA FUNCTION FOR PROGRESSIVE VALUES.
C
NUMJ = YBAR
GNUMJ = YBAR / 2.
CALL GAMMLN( GNUMJ, GAMNUMJ )
J = FLOAT(JJ) - M

C COMPUTE GAMMA. SEE TEXT FOR EQUATION DETAILS.
C
T1 = (J * (NUM - NUMJ) * ALOG(2.)) / 2.
T2 = J * ALOG(GAMNUM / GAMNUMJ)
T3 = ((NUMJ - NUM) / 2.) * SUMCHI2
Q = EXP(T1 + T2 + T3)

GAMMA = 1. / (1. + SQRT(Q))
DGAMMA = GAMOLD - GAMMA
KK = JJ * H

C---------
DO 34 I = 1, KK
    YDATA(I) = FYVAL(I)
    CONTINUE
C------------------------
CALL UNIVAR( KK, NVALS, NMISS, YBAR, YVAR, YSUM )
C------------------------
    IF( GAMOLD .EQ. FMISS ) THEN
        WRITE(OUNIT,3006) JJ, FCHI2(JJ), NUMJ, YVAR, GAMMA
        WRITE(*,3006) JJ, FCHI2(JJ), NUMJ, YVAR, GAMMA
    ELSE
        WRITE(OUNIT,3006) JJ, FCHI2(JJ), NUMJ, YVAR, GAMMA,
                        DGAMMA
        WRITE(*,3006) JJ, FCHI2(JJ), NUMJ, YVAR, GAMMA, DGAMMA
    END IF
C------------------------
    GAMOLD = GAMMA
ELSE
    WRITE(OUNIT,3007) JJ, YDATA(JJ)
    WRITE(*,3007) JJ, YDATA(JJ)
    GAMOLD = FMISS
END IF
C------------------------
30 CONTINUE
C------------------------
    WRITE(*,802)
    READ(*,*) ITYPE
    IF( ITYPE .EQ. 1 ) GOTO 1
801 FORMAT(/,' ENTER NUMBER OF VALUES IN BASE PERIOD:')
802 FORMAT(/,' ENTER 0=CONTINUE, 1=CHANGE NUMBER OF BASE VALUES:')
3000 FORMAT(/'BASE PERIOD INPUT VALUES:'
            + '/INDEX X Y'
            + '/===== ======== ========')
3001 FORMAT(15,2X,F8.3,2X,F8.2)
3002 FORMAT(/'BASE PERIOD PARAMETERS:',/,'X-VALUE RANGE = ',F5.0,'-',F5.0,/, 'NUMBER OF VALUES = ',I3,/, 'NUMBER OF MISSING VALUES = ',I3,/, 'BASE MEAN = ',F8.3,/, 'BASE VARIANCE = ',F8.3)
3003 FORMAT(' ALL VALUES IN BASE PERIOD MISSING')
3004 FORMAT('BASE CHI-SQUARE = ',F8.3,)
3005 FORMAT(/'PROGRESSIVE PARAMETERS:',/,'CHI-SQUARE DELTA',/,'INDEX OBSERVATIONS MEAN VARIANCE GAMMA GAMMA',/,
SUBROUTINE SLINEAR

SUBROUTINE TO COMPUTE CHANGE IN LINEAR TREND.

VARIABLES USED IN SUBROUTINE NOT DESCRIBED IN MAIN PROGRAM:

