Analysis of Condensation on a Horizontal Cylinder with Unknown Wall Temperature and Comparison with the Nusselt Model of Film Condensation

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NOMENCLATURE

\( A \) constant parameter defined by equation (41), \( A = \frac{-1}{4 + \Omega} \)

\( f(\Delta_0) \) optimization function, equation (37)

\( g(\Delta_0^n) \) iteration function, equation (38)

\( g \) generalized gravitational constant (magnitude of the sum of gravitational and coordinate system acceleration vectors)

\( h_i \) heat transfer coefficient inside the cylinder

\( h_{fg} \) heat of fusion

\( k_w \) thermal conductivity of the cylinder wall

\( \dot{m} \) mass flow rate

\( \dot{m}_{Nu} \) mass flow rate (per unit length of cylinder) predicted by Nusselt model

\( \text{Nu} \) Nusselt number of fluid in the cylinder, \( \text{Nu} = \frac{h_i 2r_i}{k_w} \)

\( p \) pressure

\( Q \) heat transfer rate (per unit length of half-cylinder)

\( Q_{Nu} \) heat transfer (per unit length of half-cylinder) predicted by Nusselt model

\( r_i \) cylinder inner radius

\( r_o \) cylinder outer radius

\( \text{Re} \) Reynolds number of fluid in the cylinder

\( T_b \) bulk temperature of cooling fluid inside the cylinder

\( T_w \) outer cylinder wall temperature

\( T_{sat} \) saturation temperature

\( u \) \( x \) component of velocity

\( x \) coordinate on the cylinder surface

\( y \) coordinate perpendicular to the cylinder surface
vertical distance from the uppermost point on the outer cylinder wall

\( \delta \) thickness of the liquid layer

\( \delta_{\text{Nu}} \) thickness of the liquid layer predicted by Nusselt model

\( \Theta \) dimensionless heat transfer rate defined by equation (43),

\[
\Theta = Q \left( \frac{h_{fg} \rho \tilde{p} g}{3 \mu} \right)^{-1} \left( \frac{3 \mu k (T_{\text{sat}} - T_b) r_o}{\rho \tilde{p} g h_{fg}} \right)^{3/4}
\]

\( \Theta_{\text{Nu}} \) dimensionless heat transfer rate predicted by Nusselt model, defined by equation (42),

\[
\Theta_{\text{Nu}} = Q_{\text{Nu}} \left( \frac{h_{fg} \rho \tilde{p} g}{3 \mu} \right)^{-1} \left( \frac{3 \mu k (T_{\text{sat}} - T_b) r_o}{\rho \tilde{p} g h_{fg}} \right)^{3/4}
\]

\( \Delta \) dimensionless liquid film thickness defined by equation (17),

\[
\Delta = \frac{\delta}{1 \left( \frac{3 \mu k (T_{\text{sat}} - T_b) r_o}{\rho \tilde{p} g h_{fg}} \right)^{1/4}}
\]

\( \Delta_o \) dimensionless liquid film thickness at the top of cylinder

\( \Delta_{\text{Nu}} \) dimensionless liquid film thickness predicted by Nusselt model

\( \Lambda \) limit defined by equation (36), \( \Lambda = \lim_{\phi \to \pi} \left( \Delta^3 \sin \phi \right) \)

\( \Lambda_{\text{Nu}} \) limit defined by equation (30), \( \Lambda_{\text{Nu}} = \lim_{\phi \to \pi} \left( \Delta_{\text{Nu}}^3 \sin \phi \right) \)

\( \mu \) dynamic viscosity of liquid

\( \rho \) density of liquid

\( \rho_v \) density of vapor

\( \hat{\rho} \) density difference, defined by equation (6), \( \hat{\rho} = \rho - \rho_v \)

\( \phi \) angle measured between cylinder surface normal and vertical, \( \phi = 0 \) on top of cylinder
\( \chi \) 

dimensionless temperature, defined by equation (31), \( \chi = \frac{(T_{\text{sat}} - T_w)}{(T_{\text{sat}} - T_b)} \)