'AM'  BASE PERIOD INTERCEPT FROM LLS REGRESSION.
'Bm'  BASE PERIOD SLOPE FROM LLS REGRESSION.
'DM'  CONSTANT USED IN Q(M) LIKELIHOOD RATIO.
'DMJ' CONSTANT USED IN Q(M+J) LIKELIHOOD RATIO.
'DENOM'  DENOMINATOR IN 'T21' AND 'T22' TERMS.
'DGAMMA' CHANGE IN GAMMA FROM PREVIOUS VALUE.
'GAMMA' GAMMA VALUE.
'GAMOLD' PREVIOUS VALUE OF GAMMA.
'I'  DO LOOP COUNTER.
'J'  PROGRESSIVE VALUE INDEX.
'JJ'  DO LOOP COUNTER.
'J1'  STARTING VALUE FOR DO LOOP COMPUTING PROGRESSIVE VALUES.
'J2'  ENDING VALUE FOR DO LOOP COMPUTING PROGRESSIVE VALUES.
'LNL1' NATURAL LOG LIKELIHOOD 1.
'LNL2' NATURAL LOG LIKELIHOOD 2.
'LNL3' NATURAL LOG LIKELIHOOD 3.
'LNL4' NATURAL LOG LIKELIHOOD 4.
'M'  REAL VALUE OF NUMBER OF BASE VALUES.
'MBASE' NUMBER OF BASE PERIOD VALUES.
'PREDM' PREDICTED VALUES FROM REGRESSION EQUATION USING 'M' VALUES.
'PREDMJ' PREDICTED VALUES FROM REGRESSION EQUATION USING 'M+J' VALUES.
'Q'  RATIO OF QMJ/QM.
'QM'  BASE PERIOD LIKELIHOOD RATIO.
'QMJ' PROGRESSIVE PERIOD LIKELIHOOD RATIO.
'RESIDM' ARRAY OF BASE PERIOD RESIDUALS FROM LLS REGRESSION.
'RESIDMJ' ARRAY OF PROGRESSIVE VALUE RESIDUALS FROM LLS REGRESSION.
'SSM'  SUM OF SQUARES FOR BASE PERIOD.
'SSMJ'  SUM OF SQUARES FOR 'M+J' VALUES.
'SUMT2' SUM OF 'T2' TERM IN 'LNL1' AND 'LNL4' FORMULAE.
'SUMT21' SUM OF 'T2' TERM IN 'LNL2' FORMULAE.
'SUMT22' SUM OF 'T2' TERM IN 'LNL3' FORMULAE.
'T'  REAL VALUE OF BASE PERIOD LOOP INDEX.
'T1'  WORK VARIABLE FOR FIRST TERM IN 'LNL1' AND 'LNL4' FORMULAE.
'T2'  WORK VARIABLE FOR SECOND TERM IN 'LNL1' AND 'LNL4' FORMULAE.
'T11' WORK VARIABLE FOR FIRST TERM IN 'LNL2' FORMULAE.
'T12' WORK VARIABLE FOR FIRST TERM IN 'LNL3' FORMULAE.
'T21' WORK VARIABLE FOR SECOND TERM IN 'LNL2' FORMULAE.
'T22' WORK VARIABLE FOR SECOND TERM IN 'LNL3' FORMULAE.

PARAMETER (NDIM=10000)

COMMON /WORK/ FXVAL(NDIM), FYVAL(NDIM), XDATA(NDIM),
     + YDATA(NDIM), FINDEX(NDIM), NCASE, NSUB, H,
     + OUNIT, FMISS

REAL RESIDM(NDIM), RESIDMJ(NDIM), PREDM(NDIM), PREDMJ(NDIM)
REAL M, J, LNL1, LNL2, LNL3, LNL4
INTEGER OUNIT, H
FILL WORK ARRAYS WITH BASE PERIOD VALUES, COMPUTE BASE PERIOD STATISTICS, AND OUTPUT RESULTS.

CONTINUE

WRITE(*,801)
READ(*,*) MBASE
WRITE(OUNIT,4000)
WRITE(*,4000)

DO 20 I = 1, MBASE
    WRITE(OUNIT,4001) I, FXVAL(I), FYVAL(I)
    WRITE(*,4001) I, FXVAL(I), FYVAL(I)
20 CONTINUE

DO 21 I = 1, NCASE
    XDATA(I) = FINDEX(I)
    YDATA(I) = FYVAL(I)
21 CONTINUE

CALL UNIVAR( MBASE, NVALS, NMISS, YEAR, YVAR, YSUM )

IF( NVALS .GT. 0 ) THEN
    CALL RCOEFF( MBASE, AM, BM )
    SDM = SQRT(YVAR*(12./(FLOAT(MBASE)*(FLOAT(MBASE**2)-1.))))
    SDM2 = SDM**2
    BARM = BM
    WRITE(OUNIT,4002) FXVAL(1), FXVAL(MBASE), NVALS, NMISS, YBAR, YVAR, BM
    WRITE(*,4002) FXVAL(1), FXVAL(MBASE), NVALS, NMISS, YBAR, YVAR, BM
ELSE
    WRITE(*,4003)
    RETURN
END IF

WRITE(*,802)
READ(*,*) ITYPE
IF( ITYPE .EQ. 1 ) GOTO 1

WRITE(OUNIT,4004)
WRITE(*,4004)

M = FLOAT(MBASE)
DM = (M * (M**2 - 1.)) / 12.
SSM = 0.

DO 22 I = 1, MBASE
IF( YDATA(I) .NE. FMISS ) THEN

SSM = SSM + (YDATA(I) - YBAR)**2

END IF

C---------

22 CONTINUE

C---------

DO 23 I = 1, NCASE

C---------

IF( YDATA(I) .NE. FMISS ) THEN

PREDM(I) = (XDATA(I) * BM + AM)
RESIDM(I) = (YDATA(I) - PREDM(I))

ELSE

PREDM(I) = FMISS
RESIDM(I) = FMISS

END IF

C---------

23 CONTINUE

C---------

SUMT2 = 0.
T1 = (-1.) * (M - 2.) / (2. * (SSM - (BM**2 * DM)))

C---------

DO 24 I = 1, MBASE

C---------

IF( YDATA(I) .NE. FMISS ) THEN

T = FLOAT(I)

T2 = RESIDM(I)**2 / (((M + 1.) / M) + (T - ((M + 1.) / 2.))**2 / DM)

SUMT2 = SUMT2 + T2

END IF

C---------

24 CONTINUE

C---------

LNL1 = T1 * SUMT2

C---------

C INITIALIZE AND COMPUTE FIXED VALUES; BEGIN PROGRESSIVE VALUE LOOP.

GAMOLD = FMISS
J1 = MBASE + 1
J2 = NCASE

C FILL PROGRESSIVE WORK ARRAYS, COMPUTE PROGRESSIVE STATISTICS, AND OUTPUT RESULTS.

DO 26 JJ = J1, J2

XDATA(JJ) = FINDEX(JJ)
YDATA(JJ) = FYVAL(JJ)

CALL UNIVAR(JJ, NVALS, NMISS, YBAR, YVAR, YSUM)

IF( YDATA(JJ) .EQ. FMISS ) NVALS = 0

C COMPUTE EQUATION TERMS AND GAMMA. SEE TEXT FOR EQUATION DETAILS.
C

IF( NVALS .GT. 0 ) THEN

    CALL RCOEFF(JJ, AMJ, BMJ)

    SDMJ = SQRT(YVAR*(12./(FLOAT(JJ)*(FLOAT(JJ**2)-1.))))
    SDMJ2 = SDMJ**2
    BARMJ = BMJ

    SSMJ = 0.
    J = FLOAT(JJ) - M
    DMJ = ((M + J) * ((M + J)**2 - 1.)) / 12.

DO 30 I = 1, JJ

    SSMJ = SSMJ + (YDATA(I) - YBAR)**2
    PREDMJ(I) = (XDATA(I) * BMJ + AMJ)
    RESIDMJ(I) = (YDATA(I) - PREDMJ(I))

30 CONTINUE

C

    SUMT2 = 0.
    T1 = (-1.) * (M - 2.) / (2. * (SSM - (BMJ**2 * DM)))

DO 32 I = 1, MBASE

    T = FLOAT(I)

    T2 = RESIDMJ(I)**2 / (((M + 1) / M) +
                    (T - ((M + 1.) / 2.))**2 / DM)

    SUMT2 = SUMT2 + T2

32 CONTINUE

C

    LNL4 = T1 * SUMT2

    SUMT21 = 0.
    SUMT22 = 0.

    T11 = (-1.) * (M + J - 2.)
        / (2. * (SSMJ - (BMJ**2 * DMJ)))
    T12 = (-1.) * (M + J - 2.)
        / (2. * (SSMJ - (BM**2 * DMJ)))

DO 34 I = 1, JJ

    T = FLOAT(I)

    DENOM = ((M + J + 1.) / (M + J))
            + (((T - ((M + J + 1.) / 2.))**2 / DMJ))
T21 = RESIDMJ(I)**2 / DENOM
T22 = RESIDM(I)**2 / DENOM

SUMT21 = SUMT21 + T21
SUMT22 = SUMT22 + T22

34 CONTINUE

LNL2 = T11 * SUMT21
LNL3 = T12 * SUMT22

QM = EXP(LNL4 - LNL1)
QMJ = EXP(LNL2 - LNL3)
Q = QMJ / QM
GAMMA = 1. / (1. + SQRT(Q))
DGAMMA = GAMOLD - GAMMA

T1 = M * ALOG(SDM / SDMJ)
T2 = ((M - 1.) / 2.) * (1. - (SDM2 / SDMJ2))
T3 = (M / (2. * SDM2)) * (BARMJ - BARM)**2
QM = EXP(T1 + T2 - T3)