\( \Omega \) 

dimensionless parameter defined by equation (18),

\[
\Omega = \left( \frac{r_0}{h_i r_i} + \frac{r_0 \ln \frac{r_0}{r_i}}{k_w} \right) \left( \frac{3 \mu k (T_{\text{sat}} - T_b) r_0}{\rho \dot{p} gh_{fg}} \right)^{\frac{1}{4}}
\]
ANALYSIS OF CONDENSATION ON A HORIZONTAL CYLINDER WITH UNKNOWN WALL TEMPERATURE AND COMPARISON WITH THE NUSSELT MODEL OF FILM CONDENSATION

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SUMMARY

Theoretical analysis and numerical computations are performed to set forth a new model of film condensation on a horizontal cylinder. The model is more general than the well-known Nusselt model of film condensation and is designed to encompass all the essential features of the Nusselt model. It is shown that a single parameter, constructed explicitly and without specification of the cylinder wall temperature, determines the degree of departure from the Nusselt model, which assumes a known and uniform wall temperature. It is also shown that the Nusselt model is reached for very small, as well as very large, values of this parameter. In both limiting cases the cylinder wall temperature assumes a uniform distribution and the Nusselt model is approached. The maximum deviations between the two models is rather small for cases which are representative of cylinder dimensions, materials and conditions encountered in practice.

INTRODUCTION

Condensation of saturated vapors on solid surfaces under conditions encountered terrestrially and during space flight is of great importance in application of heat and mass transfer analyses. The early heat and mass transfer literature for condensing fluids contains many insightful analysis of the physics involved, as evident in reference 1. The well known Nusselt model of film condensation on a horizontal cylinder is perhaps the most important one, owing to its fundamental geometry. It assumes that a uniform wall temperature distribution in the cylinder prevails. Predictions of the heat transfer rates are based upon this temperature which is presumed to be known, (ref. 2). However, the cylinder wall temperature for this problem is generally not known, and one would rather deal with the bulk temperature of the cooling fluid inside the cylinder. The dynamics of the heat and mass transfer processes determine the wall temperature distribution and, therefore, the wall temperature can not be specified a priori, as assumed in the Nusselt model.

The aim of the present analysis is to set forth a new and more complete model of film condensation on a horizontal cylinder, where no assumptions are made to explicitly specify the wall temperature. The new model will be compared with the Nusselt model to determine the extent of differences in the heat and mass transfer predictions for conditions most widely encountered in horizontal cylinder condensers.
THE ANALYSIS

The velocities in condensing fluids are generally very small such that, the inertial forces may be ignored in comparison to other forces, Eckert (ref. 3). Furthermore, the governing equations are distinctly boundary-layer in nature, Schlichting (ref. 4). A balance between the pressure, viscous and body forces within the condensing film prevails and the pressure within the condensing film is that of the quiescent saturated vapor away from the cylinder. The thickness of the condensing film is much smaller than the cylinder radius. Therefore, it is convenient to employ a curvilinear coordinate system in which the abscissa \( x \) lies on the cylinder surface and the ordinate \( y \) is perpendicular to and measured from the surface.

With these assumptions, the Navier–Stokes equation of motion for this problem can be reduced to:

\[
\frac{\mu}{\gamma^2} \frac{d^2 u}{d\gamma^2} - \frac{dp}{dx} + \rho g \sin \varphi = 0
\]

(1)

Denoting the distance from the uppermost point on the cylinder directly downwards by \( z \),

\[
z = r_0 (1 - \cos \varphi) = r_0 \left( 1 - \cos \frac{x}{r_0} \right)
\]

(2)

one may write,

\[
\frac{dp}{dx} = \frac{dp}{dz} \sin \varphi
\]

(3)

But, as noted earlier, the pressure within the condensing film is that of the quiescent saturated vapor away from the cylinder,

\[
\frac{dp}{dx} = \rho_v g \sin \varphi
\]

(4)

By combining equations (1) and (4),

\[
\frac{\mu}{\gamma^2} \frac{d^2 u}{d\gamma^2} = -(\rho - \rho_v) g \sin \varphi
\]

(5)

and denoting the density difference by \( \hat{\rho} \),

\[
\hat{\rho} = \rho - \rho_v
\]

(6)

equation (5) can be integrated:
The no-slip boundary condition at the outer tube wall, \( u = 0 \) at \( y = 0 \), eliminates the second integration constant, \( c_2 \). The shear force at the liquid-gas interface is taken to be negligible due to small viscosity of the vapor relative to that of the liquid, leading to evaluation of the first integration constant, \( c_1 \).

\[
\frac{du}{dy} = 0, \quad y = \delta
\]

\[
c_1 = -\frac{\hat{\rho} g}{\mu} \sin \varphi \delta
\]

\[
u = \frac{\hat{\rho} g}{2\mu} \delta^2 \sin \varphi \left( \frac{2y}{\delta} - \frac{y^2}{\delta^2} \right)
\]

The mass flow rate in the condensed layer at any \( x \) is:

\[
m = \rho \int_0^{\delta} u dy = \frac{\rho \hat{\rho} g}{3\mu} \sin \varphi \delta^3
\]

so that,

\[
d\dot{m} = \frac{\rho \hat{\rho} g}{3\mu} d\left(\delta^3 \sin \varphi\right)
\]

is the incremental increase in the mass flow rate at any \( x \). The heat transfer rate is simply,

\[
dQ = h_{fg} d\dot{m}
\]

Using the series resistance concept for the fluid layer, the cylinder wall and internal heat transfer, and assuming that the internal heat transfer coefficient \( h_i \) is an appropriate mean value, one may write,

\[
dQ = \frac{T_{sat} - T_b}{\ln \frac{r_o}{r_i} + \frac{\delta}{h_i r_i d\varphi} + \frac{1}{k_w d\varphi} + \frac{\delta}{k_r d\varphi}}
\]

By carrying out the differentiation in equation (12) and using equations (13) and (14) we get,
\[
\frac{\rho \hat{\rho}_{ghfg}}{3\mu} \left( \delta^3 \cos \phi + 3\delta^2 \sin \phi \frac{d\delta}{d\phi} \right) = \frac{(T_{\text{sat}} - T_b) r_o}{r_o \ln \frac{r_o}{r_i} + \frac{\delta}{h_i h_i}} \quad (15)
\]

The boundary condition is due to the symmetrical film thickness at the top of the cylinder, i.e.

\[
\frac{d\delta}{d\phi} = 0, \phi = 0 \quad (16)
\]

Now, by introducing a dimensionless film thickness \( \Delta \)

\[
\Delta = \frac{\delta}{\left( \frac{3\mu k(T_{\text{sat}} - T_b) r_o}{\rho \hat{\rho}_{ghfg}} \right)^{\frac{1}{4}}} \quad (17)
\]

and a dimensionless parameter \( \Omega \),

\[
\Omega = \left( \frac{r_o}{h_i h_i} + \frac{r_o \ln \frac{r_o}{r_i}}{k_w} \right) k \left( \frac{3\mu k(T_{\text{sat}} - T_b) r_o}{\rho \hat{\rho}_{ghfg}} \right)^{-\frac{1}{4}} \quad (18)
\]

equation (15) gives,

\[
3\Delta^3 \sin \phi \frac{d\Delta}{d\phi} + \Delta^4 \cos \phi = \frac{1}{1 + \frac{1}{\Omega}} \quad (19)
\]

Rearranging equation (19) results in a first order nonlinear ordinary differential equation, in a suitable form for Runge–Kutta numerical integration:

\[
\frac{d\Delta}{d\phi} = \left( 3\Delta^3 \sin \phi \right)^{-1} \left( \frac{1}{1 + \frac{1}{\Omega}} - \Delta^4 \cos \phi \right) \quad (20)
\]