T1 = (M + J)* ALOG(SDM / SDMJ)
T2 = ((M + J - 1.) / 2.) * ((SDMJ2 / SDM2) - 1.)
T3 = ((M + J) / (2. * SDM2)) * (BARMJ - BARM)**2
QMJ = EXP(T1 + T2 + T3)
Q = QMJ / QM
GAMMA = 1. / (1. + SQRT(Q))
DGAMMA = GAMOLD - GAMMA

IF( GAMOLD .EQ. FMISS ) THEN
  WRITE(OUNIT,4005) JJ, FXVAL(JJ), YDATA(JJ), YEAR, YVAR, BMJ, GAMMA
  WRITE(*,4005) JJ, FXVAL(JJ), YDATA(JJ), YVAR, BMJ, GAMMA
ELSE
  WRITE(OUNIT,4005) JJ, FXVAL(JJ), YDATA(JJ), YBAR, YVAR, BMJ, GAMMA, DGAMMA
  WRITE(*,4005) JJ, FXVAL(JJ), YDATA(JJ), YBAR, YVAR, BMJ, GAMMA, DGAMMA
END IF

GAMOLD = GAMMA
ELSE
  WRITE(OUNIT,4006) JJ, FXVAL(JJ), YDATA(JJ)
  WRITE(*,4006) JJ, FXVAL(JJ), YDATA(JJ)
END IF

26 CONTINUE
WRITE(*,802)
READ(*,*) ITYPE
IF( ITYPE .EQ. 1 ) GOTO 1

801 FORMAT(/,' ENTER NUMBER OF VALUES IN BASE PERIOD:')
802 FORMAT(/,' ENTER 0=CONTINUE, 1=CHANGE NUMBER OF BASE VALUES:')

4000 FORMAT(/,'BASE PERIOD INPUT VALUES:'
+  /'INDEX   X         Y'
+  /'======   =======   ======='

4001 FORMAT(I5,2X,F8.3,2X,F8.2)

4002 FORMAT(/,'BASE PERIOD PARAMETERS:'
+  /'X-VALUE RANGE = ',F5.0,'-','F5.0,/
+  /'NUMBER OF VALUES = ',I3,/
+  /'NUMBER OF MISSING VALUES = ',I3,/
+  /'BASE MEAN = ',F8.3,/
+  /'BASE VARIANCE = ',F8.3,/
+  /'BASE SLOPE = ',F6.3,/

4003 FORMAT( ' ALL VALUES IN BASE PERIOD MISSING'

4004 FORMAT(/,'PROGRESSIVE PARAMETERS:'
+  /'X      Y''
+  /'DELTA',/
+  /'INDEX OBSERVATIONS MEAN VARIANCE SLOPE GAMMA',
+  /' GAMMA',/
+  /'======   ===============   ===========   =============   ======
+  /'======'

4006 FORMAT(I5,2X,F8.3,1X,F7.2)

RETURN
END

SUBROUTINE RCOEFF( N, B0, B1 )

C SUBROUTINE TO COMPUTE LINEAR LEAST SQUARES (LLS) INTERCEPT AND SLOPE.
C VARIABLES USED IN SUBROUTINE NOT DESCRIBED IN MAIN PROGRAM:
C 'I'  DO LOOP COUNTER.
C 'N'  NUMBER OF VALUES.
C 'NN'  COUNTER FOR NON-MISSING VALUES.
C 'SX2'  SUMS OF SQUARED DEVIATIONS FROM THE MEAN X-VALUE.
C 'SXY'  SUM OF THE CROSS PRODUCTS OF DEVIATIONS.
C 'SUMX'  SUM OF NON-MISSING X-VALUES.
C 'SUMY'  SUM OF NON-MISSING Y-VALUES.
C 'SUMSX'  SUM OF X-VALUES Squared.
C 'SUMXY'  SUM OF X TIMES Y Squared.
C 'XBAR'  MEAN OF NON-MISSING X-VALUES.
C 'XBAR2'  NUMBER OF NON-MISSING VALUES TIMES 'XBAR' Squared.
C 'XWORK'  WORK ARRAY FOR X-VALUES.
C 'XYN'  'XBAR' TIMES 'YBAR' TIMES NUMBER OF NON-MISSING VALUES.
C 'YBAR'  MEAN OF NON-MISSING Y-VALUES.
C 'YWORK'  WORK ARRAY FOR Y-VALUES.
C
PARAMETER (NDIM=10000)
COMMON /WORK/ FXVAL(NDIM), FYVAL(NDIM), XDATA(NDIM),
B29