The boundary condition, equation (16) is transformed into,

\[
\frac{d\Delta}{d\phi} = 0, \phi = 0 \quad (21)
\]
which, when used in the differential equation (19) gives,

\[ \Delta_0 = \frac{1}{\Omega} \left( 1 + \frac{\Delta_0}{\alpha} \right) \]  

(22)

Here, \( \Delta_0 \) denotes the value of \( \Delta \) at \( \varphi = 0 \) and can be evaluated once the parameter \( \Omega \) is specified. At this point it is apparent from reference 2 that equation (20) is identical to the one obtained from the Nusselt model, when \( \Omega \rightarrow 0 \).

Recasting the Nusselt Model in Terms of the Variables of the Present Analysis

In order to make a precise comparison of the predictions of the present model to those of the Nusselt Model, one must first recast the Nusselt model in terms of the variables of the present model. If a relationship between \( T_b \) in the above analysis and \( T_w \) in the Nusselt model could be obtained, one could compare the numerical solutions of equation (20) to the results of the Nusselt model.

Applying the series-resistance concept again,

\[ Q = \frac{(T_w - T_b)}{1 + \frac{r_0}{r_i} \ln \frac{r_i}{r_o}} = \frac{(T_w - T_b) \pi r_o}{h_i \pi + \frac{r_i}{k_w \pi}} \]  

(23)

But,

\[ Q = \dot{m}_\pi h_{fg} \]  

(24)

where, \( \dot{m}_\pi \) is the mass-flow rate as \( \varphi \rightarrow \pi \), and by using equation (11),

\[ \dot{m}_\pi = \lim_{\phi \rightarrow \pi} \frac{\rho g}{3\mu} \delta^3(\phi) \sin \phi \]  

(25)

\[ Q = h_{fg} \frac{\rho g}{3\mu} \lim_{\phi \rightarrow \pi} \delta^3(\phi) \sin \phi \]  

(26)

A limit is imposed since, the mass flow crossing \( \varphi = \pi \) is zero and not a representative of the condensed mass flow rate.

Equation (26) can be written in terms of \( \Delta_{Nu} \) and \( T_w \), by noting that in the present model as \( k_w, h_{i \rightarrow \infty}, T_w \rightarrow T_b \), a constant and in effect, the Nusselt’s model is approached. Therefore, \( T_w \) replaces \( T_b \) for \( \Delta = \Delta_{Nu} \).
\[ Q_{Nu} = \left( \frac{3 \mu k (T_{sat} - T_w) r_o}{\rho \dot{\rho} g h_{fg}} \right)^{\frac{3}{4}} \left( h_{fg} \frac{\rho \dot{\rho} g}{3 \mu} \right) \lim_{\phi \to \pi} \left( \Delta \frac{3}{3} \nu \sin \phi \right) \]  

(27)

\[ Q_{Nu} = \left( \frac{\rho \dot{\rho} g h_{fg}}{3 \mu k r_o (T_{sat} - T_w)} \right)^{\frac{1}{4}} k (T_{sat} - T_w) \lim_{\phi \to \pi} \left( \Delta \frac{3}{3} \nu \sin \phi \right) = \frac{(T_w - T_b) \pi r_o}{\frac{1}{h_i} \left( \frac{r_o}{r_i} \right) + \frac{r_o}{k_w} \ln \left( \frac{r_o}{r_i} \right)} \]  

(28)

Rearranging,

\[ \frac{(T_w - T_b) \pi}{\frac{1}{h_i} \left( \frac{r_o}{r_i} \right) + \frac{r_o}{k_w} \ln \left( \frac{r_o}{r_i} \right)} = (T_{sat} - T_w) \Lambda_{Nu} \]  

(29)

Where \( \Lambda_{Nu} \) denotes the limit:

\[ \Lambda_{Nu} = \lim_{\phi \to \pi} \left( \Delta \frac{3}{3} \nu \sin \phi \right) \]  

(30)

Using the definition for \( \Omega \) and introducing a dimensionless temperature \( \chi \),

\[ \chi = \frac{(T_{sat} - T_w)}{(T_{sat} - T_b)} \]  

(31)

one obtains the following equation which will be utilized later in the analysis:

\[ 1 - \chi = \frac{\chi^4 \Omega \Lambda_{Nu}}{\pi} \]  

(32)

Writing equation (28) in terms of \( \Lambda_{Nu}, \chi \) and \( (T_{sat} - T_b) \), one obtains:

\[ Q_{Nu} = \Lambda_{Nu} \chi^\frac{3}{4} \left( h_{fg} \frac{\rho \dot{\rho} g}{3 \mu} \right)^\frac{3}{4} \left( \frac{3 \mu k (T_{sat} - T_b) r_o}{\rho \dot{\rho} g h_{fg}} \right)^\frac{3}{4} \]  

(33)

Similarly for equation (26), one obtains:
\[ Q = \Lambda \left( h_{fg} \frac{\rho \hat{p} g}{3\mu} \left( \frac{3\mu k(T_{\text{sat}} - T_b) r_0}{\rho \hat{p} g h_{fg}} \right) \right)^{\frac{3}{4}} \]  

(34)

where, in tandem with equation (30),

\[ \Lambda = \lim_{\phi \to \pi} \left( \Delta^3 \sin \phi \right) \]  

(35)

We note that comparison of \( Q_{Nu} \) and \( Q \) can now be made, once the value of \( \Omega \) is specified and the integration of equation (20) is carried out.

**Numerical Solution**

A Runge–Kutta numerical integration technique was used to integrate equation (20) from \( \varphi = 0 \) to \( \varphi = 179.99^\circ \). This method is designed to approximate the Taylor series method without requiring explicit definition or evaluations of derivatives beyond the first. The approximation is obtained at the expense of several evaluations of the function. Reference 5 describes the method in detail and provides a framework for a subroutine for solving initial value problems of ordinary differential equations. A modern modification to the classical Runge–Kutta technique has been employed to control the step size. Since only one solution value is required for calculation of the next, the method is self-starting. It requires six function evaluations per step. Four of these function values are combined with one set of coefficients to produce a fourth-order method, and all six values are combined with another set of coefficients to produce a fifth-order method. Comparison of the two values yields an error estimate which was used for step size control. The boundary condition at \( \varphi = 0 \) appears as an initial condition, where integration over a specified interval was performed in one or many subintervals. Specification of \( \frac{d\Delta}{d\varphi} = 0, \varphi = 0 \) is not an essential part of the routine, however, since the denominator of equation (20) contains a singularity at \( \varphi = 0 \), \( \frac{d\Delta}{d\varphi} \) was set to zero at this point. Once \( \Omega \) was specified, \( \Delta_0 \) was calculated and the integration was carried out. Local error of \( 10^{-5} \) and \( 10^{-8} \) yielded identical values of \( \Delta \) up to the seventh significant digit.

**Calculation of \( \Delta_0 \)**

Equation (20) is a quartic equation and in general may possess multiple roots. The roots, however, may be real or complex conjugates. Real negative roots are not physical and complex roots are not within the scope of consideration of the present work. Here interest lies with the particular solutions (i.e., positive real roots) which enable comparison of the present model with the Nusselt model of film condensation. Inspection of equation (22) reveals that two real and two complex solutions can be obtained by considering its resolvent cubic equation, as described in reference 6. Furthermore, the real positive roots are equal. During attempts to integrate equation (20) for \( \Omega > 0 \), it was discovered...
that $\Delta_0$ had to be specified to a high degree of accuracy to avoid incompatibility between the initial value and the differential equation. A successive approximation scheme was devised for this purpose by letting,

$$f(\Delta_0) = \Delta_0^\left(\frac{\Omega}{\Delta_0} + 1\right) - 1 = 0$$  \hspace{1cm} (36)$$