+ YDATA(NDIM), FINDEX(NDIM), NCASE, NSUB, H,
+ OUNIT, FMISS
REAL XWORK(NDIM), YWORK(NDIM)
INTEGER OUNIT, H

SUMX = 0.
SUMY = 0.
NN = 0

C FILL WORK ARRAYS WITH NON-MISSING VALUES.
C
C
DO 15 I = 1, N
C
IF( YDATA(I) .GT. FMISS ) THEN

   NN = NN + 1
   XWORK(NN) = XDATA(I)
   YWORK(NN) = YDATA(I)

END IF

15 CONTINUE

C COMPUTE SUMS OF X- AND Y-VALUES.
C
DO 20 I = 1, NN
C
SUMX = SUMX + XWORK(I)
SUMY = SUMY + YWORK(I)

20 CONTINUE

C COMPUTE MEANS OF X- AND Y-VALUES.
C
XBAR = SUMX / FLOAT(NN)
YBAR = SUMY / FLOAT(NN)

SUMSX2 = 0.
SUMSXY = 0.

C COMPUTE SUMS OF SQUARED DEVIATIONS AND CROSS PRODUCTS.
C
DO 22 I = 1, NN
C
SUMSX2 = SUMSX2 + XWORK(I)**2
SUMSXY = SUMSXY + (XWORK(I) * YWORK(I))

22 CONTINUE

XBAR2 = XBAR**2 * FLOAT(NN)
XYN = XBAR * YBAR * FLOAT(NN)
SX2 = SUMSX2 - XBAR2
SXY = SUMSXY - XYN
SUBROUTINE UNIVAR( N, NVALS, NMISS, YEAR, YVAR, YSUM )

C SUBROUTINE TO COMPUTE LINEAR LEAST SQUARES (LLS) INTERCEPT AND SLOPE.
C VARIABLES USED IN SUBROUTINE NOT DESCRIBED IN MAIN PROGRAM:
C 'I' DO LOOP COUNTER.
C 'N' NUMBER OF VALUES.
C 'SUMS2' SUM OF SQUARED DIFFERENCES.
C 'YWORK' WORK ARRAY FOR NON-MISSING Y-VALUES.
C
PARAMETER (NDIM=10000)
COMMON /WORK/ FXVAL(NDIM), FYVAL(NDIM), XDATA(NDIM),
+ YDATA(NDIM), FINDEX(NDIM), NCASE, NSUB, H,
+ OUNIT, FMISS
REAL YWORK(NDIM)
INTEGER OUNIT, H
YSUM = 0.
NVALS = 0
NMISS = 0

C FILL WORK ARRAY WITH NON-MISSING VALUES.
C
DO 15 I = 1, N

IF( YDATA(I) .GT. FMISS ) THEN

NVALS = NVALS + 1
YWORK(NVALS) = YDATA(I)
ELSE
NMISS = NMISS + 1
END IF

15 CONTINUE

C COMPUTE SUM OF Y-VALUES.
C
DO 20 I = 1, NVALS

YSUM = YSUM + YWORK(I)

20 CONTINUE

C COMPUTE MEAN OF Y-VALUES.
C
YBAR = YSUM / FLOAT(NVALS)

C COMPUTE SUM OF SQUARED DIFFERENCES.

SUMS2 = 0.

C-------

DO 22 I = 1, NVALS

SUMS2 = SUMS2 + (YWORK(I) - YBAR)**2

22 CONTINUE

C-------

C COMPUTE VARIANCE OF Y-VALUES.

YVAR = SUMS2 / (FLOAT(NVALS) - 1)

RETURN

END

SUBROUTINE GAMMLN( XX, GAMEXP )

C RETURNS THE VALUE LN[\text{GAMMA}(XX)] FOR XX > 0.
FROM NUMERICAL RECIPES IN FORTRAN, 2ND EDITION, 1992, PP. 206-207.

REAL COF(6)

DATA COF, STP / 76.18009172947146, -86.50532032941677,
  24.01409824083091, -1.231739572450155,
  .1208650973866179E-2, -.5395239384953E-5,
  2.5066282746310005 /

X = XX
Y = X
TMP = X + 5.5
TMP = (X + 0.5) * ALOG(TMP) - TMP
SER = 1.0000000000190015

C-------

DO 20 J = 1, 6

Y = Y + 1.
SER = SER + COF(J) / Y

20 CONTINUE

C-------

GAMMALN = TMP + ALOG(STP*SER/X)
GAMEXP = EXP(GAMMALN)

RETURN

END