and

$$\Delta_0^{n+1} = g(\Delta_0^n) = \Delta_0^n + A \left(\frac{\Omega}{\Delta_0^n} + 1\right) - 1$$  \hspace{1cm} (37)$$

where $\Delta_0^n$ denotes the n-th approximation to $\Delta_0$, the function $g(\Delta_0^n)$ is the iteration equation and A is a constant. Then,

$$\frac{dg(\Delta_0^n)}{d\Delta_0^n} = 4A(\Delta_0^n)^3 + 3A\Omega(\Delta_0^n)^2 + 1$$  \hspace{1cm} (38)$$

For $\frac{dg(\Delta_0^n)}{d\Delta_0^n} = 0$ one may choose $\Delta_0^0 = 1$ as a starting trial value, and convergence of the successive approximations can be expected.

$$4A + 3A\Omega + 1 = 0$$  \hspace{1cm} (39)$$

Therefore,

$$A = \frac{-1}{4 + 3\Omega}$$  \hspace{1cm} (40)$$

**Specification of $\Omega$**

In order to specify values of $\Omega$ which are representative of cylinder dimensions, materials and conditions encountered in practice, a simple survey was conducted. Cylinder diameters between 0.625 and 1.25 inches and wall thicknesses of 0.045 and 0.065 inches are most commonly used in terrestrial applications of horizontal tube condensers. The pipe materials are usually admiralty brass or stainless steel. It was necessary to obtain only approximate or orders of magnitude of various parameters. Standard correlations of turbulent pipe flow, Eckert (ref. 3), were used to evaluate the Nusselt number $Nu$, the cylinder internal heat-transfer coefficient $h_i$ and finally, the parameter, $\Omega$.

For conditions of atmospheric pressure for saturated water vapor and Reynolds numbers, based on cylinder diameter, ranging between $10^4$ and $10^5$, various calculations were carried out. A sample of the results is presented in Table 1 to highlight the expected range of values for $\Omega$. 


<table>
<thead>
<tr>
<th>Re</th>
<th>Nu</th>
<th>$h_i$ W/K m$^2$</th>
<th>$\Omega$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10,000</td>
<td>104</td>
<td>2.561</td>
<td>3.05</td>
</tr>
<tr>
<td>20,000</td>
<td>185</td>
<td>4.558</td>
<td>1.73</td>
</tr>
<tr>
<td>30,000</td>
<td>260</td>
<td>6.412</td>
<td>1.24</td>
</tr>
<tr>
<td>40,000</td>
<td>332</td>
<td>8.181</td>
<td>0.98</td>
</tr>
<tr>
<td>50,000</td>
<td>401</td>
<td>9.892</td>
<td>0.82</td>
</tr>
<tr>
<td>100,000</td>
<td>726</td>
<td>17.921</td>
<td>0.47</td>
</tr>
</tbody>
</table>

It should be noted that, the term in $\Omega$ containing $k_w$ is one or two orders of magnitude smaller than that containing $h_i$. Therefore, $h_i$ is the dominating parameter.

**COMPARISON AND DISCUSSION OF THE RESULTS**

In order to set forth a precise comparison of the heat transfer predicted by the present model and the Nusselt model, one may define two new heat transfer rates with the aid of equations (33) and (35) by denoting,

$$\Theta_{Nu} = Q_{Nu} \left( \frac{h_f \rho \delta g}{3 \mu} \right)^{-1} \left( \frac{3 \mu k (T_{sat} - T_b) r_0}{\rho \delta g h_f g} \right)^{-3}$$

(41)

and,

$$\Theta = Q \left( \frac{h_f \rho \delta g}{3 \mu} \right)^{-1} \left( \frac{3 \mu k (T_{sat} - T_b) r_0}{\rho \delta g h_f g} \right)^{-3}$$

(42)

These heat-transfer rates are dimensionless and are closely related to the liquid film thickness at the base of the cylinder. As described in the numerical solution section, integration of equation (20), provided the values of $\Lambda$ for parametric values of $\Omega$. As noted earlier, for $\Omega = 0$ the Nusselt model is reached with $\Lambda_{Nu} = 2.53113$. Using this value, equation (32) can be employed to obtain the corresponding values of $\chi$, for parametric values of $\Omega$. A Summary of the results is presented in
Table 2 for eight parametric values of $\Omega$ from zero to infinity. The corresponding values of $\chi$ are also listed in the table.

\[ \Theta_{\text{Nu}} = \Lambda_{\text{Nu}} \chi^\frac{3}{4} \]  
\[ \Theta = \Lambda \]  

<table>
<thead>
<tr>
<th>$\Omega$</th>
<th>$\chi$</th>
<th>$\Theta_{\text{Nu}}$</th>
<th>$\Theta$</th>
<th>% difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>1.000</td>
<td>2.53113</td>
<td>2.53113</td>
<td>0.00</td>
</tr>
<tr>
<td>0.5</td>
<td>0.693</td>
<td>1.92249</td>
<td>1.91545</td>
<td>0.36</td>
</tr>
<tr>
<td>1.0</td>
<td>0.515</td>
<td>1.53876</td>
<td>1.52224</td>
<td>1.07</td>
</tr>
<tr>
<td>1.1</td>
<td>0.486</td>
<td>1.47330</td>
<td>1.46248</td>
<td>0.74</td>
</tr>
<tr>
<td>2.0</td>
<td>0.320</td>
<td>1.07960</td>
<td>1.06326</td>
<td>1.26</td>
</tr>
<tr>
<td>3.0</td>
<td>0.221</td>
<td>0.81585</td>
<td>0.80991</td>
<td>0.73</td>
</tr>
<tr>
<td>4.0</td>
<td>0.165</td>
<td>0.65528</td>
<td>0.65135</td>
<td>0.60</td>
</tr>
<tr>
<td>$\infty$</td>
<td>0.000</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.00</td>
</tr>
</tbody>
</table>

It is apparent that the Nusselt model is in notable agreement with the present analysis. As may be expected, deviations from the Nusselt model, however small, do appear to exist. The maximum deviations are of order one percent and correspond to the moderate values of $\Omega$ in Table 2. This is not a fortuitous outcome. The range of values for various parameters were chosen to span two limiting cases. First, as the values of $\Omega$ approach zero, the deviations in heat transfer also tend to vanish. This may be attributed to progressively larger values of $h_i$ which tend to dictate $T_w$ to approach $T_b$, a constant. On the other hand, at large values $\Omega$, $h_i$ tends to be small and low heat-transfer rates are expected. The wall temperature $T_w$ again will assume a constant value, close to that of the saturated steam. These limits, of course, can be reached by the influence of all variables contained in $\Omega$ and not just $h_i$. In both limiting cases the cylinder wall temperature assumes a uniform distribution and the Nusselt model is approached. The precision by which the Nusselt model approximates the present analysis, clearly testifies to the insight often set forth by the old masters. Much can be learned by careful analysis of their assumptions and methods.
REFERENCES


Analysis of Condensation on a Horizontal Cylinder with Unknown Wall Temperature and Comparison with the Nusselt Model of Film Condensation

Theoretical analysis and numerical computations are performed to set forth a new model of film condensation on a horizontal cylinder. The model is more general than the well-known Nusselt model of film condensation and is designed to encompass all the essential features of the Nusselt model. It is shown that a single parameter, constructed explicitly and without specification of the cylinder wall temperature, determines the degree of departure from the Nusselt model, which assumes a known and uniform wall temperature. It is also shown that the Nusselt model is reached for very small, as well as very large, values of this parameter. In both limiting cases the cylinder wall temperature assumes a uniform distribution and the Nusselt model is approached. The maximum deviations between the two models is rather small for cases which are representative of cylinder dimensions, materials and conditions encountered in practice.